Production Sharing and Business Cycle Co-Movements

with Heterogeneous Firms

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Abstract

In this paper I examine whether the co-movement of business cycles can be explained by the firms’ decision to relocate the production of intermediate goods across borders through vertical FDI. First, using data from Mexico’s maquiladora industry, I show that offshore production is pro-cyclical with the U.S. manufacturing output, although the timing of the maquiladora’s response differs at the intensive and the extensive margins: hours worked respond immediately, whereas the number of offshore plants responds with a lag of three quarters. Second, I build a dynamic stochastic general equilibrium (DSGE) model with heterogeneous firms and firm entry dynamics in a two-country framework (North and South). The model examines the ability of three internationalization strategies of multinational firms - vertical FDI, horizontal FDI, and endogenous exports - to act as transmission channels of aggregate shocks across countries. I derive an asymmetric steady state in which the relatively lower wage in South acts as an incentive for the firms in North to relocate production offshore. A positive technology shock in North encourages firm entry, triggers the appreciation of relative wages and causes more firms to relocate production offshore. Higher demand for labor in North (due to firm entry) and simultaneously in South (due to offshoring) generates positive co-movement of wages and aggregate incomes. Relative to the benchmark models with either endogenous exports or horizontal FDI, offshoring through vertical FDI increases the co-movement of output and decreases the co-movement of consumption, results which come closer to the empirical correlations reported in Backus, Kehoe, Kydland (1992).

JEL classification: E32, F21, F23, F41, F42

Keywords: offshoring, vertical FDI, horizontal FDI, exports, business cycle co-movements, heterogeneous firms, firm entry, relative wages

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1 Introduction

In this paper I examine whether the firms’ decision to relocate the production of intermediate goods across borders can explain the co-movement of aggregate incomes across countries. Firms often follow strategies that involve the fragmentation of production chain and the establishment of foreign affiliates at locations with relatively lower labor costs that serve as export platforms, an action which I refer to as "offshoring through vertical FDI" (foreign direct investment) in this paper.¹ Unlike production under horizontal FDI - which implies that foreign affiliates produce mainly for the local market of the host country - this type of vertically-integrated production involves foreign affiliates that add value towards the final goods purchased by consumers in the multinationals’ country of origin or in third countries. Therefore, in my model, vertical FDI and trade are complements: The creation of production-based affiliates offshore generates trade flows, as multinationals and their foreign affiliates exchange intermediate goods with each other.

Recent literature documents empirically that country pairs that exchange intermediate goods more intensely – exchanges which often take place between offshore affiliates and the headquarters of multinational firms engaged in vertical FDI – exhibit higher correlations of aggregate incomes (Burstein, Kurtz, Tesar, 2007). Trade data also shows that, despite notable variation across sectors and industries, the extent of cross-border production under vertical FDI in manufacturing has been particularly large within the NAFTA region, Central and South America. In 2005, as much as 32, 50 and 26 percent of the manufacturing sales of U.S. affiliates from Canada, Mexico and Latin America as a whole were directed towards their U.S. parent firms, a pattern which reflects the key economic role of production sharing through vertical FDI in the region (Bureau of Economic Analysis, 2007).²

I build a dynamic stochastic general equilibrium (DSGE) model that features endogenous offshoring, heterogeneous firms and firm entry dynamics in a two-country framework (North and South). Firms in each economy are monopolistically competitive, and each firm produces a different variety of final goods using either domestic or foreign inputs. Firms also face sunk entry costs, period-by-period fixed offshoring costs, and iceberg trade costs. The sunk entry costs reflect the regulation of starting

¹I attach different meanings to "offshoring" and "outsourcing." The former concerns the integration of multinational firms across borders through vertical or/and horizontal FDI. In contrast to offshoring, outsourcing takes place when a firm purchases intermediate inputs or services from an unaffiliated supplier - either at home or abroad - rather than producing it in house. See Helpman (2006) for an ample discussion of the related literature.

²In contrast, considerably smaller shares of the sales of U.S. affiliates in Europe and Asia-Pacific were directed towards their U.S. parent multinationals (3 and 5 percent in 2005).
a business, whereas the fixed costs of producing offshore represent headquarters activities and the maintenance of production plants in the foreign country. Nevertheless, the iceberg trade costs reflect transportation, insurance and trade barriers that affect the imports of intermediate goods produced by offshore affiliates and sold to their parent firms in the country of origin. Thus, when deciding where to locate production (domestically vs. offshore), firms balance the incentive of lower labor costs against the fixed and iceberg trade costs associated with offshore production.

I derive an asymmetric steady state in which differences in firm entry regulation (i.e. the cost of starting a business) affect labor demand and generate wage differences across countries. Indeed, empirical evidence shows large variation in firm entry costs across countries: The cost of starting a business is three times higher in Mexico than in the U.S. or Canada; it is six times higher in Hungary than in the U.K. (World Bank, 2007, see Table 4). In my model, I set firm entry costs to be higher in South; consequently, labor demand and the effective wage are lower in South. Therefore, only firms originating in North have the incentive to relocate production across the border; all firms that originate and serve the South market produce domestically.

The mechanism that generates the co-movement of aggregate incomes through endogenous offshoring in my model works as follows. Firms choose endogenously to locate production either at home or overseas. A positive technology shock in the North economy encourages firm entry. In turn, firm entry causes wages to rise faster than productivity, and leads to an appreciation of the cost of effective labor in North relative to South. Subsequently, the more productive firms originating in North choose to relocate production overseas. Higher demand for labor in North (due to firm entry) and higher demand for labor in South (due to offshoring) generate positive co-movement of wages and aggregate incomes.

Throughout this paper I use the “terms of labor” as a measurement of the relative cost of effective labor between North and South. I define the terms of labor as the ratio between the South and North real costs of effective labor expressed in units of the North consumption basket, $TOL_t = \frac{w_t^S}{w_t^N} \frac{Z_t^N}{Z_t^S} Q_t$, where the cost of effective labor is the ratio between the real wage and aggregate productivity, i.e. $\frac{w_t}{Z_t}$ in the North economy. Over the business cycle, firm entry leads to an increase of the relative cost of effective labor (i.e. the terms of labor appreciate/decrease), which in turn causes more firms from North to relocate production offshore.

Firm entry and firm heterogeneity are two necessary ingredients of my model. In steady state, cross-country differences in the regulation of firm entry are translated into differences in real effective wages across countries, and therefore provide an incentive for some of the firms in North to relocate
production to South, where the cost of effective labor is relatively lower. Over the business cycle, firm entry triggered by a positive technology shock in the home economy drives the appreciation of the domestic effective wage relative to South, and thus causes more firms to relocate production to offshore.\(^3\)

Firm heterogeneity – i.e. some firms are more productive than others – is also necessary to generate endogenous offshoring in my model. If firms were identical and offshoring costs negligible, all firms would prefer to produce intermediate goods at the location with cheaper effective labor. However, with firm heterogeneity, only some of the firms can afford the fixed costs of offshoring, and their share in the number of total firms varies over the business cycle. The relocation of production by home firms affects wages in South in the same way that firm entry affects wages in North – namely, it induces upward pressure on wages due to higher demand for labor – a mechanism which then generates co-movement of aggregate incomes.\(^4\)

Given the increased complexity of multinational firms’ operations across borders (Helpman, 2006), the traditional classification of FDI into vertical and horizontal types has become increasingly difficult. Therefore, in addition to offshore production through vertical FDI (i.e. motivated by lower production costs in the foreign economy), my model nests the benchmark models with endogenous exports a la Ghironi and Melitz (2005) as well as multinational production through horizontal FDI (i.e. motivated not by low production cost, but by access to the local market) a la Contessi (2006). Thus, the model examines the ability of three different internationalization strategies of multinational firms to act as channels through which aggregate shocks are transmitted across economies. The analytical moments of my model show that endogenous offshoring through vertical FDI increases the co-movement of national incomes and decreases the co-movement of consumption relative to the benchmark models with endogenous exports a la Ghironi and Melitz (2005) and horizontal FDI a la Contessi (2006), results which are more in line with the corresponding empirical correlations.

This paper is organized as follows. Section 2 provides an empirical analysis of the cyclicality of offshoring to Mexico’s maquiladora sector relative to the U.S. manufacturing output. Section 3 introduces a DSGE model of offshoring with heterogeneous firms and firm entry dynamics. Section

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\(^3\)Without entry, the cost of effective labor in North would not appreciate relative to South after a positive technology shock, as the increase of home wage relative to its steady state level would be just as large as the increase in aggregate productivity every period; firms would have no incentive to relocate production overseas.

\(^4\)Firm heterogeneity is not only a useful feature of my model, but also represents a well-established empirical regularity. Firm productivity varies within and across sectors, and only the more productive firms import intermediate goods from overseas (Kasahara and Rodriguez, 2005; Yasar and Morrison Paul, 2006).
4 shows the impulse responses and the second moments generated by the model with vertical FDI relative to the benchmark models with endogenous exports a la Ghironi and Melitz (2005) and offshore production through horizontal FDI a la Contessi (2006). Section 5 includes an extension of the model with vertical FDI which allows for elastic labor supply, and explores the ability of the model to reproduce the empirical moments of offshoring at the intensive and extensive margins. Section 6 concludes.

2 Measuring the cyclicality of offshoring

Although the link between trade flows and business cycle co-movements has received large attention in empirical literature, there is still little known about the fluctuation of offshore production over the business cycle. Therefore, in this section I explore the cyclicality of offshore production to Mexico’s maquiladora sector relative to fluctuations in the U.S. manufacturing output and in the U.S.-Mexico relative manufacturing wage.

2.1 Trade in intermediate goods and business cycle correlations

![Figure 1. Share of intermediates in bilateral trade vs. GDP correlations. Source: Burstein, Kurtz, Tesar (2007).](image)

Recent literature documents the link between trade in intermediate goods and the co-movement of aggregate incomes, which is one of the stylized facts that my theoretical model attempts to replicate. For instance, Burstein, Kurtz, Tesar (2007) show that a positive and significant link exists between (1) the volume of intermediate goods exported by offshore affiliates back to headquarters, measured as
the share of affiliate exports in the total exports of the host economy to their country of origin in 2003, and (2) the GDP correlation between the pairs of countries involved, measured over 1985-2003 (Figure 1). This result suggests that the cross-country interdependencies generated by the involvement in production sharing are associated with a closer synchronization of business cycle fluctuations across the countries involved.

2.2 The cyclicality of offshoring with quarterly data

Next I explore the cyclicality of offshore production using data on Mexico’s maquiladora and the U.S. manufacturing industries. In particular, I explore the link between (1) variables describing Mexico’s maquiladora sector: value added, hours worked, and number of plants, and (2) variables describing the U.S. manufacturing business cycle: industrial production, and the ratio of U.S. to Mexico’s nominal hourly wage in manufacturing expressed in the same currency. The data is provided by Mexico’s National Institute of Statistics (real value added, hours worked, hourly nominal wages and number of plants in Mexico’s maquiladora sector), the Board of Governors of the Federal Reserve System (real industrial output in U.S. manufacturing) and the U.S. Bureau of Labor Statistics (hourly nominal wages in U.S. manufacturing).

The maquiladora industry in Mexico constitutes a good example to study the pattern of offshoring in U.S. manufacturing, due to several reasons. First, it represents a classic example of offshoring under vertical FDI: Plants that operate under Mexico’s maquiladora program import inputs, process them and ship them back to the country of origin. On the goods’ return to the country of origin, tariffs are paid only on the value added by the plant in Mexico (Gruben, 2001). Second, although not all plants in Mexico’s maquiladora sector are owned by U.S. firms, most of maquiladora’s value added - roughly 90 percent - is exported to the U.S. (Burstein, Kurtz, Tesar, 2007).

I employ the Baxter-King bandpass filter to the variables expressed in natural logs in order to filter out fluctuations with periodicity lower than 18 months and greater than eight years. The resulting bandpassed series are plotted in Figure 2(a). The three charts on the left in Figure 2(a) plot Mexico’s maquiladora value added (real), hours worked and number of plants together with the U.S. manufacturing output (industrial production, real) at quarterly frequency between Q1-1993 and Q4-2003. Similarly, the three charts on the right in Figure 2(a) plot Mexico’s indicators together with the relative U.S.-Mexico nominal hourly wage in manufacturing.

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5The OLS coefficient is positive (0.65) and significant at the 1 percent level.
6Data on Mexico’s maquiladora sector is provided at monthly frequency. I aggregate it into quarters.
7The data on Mexico’s maquiladora sector was provided by the Instituto Nacional de Estadistica, Geografia e Infor-
The visual inspection of the data suggests that offshoring to Mexico - measured as value added, hours worked and number of offshore plants - is pro-cyclical with the U.S. manufacturing output. More, Mexico’s value added is notably more volatile than the U.S. manufacturing output. The charts also suggest a delayed response of offshoring at the extensive margin (i.e. the number of plants in Mexico’s maquiladora sector) to fluctuations of industrial production in U.S. manufacturing.

