Disasters, Recoveries, and Predictability

François Gourio*

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Abstract

I review the disaster explanation of the equity premium puzzle, discussed in Barro (2006) and Rietz (1988). In the data, disasters are often followed by recoveries. I study how recoveries affect the implications of the model. This turns out to depend heavily on the elasticity of intertemporal substitution (IES). For a high IES, the disaster model does not generate a sizeable equity premium. I also study whether the disaster model can fit the time-series evidence on predictability of stock returns, as argued by Gabaix (2007). I find that the model has difficulties matching these facts. Finally, I propose a cross-sectional test of the disaster model: assets which do better in disasters should have lower average returns. There seems to be little evidence supporting this prediction.

1 Introduction

Twenty years ago, Thomas A. Rietz (1988) showed that infrequent, large drops in consumption make the theoretical equity premium large. Recent research has resurrected this 'disaster' explanation of the equity premium puzzle. Robert J. Barro (2006) measures disasters during the XXth century, and finds that they are frequent and large enough, and stock returns low enough relative to bond returns during disasters, to make this explanation quantitatively plausible. Xavier Gabaix (2007) extends the model to incorporate a time-varying incidence of disasters, and he argues that this simple feature can resolve many asset pricing puzzles.[1]

This paper reviews the disaster-based explanations and adds to the debate in three dimensions. First, all the papers in this literature assume that disasters are permanent. Mathematically, they model log consumption per capita as the sum of a unit root process and a Poisson jump. However a casual look at the data suggests that disasters are often followed by recoveries. The first contribution of this paper is to measure recoveries and introduce recoveries in the Barro-Rietz model. In doing so, I follow Barro's suggestion that "a worthwhile extension would deal more seriously with the dynamics of crisis regimes"

*This paper merges three short papers of mine: "Disasters and recoveries" (AER P&P May 2008), "Time-series predictability in the disaster model", and "A cross-sectional test of the disaster model". I thank Robert Barro, John Campbell, Xavier Gabaix, Ian Martin, Romain Ranciere, and Adrien Verdelhan for discussions or comments. Contact information: Boston University, Department of Economics, 270 Bay State Road, Boston MA 02215. Email: fgourio@bu.edu. Tel.: (617) 353 4534.

[1] The literature is growing rapidly, e.g. see Farhi and Gabaix (2007) for an application to the forward premium puzzle, and Martin (2008) for a nonparametric formulation.

(p. 854). I find that the effect of recoveries hinges on the intertemporal elasticity of substitution (IES): when the IES is low, recoveries may increase the equity premium implied by the model; but when it is high, the opposite happens. Hence, recoveries can have a powerful effect on the implications of this model. The implications for the pattern of P-D ratios and risk-free rates during disasters are however challenging.

My second contribution is to study if the disaster model can account for the time-series predictability of stock returns. I show theoretically that the model with a time-varying probability of disaster cannot account for the findings of stock return and excess stock return predictability, if utility is CRRA, *regardless* of the parameter values. Even using an Epstein-Zin utility function does not seem to help quantitatively much.

Finally, I study a natural cross-sectional implication: assets which do well during disasters should have low average (or expected) returns. I test this implication using as assets a variety of stocks, sorted by industries, size, and other variables, for the United States during the XXth century. I find little support for this idea - exposures to disasters are not significantly correlated with average returns.

Collectively these findings raise significant challenges for the disaster model. In particular, there is a tension between the need for a high IES to reduce the volatility of risk-free interest rates, and the need for a low IES to reduce the effect of recoveries.

The rest of the paper is organized as follows. Section 2 presents the simple disaster model. Section 3 quantifies the importance of recoveries in the data and in the model. Section 4 studies if the disaster model can account for time-variation in equity return and risk premia, and Section 5 studies the cross-sectional implications of the disaster model.

2 The Disaster model: a review

The Barro-Rietz model is a variant of the familiar Lucas tree asset pricing model. There is a representative agent with power utility:

$$E\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\gamma}}{1-\gamma}.$$

Barro and Rietz assume the following consumption process:

$$\begin{aligned}\Delta\log C_t &= \mu+\sigma\varepsilon_t,\ \text{with probability } 1-p,\\ &= \mu+\sigma\varepsilon_t+\log(1-b),\ \text{with probability } p,\end{aligned}$$

where ε_t is *iid* $N(0,1)$. Hence, each period, with probability p, consumption drops by a factor b, e.g. $b=40\%$. The realization of the disaster is *iid* and statistically independent of the "business cycle risk" ε_t at all dates. Because risk-averse agents fear large changes in consumption, a small probability of a large drop of consumption can make the theoretical equity premium large.

In this model, the P-D ratio is constant, since the consumption growth process is *iid* and utility is CRRA. We have the following expression for the risk-free and for the equity premium:

$$\log R_f=-\log\beta+\gamma\mu-\frac{\gamma^2\sigma^2}{2}-\log\left(1-p+p(1-b)^{-\gamma}\right),$$

$$\log\left(\frac{ER^e}{R^f}\right) = \sigma^2\gamma + \log\left(\frac{(1-p+p(1-b))\,(1-p+p(1-b)^{-\gamma})}{1-p+p(1-b)^{1-\gamma}}\right).$$

When $p = 0$, these formulas lead to the standard results of the *iid* lognormal model. When disasters are possible, i.e. $p > 0$, we see that the risk-free rate is lower, and the equity premium is higher.

I first follow Barro's calibration to illustrate the potential of this model to account for the equity premium puzzle. Barro uses standard values for the trend growth μ, the standard deviation of business cycle shocks σ and the discount rate ρ: $\sigma = 0.02$, $\mu = 0.025, \beta = 0.97$. He sets risk aversion equal to $\gamma = 4$, which implies an intertemporal elasticity of substitution of 1/4. Finally, the probability of a disaster in a given year is $p = 0.017$, and the disaster size b is actually random: b is drawn from the historical distribution of disasters, as measured by Barro (see the next section).[2] These parameter values leads to an arithmetic equity premium $E\left(R^e - R^f\right)$ equal to 5.6%. By contrast, without the disasters, the equity premium would be only 0.18%.

Government bonds are not risk-free in disasters. Barro argues that a reasonable calibration is that, during disasters, bonds repay with probability 0.6 the full amount and with probability 0.4 repay a fraction $1 - b$. Taking this into account reduces the risk premium to 3.5%. (Barro also studies how leverage may increase this number, and how survivorship bias may affect our estimates of the equity premium.) These numbers are driven by the very extreme disasters. For instance, if we consider only the disasters which are less than 40% (which all occurred during World War II), the equity premium is reduced to 0.8%.[3]

The disaster model has a constant P-D ratio, because disasters are *iid*. Hence it cannot address the stock market volatility puzzle or the time-series predictability puzzle. However, a natural extension is to make the probability of disasters p, or perhaps the size of disasters b, a stochastic process. Presumably, when investors' expectations of b and p change, asset prices will move, generating changes over time in the risk-free rate, the equity premium and the P-D ratio. This extension is studied in Section 4.

3 Disasters and Recoveries

I start by presenting the measure of disasters introduced by Barro (2006) and by measuring recoveries. Then I extend the Barro-Rietz model to incorporate recoveries, and I discuss the implications.

3.1 Measuring Recoveries

Barro measures disasters as the total decline in GDP from peak to through. Using 35 countries (20 OECD countries and 15 countries from Latin America and Asia), he finds 60 episodes of GDP declines greater than 15% during the XXth century. These episodes are concentrated on World War I, the Great Depression, and World War II, but there is also a significant number of disasters in Latin America since World War II. The mean decline is 29%. Figure 1 plots log GDP per capita for four countries (Germany, Netherlands, the U.S., and Chile), and figure 2 shows the data for Mexico, Peru, Urugay and Venezuela.

[2] This modifies slightly the formulas above: an expectation over the disaster size b, conditional on a disaster occuring, must be added to the formulas.

[3] This extreme sensitivity to extreme events with tiny probability is emphasized by Martin (2008).

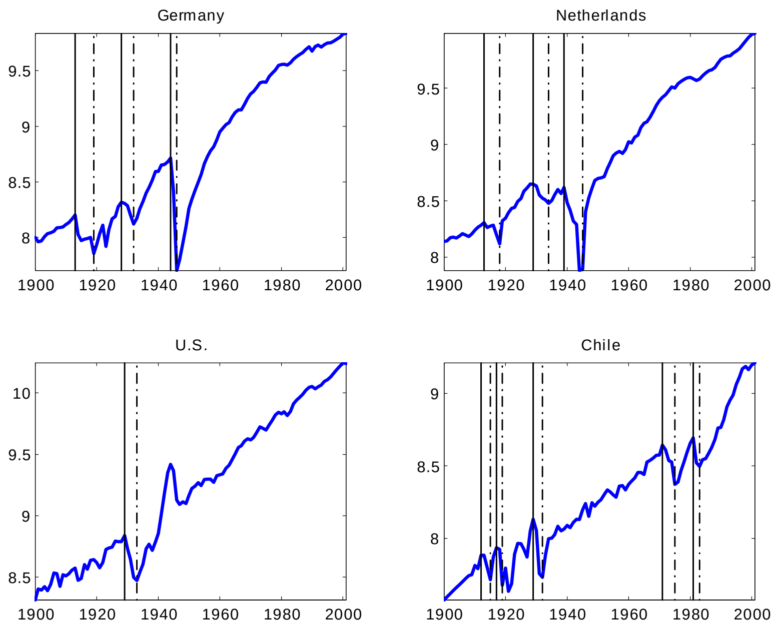

Figure 1: Log GDP per capita (in 1990 dollars) for four countries: Germany, Netherlands, the U.S. and Chile. The disaster start (resp.end) dates are taken from Barro (2006), and are shown with a vertical full (resp. dashed) line.

The vertical full lines indicate the start of disasters, and the vertical dashed lines the end of disasters, as defined by Barro.[4] In many cases, GDP bounces back just after the end of the disaster, as predicted by the neoclassical growth model following a capital destruction or a temporary decrease in productivity.[5]

To quantify the magnitude of recoveries, Table 1 present some statistics using the entire sample of disasters[6] identified by Barro. Because Barro defines disasters such that the end of the disaster is the trough, this computation implies that GDP goes up following the disaster. The key question is, How much?

The first column reports the average across countries of the cumulated growth, in each of the first five years following a disaster. The average growth rate is 11.1% in the first year after a disaster, and the total growth in the first two years amounts to 20.9%. This is of course much higher than the average growth across these countries over the entire sample, which is just 2.0%. The second column computes how much of the 'gap' from peak to trough is resorbed by this growth, i.e. how much lower is GDP per capita compared to the previous peak. At the trough, on average GDP is 29.8% less than at the previous peak. But on average, this gap is reduced after three years to 13.7%.

Of particular interest are the larger disasters, because diminishing marginal utility implies that people care enormously about them (e.g., in the basic disaster model, a .2% probability of a 20% disaster yields an equity premium of 0.5%, but a 1% probability of a 40% disaster yield an equity premium of 1.8%) Columns 3 and 4 replicate these computations for the subsample of disasters larger

[4] The data is from Maddison (2003).

[5] See however Cerra and Saxena (2007) for a different view.

[6] Except the most recent episodes in Argentina, Indonesia and Urugay, for which the next five years of data are not yet available.

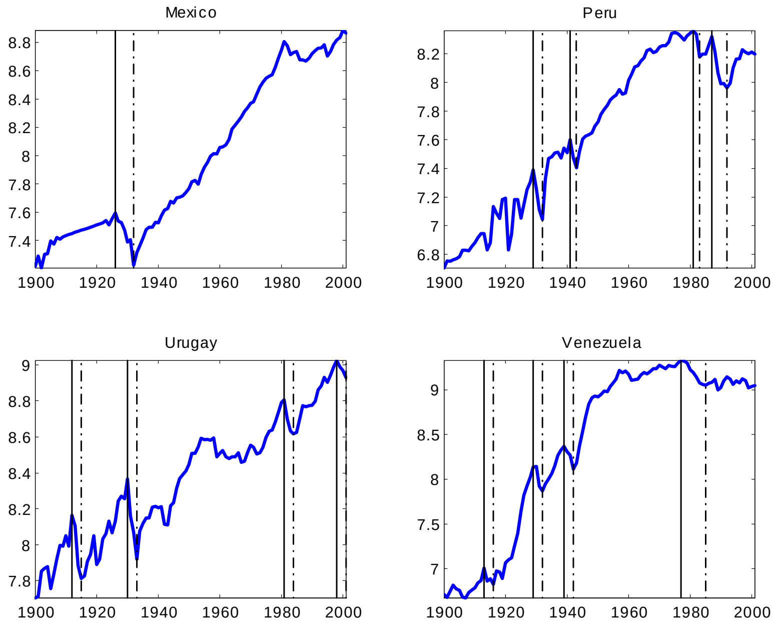

Figure 2: Log GDP per capita (in 1990 dollars) for four countries: Mexico, Peru, Urugay and Venezuela. The disaster start (resp.end) dates are taken from Barro (2006), and are shown with a vertical full (resp. dashed) line.

than 25%. These disasters are also substantially reversed, because the average growth in the first two years after the disaster is over 30%. Measuring disasters and recoveries certainly deserves more study, but it seems clear that the iid assumption is incorrect: growth is substantially larger after a disaster than unconditionally. (However, recoveries might be less strong for consumption than for GDP if people are able to smooth consumption during disasters.)

