Quantifying and Sustaining Welfare Gains from Monetary Commitment*

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Abstract

Our objectives are: to quantify the stabilization welfare gains from commitment; to examine how commitment to an optimal rule can be sustained as an equilibrium; to find a simple interest rate rule that approximates the optimal commitment one. We utilize an empirical micro-founded euro-area DSGE model, a quadratic approximation of household utility as the welfare criterion, employing a nominal interest rate lower bound. In contrast to previous studies, we find significant commitment stabilization gains of around a 0.4 – 0.5% equivalent permanent consumption increase, and with higher price stickiness gains over 2%. We find that a simple optimized commitment rule responding to inflation and the real wage mimics the optimal one.

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1 Introduction

Following the pioneering contributions of Kydland and Prescott (1977) and Barro and Gordon (1983), the credibility problem associated with monetary policy has stimulated a huge academic literature that has been influential with policymakers. The central message underlying these contributions is the existence of significant macroeconomic gains, in some sense, from ‘enhancing credibility’ through formal commitment to a policy rule or through institutional arrangements for central banks such as independence, transparency, and forward-looking inflation targets, that achieve the same outcome.

In the essentially static model used in those seminal papers and in much of the huge literature they inspired, the loss associated with a lack of credibility takes the form of a long-run inflationary bias. For dynamic models of the New Keynesian (henceforth, NK) genre, such as the DSGE model employed in this paper, the influential review of Clarida et al. (1999) emphasizes the stabilization gains from commitment which exist whether or not there is a long-run inflationary bias. But what are the size of these stabilization gains from commitment? If they are small then the credibility problem is solely concerned with the credibility of long-run low inflation.

The first objective of the paper is to quantify the stabilization gains from commitment in terms of household welfare. Previous work has addressed this question (for example, Currie and Levine (1993), McCallum and Nelson (2004), and Dennis and Söderström (2006)), but only in the context of models without micro-foundations and using an ad hoc loss function, or both, or for rudimentary NK models. The credibility issue only arises because the decisions of consumers and firms are forward looking and depend on expectations of future policy. In the earlier generation of macro-models lacking micro-foundations, many aspects of such forward-looking behavior were absent and therefore important sources of time-inconsistency were missing. Although for simple NK models a quadratic approximation of the representative consumer’s utility coincides with the standard ad hoc loss that penalizes variances of the output gap and inflation, in richer DSGE models this is far from the case. By utilizing an influential empirical micro-founded DSGE model, the euro-area model of Smets and Wouters (2003), and using a quadratic approximation of the representative household’s utility as the welfare criterion, we crucially remedy these deficiencies in earlier estimates of the gains from commitment.

A further important consideration when addressing such gains, and missing from these earlier studies, is the existence of a nominal interest rate zero lower bound (ZLB). A number of papers have studied optimal commitment policy with this constraint (for example, Eggertsson and Woodford (2003), Woodford (2003), chapter 6). In an important
contribution, Adam and Billi (2007) show that ignoring the zero lower bound constraint for the setting of the nominal interest rate can result in considerably underestimating the stabilization gain from commitment. The reason for this is that under discretion the monetary authority cannot make credible promises about future policy. For a given setting of future interest rates, the volatility of inflation is driven up by the expectations of the private sector that the monetary authority will re-optimize in the future. This means that to achieve a given low volatility of inflation the lower bound is reached more often under discretion than under commitment. These authors study a simple NK model and are able to employ non-linear techniques. Since we employ a more developed model, we necessarily choose a more tractable linear-quadratic (henceforth LQ) framework.\(^1\) We follow Woodford (2003) in approximating the effects of a zero interest rate lower bound by imposing the requirement that the interest rate volatility in commitment and discretionary equilibria are small enough to ensure that the violations of the zero lower bound constraint are very infrequent.

Our second objective is to examine how commitment to an optimal or approximately optimal rule can be sustained as a ‘reputational’ equilibrium in which reneging hardly ever occurs. We extend the incomplete information framework\(^2\) of Barro (1986) to a stochastic setting and a model with structural dynamics.

Our final objective is to search for a simple interest rate rule that closely approximates the optimal commitment (and complex) rule. This particular part of the paper resembles Levin et al. (2006), but unlike those authors incorporates a interest rate ZLB into the design of the rule.\(^3\)

The rest of the paper is organized as follows. Section 2 begins by using a simple NK model to show analytically how a stabilization bias arises in models with structural dynamics. It goes on to generalize the treatment to any linear DSGE model with a quadratic loss function and also to take into account the interest rate lower bound. We derive closed-form expressions for welfare under optimal commitment, discretion and simple commitment rules and use these to derive a ‘no-deviation condition’ for commitment.

\(^1\)A LQ framework is convenient for a number of reasons: it allows closed-form expressions for the welfare loss under optimal commitment, discretion and simple commitment rules that enable us to study the incentives to renege on commitment. Our Bayesian estimation methods use a linearized form of the dynamic model. Last but not least, the implementation of the numerical methods utilized by Adam and Billi (2007) for a simple NK model with only 2 state variables would fall foul of the “curse of dimensionality” (Judd (1998), chapter 7) in our model with 11 state variables.

\(^2\)This avoids well-established problems of trigger strategies used in Barro and Gordon (1983) – see al-Nowaihi and Levine (1994) and Persson and Tabellini (1994).

\(^3\)See Primiceri (2006) for a discussion of the importance of imposing the zero lower bound in the design of monetary rules.
to exist as an equilibrium in which reneging on commitment takes place very infrequently.