Figure 2(a). The cyclicality of offshoring to Mexico’s maquiladora sector relative to (1) U.S. manufacturing output (left) and (2) the U.S.-Mexico relative wage (right)

matica (INEGI) Mexico. Industrial production in the U.S. manufacturing sector at monthly frequency was provided by the U.S. Board of Governors of the Federal Reserve System.
2.2.1 Correlations with lags and leads of U.S. output

In order to account for the possibility of delayed responses, I compute the correlation of each of Mexico’s maquiladora indicators (value added, hours worked, numbers of plants) with various lags and leads of the U.S. business cycle indicators (industrial output and the U.S.-Mexico relative wage).

Figure 2(b). Correlations between Mexico’s maquiladora indicators and various lags and leads of (1) the U.S. manufacturing output (left) and (2) the U.S.-Mexico relative wage (right)

The charts on the left in Figure 2(b) show that offshoring to Mexico’s maquiladora industry is procyclical with U.S. manufacturing output, although the timing of maquiladora’s response differs at the intensive and the extensive margins. On one hand, correlations show a positive immediate response
of maquiladora's hours worked to fluctuations of U.S. industrial production in manufacturing. On the other hand, value added responds with a lag of one or two quarters, and the number of plants in Mexico’s maquiladora sector (i.e. offshoring at the extensive margin) responds with a lag of three quarters to the U.S. manufacturing output. The correlation of the number of maquiladora plants with the contemporaneous U.S. manufacturing output is less than 0.5, whereas the correlation with U.S. manufacturing output lagged by three quarters is positive and exceeds 0.7.

I interpret the correlations between Mexico’s maquiladora indicators and the U.S.-Mexico relative manufacturing wage ratio as evidence in favor of the cross-country transmission mechanism of business cycles that I build in my model. The charts on the right in Figure 2(b) show that the correlations of maquiladora value added, hours worked and number of plants with the lagged relative wage ratio are positive, i.e. past increases in manufacturing wage of the U.S. relative to Mexico are followed by the relocation of production from the U.S. to Mexico at both the intensive and the extensive margins. However, the correlations of each of the maquiladora indicators with the contemporaneous U.S.-Mexico wage ratio are negative, a result which shows that the relocation of production to Mexico is associated with a contemporaneous increase in Mexico’s wage, and therefore a decrease in the relative wage ratio.

2.2.2 OLS analysis

I use the following econometric specification to explore the link between the time series described in Figure 2(a):

\[
\ln(Y_{\text{MEX},t}) = \alpha + \beta \ln(\text{Output}_{\text{US},t}) + \gamma d_{\text{NAFTA},t} + \varepsilon_t, \\
\ln(Y_{\text{MEX},t}) = \alpha + \beta \ln(\frac{\text{Wage}_{US,t}}{\text{Wage}_{MEX,t}}) + \gamma d_{\text{NAFTA},t} + \varepsilon_t,
\]

where the dependent variable \(Y_{\text{MEX},t}\) represents alternatively Mexico’s manufacturing real value added, hours worked and the number of plants; the dummy variable \(d_{\text{NAFTA},t}\) controls for the impact of NAFTA membership on correlations. I expect offshore value added, hours worked and number of plants to be pro-cyclical with the U.S. manufacturing output, and negatively related to the U.S.-Mexico relative wage in manufacturing.\(^8\) I estimate the equations in both levels (to obtain the elasticities of levels) and differences (for elasticities of growth rates).

\(^8\)The results should be interpreted as correlations; they do not provide any insight on the causality between variables.
Table 1. U.S.-Mexico co-movement in manufacturing, OLS estimates

Equations in levels

<table>
<thead>
<tr>
<th>Explanatory</th>
<th>Dependent Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(OutputUS)</td>
<td>ln(VA_{MEX,t})</td>
</tr>
<tr>
<td>ln(OutputUS)</td>
<td>ln(Hours_{MEX,t})</td>
</tr>
<tr>
<td>ln(OutputUS)</td>
<td>ln(Plants_{MEX,t})</td>
</tr>
<tr>
<td>ln(WUS/WMEX)</td>
<td>-0.045 (0.074)</td>
</tr>
</tbody>
</table>

Equations in differences

<table>
<thead>
<tr>
<th>Explanatory</th>
<th>Dependent Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>dln(OutputUS)</td>
<td>dln(VA_{MEX,t})</td>
</tr>
<tr>
<td>dln(OutputUS)</td>
<td>dln(Hours_{MEX,t})</td>
</tr>
<tr>
<td>dln(OutputUS)</td>
<td>dln(Plants_{MEX,t})</td>
</tr>
<tr>
<td>dln(WUS/WMEX)</td>
<td><strong>0.152</strong>* (0.070)</td>
</tr>
</tbody>
</table>

The results in Table 1 show that Mexico’s maquiladora value added, hours worked and the number of plants are indeed pro-cyclical with U.S. manufacturing output. In the equations in levels, when each of Mexico’s value added, hours worked and number of plants alternatively represents the dependent variable, the coefficients on U.S. output are positive and statistically significant at the 1 percent level. The equations in levels also highlight the larger volatility of Mexico’s maquiladora sector relative to U.S. manufacturing. The coefficient estimates represent elasticities and are larger than unit, i.e. a 1 percent increase in U.S. manufacturing output is associated with a more than 1 percent increase in Mexico’s maquiladora sector. The coefficient estimates on the relative wage as the explanatory variable are not statistically significant.

In the equations in differences with either Mexico’s value added or hours worked as the dependent variable, the coefficients on U.S. output are positive and statistically significant at the 1 percent level. The sign and magnitude of coefficients show that faster growth in U.S. manufacturing is associated with accelerated growth in Mexico’s maquiladora value added and hours worked. With one exception (value added as the dependent variable), the coefficient estimates on the relative wage as the explanatory variable are not statistically significant.

Analyzing the cyclicality of offshoring represents a necessary step which provides empirical grounding for the offshoring-based transmission channel of business cycles that I develop in the theoretical section below.
3 Model of offshoring with heterogeneous firms

The model describes three main activities in each economy. 

(1) Households supply labor inelastically, consume an aggregate consumption basket, and finance firm entry.

(2) Firms (monopolistically-competitive, heterogeneous in productivity) produce final goods for the domestic market using intermediate goods under two possible strategies: (a) use only domestic inputs; or (b) use a mix of intermediate goods produced both domestically and offshore through vertical FDI, i.e. relocate the production of some inputs to offshore locations where labor is cheaper.

(3) Some of the firms also produce final goods for the foreign market under two possible strategies: (a) produce domestically and export the final good, like in Ghironi and Melitz (2005); or (b) use a mix of intermediate goods produced both domestically and offshore through horizontal FDI as in Contessi (2006).

Figure 3. Internationalization strategies of multinational firms

Figure 3 describes the various production strategies available to firms that serve the domestic and foreign markets. The difference between production under vertical and horizontal FDI is that the former strategy is adopted by firms serving the domestic market, is motivated by the relatively lower labor costs abroad, and involves the relocation of production offshore simultaneously with shutting down the domestic facility. In contrast, production under horizontal FDI is motivated by access to the offshore market, and involves the simultaneous production of the same variety of intermediate goods.
both at home (for the domestic market) and offshore (for the foreign market).

The following sections describe the detailed model while focussing on the North economy. The equations for South are similar unless indicated otherwise. Variables in South are denoted with a star superscript.

3.1 Households

Households in each country maximize the expected lifetime utility and provide labor inelastically, subject to the budget constraint:

$$\max_{\{B_t, x_t\}} \left[ E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma}}{1-\gamma} \right],$$

s.t.  $$B_{t+1} + \tilde{v}_t (N_t + N_{E,t}) x_{t+1} + C_t = (1 + r_t)B_t + (\tilde{d}_t + \tilde{v}_t)N_t x_t + w_t L,$$

where $\beta \in (0, 1)$ is the subjective discount factor, $C_t$ is aggregate consumption, and $\gamma > 0$ is the inverse of the inter-temporal elasticity of substitution.

The representative household in North purchases two types of assets. Every period $t$, the household buys the risk-free bond $B_{t+1}$ denominated in units of the consumption basket $C_t$. It also purchases $x_{t+1}$ shares in a mutual fund of firms in North that include: (i) $N_t$ firms that already produce at time $t$, either domestically or offshore, and (ii) $N_{E,t}$ new firms that enter the market in period $t$. Each share is worth its market value $\tilde{v}_t$, which equals the net present value of the expected stream of future profits of the average firm, measured in units of the North consumption basket.

The household enters every period $t$ with real bond holdings $B_t$ and mutual fund share holdings $x_t$ whose market value is $\tilde{v}_t N_t$. It receives interest $r_t B_t$ on bond holdings, and dividend income $\tilde{d}_t N_t$ on the mutual fund stocks equal to the profit of the average firm times the number of existing home firms in period $t$, in proportion with its stock holdings $x_t$. Finally, the representative household also receives labor income equal to the real wage $w_t$. I set $L = 1$, i.e. labor is supplied inelastically.

\[ \text{In this version of the model (complete financial autarky), stocks in the mutual fund and bonds are not traded across countries; hence, the equilibrium conditions for stock and bond holdings are } x_t = x_{t+1} = 1, B_t = B_{t+1} = 0. \]
The first-order conditions generate the Euler equations for bonds and stocks:

\[ C_t^{-\gamma} = \beta (1 + r_{t+1}) E_t \left[ C_{t+1}^{-\gamma} \right], \]
\[ \tilde{v}_t = \beta (1 - \delta) E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (d_{t+1} + \tilde{v}_{t+1}). \]

Households in North consume the consumption basket \( C_t \), which includes varieties of final goods produced by North firms \((\omega \in \Omega_t^{NN})\) using a mix of domestic and foreign inputs, through either domestic or offshore production under vertical FDI. In addition, the consumption basket \( C_t \) also includes final good varieties produced by South firms \((\omega \in \Omega_t^{SN})\) and which are either imported or produced in North under horizontal FDI:

\[ C_t = \left[ \begin{array}{c}
\int_{z_{\min}}^{z_{V,t}} y_{D,t}(\omega)^{\frac{\theta - 1}{\theta}} d\omega + \int_{z_{V,t}}^{\infty} y_{V,t}(\omega)^{\frac{\theta - 1}{\theta}} d\omega + \int_{z_{H,t}}^{\infty} y_{H,t}(\omega)^{\frac{\theta - 1}{\theta}} d\omega
\end{array} \right], \]

where \( \theta > 1 \) is the symmetric elasticity of substitution across goods. Let \( p_t(\omega) \) denote the price of each variety of final goods \( y_t(\omega) \); then the price index of the home consumption basket is \( P_t = \left[ \int p_t(\omega)^{1-\theta} d\omega \right]^{1/\theta} \) for \( \omega \in \Omega_t^{NN} \cup \Omega_t^{SN} \), and the demand for each variety of final goods \( \omega \) is \( y_t(\omega) = (p_t(\omega)/P_t)^{-\theta} C_t \). Using the home consumption basket \( C_t \) as the numeraire good and defining the real price of variety \( \omega \) as \( \rho_t(\omega) = p_t(\omega)/P_t \), I write the consumption-based price index as:

\[ 1 = \left[ \int \rho_t(\omega)^{1-\theta} d\omega \right]^{\frac{1}{1-\theta}}, \omega \in \Omega_t^{NN} \cup \Omega_t^{SN}. \]

### 3.2 Firms serving the domestic market: domestic production vs. vertical FDI

The North economy includes a continuum of monopolistically competitive firms that produce final goods. Each firm produces a different variety of final goods - therefore, the firm-specific labor productivity \( z \) also serves as an index for the existing varieties on the support interval \([z_{\min}, \infty)\). As shown in the expression for the North consumption basket \( C_t \) above, firms with productivity below...
the endogenous cutoff \( z_{V,t} \) produce their varieties of final goods using exclusively domestic inputs, whereas those above the cutoff use foreign inputs.