3.2 A Disaster model with Recoveries and Epstein-Zin utility

How do recoveries affect the predictions of the disaster model? To study this question, I extend the Barro-Rietz model and allow for recoveries. Recall that the consumption process in the Barro-Rietz model is:

$$\begin{aligned}
\Delta \log C_t &= \mu + \sigma\varepsilon_t, \text{ with probability } 1-p, \\
&= \mu + \sigma\varepsilon_t + \log(1-b), \text{ with probability } p,
\end{aligned}$$

where ε_t is iid $N(0,1)$. To allow for recoveries, I modify this process as follows: if there was a disaster in the previous period, then, with probability π, consumption goes back up by an amount $-\log(1-b)$. This is a particularly simple way of modelling recoveries. When $\pi = 0$, we have the Barro-Rietz model. When $\pi = 1$, a recovery is certain. Below I consider more complicated (and more realistic) specifications.

When $\pi > 0$, the consumption process is not iid any more, and as a result the price-dividend ratio moves over time (more details on this below). There is no useful closed form solution, but the price of

In percent	All disasters (57 events)		Disaster greater than 25% (27 events)	
Years after Trough	Growth from Trough	Loss from previous Peak	Growth from Trough	Loss from previous Peak
0	0	-29.8	0	-41.5
1	11.1	-22.8	16.1	-32.7
2	20.9	-16.8	31.3	-24.2
3	26.0	-13.7	38.6	-20.4
4	31.5	-10.2	45.5	-16.9
5	37.7	-6.1	52.2	-13.4

Table 1: Measuring Recoveries. The table reports the average of (a) the growth from the trough to 1,2,3,4,5 years after the trough and (b) the difference from the current level of output to the previous peak level, for 0,1,2,3,4,5 years after the trough.

equity can still be easily calculated using the standard recursion:

$$\frac{P_t}{C_t} = E_t \left(e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \left(1 + \frac{P_{t+1}}{C_{t+1}} \right) \right).$$

I use the same parameter values as Barro (see Section 2). Figure 3 depicts the equity premium, as a function of the probability π of a recovery. This is a comparison across different economies which are identical but for the parameter π. (In particular, I keep the same assumptions as Barro for the government bond default.) For $\pi = 0$ we have, as in Section 3, an equity premium of 3.5%. The equity premium is *higher* when the probability of a recovery is higher.

While this result may be surprising at first, it is actually easy to understand. Consider the effect on the price-dividend ratio, right after a disaster, of the possibility of recovery. Start from the present-value identity: the price of a claim to $\{C_t\}$ satisfies

$$\frac{P_t}{C_t} = E_t \sum_{k \geq 1} \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{1-\gamma},$$

hence the fact that a recovery may arise, i.e. that $C_{t+1}, C_{t+2}, ...$, is higher than would have been expected without a recovery, can increase or decrease the stock price today, depending on whether $\gamma > 1$ or $\gamma < 1$. The intuition is that good news about the future have two effects: on the one hand, they increase future dividends (equal to consumption), which increases the stock price today (the cash-flow effect), but on the other hand they increase interest rates, which lowers the stock price today (the discount-rate effect). The later effect is stronger when interest rates rise more for a given change in consumption, i.e. when the intertemporal elasticity of substitution (IES) is lower. Given a low IES, the price-dividend ratio falls more when there is a possible recovery than when there are no recoveries. This in turn means that equities are more risky ex-ante, and as a result the equity premium is larger.

This discussion clearly suggests that the IES plays a role by affecting risk-free interest rates. While Barro and Rietz used a low IES, this was merely because they required a significant risk aversion. To disentangle these two parameters, I extend the model and introduce recursive preferences, as in Epstein

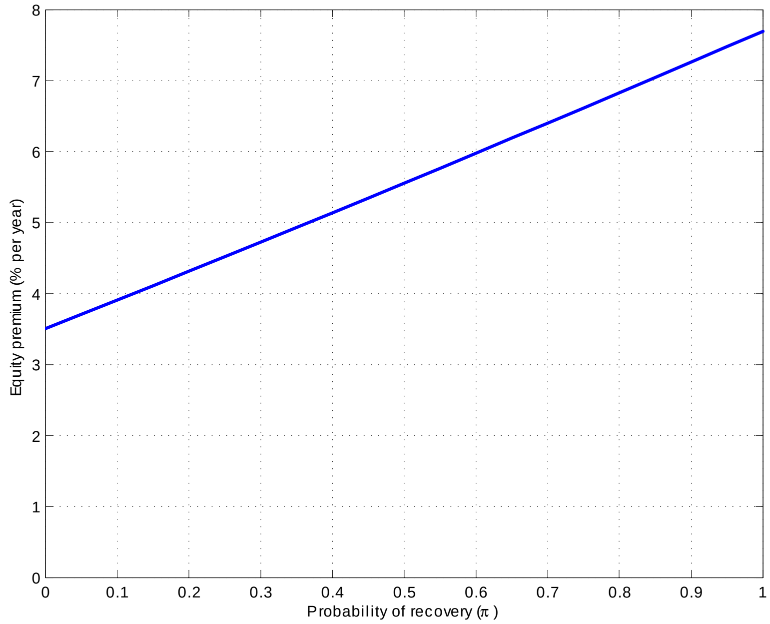

Figure 3: This figure plots the unconditional equity premium, as a function of the probability of recovery following a disaster, in the CRRA model. Parameters as in Barro (2006).

and Zin (1989). This allows me to compute the predictions of the Barro-Rietz model when risk aversion is large but the IES is not small. Utility is defined through the recursion:

$$V_t = \left(\left(1 - e^{-\rho}\right) C_t^{1-\alpha} + e^{-\rho} E_t \left(V_{t+1}^{1-\theta}\right)^{\frac{1-\alpha}{1-\theta}} \right)^{\frac{1}{1-\alpha}}.$$

With these preferences, the IES is $1/\alpha$ and the risk aversion to a static gamble is θ. The stochastic discount factor is:

$$M_{t+1} = e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \left(\frac{V_{t+1}}{E_t (V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}} \right)^{\alpha - \theta},$$

and it is straightforward to compute the equity premium and government bond price, taking default into account, with this formula (see the appendix for computational details). Table 2 shows the (log geometric unconditional) equity premium, as a function of the probability of a recovery, for four different elasticities of substitution: 1/4 (Barro's number), 1/2, 1 and 2. In this computation, the risk aversion θ is kept constant equal to 4. Note that the four lines intersect for $\pi = 0$ since in this case, consumption growth is *iid*, and the IES does not affect the equity premium.

For an IES$= 0.25$, we have the results of figure 3. When the IES is not so low however, recoveries reduce the equity premium. The intuition is that the decrease in dividends is transitory and thus in disasters stock prices fall by a smaller amount than dividends do, making equities less risky than in the iid case. These results are consistent with the literature on autocorrelated consumption growth and log-normal processes (John Y. Campbell (1999), Ravi Bansal and Amir Yaron (2004)). While Bansal and Yaron emphasize that the combination of *positively* autocorrelated consumption growth and an IES

Probability of a recovery π	0.00	0.30	0.60	0.90	1.00
IES = 0.25	3.31	4.62	5.91	7.19	7.64
IES = 0.50	3.31	3.30	3.03	2.26	1.68
IES = 1	3.31	2.69	1.94	1.00	0.54
IES = 2	3.31	2.42	1.52	0.63	0.30

Table 2: Unconditional log geometric equity premium, as a function of the intertemporal elasticity of substitution (IES) and the probability of a recovery. This table sets risk aversion 4 and the other parameters as in Barro (2006).

above unity can generate large risk premia, Campbell shows that when consumption growth is *negatively* autocorrelated, risk premia are *larger* when the IES is below unity. Recoveries induce negative serial correlation, so even though Campbell's results do not directly apply (because the consumption process is not lognormal), the intuition seems to go through.

Of course, there is no clear agreement on what is the proper value of the IES. The standard view is that it is small (e.g. Robert Hall (1988)), but this has been challenged by several authors (see among others Attanasio et al., Bansal and Yaron, Casey Mulligan (2004) Fatih Guvenen (2006), and Vissing Jorgensen (2001)). How then, can we decide which IES is more reasonable for the purpose of studying recoveries? The natural answer is to use data on asset prices during disasters.

Implications for asset prices during disasters

How can we decide which IES is more reasonable? The natural solution is to use data on asset prices during disasters. A low IES implies huge interest rates following a disaster if consumers anticipate a recovery, while a high IES implies moderately high interest rates, and a small increase in the P-D during disasters. In the data, interest rates are not huge, but the P-D ratio tends to fall during disasters, though not necessarily by a large amount. Hence, it is not clear which model fits the data best. More fundamentally, in the model, disasters are instantaneous while they are more gradual in the data, making it difficult to find an empirical counterpart to asset prices right after the disaster.

More precisely, figures 4 and 5 depict the implications of the model with recoveries ($\lambda = 1$ i.e. no waiting) for two levels of IES (.25 and 2) and for any probability of recovery. The baseline disaster model is thus the case where the probability of recovery is zero. The panels present the expected equity return, risk free rate and risk premium as well as the P-D ratio, conditional on the current state (no disaster in the previous period, or a 35% disaster just occurred).

These figures reveal that when the IES is low (the Barro calibration), the possibility of a recovery has a very large effect on risk-free interest rates following a disasters: as consumers are momentarily poor, they want to borrow against their future income, which drives the interest rate up. These huge interest rates are certainly not observed in the data. (Perhaps, people did not anticipate the recoveries.) The high IES case implies interest rates which are much smaller, because the interest rate does not need to be so high to make people willing to lend. However, the high IES case also implies that the P-D ratio increases following a disaster by a moderate amount; compare the bottom panels of Figure 5. In contrast, the low IES model implies that the P-D ratio falls.

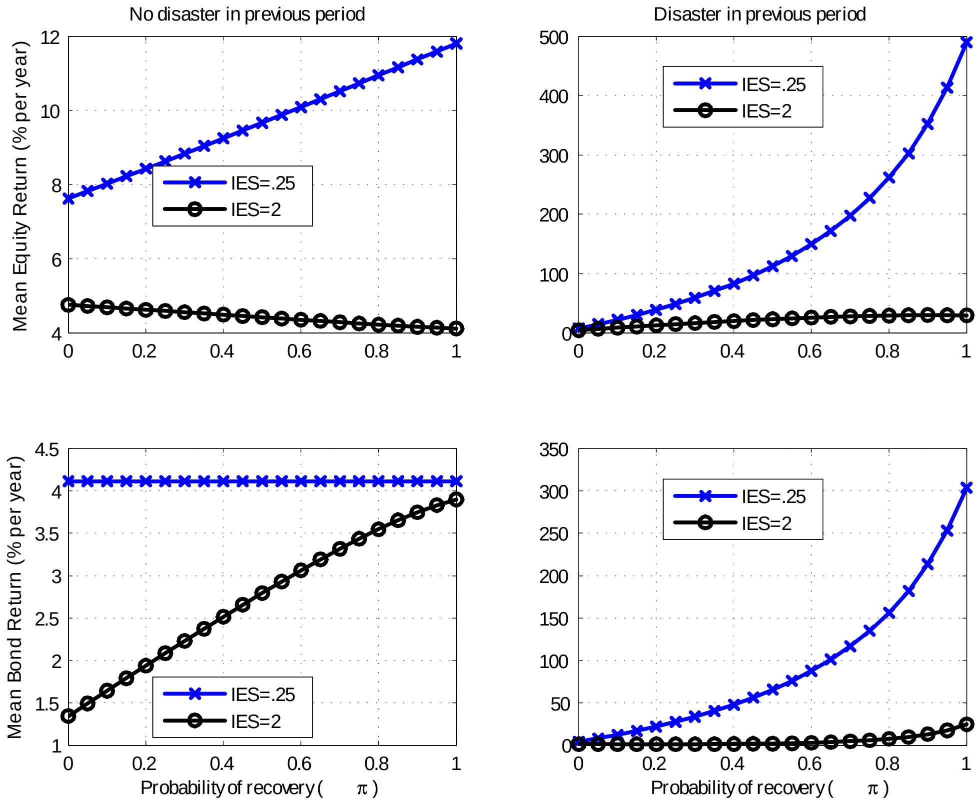

Figure 4: Asset price implications of the simple example.