Section 3 sets out a version of the Smets-Wouters model (henceforth SW) with one additional feature: the addition of a tax wedge in the steady state. Appendix A sets out the zero-interest steady state. Our welfare quadratic approximations are accurate if the zero-inflation steady state is close to the social optimum (the ‘small distortions case’ of Woodford (2003)). In Appendix A we therefore assess the quality of this approximation. A linearization of the model about the steady state and a quadratic approximation of the representative household’s utility sets up the optimization problem facing the monetary authority in the required LQ framework. In Appendix B we provide four estimates of the SW model and variants where the indexing of prices and/or wages is suppressed, and a price contract of 4 quarters is imposed.

In Section 4 we address the three central questions in the paper: how big are stabilization gains when an interest ZLB constraint is imposed, how can the fully optimal commitment rule be sustained as an equilibrium given the time-inconsistency problem and can a simple rule mimic the optimal commitment rule? Section 5 concludes. Further results and full details of our solution procedures are provided in our accompanying working paper Levine et al. (2007b), henceforth LMP.

2 The Time Inconsistency Problem

In this section, we look in depth at the time consistency problem in monetary policy. First in the context of two simple models (in New Classical and New Keynesian models), then in the more general DSGE framework. We then show how we impose the lower bound constraint in our exercises, how to derive fully optimal commitment and discretion policy, and discuss simple optimal rules. Finally, we address the issue of how commitment can be sustained as an equilibrium.

2.1 The Stabilization Bias in Two Simple DSGE Models

We first demonstrate how a stabilization bias in addition to the better known long-run inflationary bias can arise. We consider two very standard models: the first, used in seminal articles Kydland and Prescott (1977) and Barro and Gordon (1983), employs the familiar ‘New Classical Phillips Curve’

\[ \pi_t = E_{t-1} \pi_t + \lambda(y_t - \hat{y}_t) + u_t \]  

(1)

where \( \pi_t \) is the inflation rate, \( E_t(\cdot) \) is the expectations operator and \( y_t - \hat{y}_t \) is output \( y_t \), measured relative to its flexi-price value \( \hat{y}_t \), \( \lambda > 0 \) and \( u_t \) is a zero-mean shock to marginal
costs. All variables are expressed as deviations about the steady state, \( \pi_t \) as an absolute deviation, and \( y_t \) as a proportional deviation.

The second model popularized notably by Clarida et al. (1999) and Woodford (2003), is a simple NK model of the form

\[
\begin{align*}
\pi_t &= \beta E_t \pi_{t+1} + \lambda (y_t - \hat{y}_t) + u_t \quad (2) \\
y_t &= E_t y_{t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}) \quad (3)
\end{align*}
\]

In (2), \( \beta \) is the private sector’s discount factor and is derived as a linearized form of Calvo staggered price setting about a zero-inflation steady state and (3) is a linearized Euler equation with nominal interest rate \( r_t \), an absolute deviation about the steady state, and a risk aversion parameter \( \sigma \).

Kydland and Prescott (1977) and Barro and Gordon (1983) employed (1) and showed that a time-inconsistency or credibility problem in monetary policy arises when the monetary authority at time 0 sets a state-contingent inflation rate \( \pi_t \) to minimize the loss function

\[
\Omega_0 = E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left[ w_y (y_t - \hat{y}_t - k)^2 + \pi_t^2 \right] \right] \quad (4)
\]

Having set the inflation rate, the Euler equation (3) then determines the nominal interest rate that implements this target. The constant \( k \) in (4) arises because the steady state is inefficient owing to imperfect competition and other distortions, such as tax wedges. For this simple, essentially static model, optimal rules must take the form of a constant plus a stochastic shock-contingent component. These rules depend on whether the policymaker can commit, or exercises discretion and engages in period-by-period optimization. The standard results in these two cases are respectively:

\[
\begin{align*}
\pi_t &= \frac{w_y}{w_y + \lambda^2} u_t = \pi^C(u_t) \quad (5) \\
\pi_t &= \frac{w_y k}{\lambda} + \frac{w_y}{w_y + \lambda^2} u_t = \pi^D(u_t) \quad (6)
\end{align*}
\]

Thus the optimal inflation rule with commitment, \( \pi^C(u_t) \) consists of zero average inflation plus a shock-contingent component which sees inflation raised (i.e., monetary policy relaxed) in the face of a negative supply shock. The discretionary policy, \( \pi^D(u_t) \), can be implemented as a rule with the same shock-contingent component as the ex ante optimal rule. The only difference is now that it includes a non-zero average inflation or inflationary bias equal to \( \frac{w_y k}{\lambda} > 0 \) which renders the rule time-consistent. The credibility or ‘time-inconsistency’ problem, first raised by Kydland and Prescott, was simply how to eliminate the inflationary bias whilst retaining the flexibility to deal with exogenous shocks.
We have established that there are no stabilization gains from commitment in a model economy characterized by the New Classical Phillips Curve. This is not the case when we move to the NK Phillips Curve, (2): using general procedures described below in Section 2.2.2 (and in further detail in Appendix A of LMP), (5) and (6) now become

\[
\pi^*_t = \pi^*_t(u_t, u_{t-1}) = \delta \pi^*_t-1 + \delta (u_t - u_{t-1}) \quad (7)
\]

\[
\pi^D_t = \pi^D_t(u_t) = \frac{w_y k}{\lambda} + \frac{w_y}{w_y + \lambda^2} u_t \quad (8)
\]

where \( \delta = \frac{1 - \sqrt{1 - 4 \beta w^2 y^2}}{2 w^2 y^2} \). Comparing these two sets of results we see that the discretionary rule is unchanged, but the commitment rule now is a rule responding to past shocks (i.e., is a rule with memory) and therefore the stabilization component of the commitment rules now differs from that of the discretionary rule. Since the commitment rule is the ex ante optimal policy it follows that there are also now stabilization gains from commitment. The time-inconsistency problem facing the monetary authority in a NK economic environment now becomes the elimination of the inflationary bias whilst retaining the flexibility to deal with exogenous shocks in an optimal way.