Thus, each firm producing for the domestic market can choose one of two possible strategies:

1. Under *domestic production*, the home firm with idiosyncratic labor productivity \( z \) employs an amount of labor \( l_t \) and obtains output:

\[
y_{D,t}(z) = Z_t z l_t,
\]

where \( Z_t \) is the aggregate productivity of labor in North;

2. Under *vertical FDI*, the same firm uses a combination of domestic and foreign inputs produced with labor from North and South, \( l_t \) and \( l^*_t \), respectively. Following Antras and Helpman (2004), output of every variety \( z \) of final goods is a Cobb-Douglas function of the inputs:

\[
y_{V,t}(z) = \left( \frac{Z_t z l_t}{\alpha} \right)^{1-\alpha} \left( \frac{Z^*_t z l^*_t}{1-\alpha} \right)^{1-\alpha}.
\]

Under different calibrations of the model, I vary the share of foreign inputs contributed by offshore affiliates. The smaller \( \alpha \) is, the more intensive the production process is in inputs produced by offshore affiliates. Setting \( \alpha = 0 \) implies that vertical FDI firms use exclusively foreign inputs in the production of final good varieties.\(^{11}\) At the other extreme, \( \alpha = 1 \) shuts down offshore production under vertical FDI as a transmission channel for aggregate shocks.

Given the demand for final goods produced domestically, \( y_{D,t}(z) = \rho_{D,t}(z) - \theta C_t \), and the demand for final goods produced under vertical FDI, \( y_{V,t}(z) = \rho_{V,t}(z) - \theta C_t \), firms set prices by solving the following profit-maximizing problem:\(^{12}\)

\[
\begin{align*}
\max_{\{\rho_{D,t}(z)\}} \quad & d_D(z) = \rho_{D,t}(z) y_{D,t}(z) - \frac{w_t}{Z_t z} y_{D,t}(z), \\
\max_{\{\rho_{V,t}(z)\}} \quad & d_V(z) = \rho_{V,t}(z) y_{V,t}(z) - \left( \frac{w_t}{Z_t z} \right)^\alpha \left( \frac{w^*_t Q_t}{Z^*_t z} \right)^{1-\alpha} y_{V,t}(z) - f_V \left( \frac{w_t}{Z_t} \right) ^\alpha \left( \frac{w^*_t Q_t}{Z^*_t} \right)^{1-\alpha}.
\end{align*}
\]

The cost of producing one unit of output either domestically or under vertical FDI varies according to differences in labor productivity \( z \) across firms and differences in the cost of effective labor across countries. Given the domestic and offshore real wages \( w_t \) and \( w^*_t \), the unit cost of producing interme-

\(^{11}\)Using l'Hôpital's rule, \( \lim_{\alpha \to 0} \left( \frac{1}{\alpha} \right)^\alpha = \lim_{\beta \to \infty} \frac{1}{\beta} = e^\lim_{\beta \to \infty} (1/\beta) \ln \beta = e^{\lim_{\beta \to \infty} (1/\beta) \ln \beta} = e^{\lim_{\beta \to \infty} (1/\beta)} = e^0 = 1.\)

\(^{12}\)Appendix 1 shows the derivations of the aggregate price index and the demand functions of intermediate goods produced domestically and offshore.
diate goods domestically is \( \frac{w_t}{Z_t^{1,z}} \), and the unit cost of producing offshore is \( \left( \frac{w_t}{Z_t^{1,z}} \right)^\alpha \left( \frac{w_t^*Q_t}{Z_t^{1,z}} \right)^{1-\alpha} \). The real exchange rate \( Q_t = \frac{P_t^N \varepsilon_t}{P_t^S} \) is defined as the ratio of consumer price indexes expressed in North and South in the same currency, where \( \varepsilon_t \) is the nominal exchange rate.

In addition to the unit production costs, firms producing under vertical FDI also incur the period-by-period fixed offshoring costs equal to a mix of \( f_V \) units of effective labor from North and South, costs which reflect headquarter operations as well as the building and maintenance of the production facility offshore.\(^\text{14}\) They also face an iceberg trade cost (\( \tau > 1 \)) which applies to the value of inputs produced by offshore affiliates and incorporated in the final good varieties consumed in North.\(^\text{15}\) The iceberg trade costs represent transportation costs, insurance and fees, as well as language barriers and differences in the legal systems, as discussed in Anderson and Wincoop (2004).

Hence, the optimal prices per unit of output produced either domestically or offshore are:\(^\text{16}\)

\[
\rho_{D,t}(z) = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t^{1,z}}, \\
\rho_{V,t}(z) = \frac{\theta}{\theta - 1} \left( \frac{w_t}{Z_t^{1,z}} \right)^\alpha \left( \tau \frac{w_t^*Q_t}{Z_t^{1,z}} \right)^{1-\alpha}.
\]

The resulting profits from domestic and offshore production, both expressed in units of the North consumption basket \( C_t \), are:\(^\text{17}\)

\[
d_{D,t}(z) = \frac{1}{\theta} \rho_{D,t}(z)^{1-\theta} C_t, \\
d_{V,t}(z) = \frac{1}{\theta} \rho_{V,t}(z)^{1-\theta} C_t - f_V \left( \frac{w_t}{Z_t^{1,z}} \right)^\alpha \left( \tau \frac{w_t^*Q_t}{Z_t^{1,z}} \right)^{1-\alpha}.
\]

To summarize the above, the structure of production costs depends on the firm-specific productivity factor \( z \), the cost of effective labor in North and South (\( \frac{w_t}{Z_t^{1,z}} \) and \( \frac{w_t^*Q_t}{Z_t^{1,z}} \), respectively), the fixed costs

\(^\text{13}\)The latter is translated into units of the consumption basket in North. The real wage \( w_t = W_t/P_t \) in North is expressed in units of the domestic consumption basket; the offshore real wage \( w_t^* = W_t^*/P_t^* \) is expressed in units of the consumption basket in South.

\(^\text{14}\)The mix of \( f_V \) units of effective labor is equivalent to \( f_V \left( \frac{w_t}{Z_t^{1,z}} \right)^\alpha \left( \frac{w_t^*Q_t}{Z_t^{1,z}} \right)^{1-\alpha} \) units of the consumption basket in North.

\(^\text{15}\)The assumption that the iceberg trade costs apply only to the inputs produced offshore is a normalization. Domestic inputs that undergo offshore processing and then are re-imported in the country of origin, although exempt from tariffs, are still subject to transportation and insurance costs. However, I assume that the iceberg trade costs applying do domestic intermediates are less than those applying to imported offshore intermediates.

\(^\text{16}\)Appendix 2 shows the derivation of the optimal price formulas for intermediate goods produced by the monopolistically-competitive firms domestically and offshore.

\(^\text{17}\)Given the demand \( y_D(z) = \rho_D(z)^{1-\theta} C \) and optimal price \( \rho_D(z) = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t^{1,z}} \) charged for the intermediate good variety \( z \) produced domestically, the profit formula is derived as \( d_D(z) = \left( \frac{\theta}{\theta - 1} \right) \frac{\theta}{w_t} y_D(z) = \frac{1}{\theta} \rho_D(z) y_D(z) \). The profit from producing through vertical FDI is derived in a similar fashion.
of offshoring $f_V$ as well as the iceberg trade costs $\tau$. Firms that produce intermediate goods offshore benefit from the relatively lower cost of effective labor, but incur fixed offshoring costs every period and iceberg trade costs when shipping intermediates for final assembly back in North. Hence, when deciding the location of production facilities, firms compare the profits they would obtain from domestic production $d_{D,t}(z)$ to those obtained from producing intermediates offshore $d_{V,t}(z)$, a feature of the model which generates endogenous offshoring.

Every period $t$, firms with idiosyncratic productivity $z$ above a certain cutoff $z_{V,t}$ find it profitable to locate the production of intermediate inputs offshore. Given the difference between the costs of effective labor in North and South, the relatively more productive firms ($z > z_{V,t}$) obtain profits that are large enough to cover the fixed offshoring cost $f_V$ and the iceberg trade cost $\tau$ of shipping the inputs back home. On the contrary, firms with relatively low labor productivity ($z < z_{V,t}$) are unable to take advantage of the lower cost of effective labor in South, since their profits would fall short of covering the fixed and the iceberg trade costs.

As a particular case, the firm with labor productivity equal to the cutoff $z_{V,t}$ is indifferent between producing domestically or offshore: After subtracting the offshoring and trade costs, the profits it would obtain from domestic and offshore production are equal. The equality of profits allows to solve for the endogenous productivity cutoff $z_{V,t}$ that governs the location decision of production under vertical FDI:

$$z_{V,t} = \{z \mid d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t})\}.$$  

**Existence of equilibrium.** Below I show that the existence of equilibrium productivity cutoff $z_{V,t}$ requires cross-country asymmetries with respect to the cost of effective labor. In other words, firms need the incentive of a lower cost of effective labor in order to relocate production offshore through vertical FDI.

Setting $\alpha = 0$, I re-write the profit functions for domestic and offshore production through vertical FDI, respectively, as $d_{D,t}(z) = B_t \left( \frac{w_t}{Z_t} \right)^{1-\theta} z^{\theta-1}$ and $d_{V,t}(z) = B_t \left( \frac{A_t Q_t}{Z_t} \right)^{1-\theta} z^{\theta-1}$, where $\theta > 1$ and $B_t \equiv \frac{1}{\theta} \left( \frac{\theta}{1-\theta} \right)^{1-\theta} C_t$ is a measure of market size. Figure 3(b) plots profits as functions of $z^{\theta-1}$. The vertical intercepts are the annualized value of the sunk entry cost (for $d_{D,t}(z)$) and the annualized value of the sunk entry cost plus the fixed offshoring costs (for $d_{V,t}(z)$). The existence of equilibrium cutoff $z_{V,t}$ requires:

1. $d_{V,t}(z)$ to be steeper than $d_{D,t}(z)$, and
2. $z_{\min} < z_{V,t}$ must hold every period.
The first condition implies that effective wages in South must be low enough \((TOL_t < 1)\) in order to offset the iceberg trade costs \((\tau > 1)\):

\[
\tau \frac{w^*_t Q^*_t}{Z^*_t} < \frac{w_t}{Z_t z} \iff \tau TOL_t < 1.
\]

The second condition, discussed in Appendix 4, implies:

\[
d_{D,t}(z_{\text{min}}) < \Theta f_E \frac{w_t}{Z_t} + f_Y \frac{w^*_t Q^*_t}{Z^*_t},
\]

where \(\Theta = \frac{1-\beta(1-\delta)}{\beta(1-\delta)}\). The inequality shows that the profit obtained from domestic production by the firm with the minimum productivity \(z_{\text{min}}\) is less than the per-period value of the sunk entry cost plus the fixed offshoring cost. In other words, the firm that obtains zero profit from domestic production would make negative profits if it engages in offshore production, i.e. the firm with productivity \(z_{\text{min}}\) does not produce in either country.

### 3.3 Firms serving the foreign market: exports vs. horizontal FDI

Firms in the North economy also have the option to serve the foreign market through one of two possible strategies:

1. Under *endogenous exports*, firms in North use only domestic inputs in the production of final goods which they export to the South market;
(2) Under horizontal FDI, firms use a combination of domestic and foreign inputs and sell the resulting final goods to consumers in the South market. The production function for both strategies can be summarized as:

\[ y_{H,t}(z) = \left( \frac{Z_t z l_t}{\eta} \right)^{\eta} \left( \frac{Z^*_t z l^*_t}{1 - \eta} \right)^{1-\eta}, \]

where a smaller \( \eta \) corresponds to a larger content of inputs produced by offshore affiliates and that is incorporated in the final goods sold to consumers in the host market. Thus, my model of offshoring nests both Ghironi and Melitz (2005) and Contessi (2006). On one hand, setting \( \eta = 1 \) implies that firms serve the South market exclusively through endogenous exports, as in Ghironi and Melitz (2005). On the other hand, setting \( \eta = 0 \) implies that firms produce exclusively through their offshore affiliates in order to serve the South market, like in Contessi (2006).

Both exports and horizontal FDI imply a fixed cost equal to a mix of \( f_H \) units of effective labor in North and South. In addition, exports also require the iceberg trade cost \( \tau^* \) which applies to the imported content of final goods consumed in South.

Unlike offshore production through vertical FDI, which is motivated by lower labor costs, offshore production through horizontal FDI is motivated by market access, since it allows firms to avoid the trade costs associated with inputs that are produced in the host economy rather than imported. Under production through horizontal FDI, firms continue to produce in their country of origin (in order to serve the home market) at the same time with producing the same variety of final goods at the offshore location (in order to serve the market of the host economy).