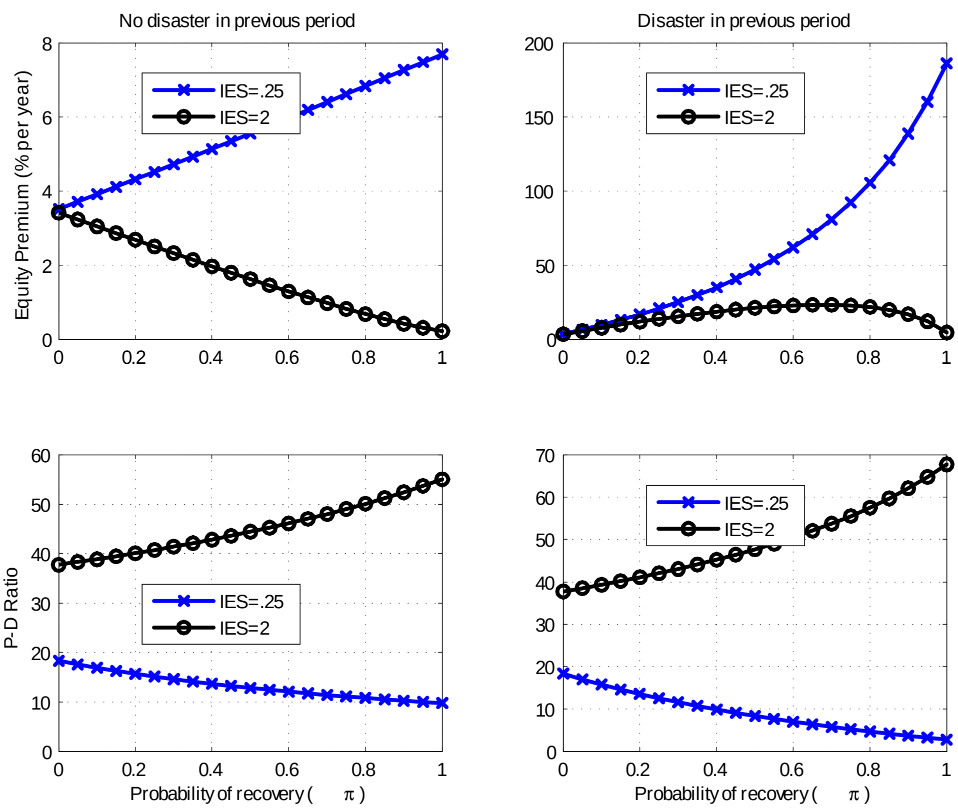

Figure 5: Asset price implications of the simple example.

These results are all driven by the changes in interest rates. The Campbell-Shiller approximation to the P-D ratio is

$$\log \frac{P_t}{D_t} = k + (1-\alpha)\, E_t \sum_{k=1}^{\infty} \beta^{k-1} \Delta c_{t+k},$$

where k is a constant and IES=$1/\alpha$.[7] Hence, future expected growth in consumption (=dividends) will increase the asset price iff $\alpha < 1$. In the data, we know that P-D ratios fall during disasters, and interest rates are high, but not on the scale implied by the low IES calibration. Hence, it is not clear which model fits the data best. There may be additional elements which affect these results. For instance, if people become more fearful in disasters (because of high uncertainty or high risk aversion), then the P-D ratio may fall as the equity premium is large, without having a large effect on the interest rate. However, this is not easy to make this explanation work quantitatively.

More realistic recovery processes

Clearly the recovery process studied above is too simple: recoveries might not occur right after a disaster, they sometimes also occur more slowly. Moreover, the size of the recovery is somewhat uncertain. To take these possibilities into account, I entertain the following Markov chain formulation. Starting in the normal growth state, with probability p, there is a disaster. Next, with probability λ each period, you stay in a 'waiting' state; with $1-\lambda$ each period, your recovery is determined: with probability π_1, consumption goes up by $-\log(1-b)$, with probability π_2 there is a partial recovery $-\log(1-b_2)$, and with probability $\pi_3 = 1 - \pi_1 - \pi_2$ there is no recovery. In any case, you then return to the 'normal state'.

Formally, the Markov matrix is:

$$\begin{pmatrix} 1-p & p & 0 & 0 & 0 \\ (1-\lambda)\,\pi_3 & 0 & \lambda & (1-\lambda)\pi_1 & (1-\lambda)\pi_2 \\ (1-\lambda)\,\pi_3 & 0 & \lambda & (1-\lambda)\pi_1 & (1-\lambda)\pi_2 \\ 1-p & p & 0 & 0 & 0 \\ 1-p & p & 0 & 0 & 0 \end{pmatrix},$$

where the first state is 'normal growth', the second state is the disaster, the third state is the 'waiting' state, and the fourth and fifth state are total or partial recoveries. Note that this formulation assumes that in states 2 and 3 (i.e. after a disaster and in the waiting period) there is no probability of disaster, but one can also consider the case where this is possible.[8]

Figure 6 depicts the effect of the speed of convergence on the equity premium. The previous computation implicitly assumed $\lambda = 1$, i.e. the decision of whether or not there is a recovery is immediate. This picture shows the results when one varies this parameter (this figure sets $\pi = 0.7$: there is a 70% chance of an eventual recovery, by a full amount, and a 30% change of no recovery). This figure clearly shows that when λ is small, the equity premium converges to the value (3.5%) that we had when there

[7] Of course, this approximation is not valid since the consumption process is not log-normal, however it is useful to understand the intuition for the results.

[8] The matrix above is written for the case of a single disaster size b. In my computations, I use the historical distribution of disaster sizes. Because the size of a recovery depends on the size of the disaster, I must actually have as many 'disaster', 'waiting' and 'recovery' states as there are sizes of disasters.

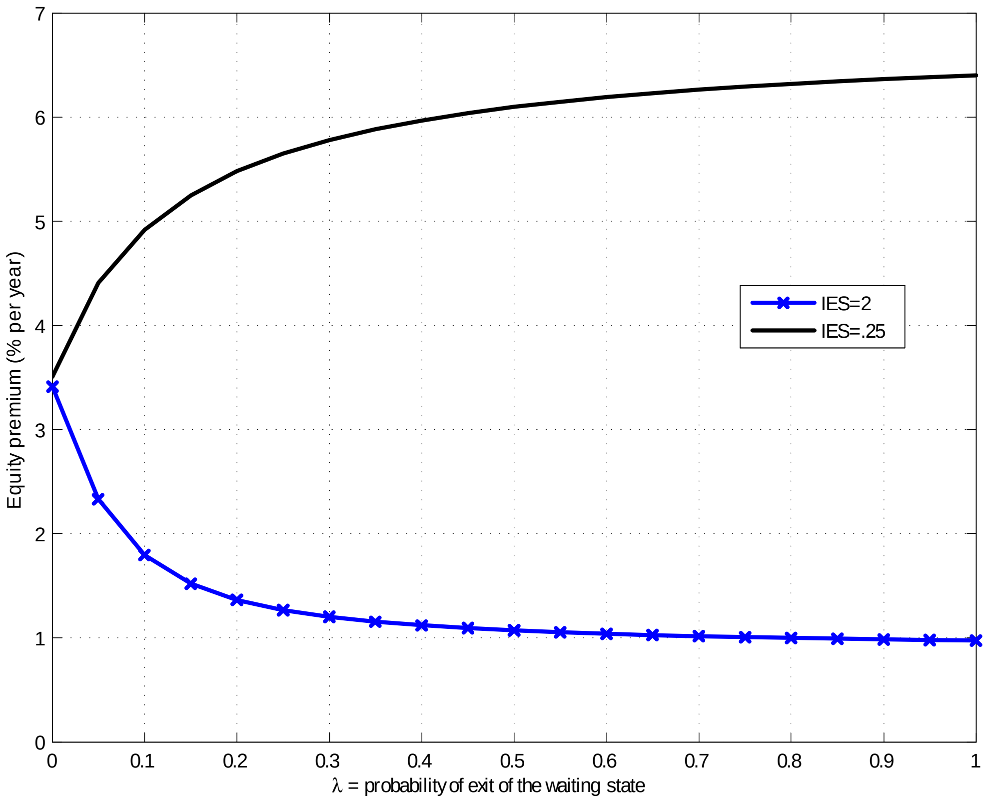

Figure 6: Impact of the speed of recovery on the unconditional equity premium in the model, for two elasticities of substitution parameters. The other parameters are as in the text (and Barro (2006)).

is no recovery at all. However, even a moderate λ (say, 0.2) shows a substantial difference between the case of a low IES, the case of a high IES, and the absence of recovery.

4 Time-Series Predictability in the Disaster Model

Given the success of the disaster model in accounting for the risk-free rate and equity premium puzzles, it is important to study if the model can also account for additional asset pricing facts. In this section I focus on the time-series predictability of stock returns and excess stock returns. Empirical research documents that both the stock return and the excess stock return are forecastable. The basic regression is:

$$R_{t+1}^e - R_{t+1}^f = \alpha + \beta \frac{D_t}{P_t} + \varepsilon_{t+1},$$

where R_{t+1}^e is the equity return, R_{t+1}^f the risk-free return, and $\frac{D_t}{P_t}$ the dividend yield. As an illustration, John Cochrane (2008) reports for the annual 1926-2004 U.S. sample: $\beta = 3.83$ (t-stat $= 2.61$, $R^2 = 7.4\%$). A key feature of the data is that using as the left-hand side the equity return R_{t+1}^e rather than the excess return $R_{t+1}^e - R_{t+1}^f$ does not change the results markedly: $\beta = 3.39$ (t-stat $= 2.28$, $R^2 = 5.8\%$). That is, equity returns are forecastable, because risk premia are forecastable, and not because of large changes in risk-free interest rates. This section studies whether a reasonable extension of the disaster model can match these empirical findings. The answer is mostly negative.

More precisely, this section extends the disaster model to incorporate a time-varying probability (or size) of disaster. The main result is that if utility is CRRA, the disaster model cannot account for *both* findings of stock return and excess stock return predictability. This is also true when utility is of the Epstein-Zin type, provided that the probability of disaster evolves in an *iid* fashion. The case of Epstein-Zin utility and persistent changes in probability of disaster, which is somewhat more promising, is analyzed through numerical simulations.

These results clarify and extend some findings of Xavier Gabaix (2007). Gabaix uses the "linearity-generating" model (Gabaix 2007b), and expresses some of his results in terms of a 'resilience' variable. Expected returns change over time because the probability of a disaster, the size of consumption disaster, or the size of dividend disaster changes over time, but my results show that only when the size of dividend disaster changes over time, and the size of consumption disaster doesn't, is the model consistent with the evidence on time-series predictability. Empirical research suggests that the expected return on many assets are correlated. In the consumption-based model, the expected returns can be positively correlated because they are all affected by properties of consumption (e.g., Campbell and Cochrane (1999)). But if variation in expected returns is due to variation in the size of dividend disaster, it is not clear why the expected returns across assets should be correlated.