### 2.2 The Stabilization Bias in General DSGE Models

The stabilization bias arose in our simple DSGE model by replacing the traditional Phillips Curve with a NK one based on staggered Calvo-type price setting. In the DSGE model presented in the next section there are a number of additional mechanisms that create price, wage and output persistence. The model also incorporates capital accumulation. All these features add structural dynamics to the model and these, together with forward-looking consumption, investment, price and wage-setting add further sources of stabilization gains from commitment.

To examine this further, consider a general linear state-space model

\[
\begin{bmatrix}
    z_{t+1} \\
    E_t x_{t+1}
\end{bmatrix}
= A
\begin{bmatrix}
    z_t \\
    x_t
\end{bmatrix}
+ B r_t + C \epsilon_{t+1}; \quad o_t = E
\begin{bmatrix}
    z_t \\
    x_t
\end{bmatrix}
\]

where \( z_t \) is a \((n - m) \times 1\) vector of predetermined variables at time \( t \) with \( z_0 \) given, \( x_t \), is a \( m \times 1 \) vector of non-predicted variables and \( o_t \) is a vector of outputs. All variables are expressed as absolute or proportional deviations about a steady state. \( A, B, C \) and \( E \) are fixed matrices and \( \epsilon_t \) is a vector of random zero-mean shocks. Rational expectations are formed assuming an information set \( \{ z_s, x_s, \epsilon_s \}, s \leq t \), the model and the monetary rule.

The linearized form of the non-linear model set out in the section 3 can be expressed in this

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See also Clarida et al. (1999)
form where \( z_t \) consists of exogenous shocks, lags in non-predetermined and output variables and capital stock; \( x_t \) consists of current inflation, the real wage, investment, Tobin’s Q, consumption and flexi-price outcomes for the latter two variables, and outputs \( o_t \) consist of real marginal costs, the marginal rate of substitution between consumption and leisure, the user cost of capital, labor supply, output, flexi-price outcomes, the output gap and other target variables for the monetary authority. Let \( s_t = M y_t \), where \( y_t = [z^T x^T]^T \), be the vector of such target variables. For both ad hoc and welfare-based loss functions discussed below, the inter-temporal loss function (4) generalizes to

\[
\Omega_0 = E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t L_t \right] 
\]

(10)

where the single-period loss function is given by \( L_t = s^T_t Q_1 s_t = y^T_t Q y_t \), where \( Q_1 \) is a fixed matrix and \( Q = M^T Q_1 M \).

### 2.2.1 Imposing an Interest Rate Zero Lower Bound Constraint

In the absence of a lower bound constraint on the nominal interest rate, the policymaker’s optimization problem is to minimize (10) subject to (9). If the variances of shocks are sufficiently large, this will lead to a large nominal interest rate variability and the possibility of the nominal rate becoming negative.\(^5\)

Define \( \bar{R} \equiv E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t R_t \right] \) to be the discounted future average of the nominal interest rate path \( \{R_t\} \). Following Woodford (2003), chapter 6, we impose an ‘approximate form’ of the ZLB constraint in the form of a requirement that the \( \bar{R} \) is at least \( k \) standard deviations above the zero lower bound \( R_t = 0 \); i.e., using discounted averages that

\[
\bar{R} \geq k \sqrt{\bar{R}^2 - (\bar{R})^2} = k \sqrt{R^2_t - (\bar{R})^2} 
\]

(11)

Squaring both sides of (11) we arrive at

\[
E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t R_t^2 \right] \leq K \left[ E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t R_t \right] \right]^2 \tag{12}
\]

where \( K = 1 + k^{-2} > 1 \).

To minimize (10) subject to (12) and the other dynamic constraints,\(^6\) we add a term \( w_r \left( \bar{R}^2 - K(\bar{R})^2 \right) \) to the Lagrangian to incorporate this extra constraint, where \( w_r > 0 \)

---

\(^5\)The same is true if the initial state is far from its steady state. We regard the initial state as a sample from the equilibrium distribution of states, so the analysis for one-off shocks applies equivalently.

\(^6\)There is another constraint \( E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t R_t \right] \geq 0 \) but it is straightforward to show that this does not bind – see Woodford (2003), page 701.
is a Lagrangian multiplier. From the first order conditions for this modified problem this is equivalent to adding terms \( E_0 (1 - \beta) \sum_{t=0}^{\infty} \beta^t w_r (\bar{R}_t - 2 K \bar{R} R_t) \) where \( \bar{R} \) is evaluated at the constrained optimum. It follows that the effect of the extra constraint is to follow the same optimization as before, except that the single period loss function is replaced with

\[
L_t = \gamma_t^T Q y_t + w_r (r_t - r^*)^2
\]

where \( r^* = (K - 1) \bar{R} > 0 \) is a nominal interest rate target for the constrained problem.\(^7\)

In what follows, we linearize around a zero-inflation steady state. With a ZLB constraint, the policymaker’s optimization problem is now to choose an unconditional distribution for \( r_t \), shifted to the right by an amount \( r^* \), about a new positive steady-state inflation rate, such that the probability of the interest rate hitting the lower bound is extremely low. As we demonstrate in Section 4.2, this is implemented by choosing an optimal combination of a sufficiently small unconditional variance, achieved by increasing \( w_r \), as well as a new sufficiently high steady-state positive inflation rate.