Following the approach described above, profit-maximizing firms from North serving the southern market set optimal prices for output that is either exported or produced under horizontal FDI:

\[ \rho_{H,t}(z) = \frac{\theta}{\theta - 1} \left( \tau^* \frac{w_l Q_l}{Z_t z} \right)^{\eta} \left( \frac{w^*_l}{Z^*_t z} \right)^{1-\eta}. \]

Every period \( t \), firms with idiosyncratic productivity \( z \) above a certain cutoff \( z_{H,t} \) find it profitable to serve the foreign market (South) - at the same time with serving their domestic market (North) - through either exports or horizontal FDI. Their profits are large enough to cover the fixed cost of serving the foreign market \( f_H \). In addition to that, firms save the iceberg trade cost when using inputs produced locally through horizontal FDI in the country which they serve (i.e. \( \eta = 0 \)). The profit function is:

\[ d_{H,t}(z) = \frac{1}{\theta} \rho_{H,t}(z)^{1-\theta} C^* Q_t - f_H \left( \frac{w_l}{Z_t} \right)^{\eta} \left( \frac{w^*_l}{Z^*_t} \right)^{1-\eta}. \]

The option to serve the foreign market (in addition to serving the domestic one) is endogenous:
the productivity cutoff $z_{H,t}$ is time-variant:

$$z_{H,t} = \inf \{ z \mid d_{H,t}(z_{V,t}) > 0 \}.$$  

### 3.4 Firm averages

**Firms serving the domestic market:** Following the approach in Melitz (2003), I define two average productivity levels that correspond to: (1) firms producing final goods using entirely domestic components (average productivity level $z_{D,t}$), and (2) firms producing final goods through vertical FDI using inputs produced both domestically and through offshore affiliates (average productivity level $z_{V,t}$). Then I solve the model in terms of two representative North firms: (1) the average home firm producing domestically, and (2) the average home firm producing offshore through vertical FDI.

As shown in Figure 4, in every period $t$ there are $N_{D,t}$ home firms producing domestically with average firm-specific labor productivity $\bar{z}_{D,t}$, and $N_{V,t}$ firms producing offshore through vertical FDI with average firm-specific labor productivity $\bar{z}_{V,t}$.\(^{18}\) Since the firm-specific labor productivities $z$ are random draws from a common distribution $G(z)$ with density $g(z)$ and support on the interval $[z_{min}, \infty)$; all firms above cutoff $z_{V,t}$ establish production facilities offshore.

\(^{18}\)One should distinguish between $z_{V,t}$ and $\bar{z}_{V,t}$. The former is the average firm-specific labor productivity of firms producing offshore, whereas the latter is the labor productivity cutoff on support interval $[z_{min}, \infty)$; all firms above cutoff $z_{V,t}$ establish production facilities offshore.
\[ z_{\text{min}}, \infty \) \), I write the domestic and offshore average productivities respectively as:

\[
\tilde{z}_{D,t} = \left[ \frac{1}{G(z_{V,t})} \int_{z_{\text{min}}}^{z_{V,t}} z^{\theta-1} g(z) \, dz \right]^{\frac{1}{\tau-1}},
\]

\[
\tilde{z}_{V,t} = \left[ \frac{1}{1 - G(z_{V,t})} \int_{z_{V,t}}^{\infty} z^{\theta-1} g(z) \, dz \right]^{\frac{1}{\tau-1}}.
\]

After defining the two average productivities, I re-write the model in terms of two representative firms, one producing domestically and the other producing offshore through vertical FDI. I start by re-writing the prices charged by each of the two representative firms as:

\[
\tilde{\rho}_{D,t} = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t \tilde{z}_{D,t}}, \quad \tilde{\rho}_{V,t} = \frac{\theta}{\theta - 1} \left( \frac{w_t}{Z_t \tilde{z}_{V,t}} \right)^\alpha \left( \frac{\tau w_t^* Q_t}{Z_t^* \tilde{z}_{V,t}} \right)^{1-\alpha}.
\]

The corresponding profits are:

\[
\tilde{d}_{D,t} = \frac{1}{\theta} (\tilde{\rho}_{D,t})^{1-\theta} C_t, \\
\tilde{d}_{V,t} = \frac{1}{\theta} (\tilde{\rho}_{V,t})^{1-\theta} C_t - f_V \left( \frac{w_t}{Z_t} \right)^\alpha \left( \frac{\tau w_t^* Q_t}{Z_t^*} \right)^{1-\alpha}.
\]

Due to the wage asymmetry across countries, firms originating in South do not engage in vertical FDI, i.e. they do not use intermediates produced offshore - in the high-wage North economy - when serving their domestic market. Their average price and profits are:

\[
\tilde{\rho}_{D,t}^* = \frac{\theta}{\theta - 1} \frac{w_t^*}{Z_t^* \tilde{z}_{D,t}^*}, \\
\tilde{d}_{D,t}^* = \frac{1}{\theta} (\tilde{\rho}_{D,t}^*)^{1-\theta} C_t^*.
\]

**Firms serving the foreign market:** As in Ghironi and Melitz (2005), the profit-maximizing representative firms in North and South that serve the foreign market - through either exports or horizontal FDI - charge the following prices:

\[
\tilde{\rho}_{H,t} = \frac{\theta}{\theta - 1} \left( \frac{\tau w_t^* Q_t}{Z_t^* \tilde{z}_{H,t}} \right)^{1-\eta} \left( \frac{w_t^*}{Z_t^* \tilde{z}_{H,t}} \right)^{1-\eta},
\]

\[
\tilde{\rho}_{H,t}^* = \frac{\theta}{\theta - 1} \left( \frac{\tau w_t^* Q_t}{Z_t^* \tilde{z}_{H,t}} \right)^{1-\eta^*} \left( \frac{w_t}{Z_t \tilde{z}_{H,t}} \right)^{1-\eta^*}.
\]
The corresponding average profits are:

\[ \bar{d}_{H,t} = \frac{1}{\theta} (\bar{\rho}_{H,t})^{1-\theta} C_t^* Q_t - f_H \left( \frac{w_t}{Z_t} \right)^{\eta} \left( \frac{w^*_t Q_t}{Z^*_t} \right)^{1-\eta}, \quad (12) \]

\[ \bar{d}^*_H,t = \frac{1}{\theta} (\bar{\rho}_t^*) \hat{C}_t Q_t^{-1} - f'_H \left( \frac{w^*_t}{Z_t} \right)^{\eta^*} \left( \frac{w^*_t Q_t^{-1}}{Z_t} \right)^{1-\eta^*}. \quad (13) \]

**Price indexes:** The consumption price index in North include the average price of the final good produced domestically, the average price of the good produced by domestic firms under vertical FDI, and the average price of the good supplied by South firms through either exports or local production under horizontal FDI, respectively:

\[ 1 = N_{D,t} (\bar{\rho}_{D,t})^{1-\theta} + N_{V,t} (\bar{\rho}_{V,t})^{1-\theta} + N_{H,t}^* (\bar{\rho}_{H,t})^{1-\theta} \quad (14) \]

In contrast to firms originating in the North economy, none of the firms in South has an incentive to establish offshore production facilities in North, where labor is relatively more expensive. Therefore, none of the firms in the South economy produces intermediates through offshore affiliates under vertical FDI. The consumption price index in South includes the average price of goods supplied by southern firms through domestic production and the price of goods provided by northern firms through either exports or horizontal FDI:

\[ 1 = N_{D,t}^* (\bar{\rho}_{D,t}^*)^{1-\theta} + N_{H,t}^* (\bar{\rho}_{H,t})^{1-\theta} \quad (15) \]

**Total profits:** The total profit of the average firm originating in North includes profits from domestic production, vertical FDI and horizontal FDI:

\[ N_t \hat{d}_t = N_{D,t} \bar{d}_{D,t} + N_{V,t} \bar{d}_{V,t} + N_{H,t} \bar{d}_{H,t}. \quad (16) \]

In contrast to northern firms, none of the firms in South engages in vertical FDI:

\[ N_{D,t}^* \bar{d}_t = N_{D,t}^* \bar{d}_{D,t} + N_{H,t}^* \bar{d}_{H,t}. \quad (17) \]

### 3.5 Pareto-distributed firm productivity

**Firms serving the domestic market:** I solve the model in terms of two representative firms - producing either domestically or through vertical FDI, respectively - under the assumption that firm-specific labor productivity \( z \) is Pareto distributed over the support interval \([z_{\text{min}}, \infty)\), with p.d.f. \( g(z) = k z_{\text{min}} / z^{k+1} \) and c.d.f. \( G(z) = 1 - (z_{\text{min}} / z)^k \) as in Melitz (2003). Parameter \( k \) reflects the dispersion of the productivity draws: A relatively larger \( k \) implies a smaller dispersion and a higher concentration of productivities \( z \) towards the lower productivity bound \( z_{\text{min}} \).
Under the Pareto assumption, I re-write the average productivity levels of the home firms producing domestically and offshore as follows:\(^{19}\)

\[ z_{D,t} = \nu z_{D,t} = \nu z_{\min} \left[ \frac{k - (\theta - 1)}{z_{V,t} - z_{\min}} \right]^{\frac{1}{\theta - 1}}, \]

\[ z_{V,t} = \nu z_{V,t}, \]

where \( \nu = \left[ \frac{k}{k - (\theta - 1)} \right]^{\frac{1}{\theta - 1}} \), \( z_{V,t} = z_{\min} \left( \frac{N_t}{N_{V,t}} \right)^{1/k} \) and \( k > \theta - 1. \)

Since firms originating in South serve their domestic market exclusively through domestic production, the average productivity of southern firms is constant:

\[ z_{D,t} = z_{\min}. \]

Finally, using that the firm at the productivity cutoff \( z_{V,t} \) is indifferent about the location of its production plants - since it obtains equal profits from domestic and offshore production, \( d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t}) \), I derive the link between the profits of the two average firms as follows:\(^{22}\)

\[ d_{V,t} = \frac{k}{k - (\theta - 1)} \left( \frac{z_{V,t}}{z_{D,t}} \right)^{\theta - 1} d_{D,t} + \frac{\theta - 1}{k - (\theta - 1)} f_{V} \left( \frac{w_t}{Z_t} \right)^{\alpha} \left( \frac{w^*_t Q_t}{Z_t^*} \right)^{1-\alpha}. \]

**Firms serving the foreign market:** Under the assumption of Pareto-distributed productivity, I re-write the average productivity levels of the firms serving the foreign market through either exports or horizontal FDI as follows:

\[ z_{H,t} = \nu z_{H,t} \left( \frac{N_t}{N_{H,t}} \right)^{1/k}, \]

\[ z^*_{H,t} = \nu z^*_{\min} \left( \frac{N^*_{H,t}}{N^*_{H,t}} \right)^{1/k}. \]

Using that the firm at the productivity cutoff \( z_{H,t} \) obtains zero profits from serving the foreign market through either exports or horizontal FDI, \( d_{H,t}(z_{H,t}) = 0 \), I derive the following profit cutoffs:

\[ d_{H,t} = \frac{\theta - 1}{k - (\theta - 1)} f_{H} \left( \frac{w_t}{Z_t} \right)^{\eta} \left( \frac{w^*_t Q_t}{Z_t^*} \right)^{1-\eta}, \]

\[ d^*_{H,t} = \frac{\theta - 1}{k - (\theta - 1)} f^*_{H} \left( \frac{w^*_t}{Z^*_t} \right)^{\eta^*} \left( \frac{w^*_t Q^*_t}{Z^*_t} \right)^{1-\eta^*}. \]

\(^{19}\)See Appendix 3.

\(^{20}\)I derive the shares of home firms producing domestically and offshore, respectively, as \( \frac{N_{D,t}}{N_t} = G(z_{V,t}) \) and \( \frac{N_{V,t}}{N_t} = 1 - G(z_{V,t}) \), where the total number of firms in every period is \( N_t = N_{D,t} + N_{V,t} \). Then I use the expression for the Pareto c.d.f. \( G(z_{V,t}) \) to derive the productivity cutoff \( z_{V,t} \) from the share of offshoring firms, \( z_{V,t} = z_{\min} \left( \frac{N_t}{N_{V,t}} \right)^{1/k} \).

\(^{21}\)The condition that \( k > \theta - 1 \) ensures that the variance of firm size is finite, since the average productivities of the domestic-based and outsourcing firms are, respectively, \( \bar{z}_{V,t} = \left[ \frac{k}{k - (\theta - 1)} \right]^{\frac{1}{\theta - 1}} z_{V,t} \).

\(^{22}\)See Appendix 4.
3.6 Firm entry and exit

Firm entry and exit take place every period. Following Gironi and Melitz (2005), firm entry requires a sunk entry cost – i.e. which reflects the cost of starting a business – equal to $f_E$ units of home effective labor. Potential entrants do not become aware of their idiosyncratic labor productivity $z$ until after they enter the market. After paying the sunk entry cost, each firm is randomly assigned an idiosyncratic labor productivity $z$ drawn independently from a common distribution $G(z)$ with support on the interval $[z_{min}, \infty)$, which the firm keeps for the entire duration of its life.