4.1 Time-varying Disaster Probability with Power utility

The model is a standard "Lucas tree" endowment economy. There is a representative consumer who has power utility (constant relative risk aversion):

$$E\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\gamma}}{1-\gamma}.$$

To generate variation in expected returns over time, we need to introduce some variation over time in the riskiness of stocks. The natural idea is to make the probability of disaster-time varying. (The case of a time-varying size of disasters is tackled below.) Assume, then, that the endowment follows the following stochastic process:

$$\begin{aligned}\Delta\log C_{t+1} &= \mu+\sigma\varepsilon_{t+1},\ \text{with probability } 1-p_t,\\ &= \mu+\sigma\varepsilon_{t+1}+\log(1-b),\ \text{with probability } p_t,\end{aligned}$$

where ε_{t+1} is *iid* $N(0,1)$ and $0<b<1$ is the size of the disaster. Hence, in period $t+1$, with probability p_t, consumption drops by a factor b. The disaster probability $p_t\in[0,1]$ evolves over time according to a first-order Markov process, governed by the transition probabilities $F(p_{t+1}|p_t)$. Note that p_t is the probability of a disaster at time $t+1$, and it is drawn at time t. The Markov process is assumed to be monotone, i.e. $F(x|y_1)\le F(x|y_2)$ for any $x\in[0,1]$ and for any $y_1\ge y_2$. This assumption means that p_t is positively autocorrelated. The Markov process is also assumed to have the Feller property.[9] The realization of the disaster, and the process $\{p_t\}$ are further assumed to be statistically independent of ε_t at all dates. Finally, I assume that the realization of p_{t+1} is independent of the realization of disasters

[9] That is, the conditional expectation of a continuous function of the state tomorrow is itself a continuous function of the state today.

at time $t+1$, conditional on p_t. That is, the new draw for the probability of disaster at time $t+2$, labelled p_{t+1}, is independent of whether there is a disaster at time $t+1$. This simplification is crucial to obtain analytical results. It implies that the P-D ratio is conditionally uncorrelated with current dividend growth or consumption growth.

This simple economy has a single state variable, the probability of disaster p. We can express the asset prices as a function of this state variable, which is assumed to be perfectly observed by the agents in the economy. The risk-free rate satisfies the usual Euler equation:

$$E_t\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right)R_{t+1}^f = 1.$$

Computing this conditional expectation[10] yields:

$$\log R^f(p) = -\log\beta + \gamma\mu - \frac{\gamma^2\sigma^2}{2} - \log\left(1-p+p(1-b)^{-\gamma}\right).$$

When $p=0$, this formula collapses to the well-known result of the iid lognormal model. Because $b<1$, we see that the risk-free rate is lower when $p>0$, and the higher the probability of a disaster, the lower the risk-free rate. This reflects that a higher probability of disaster reduces expected growth and increases risk, and thus leads agents to save, both for intertemporal substitution and for precautionary reasons. This drives the risk-free rate down.[11]

The second asset we consider is a "stock", i.e. an asset which pays out the consumption process. The stock price satisfies the standard recursion:

$$\frac{P_t}{D_t} = E_t\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\left(\frac{P_{t+1}}{D_{t+1}}+1\right)\right).$$

As usual, the price-dividend ratio depends only on the state variable, in this case p. Denote $q(p)$ the P-D ratio when $p_t = p$. Given the assumption that the realization of disaster is independent of the new draw for p, conditional on the current value of p, q satisfies the functional equation:

$$q(p) = \beta e^{(1-\gamma)\mu+(1-\gamma)\frac{\sigma^2}{2}}\left(1-p+p(1-b)^{1-\gamma}\right)\int_0^1 \left(q(p')+1\right)dF(p'|p). \tag{1}$$

Let $g(p) = \beta e^{(1-\gamma)\mu+(1-\gamma)\frac{\sigma^2}{2}}\left(1-p+p(1-b)^{1-\gamma}\right)$. The function g is increasing if $\gamma>1$ and is decreasing if $\gamma<1$. This equation can be analyzed using standard tools from Stokey, Lucas and Prescott (1989), leading to the following result:

Proposition 1 *Assume that* $\xi \overset{def}{=} \max_{0\le p\le 1} g(p) < 1$. *Then there exists a unique solution* q^* *to equation (1). Moreover,* q^* *is nondecreasing if* g *is nondecreasing and is nonincreasing if* g *is nonincreasing.*

Proof. Define B the set of continuous (and thus bounded) functions mapping $[0,1]$ into $\mathbb{R}^+$. Define the operator $T: B \to B$, which maps a function $q \in B$ into a new function $\widetilde{q} \in B$, defined by the

[10] Note that this conditional expectation is an integral over three random variables: (1) the business cycle shock ε_{t+1}, which is $N(0,\sigma^2)$, (2) the realization of the disaster, which is a binomial variable determined by p_t, (3) the new draw of p_{t+1}, given p_t. The assumptions above imply that these three variables are conditionnally independent, which is why the computation of the integral is simple.

[11] An extension of the disaster model would have positive as well as negative disasters, thus creating a pure precautionary savings effect. However, diminishing marginal utility implies that the positive disasters typically do not matter much.

right-hand-side of (1), i.e.:

$$\widetilde{q}(p) = (Tq)(p) = g(p) \int_0^1 \left(q(p') + 1\right) dF(p'|p) = g(p) + g(p) \int_0^1 q(p') dF(p'|p). \tag{2}$$

Since the Markov process F has the Feller property, T indeed maps B into B. Next we show that T is a contraction. To see this, pick any two $q_1, q_2 \in B$, then for any $p \in [0,1]$:

$$\begin{aligned}
(Tq_1)(p) - (Tq_2)(p) &= g(p) \int_0^1 \left(q_1(p') - q_2(p')\right) dF(p'|p), \\
|(Tq_1)(p) - (Tq_2)(p)| &\leq \xi \int_0^1 |(q_1(p') - q_2(p'))| \, dF(p'|p) \\
&\leq \xi \left\| q_1 - q_2 \right\|_\infty ,
\end{aligned}$$

hence $\sup_{p\in[0,1]} |(Tq_1)(p) - (Tq_2)(p)| = \|Tq_1 - Tq_2\|_\infty \leq \xi \|q_1 - q_2\|_\infty$, where $\|f\|_\infty = \sup_{x\in[0,1]} |f(x)|$ is the sup norm. Since $\xi < 1$, this shows that T is a contraction.[12] The contraction mapping theorem implies that there exists a unique solution q^* to the fixed point problem $Tq = q$. Because the Markov process F is monotone, T satisfies a monotonicity property. More precisely, if g is nondecreasing, then T maps nondecreasing functions into nondecreasing functions; and if g is nonincreasing, then T maps nonincreasing functions into nonincreasing functions. This can be seen from (3). For instance if g is nondecreasing: we know that if q is nondecreasing, the function $p \to \int_0^1 q(p')dF(p'|p)$ is nondecreasing; because both g and q are nonnegative and increasing, the product $g(p)\int_0^1 q(p')dF(p'|p)$ is nondecreasing, and thus $g(p) + g(p)\int_0^1 q(p')dF(p'|p)$ is nondecreasing. Because the set of nondecreasing (resp. nonincreasing) functions is closed under the sup norm, this result implies that the fixed point q^* is nondecreasing if g is nondecreasing and is nonincreasing if g is nonincreasing (see Theorem 4.7 in Stokey, Lucas and Prescott (1989)). ∎

We are now in position to compute the expected return on equity. Given the definition $E_t R^e_{t+1} = E_t \left(P_{t+1} + C_{t+1}\right)/P_t$, we have:

$$ER^e(p) = E_t \left(\frac{C_{t+1}}{C_t}\right) E_{p'|p} \left(\frac{q(p') + 1}{q(p)}\right).$$

The Euler equation (1) implies that $E_{p'|p}\left(\frac{q(p')+1}{q(p)}\right) = \frac{1}{g(p)}$, hence:

$$ER^e(p) = \frac{e^{\mu + \frac{\sigma^2}{2}} \left(1 - p + p(1-b)\right)}{\beta e^{(1-\gamma)\mu + (1-\gamma)^2 \frac{\sigma^2}{2}} \left(1 - p + p(1-b)^{1-\gamma}\right)}.$$

Rearranging and taking logs yields:

$$\log ER^e(p) = \gamma\mu - \frac{\gamma^2\sigma^2}{2} + \gamma\sigma^2 - \log\beta + \log \frac{\left(1 - p + p(1-b)\right)}{\left(1 - p + p(1-b)^{1-\gamma}\right)}.$$

Again we recognize the first four terms as the iid lognormal model. The last term, which varies over time with p, is decreasing in p, as can be easily verified by taking derivatives. A higher probability of disaster reduces expected growth, reducing the expected return on equity.

The log equity premium is obtained as:

$$\log \frac{ER^e(p)}{R^f(p)} = \gamma\sigma^2 + \log \frac{\left(1 - p + p(1-b)\right)\left(1 - p + p(1-b)^{-\gamma}\right)}{\left(1 - p + p(1-b)^{1-\gamma}\right)}.$$

[12] When g is increasing, we can alternatively use the Blackwell sufficient conditions (see Stokey, Lucas and Prescott (1989), chapter 4) to establish this result, but when g is decreasing the Blackwell sufficient conditions do not hold.

Taking derivatives in this expression shows that this is an increasing function of p when p is small enough.[13] The following proposition summarizes the results:

Proposition 2 *Assume that the Markov process F is monotone and satisfies the Feller property, and that* $\max_{p\in[0,1]} \beta e^{(1-\gamma)\mu+(1-\gamma)\frac{\sigma^2}{2}} \left(1-p+p(1-b)^{1-\gamma}\right) < 1$. *Then, (a) the risk-free rate and the expected equity return are decreasing in p; (b) the price-dividend ratio is increasing in p if and only if* $\gamma > 1$; *(c) for p small enough, the equity premium is increasing in p.*

It is interesting to relate these results to the empirical evidence outlined in the introduction, i.e. $Cov_t(P_t/D_t, E_t R^e_{t+1}) < 0$ and $Cov_t(P_t/D_t, E_t R^e_{t+1} - R^f_{t+1}) < 0$. Proposition 2 implies that, whatever the value of γ, the model cannot match both facts. If $\gamma > 1$, then the equity premium and the P-D ratio are both increasing in p, hence a high P-D ratio forecasts a high excess stock return, contrary to the data. (The fact that a high P-D ratio corresponds to a high probability of disaster p is also counterintuitive.) If $\gamma < 1$, then the P-D ratio and the equity return are both decreasing in p, and hence a high P-D ratio forecasts a high equity return, contrary to the data. The reason why the disaster model generates these counterfactual implications is that it predicts large variations in risk-free interest rates.

It is straightforward to allow for leverage. If we use the standard formulation, $\Delta \log D_t = \lambda \Delta \log C_t$, we simply need to replace in the formulas above the factor $(1-b)^{1-\gamma}$ by $(1-b)^{\lambda-\gamma}$. As a result, the P-D ratio is increasing in p if and only if $\gamma > \lambda$. But the fundamental conundrum remains: neither $\gamma > \lambda$ nor $\lambda > \gamma$ allows the model to generate both the stock return and the excess stock return predictability.

Rather than having the probability of disaster p change over time, one may assume that it is the size of disasters b that changes over time, i.e.

$$\begin{aligned}
\Delta \log C_{t+1} &= \mu + \sigma\varepsilon_{t+1}, \text{ with probability } 1-p, \\
\Delta \log C_{t+1} &= \mu + \sigma\varepsilon_{t+1} + \log(1-b_t), \text{ with probability } p.
\end{aligned}$$

If we make the same assumptions for b as the ones we did for p above, it is straightforward to prove the following analogous result. Define $g(b) = \beta e^{(1-\gamma)\mu+(1-\gamma)\frac{\sigma^2}{2}} \left(1-p+p(1-b)^{1-\gamma}\right)$. Assume b follows a Markov process with support $\left[\underline{b}, \overline{b}\right]$ with transition function F, and assume that the Markov process is monotone and satisfies the Feller property, and that the independence assumptions hold.