This approach to imposing the ZLB constraint in effect replaces the constraint \( R_t \geq 0 \) with an nominal interest rate variability constraint which ensures the ZLB is hardly ever hit. By contrast, the work of a number of authors including Adam and Billi (2006, 2007), Coenen and Wieland (2003), Eggertsson and Woodford (2003) and Eggertsson (2006) study optimal monetary policy with commitment in the face of an exact non-linear constraint \( R_t \geq 0 \) which allows for the effect of liquidity traps in the form of \( R_t = 0 \) on both policy design and expectations. In our LQ framework, if the ZLB actually reached, the interest rate is then allowed to become negative, possibly using a scheme proposed by Gesell (1934) and Keynes (1936). We mitigate that by choosing a very low probability of this happening, \( p = 0.025 \); i.e., in our quarterly model, an average frequency of once every forty quarters. Comparing the results of Eggertsson and Woodford (2003) with his LQ approach, Woodford (2003), page 427, notes the optimal policy with commitment in both cases involves a higher average inflation than would be the case in the absence of ZLB considerations. What are lacking in the approximate approach is the asymmetric nature of the distribution of the nominal interest rate and the wedge between the steady state and the average. Since the ZLB constraint \( R_t \geq 0 \) only applies when the interest rate has to adjust downwards (say in response to a negative demand shock) and not upwards in response to shocks, the corresponding upward movements of the inflation rate are constrained, but not the downward movements. It follows that the average inflation rate under an exact ZLB is below the steady state, whereas with the approximate form

\[^7\text{Recall that } r_t \text{ is an absolute deviation defined as } r_t \equiv R_t - R \text{ where } R \text{ is the steady state value of } R_t.\]

Note also that the steady state of \( \bar{R}_t = R \).
they coincide. If the steady state inflation rate is zero or small this gives the deflationary bias associated with the exact ZLB highlighted by Krugman (1998), Eggertsson (2006) and Adam and Billi (2007).

Of these papers, Adam and Billi (2007) is the only one to also study discretion and to address the issue of stabilization gains from commitment, but only for the simplest possible New Keynesian model, (2) and (3). The computation of Markov-perfect discretionary policy (see below) is only possible for small non-linear models (even without a ZLB). Owing to the “curse of dimensionality” mentioned in footnote 1, the application of their numerical methods to a model with higher order dynamics, such as the one we study here, is not feasible. To summarize, we impose ZLB considerations in an approximate form that allows us to stay within a LQ framework. This allows us to compute solutions for a medium sized DSGE model. The price we have to pay for these computational opportunities are first, when the ZLB is actually reached we do not allow for that effect on policy and expectations and second, the linearity of the structure imposes symmetric distributions of variables under optimal policy removing the wedge between the average outcomes and the steady state. This in turn removes the deflationary bias when the steady state is close to zero.

2.2.2 Commitment Versus Discretion

First consider the optimization problem without a ZLB constraint the consideration of which we defer to section 4.2. Then \( r^* = 0 \) in the loss function (13) and we minimize a discounted sum of this function subject to the constraint of the linearized model. To derive the ex ante optimal policy with commitment (OP), following Currie and Levine (1993), we then minimize the Lagrangian

\[
L_0 = E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left[ (y_t Q y_t + w_t r_t^2 + p_{t+1} (A y_t + B r_t - y_{t+1}) \right] \right]
\]

with respect to \( \{r_t\}, \{y_t\} \) and the row vector of co-state variables, \( p_t \), given \( z_0 \). Following Appendix A of LMP (where more details are provided), this leads to an optimal rule

\[
r_t = D \begin{bmatrix} z_t \\ p_{2t} \end{bmatrix}
\]

\[^8\text{Perturbation methods for producing second-order approximate solutions to the first-order conditions of the Ramsey problem also avoid this curse. However as Schmitt-Grohe and Uribe (2007) report, this method “is ill-suited to handle non-negativity constraint” and they approximate the ZLB in the same way as we do here.}
where
\[
\begin{bmatrix}
z_{t+1} \\
p_{2t+1}
\end{bmatrix} = H \begin{bmatrix}
z_t \\
p_{2t}
\end{bmatrix}
\] (16)
and the optimality condition\(^9\) at time \(t = 0\) imposes \(p_{20} = 0\). In (15) and (16) \(p_{T}^T = [p_{1T}^T \ p_{2T}^T]\) is partitioned so that \(p_{1T}\), the co-state vector associated with the predetermined variables, is of dimension \((n - m) \times 1\) and \(p_{2T}\), the co-state vector associated with the non-predetermined variables, is of dimension \(m \times 1\). The (conditional) loss function is given by
\[
\Omega_t^{OP} = - (1 - \beta) \text{tr} \left( N_{11} \left( Z_t + \frac{\beta}{1 - \beta} \Sigma \right) + N_{22} p_{2t} p_{2t}^T \right)
\] (17)
where \(Z_t = z_t z_t^T\), \(\Sigma = \text{cov}(C \epsilon_t)\),
\[
N = \begin{bmatrix}
S_{11} - S_{12} s_{22}^{-1} s_{21} & S_{12} s_{22}^{-1} \\
-S_{22}^{-1} s_{21} & S_{22}^{-1}
\end{bmatrix}
= \begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix}
\] (18)
and \(n \times n\) matrix \(S\) is the solution to the steady-state Ricatti equation. In (18) matrices \(S\) and \(N\) are partitioned conformably with \(y_t = [z_t^T x_t^T]^T\) so that \(S_{11}\) for instance has dimension \((n - m) \times (n - m)\).

Note that in order to achieve optimality the policy-maker sets \(p_{20} = 0\) at time \(t = 0\). At time \(t > 0\) there then exists a gain from reneging by resetting \(p_{2t} = 0\). It can be shown that matrices \(N_{11}\) and \(N_{22}\) are negative definite, so the loss in (17) is positive and an incentive to renege exists at all points along the trajectory of the optimal policy by resetting \(p_{2t} = 0\). This essentially is the time-inconsistency problem facing stabilization policy in a model with structural dynamics.