Potential entrants are forward looking and correctly anticipate their expected profits $\bar{d}_t$ as well as the probability $\delta$ of receiving an exit-inducing shock every period. Hence, potential entrants do anticipate their expected post-entry value given by the present discounted value of their expected stream of profits:

$$\bar{v}_t = \mathbb{E}_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left( \frac{C_s}{C_t} \right)^{-\gamma} \bar{d}_s.$$ 

Therefore, firm entry takes place until the value of the average firm equals the sunk entry cost:

$$\bar{v}_t = f_E \frac{w_t}{Z_t}. \quad (26)$$

Each of the $N_{E,t}$ firms entering at time $t$ does not produce until period $t+1$, which introduces a one period time-to-build into the model. Irrespective of their productivity, firms - including the new entrants - are also subject to a random exit shock that occurs with probability $\delta$ at the end of every period after production has taken place. Thus, the law of motion for the number of producing firms is:

$$N_{t+1} = (1-\delta)(N_t + N_{E,t}). \quad (27)$$

3.7 Aggregate demand and value added

In order to quantify the business cycle co-movements generated by my model, I use value added as a measure of the aggregate incomes of the two countries. Thus, the model takes into account the fact that inputs produced in South and used in the production of the final good consumed in North are part of the value added of the South economy, and therefore must not be double-counted in the value added of North. Under financial autarky on the market for both bonds and stocks (i.e. $B_{t+1} = B_t = 0$ and $x_{t+1} = x_t = 1$ in equilibrium), value added is equal to the wage bill plus the total amount of stock

\footnote{Or $f_E w_t / Z_t$ units of the consumption basket in North.}
dividends derived by households in each country:

\[ Y_t = w_t L + N_t \tilde{d}_t. \]

Finally, aggregate accounting under financial autarky implies that households spend their income from labor and stock holdings on consumption and investment in new firms:

\[ C_t + N_{E,t} \tilde{v}_t = w_t L + N_t \tilde{d}_t. \]  (28)

### 3.8 Balanced trade

Under financial autarky, the real exchange rate \( Q_t \) is pinned down by the balanced current account condition for North (which reflects the corresponding balance for South):

\[
\begin{align*}
\left(1 - \alpha\right) N_{V,t} (\tilde{\rho}_{V,t})^{1-\theta} C_t &+ \eta^* N_{H,t} (\tilde{\rho}_{H,t})^{1-\theta} C_t + (1 - \eta^*) N_{H,t} \tilde{d}_{H,t} Q_t \\
\text{(1) VFDI imports} &+ \text{(2) HFDI imports} &+ \text{(3) HFDI profits*} \\
= \eta N_{H,t} (\tilde{\rho}_{H,t})^{1-\theta} C_t^* Q_t &+ (1 - \alpha) N_{V,t} \tilde{d}_{V,t} + (1 - \eta) N_{H,t} \tilde{d}_{H,t} \\
\text{(4) HFDI exports} &+ \text{(5) VFDI profits} &+ \text{(6) HFDI profits} 
\end{align*}
\]

The equations shows that the sum of (1) North’s imports of inputs manufactured by offshore affiliates under vertical FDI, (2) North’s imports of inputs used by South firms for local production under horizontal FDI, and (3) the repatriated profits of South firms active in North through horizontal FDI must equal the sum of (4) North’s exports of inputs used by North firms serving the South market through horizontal FDI, (5) and (6), i.e. the repatriated profits of North firms obtained from vertical and horizontal FDI, respectively.

### 3.9 Model summary and asymmetries between North and South

As shown in Table 2, the model for the North economy is summarized by 16 equations in 16 endogenous variables: \( N_t, N_{D,t}, N_{V,t}, N_{H,t}, N_{E,t}, \tilde{d}_t, \tilde{d}_{D,t}, \tilde{d}_{V,t}, \tilde{d}_{H,t}, \tilde{z}_{D,t}, \tilde{z}_{V,t}, \tilde{z}_{H,t}, \tilde{v}_t, \tilde{r}_t, \sigma_t, w_t \) and \( C_t \), plus the equation describing aggregate productivity \( Z_t \). Firms in the South economy produce domestically for the domestic market, and serve the foreign market through either exports or local production under horizontal FDI, but have no incentive to engage in vertical FDI since effective labor in North is relatively more expensive. Therefore, the South economy is described by only 11 equations in 11 endogenous variables, i.e. there are no counterparts for \( N_t, N_{V,t}, \tilde{d}_{V,t}, \tilde{z}_{D,t} \) and \( \tilde{z}_{V,t} \). In particular, since no firms from South produce offshore, the average labor productivity of the Southern firms is a constant, \( \tilde{z}_D = \nu z_{min}^* \), where \( \nu = \left[ \frac{k}{k-\theta-11} \right]^{\frac{1}{\theta-1}} \). Finally, I use the pricing and profit formulas defined by equations (6) - (15) and summarized in Table 3.
### Table 2. Model summary

<table>
<thead>
<tr>
<th>Equation Type</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler equation, bonds</td>
<td>$C_t^{-\gamma} = \beta (1 + r_{t+1}) E_t \left[ C_{t+1}^{-\gamma} \right]$</td>
</tr>
<tr>
<td></td>
<td>$C_t^{\ast -\gamma} = \beta (1 + r_{t+1}^{<em>}) E_t \left[ C_{t+1}^{</em> -\gamma} \right]$</td>
</tr>
<tr>
<td>Euler equation, stocks</td>
<td>$\tilde{\nu}<em>t = \beta(1 - \delta) E_t \left( \frac{C</em>{t+1}}{C_t} \right)^{-\gamma} (\tilde{d}<em>{t+1} + \tilde{\nu}</em>{t+1})$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\nu}<em>t^{*} = \beta(1 - \delta) E_t \left( \frac{C</em>{t+1}^{<em>}}{C_t^{</em>}} \right)^{-\gamma} (\tilde{d}<em>{t+1}^{*} + \tilde{\nu}</em>{t+1}^{*})$</td>
</tr>
<tr>
<td>Free entry</td>
<td>$\tilde{\nu}_t = \frac{I_E w_t}{Z_t}$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\nu}_t^{<em>} = \frac{I_E^{</em>} w_t^{<em>}}{Z_t^{</em>}}$</td>
</tr>
<tr>
<td>Rule of motion, # firms</td>
<td>$N_{t+1} = (1 - \delta)(N_t + N_{E,t})$</td>
</tr>
<tr>
<td></td>
<td>$N_{D,t+1}^{<em>} = (1 - \delta)(N_{D,t}^{</em>} + N_{E,t}^{*})$</td>
</tr>
<tr>
<td>Aggregate accounting</td>
<td>$C_t + N_{E,t} \tilde{\nu}_t = w_t L + N_t \tilde{d}_t$</td>
</tr>
<tr>
<td></td>
<td>$C_t^{<em>} + N_{E,t}^{</em>} \tilde{\nu}<em>t^{<em>} = w_t L^{</em>} + N</em>{D,t}^{<em>} \tilde{d}_t^{</em>}$</td>
</tr>
<tr>
<td>Consumption price index</td>
<td>$1 = N_{D,t} (\tilde{p}<em>{D,t})^{1-\theta} + N</em>{V,t} (\tilde{p}<em>{V,t})^{1-\theta} + N</em>{H,t}^{<em>} \left( \tilde{p}_{H,t}^{</em>} \right)^{1-\theta}$</td>
</tr>
<tr>
<td></td>
<td>$1 = N_{D,t}^{<em>} \left( \tilde{p}_{D,t}^{</em>} \right)^{1-\theta} + N_{H,t} \left( \tilde{p}_{H,t} \right)^{1-\theta}$</td>
</tr>
<tr>
<td>Total profits</td>
<td>$N_{t+1} \tilde{d}<em>t = N</em>{D,t} \tilde{d}<em>{D,t} + N</em>{V,t} \tilde{d}<em>{V,t} + N</em>{H,t} \tilde{d}_{H,t}$</td>
</tr>
<tr>
<td></td>
<td>$N_{D,t}^{<em>} \tilde{d}<em>t = N</em>{D,t}^{</em>} \tilde{d}<em>{D,t} + N</em>{H,t}^{*} \tilde{d}_{H,t}$</td>
</tr>
<tr>
<td>Number of firms (Home)</td>
<td>$N_t = N_{D,t} + N_{V,t}$</td>
</tr>
<tr>
<td>VFDI profits link (Home)</td>
<td>$\tilde{d}<em>{V,t} = \frac{k}{k - (\theta - 1)} \left( \frac{w_t}{Z_t} \right)^{\theta-1} \tilde{d}</em>{D,t} + \frac{\theta-1}{k - (\theta - 1)} f_V \left( \frac{w_t}{Z_t} \right)^{\alpha} \left( \frac{w_t Q_t}{Z_t^{*}} \right)^{1-\alpha}$</td>
</tr>
<tr>
<td>HFDI profits link</td>
<td>$\tilde{d}_{H,t} = \frac{\theta-1}{k - (\theta - 1)} f_H \left( \frac{w_t^{<em>}}{Z_t^{</em>}} \right)^{\eta} \left( \frac{w_t Q_t}{Z_t} \right)^{1-\eta}$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{d}_{H,t}^{<em>} = \frac{\theta-1}{k - (\theta - 1)} f_H^{</em>} \left( \frac{w_t^{<em>}}{Z_t^{</em>}} \right)^{\eta^{<em>}} \left( \frac{w_t^{</em>} Q_t}{Z_t^{<em>}} \right)^{1-\eta^{</em>}}$</td>
</tr>
<tr>
<td>Dom. productivity (Home)</td>
<td>$\tilde{z}<em>{D,t} = \nu z</em>{\min} \tilde{z}<em>{V,t} \left[ \frac{k^{-(\theta-1)} - k^{-(\theta-1)}}{\tilde{z}</em>{V,t} - z_{\min}} \right]^{\frac{1}{\theta - 1}}$</td>
</tr>
<tr>
<td>VFDI productivity (Home)</td>
<td>$\tilde{z}<em>{V,t} = \nu z</em>{\min} \left( \frac{N_t}{N_{V,t}} \right)^{1/k} \tilde{z}_{V,t}$</td>
</tr>
<tr>
<td>HFDI productivity</td>
<td>$\tilde{z}<em>{H,t} = \nu z</em>{\min} \left( \frac{N_t}{N_{H,t}} \right)^{1/k} \tilde{z}_{H,t}$</td>
</tr>
<tr>
<td>Balanced trade</td>
<td>$(1 - \alpha) N_{V,t} \left( \tilde{p}<em>{V,t} \right)^{1-\theta} C_t + \eta N</em>{H,t}^{<em>} \left( \tilde{p}_{H,t}^{</em>} \right)^{1-\theta} C_t + (1 - \eta^{<em>}) N_{H,t}^{</em>} \tilde{d}_{H,t} Q_t =$</td>
</tr>
<tr>
<td></td>
<td>$= \eta N_{H,t} \left( \tilde{p}<em>{H,t} \right)^{1-\theta} C_t + (1 - \alpha) N</em>{V,t} \tilde{d}<em>{V,t} + (1 - \eta) N</em>{H,t} \tilde{d}_{H,t}$</td>
</tr>
</tbody>
</table>
### Table 3. Pricing and profit formulas