Proposition 3 *Assume* $\max_{\underline{b}\leq b\leq \bar{b}} g(b) < 1$. *Then there exists a unique solution q^* to the functional equation defining the price-dividend ratio. Moreover, (a) the risk-free rate and the expected equity return are decreasing in b; (b) the price-dividend ratio is increasing in b if and only if* $\gamma > 1$; *(c) if p is small enough, the equity premium is increasing in b.*

This extension thus does not resolve the previously noted conundrum. Finally, a last extension is to only allow the size of dividend disasters to change over time. Assume, then, that the stochastic processes

[13] Because disasters are a binomial variable, the uncertainty is highest for intermediate values of p, and hence the risk premium is not increasing over the entire range of values: if p is large enough, a further increase reduces the uncertainty and thus the risk premium. This remark is not important in practice because disasters are always calibrated as rare events.

for consumption and dividends satisfy:

$$\begin{aligned}
\Delta \log C_{t+1} &= \mu + \sigma\varepsilon_{t+1}, \\
\text{and } \Delta \log D_{t+1} &= \mu + \sigma\varepsilon_{t+1}, \text{with probability } 1-p; \\
\\
\text{or } \Delta \log C_{t+1} &= \mu + \sigma\varepsilon_{t+1} + \log(1-b), \\
\text{and } \Delta \log D_{t+1} &= \mu + \sigma\varepsilon_{t+1} + \log(1-d_t), \text{ with probability } p,
\end{aligned}$$

where d_t follows a monotone Feller Markov process with support $\left[\underline{d}, \overline{d}\right]$, and the independence assumptions hold. Let $g(d) = \beta e^{(1-\gamma)\mu+(1-\gamma)\frac{\sigma^2}{2}} \left(1-p+p(1-d)(1-b)^{-\gamma}\right)$. In this case, g is always nonincreasing.

Proposition 4 *Assume* $\max_{\underline{d}\leq d\leq \overline{d}} g(d) < 1$. *Then there exists a unique solution* q^* *to the functional equation defining the price-dividend ratio. Moreover, (a) the risk-free rate is constant over time, (b) the price-dividend ratio is decreasing in* d*, (b) the expected equity return and equity premium are increasing in* d*.*

Hence, this model is consistent with the two pieces of evidence on predictability. It also generates the intuitive result that a high expected size of disaster leads to a low P-D ratio. However, it is unclear if the assumption of that the size of the dividend disaster changes over time, but the size of a consumption disaster does not, is empirically reasonable.

4.2 Time-varying Disaster Probability with Epstein-Zin utility

Since the failure of the disaster model in Proposition 2 is due to the fact that interest rates vary too much, it is interesting to allow for Epstein-Zin utility so as to separate intertemporal elasticity of substitution (IES) from risk aversion, and to use the IES parameter to reduce the volatility of the risk-free rate. Utility is defined recursively as

$$V_t = \left((1-\beta)\, C_t^{1-\alpha} + \beta E_t \left(V_{t+1}^{1-\theta}\right)^{\frac{1-\alpha}{1-\theta}} \right)^{\frac{1}{1-\alpha}}.$$

With these preferences, the IES is $1/\alpha$ and the risk aversion to a static gamble is θ. When $\theta = \alpha$, or if there is no risk, these preferences collapse to the familiar case of expected utility. In general we cannot reduce compound lotteries, so that the intertemporal composition of risk matters: the agent prefers an early resolution of uncertainty if $\theta > \alpha$. The stochastic discount factor is:

$$M_{t+1} = e^{-\rho} \left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} \left(\frac{V_{t+1}}{E_t(V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}}\right)^{\alpha-\theta}.$$

I assume that the stochastic process for the endowment is the same as in Proposition 2. The main result is:

Proposition 5 *Assume that the disaster probability is iid, i.e.* $F(p_{t+1}|p_t) = F(p_{t+1})$. *Then (a) if* $\theta \geq 1$*, the P-D ratio is increasing in* p *if and only if* $\alpha > 1$*; (b) the risk-free rate and the expected return on equity are both decreasing in* p*, and (c) the equity premium is increasing in* p*, for* p *small enough.*

Proof. First, rewrite the utility recursion as:

$$\frac{V_t}{C_t} = \left(1 - e^{-\rho} + e^{-\rho} E_t \left(\left(\frac{V_{t+1}}{C_{t+1}}\right)^{1-\theta} \left(\frac{C_{t+1}}{C_t}\right)^{1-\theta} \right)^{\frac{1-\alpha}{1-\theta}} \right)^{\frac{1}{1-\alpha}}.$$

The state variable is the probability of a disaster p. Let $g(p_t) = V_t/C_t$. Then g satisfies the functional equation:

$$g(p)^{1-\alpha} = 1 - e^{-\rho} + e^{-\rho} \left(Eg(p')^{1-\theta}\right)^{\frac{1-\alpha}{1-\theta}} \left(1 - p + p(1-b)^{1-\theta}\right)^{\frac{1-\alpha}{1-\theta}} e^{(1-\alpha)\mu + (1-\alpha)(1-\theta)\frac{\sigma^2}{2}},$$

where the expectation is over p' next period; given the *iid* assumption, this expectation is independent of p. The price dividend ratio satisfies $P_t/C_t = q(p_t)$ with:

$$\begin{aligned} q(p) &= E_t\left(M_{t+1}\frac{C_{t+1}}{C_t}\left(1 + \frac{P_{t+1}}{C_{t+1}}\right)\right) \\ q(p) &= E_t\left(e^{-\rho}\left(\frac{C_{t+1}}{C_t}\right)^{1-\alpha}\left(\frac{V_{t+1}}{E_t(V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}}\right)^{\alpha-\theta}\left(1 + \frac{P_{t+1}}{C_{t+1}}\right)\right). \end{aligned}$$

Straightforward computations yield

$$q(p) = e^{-\rho} e^{(1-\alpha)\mu + (1-\theta)(1-\alpha)\frac{\sigma^2}{2}} \left(1 - p + p(1-b)^{1-\theta}\right)^{\frac{1-\alpha}{1-\theta}} \frac{E\left[g(p')^{\alpha-\theta}\left(1 + q(p')\right)\right]}{E\left(g(p')^{1-\theta}\right)^{\frac{\alpha-\theta}{1-\theta}}}.$$

The expectation on the right-hand side is constant, given our *iid* assumption. Hence, if $\theta > 1$, the price-dividend ratio $q(p)$ is increasing in p if and only if $(1-\alpha)/(1-\theta) > 0$ i.e. $1 - \alpha < 0$ or $\alpha > 1$, i.e. an elasticity of substitution less than unity. The expected equity return is thus:

$$\begin{aligned} E_t R^e_{t+1}(p) &= E_t\left(\frac{q(p_{t+1}) + 1}{q(p)}\frac{C_{t+1}}{C_t}\right) \\ &= \frac{E\left(q(p')\right) + 1}{q(p)}\left(1 - p + p(1-b)\right) e^{\mu + \frac{\sigma^2}{2}} \\ &= \frac{\left(1 - p + p(1-b)\right) e^{\mu + \frac{\sigma^2}{2}}}{e^{-\rho} e^{(1-\alpha)\mu + (1-\theta)(1-\alpha)\frac{\sigma^2}{2}} \left(1 - p + p(1-b)^{1-\theta}\right)^{\frac{1-\alpha}{1-\theta}}} \frac{E\left(q(p') + 1\right) E\left(g(p')^{1-\theta}\right)^{\frac{\alpha-\theta}{1-\theta}}}{E\left(g(p')^{\alpha-\theta}\left(1 + q(p')\right)\right)} \end{aligned}$$

Hence,

$$\log ER^e(p) = \text{constant } + \alpha\mu + \rho + \frac{\sigma^2}{2}\left(1 - (1-\theta)(1-\alpha)\right) + \log\frac{\left(1 - p + p(1-b)\right)}{\left(1 - p + p(1-b)^{1-\theta}\right)^{\frac{1-\alpha}{1-\theta}}}.$$

This expression is decreasing in p. The risk-free rate is:

$$\begin{aligned} R^f(p) &= \frac{E_t\left(\left(\frac{V_{t+1}}{C_{t+1}}\right)^{1-\theta}\left(\frac{C_{t+1}}{C_t}\right)^{1-\theta}\right)^{\frac{\alpha-\theta}{1-\theta}}}{e^{-\rho} E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\theta}\left(\frac{V_{t+1}}{C_{t+1}}\right)^{\alpha-\theta}\right)} = \frac{\left(1 - p + p(1-b)^{1-\theta}\right)^{\frac{\alpha-\theta}{1-\theta}} e^{(\alpha-\theta)\mu + (\alpha-\theta)(1-\theta)\frac{\sigma^2}{2}} E\left(g(p')^{1-\theta}\right)^{\frac{\alpha-\theta}{1-\theta}}}{e^{-\rho} e^{-\theta\mu + \theta^2\frac{\sigma^2}{2}}\left(1 - p + p(1-b)^{-\theta}\right) E\left(g(p')^{\alpha-\theta}\right)} \\ \log R^f(p) &= \text{constant } + \alpha\mu + \rho + (\alpha - \theta - \theta\alpha)\frac{\sigma^2}{2} + \log\frac{\left(1 - p + p(1-b)^{1-\theta}\right)^{\frac{\alpha-\theta}{1-\theta}}}{\left(1 - p + p(1-b)^{-\theta}\right)}. \end{aligned}$$

Taking derivatives with respect to p shows that this is a decreasing function of p for any θ, α. Finally, the log equity premium is:

$$\begin{aligned} \log\frac{E_t R^e_{t+1}(p)}{R^f(p)} &= \text{constant } + \frac{\sigma^2}{2}\left(1 - (1-\theta)(1-\alpha)\right) - (\alpha - \theta - \theta\alpha)\frac{\sigma^2}{2} + \log\frac{\left(1 - p + p(1-b)\right)\left(1 - p + p(1-b)^{-\theta}\right)}{\left(1 - p + p(1-b)^{1-\theta}\right)} \\ &= \text{constant } + \frac{\sigma^2}{2}\theta + \log\frac{\left(1 - p + p(1-b)\right)\left(1 - p + p(1-b)^{-\theta}\right)}{\left(1 - p + p(1-b)^{1-\theta}\right)}. \end{aligned}$$

As in Section 2, it is easy to see by taking derivatives that this term is increasing in p for small p. ∎

The conclusion from this result is that introducing Epstein-Zin preferences does not solve the conundrum when p is *iid*: the equity premium is increasing in p, while the expected equity return is decreasing in p. Hence, no matter the values of θ and α, it is impossible to generate the findings of stock return and excess return predictability.

Numerical Experiments with persistent shocks to p

Of course, it may be that the iid assumption is too restrictive. It is not possible to obtain analytical results with Epstein-Zin utility when the probability p is serially correlated. However, it is easy to solve the model numerically (details are available upon request). For these computations, I set the parameters as follows: $\beta = e^{-0.03}$, $\mu = 0.025$, $\sigma = 0.02$, $\theta = 4$. The process for consumption growth is the sum of the iid normal shock ε_t and a Markov chain, described by the transition matrix:

$$Q = \begin{pmatrix} (1-p_1)(1-\pi) & (1-p_1)\pi & p_1 \\ (1-p_2)\pi & (1-p_2)(1-\pi) & p_2 \\ (1-\overline{p})\frac{1}{2} & (1-\overline{p})\frac{1}{2} & \overline{p} \end{pmatrix},$$

where the third state is the disaster state, the first state is the low disaster probability state (i.e., $p_1 < p_2$), and the second state is the high probability of disaster state. The parameter π describes the transition from the low-probability to the high-probability state. I set $\pi = .2$, $p_1 = .01$ and $p_2 = .024$ (so that on average the probability of disaster is 1.7 as in Barro (2006)) and I set the disaster size b equal to 40%. Finally, I try different values for α, from 0.5 to 4 (i.e. ranging from an IES equal to .25 to one equal to 2).[14]

The main result is that the P-D ratio does not change a lot as the economy switches from state 1 to state 2. If the IES is equal to 2, the P-D ratio decreases from 36.0 to 35.4; if the IES is equal to 0.25, the P-D ratio increases from 21.1 to 23.3. The equity premium always increases, going from roughly 1.8% to 3.6% (plus or minus 0.2%) for all values of the IES. The expected equity return increases by a small amount if the IES is equal to 2, from 4.6% to 4.8%, but if the IES is equal to .25 it decreases significantly from 9.2% to 3.9%. These numerical results show that the low IES model still fails to generate the fact that a high P-D forecasts low equity premia. The high IES model fits qualitatively both the predictability of stock returns and of equity risk premia. However, the success is not quantitative: the model predicts tiny variations of P-D ratios and equity return and much larger variations in risk premia. Hence, even in this version of the model, the movements in risk-free interest rates are still very large compared to the data.[15]

Overall, my results suggest that it is difficult for the disaster model to fit the facts on predictability of stock returns and excess stock returns. With Epstein-Zin utility and with an IES above unity, the model can fit the facts qualitatively, but not quantitatively.