To evaluate the discretionary (time-consistent) policy we write the expected loss \(\Omega_t\) at time \(t\) as
\[
\Omega_t = E_t \left[(1 - \beta) \sum_{\tau=t}^{\infty} \beta^{\tau-t} L_{\tau}\right] = (1 - \beta) \langle y_t^T Q y_t + w_t r_t^2 \rangle + \beta \Omega_{t+1}
\] (19)
The dynamic programming solution then seeks a stationary solution of the form \(r_t = -F z_t\), \(\Omega_t = z_t^T S z\) and \(x = -N z\) where matrices \(S\) and \(N\) are different matrices from those under commitment (unless there is no forward-looking behavior), now of lower dimension \((n - m) \times (n - m)\) and \(m \times (n - m)\) respectively. The value function \(\Omega_t\) is minimized at time \(t\), subject to (9), in the knowledge that a similar procedure will be used to minimize \(\Omega_{t+1}\) at time \(t + 1\).\(^{10}\) Both the instrument \(r_t\) and the forward-looking variables \(x_t\) are now proportional to the predetermined component of the state-vector \(z_t\) and the equilibrium

\(^9\)Optimality from a ‘timeless perspective’ imposes a different condition at time \(t = 0\) (see Appendix A.1.2 of LMP), but this has no bearing on the stochastic component of policy.

\(^{10}\)See Currie and Levine (1993) and Söderlind (1999).
we seek is therefore *Markov Perfect*. In Appendix A of LMP we set out an iterative process for \( F_t, N_t, \) and \( S_t \) starting with some initial values. If the process converges to stationary values independent of these initial values,\(^{11}\) \( F, N \) and \( S \) say, then the time-consistent (TC) feedback rule is \( r_t = -Fz_t \) with loss at time \( t \) given by

\[
\Omega_{t}^{TC} = (1 - \beta) \text{tr} \left( S \left( Z_t + \frac{\beta}{1 - \beta} \Sigma \right) \right)
\]  

(20)

### 2.2.3 Simple Commitment Rules

We now address a problem with the optimal commitment rule (15): in all but very simple models it is extremely complex, with the interest rate feeding back at time \( t \) on the full state vector \( z_t \) and all past realizations of \( z_t \) back to the initiation of the rule at \( t = 0 \). We therefore seek to mimic the optimal commitment rule with simple (SIM) rules of the form

\[
r_t = Dy_t = D \begin{bmatrix} z_t \\ x_t \end{bmatrix}
\]  

(21)

where \( D \) is constrained to be sparse in some specified way. In Appendix A of LMP we show that the loss at time \( t \) is given by

\[
\Omega_{t}^{SIM} = (1 - \beta) \text{tr} \left( V \left( Z_t + \frac{\beta}{1 - \beta} \Sigma \right) \right)
\]  

(22)

where \( V = V(D) \) satisfies a Lyapunov equation. \( \Omega_{t}^{SIM} \) can now be minimized with respect to \( D \) to give an *optimized simple rule* of the form (21) with \( D = D^* \). An important feature of optimized simple rules is that unlike their optimal commitment or optimal discretionary counterparts they are *not certainty equivalent*. In fact if the rule is designed at time \( t = 0 \) then \( D^* = f^* \left( Z_0 + \frac{\beta}{1 - \beta} \Sigma \right) \) and so depends on the displacement \( z_0 \) at time \( t = 0 \) and on the covariance matrix of innovations \( \Sigma = \text{cov}(\epsilon_t) \). From non-certainty equivalence it follows that if the simple rule were to be re-designed at any time \( t > 0 \), since the re-optimized \( D^* \) will then depend on \( Z_t \) the new rule will differ from that at \( t = 0 \). This feature is true in models with or without rational forward-looking behavior and it implies that *simple rules are time-inconsistent even in non-RE models.*

### 2.3 Sustaining the Commitment Outcome as An Equilibrium

Suppose that there are two types of monetary policymaker, a ‘strong’ type who perceives substantial off-model costs from reneging on any such commitment, and a ‘weak’ type who optimizes in an opportunistic fashion on a period-by-period basis. The ‘strong type’ could be a policymaker with a modified loss function as in Rogoff (1985), Walsh (1995),

\(^{11}\)Indeed we find this is the case in the results reported in the paper.
Svensson and Woodford (2005), though for the Rogoff-delegation case the outcome is second-best. In a complete information setting, these types would be observed by the public and the strong type would pursue the optimal commitment or optimized simple monetary rule, and the weak type would pursue the discretionary policy. We assume there is uncertainty about the type of policymaker and the weak type is trying to build a reputation for commitment.\textsuperscript{12} The game is now one of incomplete information and we examine the possibility that commitment rules can be sustained as a perfect Bayesian equilibrium.

Consider the following strategy profile:

1. A strong type always follows an optimal or simple commitment rule.

2. In period $t$ a weak type acts as strong and follows the commitment rule with probability $1 - q_t$, if it has acted strong ($q_t = 0$) in all previous periods. Otherwise it pursues the discretionary rule and reveals its type.

3. Let $\rho_t$ be the probability assigned by the private sector to the event that the policymaker is of the strong type. We can thus regard $\rho_t$ as a measure of reputation. At the beginning of period 0 the private sector chooses its prior $\rho_0 > 0$. In period $t$ the private sector receives the ‘signal’ consisting of the inflation set by the policymaker. At the end of the period it updates $\rho_t$, using Bayes rule, and then forms expectations of the next period’s inflation rate.