<table>
<thead>
<tr>
<th>Production Prices</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Domestic</strong></td>
<td></td>
</tr>
<tr>
<td>$\tilde{\rho}<em>{D,t}$ = $\frac{\theta}{\theta - 1} \frac{w_t}{Z_t z</em>{D,t}}$</td>
<td>$\tilde{d}<em>{D,t}$ = $\frac{1}{\theta} (\tilde{\rho}</em>{D,t})^{1-\theta} C_t$</td>
</tr>
<tr>
<td>$\tilde{\rho}<em>{D,t}^<em>$ = $\frac{\theta}{\theta - 1} \frac{w_t^</em>}{Z_t^* z</em>{D,t}^*}$</td>
<td>$\tilde{d}<em>{D,t}^*$ = $\frac{1}{\theta} (\tilde{\rho}</em>{D,t}^<em>)^{1-\theta} C_t^</em>$</td>
</tr>
<tr>
<td><strong>VFDI</strong></td>
<td></td>
</tr>
<tr>
<td>$\tilde{\rho}<em>{V,t}$ = $\frac{\theta}{\theta - 1} \left( \frac{w_t}{Z_t z</em>{V,t}} \right)^\alpha \left( \frac{w_t^* Q_t}{Z_t^* z_{V,t}^*} \right)^{1-\alpha}$</td>
<td>$\tilde{d}<em>{V,t}$ = $\frac{1}{\theta} (\tilde{\rho}</em>{V,t})^{1-\theta} C_t - f_V \left( \frac{w_t}{Z_t^<em>} \right)^\alpha \left( \frac{w_t^</em> Q_t}{Z_t^*} \right)^{1-\alpha}$</td>
</tr>
<tr>
<td><strong>HFDI</strong></td>
<td></td>
</tr>
<tr>
<td>$\tilde{\rho}<em>{H,t}$ = $\frac{\theta}{\theta - 1} \left( \frac{w_t^* Q_t}{Z_t^* z</em>{H,t}^*} \right)^\eta \left( \frac{w_t}{Z_t z_{H,t}} \right)^{1-\eta}$</td>
<td>$\tilde{d}<em>{H,t}$ = $\frac{1}{\theta} (\tilde{\rho}</em>{H,t})^{1-\theta} C_t^* Q_t - f_H \left( \frac{w_t}{Z_t^<em>} \right)^\eta \left( \frac{w_t^</em> Q_t}{Z_t^*} \right)^{1-\eta}$</td>
</tr>
<tr>
<td>$\tilde{\rho}<em>{H,t}^<em>$ = $\frac{\theta}{\theta - 1} \left( \frac{w_t^</em> Q_t}{Z_t^* z</em>{H,t}^<em>} \right)^\eta^</em> \left( \frac{w_t}{Z_t z_{H,t}} \right)^{1-\eta^*}$</td>
<td>$\tilde{d}<em>{H,t}^*$ = $\frac{1}{\theta} (\tilde{\rho}</em>{H,t}^<em>)^{1-\theta} C_t^</em> Q_t^{-1} - f_H^* \left( \frac{w_t}{Z_t^<em>} \right)^\eta^</em> \left( \frac{w_t^* Q_t}{Z_t^<em>} \right)^{1-\eta^</em>}$</td>
</tr>
</tbody>
</table>

### 4 Results

#### 4.1 Steady state

Intuitively, firms from North have an incentive to relocate input production to South only if the difference in the cost of effective labor is large enough to cover both the fixed costs of offshoring and the iceberg trade costs of shipping the inputs produced offshore back home. In line with this intuition, in this section I demonstrate analytically the need for an asymmetric steady state with respect to wages.

Under $\alpha = 0$, the steady-state solutions for all variables of my model can be obtained analytically after solving for the labor productivity of the average firm that produces intermediate goods offshore, $z_V$. The solution is described by the hyperbola:

$$\xi_1 z_V^{-\theta} + \xi_2 z_V^{-k} = \xi_3,$$

with parameters $\xi_2 > 0$ and $\xi_3 > 0$ for all plausible calibrations, $k > 0$ and $\theta > 1$.

In Figure 5, I plot the hyperbola described above for the cases of $\xi_1 > 0$ (squares) and $\xi_1 < 0$ (diamonds). The figure shows that the steady-state solution for $z_V$ lies within the support interval $[z_{\text{min}}, \infty)$, with $z_{\text{min}} = 1$, if and only if:

$$\xi_1 = z_{\text{min}}^{\theta - 1} \left( \frac{k}{k - (\theta - 1)} \right)^2 \frac{(TOL)^{\theta - 1}}{1 - (TOL)^{\theta - 1}} > 0,$$

where the terms of labor, $TOL = \frac{w^* / Z^*}{w / Z}$, represent a measure of the relative cost of effective labor

---

$^{24}z_V$ is on the horizontal axis, $\xi_3$ is on the vertical axis.
between the two countries.

Figure 5. Steady state solution for $\tilde{z}_V$

The resulting condition, $\tau TOL < 1$, can be satisfied in two different ways:

(a) $TOL = 1$ and $\tau < 1$;
(b) $\tau \geq 1$ and $TOL < 1$.

With method (a), the model would have a symmetric steady state in which real effective wages in North and South are identical, i.e. $TOL = 1$. In absence of cross-country differences in wages, firms would be motivated to relocate production offshore by the existence of an “iceberg subsidy,” $\tau < 1$, that reduces the marginal cost of intermediate goods manufactured offshore and shipped home.

With method (b), the model would have an asymmetric steady state in which the cost of effective labor is relatively higher in North than in South, i.e. $TOL < 1$. The cross-country difference in the cost of effective labor acts as an incentive for firms originating in North to relocate production to South. The difference in the cost of effective labor must be large enough to cover the fixed offshoring cost and the iceberg trade cost $\tau > 1$.

I follow method (b): I impose $f_E^S > f_E^N$, i.e. the cost of starting a business is higher in South than in North, to obtain an asymmetric steady of effective wages. The calibration is supported empirically by the data on cross-country differences in the cost of starting a business described in Table 4.
4.2 Calibration

I set the cost of starting a business to be 2 times larger in South than in North \(f_E^* = 2f_E\), where I assume \(f_E = 1\) without loss of generality. As shown in Table 4, this calibration reflects the considerable variation in the cost of starting a business across countries: the monetary cost is 3 times higher in Mexico than in the U.S. or Canada; it is 6 times higher in Hungary than in the U.K.

Table 4. Firm entry costs, selected economies

<table>
<thead>
<tr>
<th>Economy</th>
<th>Procedures (number)</th>
<th>Duration (days)</th>
<th>Min. Capital Req. (% of GNI/capita)</th>
<th>Cost (% of GNI/capita)</th>
<th>Cost (US$)</th>
<th>Relative Cost (U.S. = 1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>0.7</td>
<td>314.79</td>
<td>1.0</td>
</tr>
<tr>
<td>Canada</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0.9</td>
<td>325.53</td>
<td>1.0</td>
</tr>
<tr>
<td>Mexico</td>
<td>8</td>
<td>27</td>
<td>11.6</td>
<td>13.3</td>
<td>1,046.71</td>
<td>3.3</td>
</tr>
<tr>
<td>Germany</td>
<td>9</td>
<td>18</td>
<td>42.8</td>
<td>5.7</td>
<td>2,087.34</td>
<td>6.6</td>
</tr>
<tr>
<td>U.K.</td>
<td>6</td>
<td>13</td>
<td>0</td>
<td>0.8</td>
<td>321.44</td>
<td>1.0</td>
</tr>
<tr>
<td>France</td>
<td>5</td>
<td>7</td>
<td>0</td>
<td>1.1</td>
<td>402.05</td>
<td>1.3</td>
</tr>
<tr>
<td>Italy</td>
<td>9</td>
<td>13</td>
<td>9.8</td>
<td>18.7</td>
<td>5,987.74</td>
<td>19.0</td>
</tr>
<tr>
<td>Spain</td>
<td>10</td>
<td>47</td>
<td>13.7</td>
<td>15.1</td>
<td>4,163.07</td>
<td>13.2</td>
</tr>
<tr>
<td>Poland</td>
<td>10</td>
<td>31</td>
<td>196.8</td>
<td>21.2</td>
<td>1,736.28</td>
<td>5.5</td>
</tr>
<tr>
<td>Czech Rep.</td>
<td>10</td>
<td>17</td>
<td>34.9</td>
<td>10.6</td>
<td>1,344.08</td>
<td>4.3</td>
</tr>
<tr>
<td>Hungary</td>
<td>6</td>
<td>16</td>
<td>65.1</td>
<td>17.7</td>
<td>1,938.15</td>
<td>6.2</td>
</tr>
</tbody>
</table>


The asymmetric sunk entry costs - together with the fixed offshoring cost \(f_V = 0.002\), the trade iceberg cost \(\tau = 1.3\), and the fixed cost of serving the foreign market \(f_H = 0.006\) - generate \(TOL = 0.74\):

In steady state, the cost of effective labor in South – defined as real wage over aggregate productivity – is 74 percent the value of the corresponding measure in North.

Interpreting periods as quarters, I set the subjective rate of time discount \(\beta = 0.99\) and the coefficient of relative risk aversion \(\sigma = 2\). Following Ghironi and Melitz (2005), I set the probability of firm exit \(\delta = 0.025\) to match annual 10 percent job destruction in the U.S. Following Bernard, Eaton, Jensen and Kortum (2003), I also set the intra-temporal elasticity of substitution \(\theta = 3.8\), calibrated after U.S. plant and macro trade data.

Finally, I set the Pareto distribution coefficient \(k = 5.5\) and the lower bound of the support interval of idiosyncratic firm productivities \(z_{min} = 1\) without loss of generality. In future work, I aim to fine-tune the calibration of \(k, f_V\) and \(f_H\) in order to match empirical patterns such as the fraction of firms that use imported inputs in production (14 percent, as documented by Bernard, Jensen, Redding and
Schott, 2007), and the fraction of firms that serve the foreign market (21 percent through exports, as documented by Bernard, Eaton, Jensen and Kortum, 2003).

4.3 Impulse responses

I log-linearize the model around the steady state and compute the impulse responses to a transitory 1 percent increase in aggregate productivity in the North economy assuming $\log Z_{t+1} = \rho \log Z_t + u_t$ with $\rho = 0.9$.

![Figure 6. Endogenous offshoring with Ghironi and Melitz (2005) as benchmark: impulse responses to a transitory 1 percent technology shock](image)

Figure 6 shows the impulse responses of the model with both exports and endogenous offshoring.
through vertical FDI (solid line, $\alpha = 0, \eta = 1$), and contrasts them with the impulse responses of the benchmark model with endogenous exports a la Ghironi and Melitz (2005) (dotted line, $\alpha = 1, \eta = 1$). For each variable, the horizontal axis illustrates quarters after the initial shock, and the vertical axis shows the percentage deviations from the original steady state in each quarter.

On impact, the 1 percent increase in aggregate labor productivity in North generates an equal increase in the real wage $w_t$. The increase in demand for inputs produced in both North and South also generates a jump of real wages in South $w_t^*$ and of the terms of labor $TOL_t$: Since the increase of aggregate productivity in North is not replicated in South, on impact there is excess demand for units of effective labor in South. Therefore, the rise of South wages above aggregate productivity deters some of the North firms from producing offshore, due to: (1) the increase in the unit cost of producing offshore, and (2) the increase in the fixed cost of relocation, both sensitive to the cost of effective labor in South. Hence, the number of North firms that operate offshore affiliates in South ($N_{V,t}$) drops on impact.

Over the business cycle, the increase in aggregate labor productivity and higher expected profits in North stimulate firm entry ($N_t$ increases). Firm entry generates higher labor demand in North, and thus leads to an appreciation of the cost of effective labor in North relative to South (as shown by the decline of $TOL_t$ below the initial steady state). Following the appreciation of the terms of labor, more firms from North have an incentive to relocate production to South. Hence, after the initial drop triggered by the spike in South real wages, the number of firms from North that relocate production to South ($N_{V,t}$) rises above the original steady state in the medium run.

Thus, the initial jump of the South real wage, caused by the higher demand for existing offshore varieties on impact (i.e. offshoring at the intensive margin), is followed by an additional increase once more firms from North start re-locating production to South over the business cycle (i.e. offshoring at the extensive margin).

In comparison with Ghironi and Melitz (2005), the model with vertical FDI generates a larger increase in real wages in South, which is generated by the relocation of production by firms originating in North.\(^{25}\) Hence, higher demand for labor in North due to firm entry and higher demand for labor in South due to offshoring through vertical FDI generate a co-movement of wages and aggregate incomes which is notably larger than in Ghironi and Melitz (2005).

\(^{25}\)Over the business cycle, the terms of labor in the model with vertical FDI appreciate by less (i.e. $TOL$ decreases by less) than in Ghironi and Melitz (2005) due to the stronger increase in real wages in South caused by offshoring.
4.4 Second moments

I simulate the model and calculate the cross-country correlations of national income and consumption in North and South using the shocks to aggregate productivities \( Z_t \) and \( Z_t^* \) as the only sources of business cycle fluctuations, as in Backus, Kehoe, and Kydland (1992):

\[
\begin{bmatrix}
    Z_t \\
    Z_t^*
\end{bmatrix} = \begin{bmatrix}
    \phi_Z & \phi_{ZZ^*} \\
    \phi_{Z^*Z} & \phi_{Z^*}
\end{bmatrix} \begin{bmatrix}
    Z_{t-1} \\
    Z_{t-1}^*
\end{bmatrix} + \begin{bmatrix}
    \xi_t^Z \\
    \xi_t^{Z^*}
\end{bmatrix},
\]

where \( \phi_Z = \phi_{Z^*} = 0.906 \) and \( \phi_{ZZ^*} = \phi_{Z^*Z} = 0.088 \). The variances of innovations are 0.73% and the covariance is 0.19%.