[14] I also follow Barro's assumptions regarding bond default; i.e. in disasters, with probability .4, the government defaults and repays $1 - b = 60$ cents on the dollar.

[15] Note that this technical 'trick' can be used to analyze other models: one can obtain exact analytical results, without assuming log-linearity or log-normality. The critical assumption is that the state variable which determines the risk premium is conditionally independent of the variable determining dividend growth or consumption. This assumption may not be appealing for some models, but it is reasonable for the disaster model.

5 Cross-Sectional Implications of the Disaster Model

In this section I test whether the disaster model can make sense of the heterogeneity of expected returns across stocks. As is well known from the empirical finance literature, (1) small market capitalization stocks earn higher returns than large stocks on average, (2) value stocks[16] earn higher returns than growth stocks on average (and this effect is stronger among small firms), and (3) stocks which went up recently (winners) earn higher returns than past losers on average - the momentum anomaly. These strategies have low risk by standard measures, such as the Capital Asset Pricing Model (Fama and French (1996)), or the consumption-based model. Similarly, the cross-section of industry expected returns is not well described by standard asset pricing models (Fama and French (1997)). Can we make sense of these asset pricing puzzles using as measure of risk the "exposure to disaster"?

Of course, the difficulty is that it is hard to measure the exposure of a stock to a disaster. The present note proposes two tests, corresponding to two measures of this exposure. First, I use 9-11 as a natural experiment to measure the exposure of a stock to disasters. Second, I proxy the exposure to disaster by the exposure to large downward market returns.

While there does not appear to be previous work which attempts to fit the disaster model to the cross section of expected stock returns, a few papers conduct related empirical exercises. However, several papers test for an asymmetry between upside and downside risk. Ang, Chen and Xing (2006) find that downside risk is more important than upside risk. However, they do not distinguish between large downside risk and small downside risk. Closer to this study is the work of Hollifield, Routledge and Zin (2004) and Polkovinchenko (2006) derive from "exotic" preferences consumption-based models which lead agents to overweights bad outcomes.

5.1 Theory

The theory is a simple consequence of the Barro-Rietz model. Consider a Lucas tree economy with a representative consumer who has power utility. Following Rietz (1988) and Barro (2006), assume that the exogenous process for aggregate consumption is:

$$
\begin{aligned}
\Delta \log C_t &= \mu + \sigma_c \varepsilon_t, \text{ with probability } 1-p \text{ (no disaster)}, \\
&= \mu + \sigma_c \varepsilon_t + \log(1-b), \text{ with probability } p \text{ (disaster)},
\end{aligned}
$$

where ε_t is *iid* $N(0,1)$ and is independent of the realization of the disaster. Assume that there are some assets, indexed by i, with dividends $\{D_{it}\}$ satisfying:

$$
\begin{aligned}
\Delta \log D_{it} &= \mu_i + \lambda_i \sigma \varepsilon_t + \upsilon_{it}, \text{ if there is no disaster}, \\
&= \mu_i + \lambda_i \sigma \varepsilon_t + \eta_i \log(1-b) + \upsilon_{it} \text{ if there is a disaster.}
\end{aligned}
$$

Hence, assets differ in (1) the drift of their dividends μ_i, (2) their exposure λ_i to 'standard business shocks' ε_t, (3) their exposure η_i to disasters, and (4) some idiosyncratic shock υ_{it} which is *iid* and independent of ε_t and the disaster. Standard computations lead to the log risk premium on asset i :

$$
\log\left(\frac{\mathbb{E}R_i}{R_f}\right) = \lambda_i \gamma \sigma^2 + \log\left(\frac{\left(1-p+p(1-b)^{-\gamma}\right)\left(1-p+p(1-b)^{\eta_i}\right)}{1-p+p(1-b)^{\eta_i-\gamma}}\right), \quad (3)
$$

[16] i.e., stocks with high book-to-market ratios.

where γ is the risk aversion coefficient. The first term in this formula is the standard result of the *iid* lognormal case. The second term is new and reflects the exposure to disasters η_i. It is easy to verify using this formula that assets with higher η_i will have higher average returns. This is true in a long sample that includes disasters and especially true in a small sample which does not include disasters. Empirical research in finance documents that the first term has little explanatory power, i.e. average returns of stocks are not systematically correlated with consumption betas in the cross-section. However, the second term could be much larger than the first one, by the same logic as the results of Rietz and Barro. To illustrate this possibility, consider the following parameter values, similar to Barro (2006): risk aversion $\gamma = 4$, probability of disaster $p = 0.02$, disaster size $b = 0.4$, volatility of consumption growth outside disasters $\sigma_c = .02$. A claim to consumption ($\eta_i = \lambda_i = 1$) has a risk premium of 2.73%, i.e. its expected return is 2.73% larger than that of a perfectly risk-free asset.[17] An asset which has a consumption beta equal to 1, except during disasters, when it has a beta of 2, (i.e. $\eta_i = 2$ and $\lambda_i = 1$) has a risk premium of 4.60%. The second asset would thus earn substantially higher average returns, and in a sample without disaster would earn higher returns in any period (if υ_{it}=0). It would appear to be an arbitrage opportunity. During a disaster however, the second asset would have a return of (approximately) −80%, while the first asset would have a return of (approximately) −40%. I now proceed to test this implication of the disaster story.

5.2 The 9/11 Natural Experiment

The terrorist attacks of 9/11 offer an interesting example. On 9/17/01 (the first day of trading on the NYSE after 9/11), the S&P 500 dropped 4.9%, but some industries fared very differently: the S&P 500 consumer discretionary index fell 9.8%, the energy index 2.9%, the health care index 0.6%, while defense industry stocks soared: Northrop Grumman was up 15.6% and Lockheed Martin 14.7%. Tables A1 and A2 reports the list of the 50 stocks traded on the NYSE with the largest decline and largest increase that day, together with their 2-digit SIC code.

While 9/11 is not a disaster according to Barro's definition, many people feared at the time that it marked the beginning of a disaster. The heterogeneity across stocks and industries in response to 9/11 is impressive and suggests a natural test: if we take these responses to the 9/11 'shock' as proxies for the responses to a true disaster, do they justify the differences in expected returns?

Figure 7 plots the mean monthly excess returns against the return on 9/17 for 48 industry portfolios.[18] If the disaster story is true, industries which did well on 9/17 (e.g. defense, tobacco, gold, shipping and railroad, coal) should have low average returns, and industries which did poorly (e.g. transportation, aerospace, cars, leisure) should have high average returns, so we should see a negative relationship. However, the correlation is slightly positive (0.20).[19] Table 3 presents the 48 industries, sorted by their return on 9/17, and the mean return, volatility of return, market beta, and disaster beta (to be defined below).

[17] Barro (2006) also incorporates default of government bonds.

[18] The 48 industry portfolios are taken from Prof. French's website. These portfolios correspond roughly to two-digit SIC industries. The mean excess return is calculated over the sample 1970-2004 using monthly data.

[19] More formally, one could try and impose the formula (1) on this cross-section, but this does not work at all.

Figure 8 performs the same test for the 25 size and book-to-market sorted portfolios introduced by Fama and French. In this case, the correlation is moderately negative (-0.16). Looking at the other puzzles in the finance literature, the Fama-French factor SMB was up 0.24%, HML was down -0.93%, UMD was up 2.72%, and the small-value/small-growth excess return was -0.20%.[20] Hence, only HML and the small-value/small-growth excess return have the correct sign, and the magnitude is not large. For instance, the value premium is roughly equal to the equity premium, so we would need value stocks to do twice worse than growth stocks in disasters, i.e. we would require that HML was down by approximately 10% on 9-11. This is ten times more than the data (-0.93%).

Of course one possible answer is that 9/11 is not the ideal experiment, and some stocks or industries, such as aerospace may have been especially affected by 9/11, more than they would be by another type of disaster.

5.3 Measuring the Exposure to Large Downside Risk

The second test I perform is to compute the sensitivity of various portfolios of stocks to large declines in the stock market more generally. Can the disaster explanation account for the cross-sectional puzzles like value-growth, momentum, small-big which have attracted so much attention in the empirical finance literature?

Instead of taking the risk factor to be the market return, as in the CAPM, I pick the factor to be the market return conditional on a large negative market return. Formally, I test the linear factor model $M_t = 1 - bx_t$ where $x_t = \left(R^m_{t+1} - R^f_{t+1}\right) \times 1_{\left(R^i_{t+1} - R^f_{t+1}\right) < t}$, where R^m_{t+1} is the market return and R^f_{t+1} is the risk-free return, and $1_{(.)}$ is a characteristic function. I use monthly data and set arbitrarily $t = -10\%$. Since 1926 there have been 29 months where the excess market return is less than -10%. This is equivalent to measuring the exposure of assets to large negative events by running the following time series regression:

$$R^i_{t+1} - R^f_{t+1} = \alpha_i + \beta^d_i \left(R^m_{t+1} - R^f_{t+1}\right) \times 1_{\left(R^i_{t+1} - R^f_{t+1}\right) < t} + \varepsilon_{it+1}, \tag{4}$$

Securities which have a large "disaster beta" β^d_i do badly when the stock market does very badly. (This can be justified as an approximation to the model of section 2, since the market return in this case is proportional to consumption growth.) I call this model the "Disaster CAPM".

Following Cochrane (2001), I use GMM to estimate the factor model. For comparison, I also estimate with the same procedure the CAPM. Table 4 presents the results, using three weighting matrix (the identity matrix for the 1st stage, the optimal weighting matrix for the 2nd stage, and the Hansen-Jagannathan recommended weighting matrix equal to the inverse of the covariance of returns). I use three sets of test assets: the 6 Fama-French benchmark portfolios (intersection of 3 categories of size and 2 categories of book-to-market), the 25 Fama-French size and book-to-market sorted portfolios, and the 17 industry portfolios.[21] Figure 9 presents the standard cross-sectional plots. This figure makes

[20] SMB is a portfolio long small firms and short large firms; HML is long in firms with high book-to-market and short firms in low book-to-market; UMD is long winners (firms with high return in the past month) and short losers. All these strategies generate significant excess returns (see Table 4), which are not accounted for by CAPM or CCAPM betas.

[21] Note that the 25 portfolios sample start only in July 1931, after the beginning of the Depression, due to missing data.

clear that the disaster model does not improve on the standard CAPM: the fit of the two models is very similar. The statistics of Table 4 confirm this: the J-statistic or the mean absolute pricing errors are comparable across the two models, and are often worse for the disaster model.

The reason for the similarity of fit is simply that the disaster beta is highly correlated with the market beta: for the 25 Fama-French portfolios, the correlation is over .96, and for the 17 industries portfolios, it is over .95.

In Table 5, I perform the time-series regressions (4) for the excess returns HML, SMB, UMD and small-value-small-growth. We see that the coefficient β^d does not explain very well the mean returns: for UMD, it has the wrong sign, it is insignificant for SV-SG; for HML it is small and borderline significant. Only for SMB is there some empirical support for the disaster story: small firms have indeed more negative returns than large firms when there is a big negative stock return.

The main problem with this section is that measuring the exposure to disasters is hard. However, the preliminary conclusion is that there is little support in the data for disaster explanation when looking at the cross-section of stock returns (value, momentum, industries), with the exception of size effects.

[To be added: tests with consumption growth annual data during the Depression.]

Overall, there appears to be little support for this cross-sectional implication of the disaster model. Of course, one may argue that the measures of exposure to disaster used in this note are highly imperfect. It would be valuable to learn more about the historical properties of asset returns during disasters in other countries and in other periods.

6 Conclusions

The disaster explanation of asset prices is attractive on several grounds: there are reasonable calibrations which generate a sizeable equity premium. Disasters can easily be embedded in standard macroeconomic models. Moreover, the explanation is consistent with the empirical finance literature which documents deviations from log-normality. Inference about extreme events is hard, so it is possible that investors' expectations do not equal an objective probability. But precisely because the disaster explanation is not rejected on a first pass, we should be more demanding, and study if it is robust and if it can account quantitatively for other asset pricing puzzles. The current paper points toward some areas which would benefit from further study.

This is why I also consider the 6 benchmark portfolios, for which data is available since 1926.