In principle there are three types of equilibria to these games. If both strong and weak governments send the same message (i.e. implement the same interest rate) we have a pooling equilibrium. If they send different messages this gives a separating equilibrium. If one or both players randomizes with a mixed strategy we have a hybrid equilibrium. Thus in the above game, $q_t = 0$ gives a pooling equilibrium, $q_t = 1$ a separating equilibrium and $0 < q_t < 1$ a hybrid equilibrium. If $q_t = 0$ is a Perfect Bayesian Equilibrium to this game, then we have solved the time-inconsistency problem.

To show that $q_t = 0$ is such an equilibrium, it is sufficient to show that, given beliefs by the private sector, there is no incentive for a weak government to ever deviate from acting strong. To show this we must compare the welfare if the policymaker continues with the optimal commitment policy at time $t$ with that if he/she reneges, re-optimizes and then suffers a loss of reputation.

Consider the optimal commitment rule (0P) first. At time $t$ the single-period loss

\textsuperscript{12}Note that the type of planner is chosen once and for all, and at random, at time $t = 0$. 
function is $L(z_t, p_{2t})$ and the inter-temporal loss function can be written

$$\Omega^OP_t(z_t, p_{2t}) = (1 - \beta)L(z_t, p_{2t}) + \beta\Omega^OP_{t+1}(z^OP_{t+1}, p_{2,t+1})$$

(23)

where $(z^OP_{t+1}, p_{2,t+1})$ is given by (16). If the policymaker re-optimizes (R) at time $t$ the corresponding loss is

$$\Omega^R_t(z_t, 0) = (1 - \beta)L(z_t, 0) + \beta\Omega^TC_{t+1}(z^R_{t+1})$$

(24)

where from (16) we now have that $z^R_{t+1} = H_{11}z_t$.

The condition for a perfect Bayesian pooling equilibrium is that for all realizations of shocks to $(z_t, p_{2t})$ at every time $t$ the no-deviation condition

$$\Omega^OP_t(z_t, p_{2t}) < \Omega^R_t(z_t, 0)$$

(25)

holds. If this condition holds, then the weak authority always mimics the strong one and follows the commitment rule thus sustaining average zero inflation coupled with optimal stabilization.

Using (23), (24), (17) and (20) the no-deviation condition (25) can be re-expressed as

$$L(z_t, p_{2t}) - L(z_t, 0) - \beta E_t [\text{tr}(SZ^R_{t+1} + N_{11}Z^OP_{t+1} + N_{22}p^T_{2,t+1}p_{2,t+1})] < \frac{\beta^2}{1 - \beta} \text{tr}((S + N_{11})\Sigma)$$

(26)

The first term on the left-hand-side of (26) is the single-period gain from reneging and putting $p_{2t} = 0$. The second term on the left-hand-side of (26) are the possible one-off stabilization gains since the state of the economy after reneging reflected in $z^R_{t+1}$ will be closer to the long-run than that along the commitment policy reflected in $z^OP_{t+1}, p_{2,t+1}$. These two terms together constitute the temptation to renege. Since $\text{tr}((S + N_{11}) > 0$, the right-hand-side is always positive and constitutes the penalty in the shape of the stabilization loss when dealing with future shocks following a loss of reputation.

If the time-period is small (i.e. $\beta \simeq 1$), then the single-period gains are also relatively small and we can treat the loss of reputation as if it were instantaneous. Then the no-deviation condition becomes simply

$$\Omega^OP_t < \Omega^TC_t$$

(27)

for all realizations of exogenous stochastic shocks. From (17) and (20) this becomes

$$\text{tr}((N_{11} + S)(Z_t + \frac{\beta}{1 - \beta}\Sigma) > -\text{tr}(N_{22}p_{2t}p^T_{2t})$$

(28)

Note that both $-N_{22}$ and $(N_{11} + S)$ are positive definite (see Currie and Levine (1993), chapter 5 for a continuous-time analysis on which the discrete-time analysis here is based).
It follows that both the right-hand and left-hand side are positive, so (28) is not automatically satisfied.\footnote{The analysis of this section assumes that the steady state is the same under commitment and discretion. When the interest rate lower bound constraint is introduced this is no longer the case because the steady-state inflation rates differ. Let the new steady-state inflation rates for the two cases be \((\pi^*)^{OP}\) and \((\pi^*)^{TC}\) respectively and the increase in the steady-state welfare loss arising from an positive inflation rate be \(W'(\pi^*)\). Then a term \(\frac{W((\pi^*)^{TC}) - W((\pi^*)^{OP})}{1-\beta}\) is added to the left-hand-side of (26) and (28).}

The no-deviation condition for a simple rule follows in a similar fashion (see LMP for details). In these two no-deviation conditions (26) and (28), since \(Z_t\) or \(p_{2t}\) are unbounded stochastic variables there will inevitably be some realizations for which they are not satisfied. In other words the Bayesian equilibrium must be of the mixed-strategy type with \(q_t > 0\). What we must now show that \(q_t\) is very small, so we only experience very occasional losses of reputation. We examine this in Section 4.4.

3 The Model

In this section, we examine the Smets-Wouters micro-founded DSGE model that we use for our exercises.