Before computing the co-movement of national incomes and consumption across countries, I deflate home and foreign variables by the average price indices in each country in order to eliminate the variety effect generated by differences in the number of varieties of final goods available in each country. For instance, I deflate the value added of North as \( Y_{R,t} = P_t Y_t / \bar{P}_t \), where \( P_t = N_t^{1-r} \bar{P}_t \). As discussed in Ghironi and Melitz (2005), the empirical price deflators are best represented by the average price index \( \bar{P}_t \) rather than the welfare-based price index \( P_t \), as the latter includes the welfare effect generated by the availability of \( N_t \) product varieties.

Table 5 shows the effect of vertical FDI on the cross-country co-movement of national incomes and consumption. I take as benchmarks the model with exogenous exports a la Ghironi and Melitz (2005) and the model with local production through horizontal FDI a la Contessi (2006), both of which are nested by my model.

<table>
<thead>
<tr>
<th>Correlations</th>
<th>Calibration</th>
<th>( Y_{R}, Y_{R}^* )</th>
<th>( C_{R}, C_{R}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic production + VFDI + exports</td>
<td>( \alpha = 0, \eta = 1 )</td>
<td>0.50</td>
<td>0.76</td>
</tr>
<tr>
<td>Only domestic production + exports (GM 2005)</td>
<td>( \alpha = 1, \eta = 1 )</td>
<td>0.23</td>
<td>0.87</td>
</tr>
<tr>
<td>Domestic production + VFDI + HFDI</td>
<td>( \alpha = 0, \eta = 0 )</td>
<td>0.45</td>
<td>0.71</td>
</tr>
<tr>
<td>Only domestic production + HFDI (Contessi 2006)</td>
<td>( \alpha = 1, \eta = 0 )</td>
<td>0.28</td>
<td>0.94</td>
</tr>
<tr>
<td>Data (BKK 1992)</td>
<td></td>
<td>0.70</td>
<td>0.46</td>
</tr>
</tbody>
</table>

The results show that endogenous offshoring through vertical FDI increases the co-movement of output and decreases the co-movement of consumption relative to the benchmark models with either endogenous exports or horizontal FDI. Although the rank of correlations is inverted (i.e. the co-
movement is still greater for consumption than for national income), the results are closer to the corresponding empirical correlations reported in Backus, Kehoe, Kydland (1992).

5 Extended model: elastic labor supply

In this section I extend my benchmark model of offshoring by introducing elastic labor supply. Then I explore the ability of the extended model to reproduce the correlation of the maquiladora indicators with U.S. output measured at various lags and leads. In particular, I am interested in the ability of the model to reproduce the time pattern of offshoring at the intensive and extensive margins, as described by the data in Figure 2(b) and summarized in Figure 8 (first panel).

Figure 8. The co-movement between U.S. manufacturing (at lags and leads) and Mexico’s maquiladora manufacturing: the intensive vs. extensive margins

The first panel in Figure 8 shows the correlation between Mexico’s maquiladora indicators (i.e. value added, hours worked, and plants) and various lags and leads of U.S. industrial output in manufacturing, measured in real terms at quarterly frequency. It shows that, among the three maquiladora
indicators, the number of offshore plants (i.e. offshoring at the extensive margin) has the lowest contemporaneous correlation with U.S. output. Whereas the correlation of the number of plants with contemporaneous U.S. manufacturing output is below 0.5, the correlation with U.S. output lagged by three quarters exceeds 0.75. Unlike the number of plants, the maquiladora value added and hours worked respond faster to fluctuations in U.S. manufacturing output (i.e. hours worked respond immediately, and value added responds with a lag of one or two quarters). The pattern suggests that the relocation of production offshore across the business cycle takes place first at the intensive margin (the number of hours worked per firm reacts first), followed by the relocation at the extensive margin (the number of plants reacts with a lag of three quarters).

In the extended model I introduce elastic labor supply: The representative household consumes the basket of goods $C_t$ and supplies hours of work $L_t$ in a competitive labor market each period $t$, at the going real wage $w_t$. The household maximizes the expected intertemporal utility:

$$
\max_{\{C_t, B_t, x_t\}} \left[ \sum_{s=t}^{\infty} \beta^{s-t} \left( \ln C_s - \chi \frac{L_s^{1+\psi}}{1+\psi} \right) \right],
$$

s.t. $B_{t+1} + \tilde{v}_t (N_t + N_{E,t}) x_{t+1} + C_t = (1 + r_t) B_t + (\tilde{d}_t + \tilde{v}_t) N_t x_t + w_t L_t$,

where $\chi > 0$ is the weight of disutility from labor in the period utility function, and $\psi \geq 0$ is the Frisch elasticity of labor supply to wages and the intertemporal elasticity of substitution in labor supply. Following King, Plosser and Rebello (1988) and the discussions in Campbell (1994) and Bilbie et al. (2006), log utility for consumption is required to obtain constant steady state labor supply (balanced growth) in a model with utility additively separable over consumption and hours.

In order to account for elastic labor supply in the model, I add the first order condition for hours worked in North and South to the equations in Table 2. I also adjust the aggregate accounting equations to allow for variation in hours worked, as shown in Table 6.

**Table 6. Extended model, elastic labor supply**

<table>
<thead>
<tr>
<th>Euler equation, labor</th>
<th>$\chi(L_t)^{\frac{1}{\psi}} = w_t C_t^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\chi(L_t)^{\frac{\psi}{\psi}} = w_t^{\frac{\psi}{\psi}} C_t^{*-1}$</td>
</tr>
<tr>
<td>Aggregate accounting</td>
<td>$C_t + N_{E,t} \tilde{v}_t = w_t L_t + N_t \tilde{d}_t$</td>
</tr>
<tr>
<td></td>
<td>$C_t^* + N_{E,t}^* \tilde{v}_t^* = w_t^* L_t^* + N_t^* \tilde{d}_t^*$</td>
</tr>
</tbody>
</table>

In my model, the value added by firms originating in North that produce through vertical FDI in South is the theoretical counterpart of the empirical measure of value added in Mexico’s maquiladora.
sector:

\[ VA_t = (1 - \alpha)N_{V,t} (\tilde{p}_{V,t})^{1-\theta} C_t. \]

I also deflate value added by the average CPI in the North economy in order to eliminate the variety effect, i.e. \( V_{AR,t} = P_t \left( VA_t / \bar{P}_t \right) \), where \( P_t = N_t^{1-\theta} \bar{P}_t \).

The theoretical counterpart of the number of hours worked in Mexico’s maquiladora sector is \( L_{V,t}^* \): The variable represents hours worked by the South workers employed in the \( N_{V,t} \) offshore plants of firms originating in North that produce under vertical FDI in the South economy. Their labor is associated with both fixed and marginal costs activities:

\[
L_{V,t}^* = N_{V,t} \left[ \frac{f_V}{(Z_t)^{\alpha} (\bar{Z}_t)^{1-\alpha}} + \frac{(1-\alpha)(\tilde{p}_{V,t})^{-\theta} C_t}{Z_t^{\psi} \bar{Z}_{V,t}^{\psi}} \right].
\]

I calibrate the extended model to allow for endogenous offshoring through vertical FDI (\( \alpha = 0 \)) while nesting the model with endogenous exports (\( \eta = 1 \)) a la Ghironi and Melitz (2005). I derive an asymmetric steady state using differences in firm entry regulations (\( f^E_E = 4f^E_E \)) in order to generate wage differences across countries. As in Bilbey et al. (2006), I set \( \chi = 0.924271 \) in order to obtain the steady-state level of hours worked being equal to unit irrespective of the value of \( \psi \), i.e. \( L = \left\{ \frac{1}{\chi} \left[ 1 - \frac{r}{\theta(1+\psi)} \right] \right\}^{\psi} \) \( \frac{1}{1+\psi} \). I also set the elasticity of labor supply to wages in North at \( \psi = 2 \), and consider different values for the elasticity of labor supply in South, i.e. low elasticity when \( \psi = 2 \), and high elasticity when \( \psi = 10 \).

Figure 8 contrasts the empirical second moments (first panel) with the corresponding analytical moments. The second panel shows the correlations of maquiladora’s value added, hours worked, and number of plants generated by the model with inelastic labor supply. Although the model replicates the rank of contemporaneous correlations between the maquiladora indicators and U.S. output, the magnitudes and signs are not in line with the empirical moments. In the model, the number of plants has a negative correlation with contemporaneous U.S. manufacturing output, rather than positive as in the data. Moreover, the rank of correlations of value added and number of plants with lags and leads of U.S. output departs from the empirical moments: In the model, the correlations with lagged U.S. output are too small, and the correlations with lead values of U.S. output are too large (particularly for offshore value added).

The third and fourth panels show the simulated moments generated by the model with elastic labor supply when the elasticity of labor supply to wage in the South economy is either low (\( \psi = 2 \) or
high ($\psi = 10$) respectively. In particular, the model with high elasticity of labor supply in the South is partially successful in reproducing the empirical moments of offshoring. The number of offshore plants (i.e. offshoring at the extensive margin) responds to fluctuations in U.S. manufacturing output with a lag of three quarters, a result which is in line with the empirical moments. The magnitudes are also respected, i.e. the correlation of the number of plants with U.S. output lagged by three quarters is 0.6 in the model and 0.75 in the data.

However, the model with elastic labor supply does not reproduce the immediate response of hours worked shown by the data (i.e. offshoring at the intensive margin). Instead, the correlations between hours worked and U.S. output follow the pattern of value added and the number of plants, variables which respond with a lag to fluctuations in U.S. output. Therefore, in further work I aim to disconnect the responses of offshoring at the intensive and extensive margins to fluctuations in U.S. manufacturing output.

6 Conclusion

In this paper, I aim to rationalize the link between offshoring and the co-movement of business cycles in a DSGE model in which trade in intermediate goods and offshore production under vertical FDI play a special role. The model is successful in replicating the stylized facts described in the empirical section: Over the business cycle, an economic boom and the appreciation of wages in North are followed by the relocation of production offshore to the country with lower labor costs. In turn, higher demand for labor in South exercises upwards pressure on wages. Higher demand for labor in North (due to firm entry) and simultaneously in South (due to offshoring) results into the positive co-movement of wages and national incomes.

Empirical data from Mexico’s maquiladora industry also suggests that the timing of relocation differs at the intensive and extensive margins: Hours worked in the maquiladora sector respond immediately to fluctuations in U.S. manufacturing output, whereas the number of plants responds with a lag of three quarters. My model with elastic labor supply is partially successful in replicating this pattern. The model does generate a delayed response in the number of offshore plants; the sign and magnitude of correlations with lagged North output are in line with the empirical moments. However, hours worked in the offshore sector also respond with a lag – therefore, in future work, I aim to disconnect the timing of relocation at the intensive and extensive margins.
References


A Appendix

A.1 The price index of $C_t$ and the demand for intermediate goods

Under $\alpha = 0$, households in North minimize total expenditure subject to the aggregator constraint:

$$\min_{\{y_{D,t}(z), y_{V,t}(z)\}} P_t C_t = \int_{z_{\min}}^{z_{V,t}} p_{D,t}(z) y_{D,t}(z) dz + \int_{z_{V,t}}^{\infty} p_{V,t}(z) y_{V,t}(z) dz,$$

subject to

$$C_t = \left[ \int_{z_{\min}}^{z_{V,t}} y_{D,t}(z)^{\theta - 1} \frac{dz}{z^{1-\theta}} + \int_{z_{V,t}}^{\infty} y_{V,t}(z)^{\theta - 1} \frac{dz}{z^{1-\theta}} \right]^{\theta - 1}.\]

Given the first-order conditions:

$$p_{D,t}(z) = \lambda C_t^{\frac{1}{\theta}} y_{D,t}(z)^{-\frac{1}{\theta}},$$

$$p_{V,t}(z) = \lambda C_t^{\frac{1}{\theta}} y_{O,t}(z)^{-\frac{1}{\theta}},$$

the amount of total expenditures can be re-written as:

$$P_t C_t = \int_{z_{\min}}^{z_{V,t}} p_{D,t}(z) y_{D,t}(z) dz + \int_{z_{V,t}}^{\infty} p_{V,t}(z) y_{V,t}(z) dz =$$

$$= \lambda C_t^{\frac{1}{\theta}} \left[ \int_{z_{\min}}^{z_{V,t}} y_{D,t}(z)^{\frac{\theta - 1}{\theta}} \frac{dz}{z^{1-\theta}} + \int_{z_{V,t}}^{\infty} y_{V,t}(z)^{\frac{\theta - 1}{\theta}} \frac{dz}{z^{1-\theta}} \right] =$$

$$= \lambda C_t^{\frac{1}{\theta}} C_t^{\frac{\theta - 1}{\theta}} = \lambda C_t.$$