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7 Appendix: Computational Method

This appendix details the calculations required to solve the model with Epstein-Zin utility. I will assume the following processes for consumption growth and dividend growth:

$$\begin{aligned}\Delta \log C_t &= \mu_c(s_t) + \sigma_c(s_t)\varepsilon_t,\\ \Delta \log D_t &= \mu_d(s_t) + \sigma_d(s_t)\varepsilon_t,\end{aligned}$$

with s_t a Markov state and ε_t an *iid* $N(0,1)$ random variable, and ε_t and s_t are independent. This encompasses the standard Mehra-Prescott model as well as the disaster model, and can easily accommodate additional dynamics.

Normalize utility by consumption to write the utility-consumption ratio V_t/C_t :

$$\frac{V_t}{C_t} = \left(1 - e^{-\rho} + e^{-\rho} E_t\left(\left(\frac{V_{t+1}}{C_{t+1}}\right)^{1-\theta}\left(\frac{C_{t+1}}{C_t}\right)^{1-\theta}\right)^{\frac{1-\alpha}{1-\theta}}\right)^{\frac{1}{1-\alpha}}. \tag{5}$$

Let $f(s_t) = \left(\frac{V_t}{C_t}\right)^{1-\alpha}$. Then:

$$f(s) = 1 - e^{-\rho} + e^{-\rho} E_{\varepsilon', s'|s}\left(f(s')^{\frac{1-\theta}{1-\alpha}}\left(\frac{C_{t+1}}{C_t}\right)^{1-\theta}\right)^{\frac{1-\alpha}{1-\theta}}.$$

$$f(s) = 1 - e^{-\rho} + e^{-\rho}\left[\sum_{s'\in S} Q(s,s') f(s')^{\frac{1-\theta}{1-\alpha}} \exp\left((1-\theta)\mu_c(s') + (1-\theta)^2\frac{\sigma_c(s')^2}{2}\right)\right]^{\frac{1-\alpha}{1-\theta}}.$$

I solve this by iterating, starting with the guess $f(s) = 1$. Next, let $h(s_t) = V_t/C_t = f(s_t)^{1/(1-\alpha)}$. The SDF is:

$$\begin{aligned}M_{t+1} &= e^{-\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\left(\frac{V_{t+1}}{E_t(V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}}\right)^{\alpha-\theta},\\ &= e^{-\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\theta}\frac{h(s_{t+1})^{\alpha-\theta}}{E_t\left(h(s_{t+1})^{1-\theta}\left(\frac{C_{t+1}}{C_t}\right)^{1-\theta}\right)^{\frac{\alpha-\theta}{1-\theta}}},\\ M_{t+1} &= e^{-\rho}\exp(-\theta\mu_c(s_{t+1}) - \theta\sigma_c(s_{t+1})\varepsilon_{t+1})h(s_{t+1})^{\alpha-\theta}\\ &\quad\times\left[\sum_{s'\in S} Q(s_t,s') f(s')^{\frac{1-\theta}{1-\alpha}}\exp\left((1-\theta)\mu_c(s') + (1-\theta)^2\frac{\sigma_c(s')^2}{2}\right)\right]^{\frac{\theta-\alpha}{1-\theta}}.\end{aligned}$$

To compute the bond price, I use the condition:

$$P_t^b = E_t\left(M_{t+1}\times x_{t+1}\right),$$

with x_{t+1} =payoff of bond, which given the default is 1 if there is no disaster and $(1-def)\,prdef + (1-prdef)$ if there is a disaster, where def = amount of default and $prdef$ = probability of default conditional on disaster. Hence,

$$\begin{aligned}P^b(s) &= \zeta(s)e^{-\rho}\sum_{s'\in S} Q(s,s')E_\varepsilon \exp(-\theta\mu_c(s') - \theta\sigma_c(s')\varepsilon)h(s')^{\alpha-\theta}x(s'),\\ &= \zeta(s)e^{-\rho}\sum_{s'\in S} Q(s,s')\exp\left(-\theta\mu_c(s') + \frac{\theta^2}{2}\sigma_c^2(s')\right)h(s')^{\alpha-\theta}x(s'),\end{aligned}$$

$$\zeta(s) = \left[\sum_{s' \in S} Q(s,s') f(s')^{\frac{1-\theta}{1-\alpha}} \exp\left((1-\theta)\mu_c(s') + (1-\theta)^2 \frac{\sigma_c(s')^2}{2}\right)\right]^{\frac{\theta-\alpha}{1-\theta}}.$$

The realized bond return is $x_{t+1}/P_t^b = x(s_{t+1})/P^b(s_t)$; the expected bond return conditional on the current state is $cbr(s) = E_t\left(x_{t+1}\right)/P^b(s_t) = \sum_{s' \in S} Q(s_t, s') x(s')/P^b(s_t)$; and the unconditional bond return is $E\left[E_t\left(x_{t+1}\right)/P^b(s_t)\right] = \sum_{s \in S} \mu(s) cbr(s)$ where μ is the ergodic distribution of the markov chain $\{s_t\}$.

Finally, I calculate the value of equity with the standard recursion:

$$\frac{P_t}{D_t} = E_t\left(M_{t+1}\left(1 + \frac{P_{t+1}}{D_{t+1}}\right)\frac{D_{t+1}}{D_t}\right),$$

which shows that $\frac{P_t}{D_t}$ is a function of the state variable $s_t : \frac{P_t}{C_t} = g(s_t)$. I again find g by iterating on this recursion:

$$g(s_t) = E_t\left(e^{-\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\theta}\left(\frac{D_{t+1}}{D_t}\right)\left(\frac{h(s_{t+1})}{E_t\left(h(s_{t+1})^{1-\theta}\left(\frac{C_{t+1}}{C_t}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}}\right)^{\alpha-\theta} . \left(g(s_{t+1}) + 1\right)\right),$$

$$g(s) = e^{-\rho}\zeta(s)\sum_{s' \in S} Q(s,s') \exp\left(\mu_d(s') - \theta\mu_c(s') + \frac{(\sigma_d(s') - \theta\sigma_c(s'))^2}{2}\right) h(s')^{\alpha-\theta}\left(g(s') + 1\right).$$

The conditional equity return is then:

$$ecr(s) = \frac{E_{\varepsilon', s'|s}\left(\left(g(s') + 1\right)\frac{D_{t+1}}{D_t}\right)}{g(s)} = \frac{\sum_{s' \in S} Q(s,s')\left(g(s') + 1\right)\exp(\mu_d(s') + \sigma_d(s')^2/2)}{g(s)},$$

and the unconditional equity return is simply $\sum_{s \in S} \mu(s) ecr(s)$.

Industry	R_{9-17}	$E(R)$	$\sigma(R)$	β	β_d	Industry	R_{9-17}	$E(R)$	$\sigma(R)$	β	β_d
Aero	-18.37	1.26	7.06	1.13	1.19	Mines	-4.64	0.99	6.81	0.98	1.08
Trans	-14.01	1.04	5.97	1.04	1.08	RlEst	-4.59	0.59	7.00	1.05	1.14
Fun	-13.38	1.40	7.56	1.26	1.52	Insur	-4.43	1.18	5.50	0.89	0.83
Autos	-12.68	0.96	6.44	1.00	1.18	FabPr	-4.41	0.61	7.06	1.07	1.20
Other	-10.22	0.54	7.17	1.17	1.25	Books	-4.07	1.13	5.74	1.02	0.98
Fin	-9.70	1.20	5.65	1.09	1.12	Banks	-4.05	1.27	5.80	1.01	1.03
Clths	-9.50	1.11	6.88	1.11	1.21	Telcm	-3.82	1.00	4.93	0.77	0.72
Txtls	-9.07	0.98	6.31	0.96	1.16	Whlsl	-3.80	0.97	5.81	1.06	1.23
Chems	-8.81	1.06	5.52	0.98	0.99	Soda	-3.42	1.13	6.87	0.84	0.80
Steel	-8.51	0.80	7.26	1.21	1.38	Agric	-3.15	0.99	6.27	0.86	1.09
Rtail	-7.34	1.13	5.94	1.05	1.07	Hlth	-3.10	0.98	9.01	1.19	1.22
Chips	-7.17	1.09	8.18	1.48	1.59	PerSv	-2.94	0.72	6.92	1.13	1.19
Toys	-6.96	0.87	7.31	1.15	1.31	Oil	-2.75	1.18	5.42	0.75	0.82
Cnstr	-6.96	1.08	7.30	1.25	1.23	MedEq	-2.61	1.05	5.49	0.91	0.84
Meals	-6.92	1.04	6.61	1.10	1.28	Hshld	-2.22	0.93	5.01	0.84	0.80
BusSv	-6.90	1.12	7.13	1.37	1.41	Util	-1.68	0.97	4.21	0.52	0.39
Mach	-6.88	0.98	6.19	1.17	1.24	Drugs	-1.07	1.12	5.42	0.83	0.78
ElcEq	-6.86	1.27	6.32	1.17	1.16	Food	-0.66	1.24	4.84	0.71	0.73
Boxes	-6.68	1.00	5.86	0.93	1.11	Beer	0.57	1.18	5.78	0.82	0.99
Paper	-6.66	1.12	5.62	0.94	0.96	Coal	0.62	1.38	9.82	1.09	1.05
BldMt	-5.93	1.05	5.92	1.09	1.14	Smoke	2.75	1.50	6.66	0.67	0.68
Comps	-5.84	0.87	7.58	1.26	1.34	Gold	4.61	1.17	10.93	0.65	1.01
LabEq	-5.71	0.96	7.76	1.39	1.40	Ships	6.62	0.97	7.05	0.99	0.91
Rubbr	-5.65	1.02	5.99	1.02	1.19	Guns	14.92	1.33	6.95	0.80	0.94

Table 3: Return on 9-17, average monthly excess return, monthly return volatility, market beta, and downward beta, for the 48 industries of Fama and French. returns.

	$E(R)$	$E(R \mid R^m < -.1)$	$E(R \mid R^m < -.15)$	β_d
HML	0.40	-0.68	0.15	0.08
t-stat	3.47	-0.53	0.09	1.90
SMB	0.24	-2.69	-2.68	0.18
t-stat	2.19	-3.63	-1.90	4.59
UMD	0.76	4.26	5.97	-0.24
t-stat	5.01	3.24	2.56	4.23
SV-SG	0.49	0.48	0.46	0.02
t-stat	4.14	0.41	0.33	0.41

Table 4: Mean returns on the HML, SMB, UMD and small growth-small value portfolios, for the full sample, the sample of market declines greater than ten percent or fifteen percent, and the disaster beta.

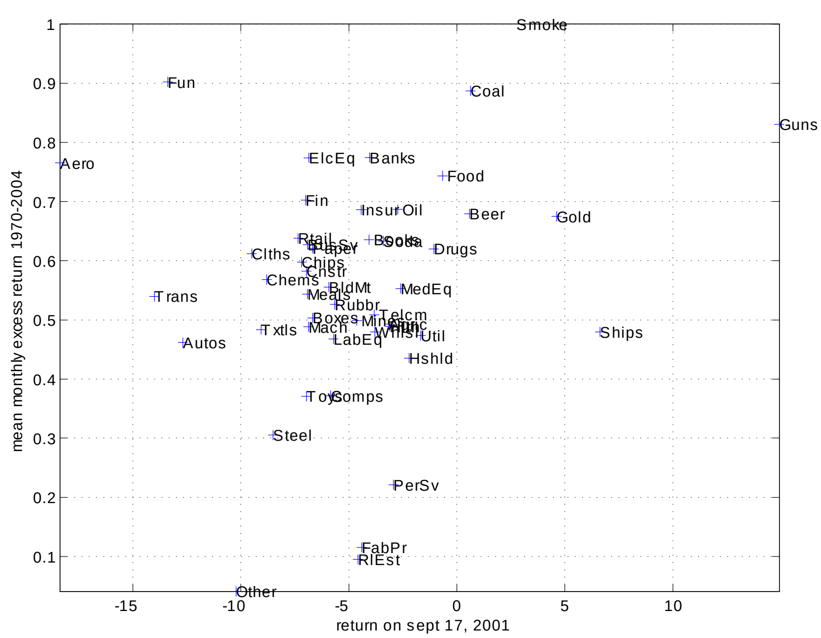

Figure 7: Mean monthly excess return (1970-2004) and return on 9-17-01, for 48 portfolios of stocks sorted by industry. Data from prof. French's website.