3.1 The Smets-Wouters Model

The Smets-Wouters (SW) model is an extended version of the standard New-Keynesian DSGE closed-economy model with sticky prices and wages estimated by Bayesian techniques. The model features three agents: households, firms and the monetary policy authority. Households maximize a utility function with two arguments (consumption and leisure) over an infinite horizon. Consumption appears in the utility function relative to a time-varying external habit-formation variable. Labor is differentiated over households, so that there is some monopoly power over wages, which results in an explicit wage equation and allows for the introduction of sticky nominal Calvo-type wages contracts. Households also rent capital services to firms and decide how much capital to accumulate given adjustment costs. Firms produce differentiated goods, decide on factor inputs, and set Calvo-type price contracts. Wage and price setting is augmented by the assumption that those prices and wages that can not be freely set are partially indexed to past inflation. Prices are therefore set as a function of current and expected real marginal cost, but are also influenced by past inflation. Real marginal cost depends on wages and the rental rate of capital. The short-term nominal interest rate is the instrument of monetary policy. The stochastic behavior of the model is driven by ten exogenous shocks: five shocks arising from technology and preferences, three cost-push shocks and two monetary-policy shocks.
Consistent with the DSGE set up, potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of cost-push shocks.

We incorporate one important modification to the SW model: the addition of distortionary taxes at the steady state. As we will see this has a bearing on the inefficiency at the steady state, the quadratic approximation of the utility function used for the welfare analysis and the existence of an inflationary bias.

3.2 Households

In a cashless economy version of the model, a representative household \( r \) maximizes

\[
E_0 \sum_{t=0}^{\infty} \beta^t U_{C,t} \left[ \frac{(C_t(r) - H_{C,t})^{1-\sigma}}{1-\sigma} - U_{L,t} \frac{L_t(r)^{1+\phi}}{1+\phi} + u(G_t) \right] 
\]

where \( \beta \) is the household’s discount factor, \( U_{C,t} \) and \( U_{L,t} \) are preference shocks common to all households, \( C_t(r) \) is an index of consumption, \( L_t(r) \) are hours worked, \( H_{C,t} \) represents ‘external habit’ in consumption, or the desire not to differ too much from other households, and we choose \( H_{C,t} = h C_{t-1} \), where \( C_t = \frac{1}{T} \sum_{t=1}^{T} C_t(r) \) is the average consumption index, \( h \in [0, 1] \). When \( h = 0, \sigma > 1 \) is the risk aversion parameter (or the inverse of the inter-temporal elasticity of substitution). \( u(G_t) \) is the utility from exogenous real government spending \( G_t \). We normalize the household number to unity.

The representative household \( r \) must obey a budget constraint:

\[
(1 + T_{C,t}) P_t(C_t(r) + I(r)) + E_t[D_{t+1} B_{t+1}(r)] = (1 - T_{Y,t}) P_t Y_t(r) + B_t(r) + TR_t \tag{30}
\]

where \( P_t \) is the GDP price index and \( I_t(r) \) is investment. Assuming complete financial markets, \( B_{t+1}(r) \) is a random variable denoting the payoff of the portfolio \( B_t(r) \), purchased at time \( t \), and \( D_{t+1} \) is the stochastic discount factor over the interval \([t, t+1]\) that pays one unit of currency in a particular state of period \( t+1 \) divided by the probability of an occurrence of that state given information available in period \( t \). The nominal rate of return on bonds (the nominal interest rate), \( R_t \), is then given by the relation \( E_t[D_{t+1}] = \frac{1}{1 + R_t} \). The tax structure is as follows: \( TR_t \) are lump-sum transfers to households by the government net of lump-sum taxes, \( T_{C,t} \) and \( T_{Y,t} \) are consumption and income tax rates respectively. The income tax rate is paid on total income, \( P_t Y_t(r) \), given by

\[
P_t Y_t(r) = W_t(r) L_t(r) + (R_{K,t} Z_t(r) - \Psi(Z_t(r))) P_t K_{t-1}(r) + \Gamma_t(r) \tag{31}
\]

where \( W_t(r) \) is the nominal wage rate, \( R_{K,t} \) is the real return on beginning-of period \( t \) capital stock, \( K_{t-1} \), owned by households, \( Z_t(r) \in [0, 1] \) is the degree of capital utilization with costs \( P_t \Psi(Z_t(r)) K_{t-1}(r) \) where \( \Psi', \Psi'' > 0 \), and \( \Gamma_t(r) \) is income from dividends.
derived from the imperfectly competitive intermediate firms plus the net cash inflow from state-contingent securities. We first consider the case of flexible wages and introduce wage stickiness later.

Capital accumulation is given by

\[ K_t(r) = (1 - \delta)K_{t-1}(r) + (1 - S(X_t(r))) I_t(r) \]  

(32)

where \( \delta \) is the depreciation rate, \( X_t(r) = \frac{U_{I,t} I_t(r)}{I_{t-1}(r)} \), \( U_{I,t} \) is a shock to investment costs and the investment adjustment cost function, \( S(\cdot) \), has the properties \( S(1) = S'(1) = 0 \). As seen below, intermediate firms employ differentiated labor with a CES aggregator with elasticity of substitution \( \eta \). Then the demand for each consumer’s labor is given by

\[ L_t(r) = \left( \frac{W_t(r)}{W_t} \right)^{-\eta} L_t \]

(33)

where \( W_t = \left[ \int_0^1 W_t(r)^{1-\eta} dr \right]^{1-\eta} \) is an average wage index and \( L_t = \left[ \int_0^1 L_t(r)^{\eta-1} dr \right]^{\eta-1} \) is average employment.

Household \( r \) chooses \( \{C_t(r)\}, \{M_t(r)\}, \{K_t(r)\}, \{Z(r)\} \) and \( \{W_t(r)\} \) to maximize (29) subject to (30)–(33), taking external habit \( H_{C,t} \), interest rates and prices and as given. The insurance provided by state-contingent securities (the complete financial markets assumption) enables us to impose symmetry on households (so that \( C_t(r) = C_t \), etc).