The resulting identity $\lambda_t = P_t$ leads to the price index and the demand formulas for intermediate goods produced domestically and offshore:

$$P_t = \left[ \int_{z_{\min}}^{z_{V,t}} p_{D,t}(z)^{1-\theta} dz + \int_{z_{V,t}}^{\infty} p_{V,t}(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}},$$

$$y_{D,t}(z) = \left( \frac{p_{D,t}(z)}{P_t} \right)^{-\theta} C_t = \rho_{D,t}(z)^{-\theta} C_t,$$

$$y_{V,t}(z) = \left( \frac{p_{V,t}(z)}{P_t} \right)^{-\theta} C_t = \rho_{V,t}(z)^{-\theta} C_t.$$
A.2 Optimal pricing of intermediate goods produced domestically and offshore

The profit maximization problem for the home firm producing intermediate good variety \( z \) domestically is:

\[
\max_{\{\rho_{D,t}(z)\}} \rho_{D,t}(z)y_{D,t}(z) - \frac{w_t}{Z_t z} y_{D,t}(z).
\]

Given the demand function \( y_{D,t}(z) = \rho_{D,t}(z)^{-\theta} C_t \), the first-order condition leads to the pricing formula:

\[
y_{D,t}(z) + \rho_{D,t}(z) \frac{\partial y_{D,t}(z)}{\partial \rho_{D,t}(z)} - \frac{w_t}{Z_t z} \frac{\partial y_{D,t}(z)}{\partial \rho_{D,t}(z)} = 0 \Rightarrow \rho_{D,t}(z) = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z}.
\]

The home firm that produces offshore the intermediate good variety \( z \) solves the following profit maximizing problem:

\[
\max_{\{\rho_{V,t}\}} \rho_{V,t}(z) y_{V,t}(z) - \left( \frac{w_t}{Z_t z} \right)^\alpha \left( \tau \frac{w_t^*}{Z_t^* Q_t} \right)^{1-\alpha} y_{D,t}(z) - f_V \left( \frac{w_t}{Z_t} \right)^\alpha \left( \frac{w_t^*}{Z_t^* Q_t} \right)^{1-\alpha} - f_S \left( \frac{w_t}{Z_t} \right)^\alpha \left( \frac{w_t^*}{Z_t^* Q_t} \right)^{1-\alpha}
\]

Given the demand function \( y_{V,t}(z) = \rho_{V,t}(z)^{-\theta} C_t \), the first-order condition generates the pricing formula:

\[
\rho_{V,t}(z) = \frac{\theta}{\theta - 1} \left( \frac{w_t}{Z_t z} \right)^\alpha \left( \tau \frac{w_t^*}{Z_t^* Q_t} \right)^{1-\alpha}.
\]

A.3 Implications of firm heterogeneity

My model implies that the more productive firms use imported intermediate goods in their production process. Moreover, following Melitz (2003), the more productive firms have lower marginal costs, charge lower prices, obtain larger revenues and profits. Given two firms with firm-specific productivity factors \( z_2 > z_1 \),

\[
y(\frac{z_2}{z_1}) = \left( \frac{z_2}{z_1} \right)^\theta > 1, \quad r(\frac{z_2}{z_1}) = \left( \frac{z_2}{z_1} \right)^{\theta-1} > 1,
\]

i.e., the more productive firm has larger output and revenue, where output and revenue are given by \( y_t(z) = [\rho_t(z)]^{-\theta} V_t \) and \( r_t(z) = \rho_t(z) y_t(z) \), respectively.

Both implications of firm heterogeneity are in line with the results of empirical literature. This property matches the stylized fact that firms using imported components are more productive and larger than non-importing firms, as documented in Kurz (2006), Kasahara and Lapham (2006), Ramanarayanan (2006).
A.4 Existence of equilibrium productivity cutoff $z_{V,t}$

As discussed in the text, two conditions must hold every period in order to ensure for the existence of equilibrium productivity cutoff $z_{V,t}$: (1) $d_{V,t}(z)$ is steeper than $d_{D,t}(z)$, and (2) $z_{\text{min}} < z_{V,t}$. The first condition implies that the effective wage in South must be low enough relative to the effective wage in North ($TOL_t < 1$) in order to offset the iceberg trade cost ($\tau > 1$):

$$\tau \frac{w_t^*Q_t}{Z_t^*} < \frac{w_t}{Z_t} \iff \tau TOL_t < 1.$$ 

The second condition, $z_{\text{min}} < z_{V,t}$, requires that:

$$\text{Slope}(d_{V,t}(z)) < \frac{\Theta f_E \frac{w_t}{Z_t} + f_V \frac{w_t^*Q_t}{Z_t^*}}{z_{\text{min}}^\theta},$$

$$z_{\text{min}}^{\theta-1} \left( \frac{\theta}{\theta - 1} \frac{w_t}{Z_t} \right)^{1-\theta} C_t < \Theta f_E \frac{w_t}{Z_t} + f_V \frac{w_t^*Q_t}{Z_t^*},$$

$$\frac{1}{\theta} \left( \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z_{\text{min}}} \right)^{1-\theta} C_t < \Theta f_E \frac{w_t}{Z_t} + f_V \frac{w_t^*Q_t}{Z_t^*},$$

$$d_{D,t}(z_{\text{min}}) < \Theta f_E \frac{w_t}{Z_t} + f_V \frac{w_t^*Q_t}{Z_t^*},$$

where $\Theta = \frac{1-\beta(1-\delta)}{\beta(1-\delta)}$. The last inequality shows that the profit obtained from domestic production by the firm with the minimum productivity $z_{\text{min}}$ must be smaller than the sum of the per-period value of the sunk entry cost and the fixed cost of offshoring. In other words, the firm that obtains zero profit from domestic production would make negative profits if it engages in offshore production, i.e. the firm with productivity $z_{\text{min}}$ does not produce in either country.

A.5 Average productivities under the Pareto distribution

The average productivity of home firms that produce offshore is:

$$\bar{z}_{V,t} = \left[ \frac{1}{1 - G(z_{V,t})} \int_{z_{V,t}}^{\infty} z^{\theta-1} g(z) dz \right]^{\frac{1}{\theta-1}} = \left[ \left( \frac{z_{V,t}}{z_{\text{min}}} \right)^k \frac{k z_{\text{min}}^{k-1} z_{V,t}^{\theta-1-k}}{k - (\theta - 1) z_{V,t}} \right]^{\frac{1}{\theta-1}} = \left[ \frac{k}{k - (\theta - 1)} \right]^{\frac{1}{\theta-1}} z_{V,t} = \nu z_{V,t},$$

where $\nu = \left[ \frac{k}{k - (\theta - 1)} \right]^{\frac{1}{\theta-1}}$. 

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The average productivity of home firms producing domestically is:

\[
\tilde{z}_{D,t} = \left[ \frac{1}{G(z_{V,t})} \int_{z_{\text{min}}}^{z_{V,t}} z^{\theta-1} g(z) dz \right]^{\frac{1}{\theta}} = \left[ \frac{z_{V,t}^{k} - z_{\text{min}}^{k}}{z_{V,t}^{k} - z_{\text{min}}^{k}} \int_{z_{\text{min}}}^{z_{V,t}} \frac{z^{\theta-1} k_{\text{min}}}{z^{k+1}} dz \right]^{\frac{1}{\theta}} = \\
\left[ \frac{z_{V,t}^{k} - k_{\text{min}}^{k}}{z_{V,t}^{k} - z_{\text{min}}^{k}} \left( z_{V,t}^{\theta-1} - z_{\text{min}}^{\theta-1} \right) \right]^{\frac{1}{\theta}} = \\
\left[ \frac{k}{k - (\theta - 1)} \left( z_{V,t}^{\theta-1} - z_{\text{min}}^{\theta-1} \right) \right]^{\frac{1}{\theta}} = \\
\nu z_{\text{min}} z_{V,t} \left[ \frac{k^{k} - k_{\text{min}}^{k}}{z_{V,t}^{k} - z_{\text{min}}^{k}} \right]^{\frac{1}{\theta}}.
\]

A.6 Link between average profits with fixed costs \( f O \frac{w_{t}^{*} Q_{t}}{Z_{t}^{*}} \)

Under \( \alpha = 0 \), the average profit of the home firms producing domestically is:

\[
\tilde{d}_{D,t} = d_{D,t}(\tilde{z}_{D,t}) = \frac{1}{\theta} \left[ \frac{d_{t} + w_{t}^{*} Q_{t}}{\theta - 1 Z_{t}^{*} \tilde{z}_{D,t}} \right]^{1-\theta} = \frac{1}{\theta} \left[ \frac{d_{t} + w_{t}^{*} Q_{t}}{\theta - 1 Z_{t}^{*}} \right]^{1-\theta} C_{t}^{\frac{1}{\theta}} C_{t}^{\frac{1}{\theta}} = \\
= \frac{1}{\theta} \left[ \frac{d_{t} + w_{t}^{*} Q_{t}}{\theta - 1 Z_{t}^{*}} \right]^{1-\theta} C_{t}^{\frac{1}{\theta}} (\nu z_{\text{min}} z_{V,t}^{\theta-1})^{\theta-1} = \\
= \frac{1}{\theta} \left[ \frac{d_{t} + w_{t}^{*} Q_{t}}{\theta - 1 Z_{t}^{*}} \right]^{1-\theta} C_{t}^{\frac{1}{\theta}} (\nu z_{\text{min}} z_{V,t}^{\theta-1})^{\theta-1} = \\
= d_{D,t} \left( \nu z_{\text{min}} \right)^{\theta-1} \left[ k^{k} - k_{\text{min}}^{k} \right]^{\theta-1} z_{V,t}^{k} - z_{\text{min}}^{k} z_{V,t}^{k} - z_{\text{min}}^{k}.
\]

Similarly, the average profit of the home firm producing offshore is:

\[
\tilde{d}_{V,t} = d_{V,t}(\tilde{z}_{V,t}) = \frac{1}{\theta} \left[ \frac{d_{t} + w_{t}^{*} Q_{t}}{\theta - 1 Z_{t}^{*} \tilde{z}_{V,t}} \right]^{1-\theta} C_{t}^{\frac{1}{\theta}} - f_{V} \frac{w_{t}^{*} Q_{t}}{Z_{t}^{*}} = \\
= \frac{1}{\theta} \left[ \frac{d_{t} + w_{t}^{*} Q_{t}}{\theta - 1 Z_{t}^{*}} \right]^{1-\theta} C_{t}^{\frac{1}{\theta}} - f_{V} \frac{w_{t}^{*} Q_{t}}{Z_{t}^{*}} = \\
= \left( \frac{1}{\theta} \left[ \frac{d_{t} + w_{t}^{*} Q_{t}}{\theta - 1 Z_{t}^{*} \tilde{z}_{V,t}} \right]^{1-\theta} C_{t}^{\frac{1}{\theta}} - f_{V} \frac{w_{t}^{*} Q_{t}}{Z_{t}^{*}} \right) \nu^{\theta-1} + \left( \nu^{\theta-1} - 1 \right) f_{V} \frac{w_{t}^{*} Q_{t}}{Z_{t}^{*}} = \\
= d_{V,t} \left( \nu z_{\text{min}} \right)^{\theta-1} + \frac{\theta - 1}{k - (\theta - 1)} f_{V} \frac{w_{t}^{*} Q_{t}}{Z_{t}^{*}}.
\]

The home firm with productivity equal to the cutoff \( z_{V,t} \) is indifferent between locating its production plants domestically or offshore. Hence, the identity of profits at the productivity cutoff,
\[ d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t}), \] and the two expressions above allow me to derive the following link between the profits to the two representative firms in the home economy:

\[
\tilde{d}_{V,t} = \left( \frac{1}{\nu z_{\min}} \right)^{\theta-1} \left[ \frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^{k} - z_{\min}^{k}} \right]^{-1} \tilde{d}_{D,t}^{\theta-1} + \frac{\theta - 1}{k - (\theta - 1)} f_{V} w_{t}^{*} q_{t} \frac{Z_{t}^{*}}{Z_{t}^{*}} =
\]

\[
= z_{\min}^{1-\theta} \left[ \frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^{k} - z_{\min}^{k}} \right]^{-1} \tilde{d}_{D,t} + \frac{\theta - 1}{k - (\theta - 1)} f_{V} w_{t}^{*} q_{t} \frac{Z_{t}^{*}}{Z_{t}^{*}} =
\]

\[
= \frac{k}{k - (\theta - 1)} \left( \frac{z_{V,t}}{z_{D,t}} \right)^{\theta-1} \tilde{d}_{D,t} + \frac{\theta - 1}{k - (\theta - 1)} f_{V} w_{t}^{*} q_{t} \frac{Z_{t}^{*}}{Z_{t}^{*}}.
\]