	CAPM			Disaster CAPM		
	1st stage	2nd stage	HJ weights	1st stage	2nd stage	HJ weights
Test assets: 6 benchmark portfolios (book-to-market,size), sample: 7-26 to 12-06						
b	0.0251	0.0249	0.0219	0.1025	0.1076	0.0901
s.e.	0.0076	0.0074	0.0072	0.0430	0.0414	0.0390
J-test	12.5031	12.5032	12.8243	11.6305	11.6305	12.0926
p-val	0.0285	0.0285	0.0251	0.0402	0.0402	0.0335
lambda	0.7428	0.7359	0.6485	0.7261	0.7619	0.6381
std(lambda)	0.2255	0.2200	0.2118	0.3043	0.2933	0.2759
MAE	0.1163	0.1169	0.1351	0.1287	0.1286	0.1441
R2	54.6876	54.4809	22.7279	46.7913	42.1493	17.9653
Test assets: 25 size and book-to-market portfolios, sample: 7-31 to 12-06						
b	0.0276	0.0252	0.0243	0.1213	0.0675	0.0823
s.e.	0.0079	0.0056	0.0053	0.0503	0.0405	0.0336
J-test	49.3752	49.3753	49.5270	42.8140	42.8140	46.9350
p-val	0.0017	0.0017	0.0016	0.0104	0.0104	0.0034
λ	0.7879	0.7190	0.6920	0.7667	0.4265	0.5201
s.e.	0.2239	0.1597	0.1522	0.3181	0.2557	0.2127
MAE	0.1790	0.2011	0.2132	0.1876	0.4529	0.3470
R2	13.0149	1.1419	-9.8450	4.0903	-294.8382	-153.0520
Test assets: 17 industry portfolios, sample 7-26 to 12-06						
b	0.0244	0.0251	0.0227	0.1003	0.1052	0.0834
s.e.	0.0073	0.0069	0.0068	0.0420	0.0384	0.0355
J-test	2.2550	2.2550	2.3901	2.7121	2.7121	3.1604
p-val	0.9722	0.9722	0.9667	0.9511	0.9511	0.9239
λ	0.7226	0.7429	0.6733	0.7102	0.7452	0.5903
s.e.	0.2149	0.2045	0.2009	0.2977	0.2722	0.2513
MAE	0.0255	0.0297	0.0629	0.0405	0.0503	0.1417
R2	93.4289	90.1938	71.1187	84.6576	73.5754	-46.0484

Table 5: Results from the estimation of the linear factor model, with the excess return as the factor (CAPM) or the excess return conditional on a large negative return (Disaster CAPM). The table reports the coefficient b of the SDF (M =1-bf), the Hansen J-test, and the price of risk lambda, as well as the mean absolute pricing error and the R2 (which does not include a constant).

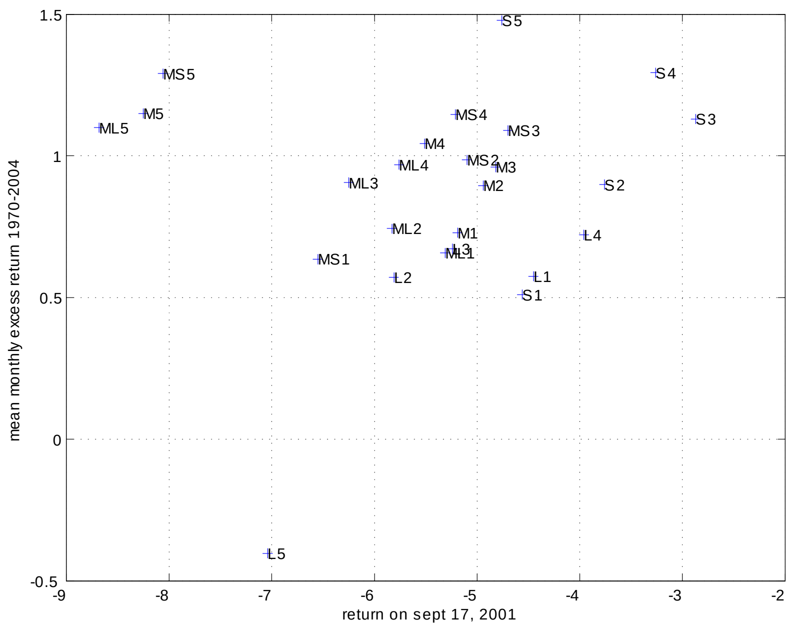

Figure 8: Mean monthly excess return (1970-2004), and return on 9-17-01, for 25 portfolios of firms sorted by size and book-to-market. Data from prof. French's website.

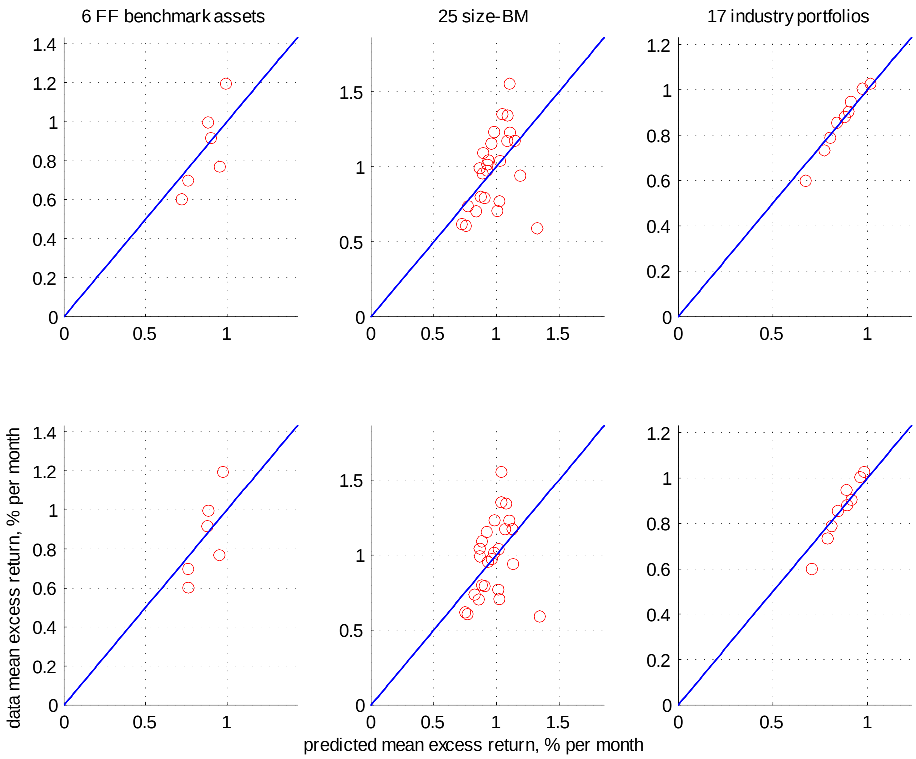

Figure 9: Estimation of two single-factor model: the CAPM (top panel) and the Disaster CAPM (bpttom panel). The test assets are (a) left column: the 6 benchmark portfolios of Fama and French (sorted by size and book-to-market), sample July 1926 to December 2006; (b) center column: the 25 Fama-French size and book-to-market sorted portfolios, sample July 1931 to December 2006; (c) right column: the 17 industry portfolios, sample July 2006 to December 2006. The figure displays the GMM 1st stage results. The factors are demeaned prior to estimation.

Table A1: The Fifty Lowest Returns on the NYSE on September 17, 2001

Company Name	SIC	% return on 9-17-01
AMERICA WEST HOLDINGS CORP	45	-65
U S AIRWAYS GROUP INC	45	-52
CONTINENTAL AIRLINES INC	45	-49
WYNDHAM INTL INC	70	-48
DELTA AIR LINES INC	45	-45
U A L CORP	45	-43
SABRE HOLDINGS CORP	73	-41
AIRTRAN HOLDINGS INC	45	-40
ROYAL CARIBBEAN CRUISES LTD	44	-40
A M R CORP DEL (AA)	45	-39
TITANIUM METALS CORP	33	-39
P O PRINCESS CRUISES P L C	44	-39
MIDWEST EXPRESS HOLDINGS INC	67	-36
AVIATION SALES CO	50	-35
TRENWICK GROUP LTD	64	-34
ROGERS WIRELESS COMMS INC	48	-33
RESORTQUEST INTERNATIONAL INC	65	-33
CARNIVAL CORP	44	-32
BUDGET GROUP INC	75	-30
I C I C I BANK LTD	60	-30
GALILEO INTERNATIONAL INC	73	-29
ALASKA AIRGROUP INC	45	-29
STARWOOD HOTELS	70	-28
UNITED TECHNOLOGIES CORP	37	-28
PRECISION CASTPARTS CORP	33	-28
CHINA SOUTHERN AIRLINES CO LTD	45	-28
ODYSSEY RE HOLDINGS CORP	63	-28
FINOVA GROUP INC	61	-28
A A R CORP	50	-27
SATYAM COMPUTER SERVICES LTD	73	-27
ORIENT EXPRESS HOTELS INC	70	-26
GROUP 1 AUTOMOTIVE INC	55	-26
BRITISH AIRWAYS PLC	45	-25
LASALLE HOTEL PROPERTIES	67	-25
CHART HOUSE ENTERPRISES INC	58	-25
GALEY LORD INC	22	-25
HOST MARRIOTT CORP NEW	67	-25
AVIALL INC NEW	50	-25
SCOR	63	-25
HIGHLANDS INSURANCE GROUP INC	63	-24
SOUTHWEST AIRLINES CO	45	-24
COACH INC	31	-24
HILTON HOTELS CORP	70	-24
SEA CONTAINERS LTD	73	-23
FOUR SEASONS HOTELS INC	70	-23
K L M ROYAL DUTCH AIRLINES	45	-23
PENTON MEDIA INC	27	-22
WIPRO LTD	73	-22
M G M MIRAGE	70	-22
PARTNERRE LTD	63	-22

Notes: the most frequent SIC codes in this table are: 45 (Air Transportation), 70 (Hotels), 73 (advertising, business services, …), 37 (aircrafts, ships, motor vehicles, …), 63 (Insurance).

Table A2: The Fifty Highest Returns on the NYSE on September 17, 2001

Company Name	SIC	% return on 9-17-01
COMDISCO INC	73	6.3
NELSON THOMAS INC	27	6.4
CARBO CERAMICS INC	32	6.4
JACOBS ENGINEERING GROUP INC	87	6.6
GOLDEN WEST FINANCIAL CORP	67	6.6
MACK CALI REALTY CORP	67	6.6
LORAL SPACE COMMUNICATIONS	48	7.1
HILB ROGAL HAMILTON CO	64	7.2
T V X GOLD INC	10	7.5
TRITON P C S HOLDINGS INC	48	7.8
VINTAGE PETROLEUM INC	13	7.9
BLACKROCK INC	62	7.9
FIRST AMERICAN CORP CALIF	63	7.9
GENCORP INC	30	8
C N O O C LTD	13	8.1
FEDERAL SIGNAL CORP	37	8.8
GENERAL DYNAMICS CORP	37	9.1
ORBITAL SCIENCES CORP	37	9.4
WILLIS GROUP HOLDINGS LIMITED	64	9.4
BROWN BROWN INC	64	9.7
MERIDIAN GOLD INC	10	9.7
HARRIS CORP	36	9.8
TELEFONICA MOVILES S A	48	10
CHINA MOBILE HONG KONG LTD	48	10
FLAGSTAR BANCORP INC	60	11
STURM RUGER CO INC	34	11
COMPANIA DE MINAS BV SA	10	11
NOKIA CORP	36	12
CAMPBELL RESOURCES INC NEW	10	13
U R S CORP NEW	87	14
TITAN CORP	36	14
LOCKHEED MARTIN CORP	37	15
TEFRON LTD	23	15
HECLA MINING CO	10	15
UNITED INDUSTRIAL CORP	87	15
GALLAGHER ARTHUR J CO	64	16
NORTHROP GRUMMAN CORP	37	16
COEUR D ALENE MINES CORP ID	10	16
TELEDYNE TECHNOLOGIES	87	18
GLAMIS GOLD LTD	10	18
U S AGGREGATES INC	14	19
WACKENHUT CORP	73	19
AMERICAN WATER WORKS INC	49	19
ALLIANT TECHSYSTEMS INC	36	21
EDO CORP	36	23
WACKENHUT CORP	73	27
RAYTHEON CO	38	27
BETHLEHEM STEEL CORP	33	28
DELTA WOODSIDE INDS INC NEW	50	29
L 3 COMMUNICATIONS HLDGS INC	36	38
ARMOR HOLDINGS INC	73	39