Then we have the first-order necessary conditions:

\[ 1 = \beta(1 + R_t)E_t \left[ \frac{MU_{C,t+1} I_{t+1}}{MU_t} \frac{P_t}{P_{t+1}} \right] \]

(34)

\[ Q_t = E_t \left[ \beta \left( \frac{MU_{C,t+1}}{MU_t} \right) (Q_{t+1}(1 - \delta) + R_{K,t+1} Z_t - \Psi(Z_{t+1})) \right] \]

(35)

\[ 1 = Q_t[1 - (1 - S(X_t) - S'(X_t)X_t)] + \beta E_t Q_{t+1} \left( \frac{(C_{t+1} - H_{C,t+1})}{(C_t - H_{C,t})} \right)^{-\sigma} S'(X_t) \frac{U_{I,t+1} I_{t+1}^2}{I_t^2} \]

(36)

\[ R_{K,t} = \Psi'(Z_t) \]

(37)

\[ \frac{W_t(1 - T_{C,t})}{(1 + T_{C,t})P_t} = -\frac{1}{1 - \frac{1}{\eta}} \frac{MU_t^L}{MU_t^C} \equiv \frac{1}{1 - \frac{1}{\eta}} MRS_t = \frac{U_{L,t} I_t}{(1 - \frac{1}{\eta})} L_t^\phi (C_t - H_{C,t})^{\sigma} \]

(38)

where \( MU_t^C = U_{C,t}(C_t - H_{C,t})^{-\sigma} \) and \( MU_t^L = -U_{L,t} L_t^\phi \) are the marginal utilities of consumption and work respectively. Condition (34) is the familiar Keynes-Ramsey rule adapted to incorporate habit in consumption. In (35) and (36), \( Q_t \) is the real value of capital (Tobin’s Q) and these conditions describe optimal investment behavior. (37) describes optimal capacity utilization and (38) equates the real disposable wage with
the marginal rate of substitution \((MRS_i)\) between consumption and leisure and reflects the monopolistic market power of households supplying a differentiated factor input with elasticity \(\eta\).

### 3.3 Firms

Competitive final goods firms use a continuum of intermediate goods according to a constant returns CES technology to produce aggregate output

\[
Y_t = \left( \int_0^1 Y_t(f)^{(\zeta-1)/\zeta} df \right)^{\zeta/(\zeta-1)}
\]

where \(\zeta\) is the elasticity of substitution and the firm number is normalized to unity. This implies a set of demand equations for each intermediate good \(f\) with price \(P_t(f)\) of the form

\[
Y_t(f) = \left( \int_0^1 P_t(f)^{1-\zeta} df \right)^{-1/\zeta}
\]

is an aggregate price index.

In the intermediate goods sector each good \(f\) is produced by a single firm \(f\) using differentiated labor and capital with a Cobb-Douglas technology:

\[
Y_t(f) = A_t(Z_t(f)K_t-1(f))^\alpha L_t(f)^{1-\alpha} - F
\]

where \(Z_t(f)\) denotes capacity utilization, \(F\) are fixed costs of production and

\[
L_t(f) = \left( \int_0^1 L_t(r,f)^{\eta-1}/\eta df \right)^{\eta/(\eta-1)}
\]

is an index of differentiated labor types used by the firm, where \(L_t(r,f)\) is the labor input of type \(r\) by firm \(f\). \(A_t\) is an exogenous shock capturing shifts to trend total factor productivity in this sector. The cost of labor is \((1+T_{L,t})W_t\) where \(T_{L,t}\) is a payroll tax paid by the firm. Minimizing costs \(P_tR_{K,t}Z_t(f)K_t-1(f) + (1+T_{L,t})W_tL_t(f)\) and aggregating over firms leads to the demand for labor as in (33), where \(\int_0^1 L_t(r,f)df = L_t(r)\), and to

\[
\frac{(1+T_{L,t})W_tL_t(f)}{Z_tP_tR_{K,t}K_t-1(f)} = \frac{1-\alpha}{\alpha}
\]

In an equilibrium of equal households and firms, all wages adjust to the same level \(W_t\) and it follows that \(Y_t = A_t(Z_tK_t-1)^\alpha L_t^{1-\alpha} - F\). The firm’s cost-minimizing real marginal cost is therefore given by

\[
MC_t = \frac{1}{A_t} \left( \frac{(1+T_{L,t})W_t}{P_t} \right)^{1-\alpha} R_{K,t}^\alpha \alpha^{-\alpha}(1-\alpha)^{(1-\alpha)}
\]
3.4 Price and Wage-Setting

Turning to price and wage-setting, we follow the standard Calvo framework supplemented with indexation. Thus, at each period there is a probability of $1 - \xi_p$ and $1 - \xi_w$ that the price and wage is set optimally. The optimal price derives from maximizing discounted profits whilst wages are set such as to maximize discounted the utility from labor consumption minus the disutility of labor effort. For those firms and workers unable to reset, prices and wages are indexed to last period’s aggregate inflation, with indexation parameters indicated by $\gamma_p$ and $\gamma_w$ respectively. It can be shown that this leads to the following first-order conditions:

$$E_t \sum_{k=0}^{\infty} \xi^k_p D_{t+k}Y_{t+k} (f) \left[ P_t^0 (f) \left( \frac{P_{t+k}^0 - 1}{P_{t-1}^0} \right)^{\gamma_p} - \frac{\zeta}{(\zeta - 1)} P_{t+k}MC_{t+k} \right] = 0 \quad (44)$$

$$E_t \sum_{k=0}^{\infty} (\xi \beta^k) W_{t+k}^{-\eta} \left( \frac{P_{t+k} - 1}{P_{t-1}} \right)^{-\gamma_w \eta} L_{t+k}^{\gamma_p^C(r)} \left[ W_t^0 (r) (1 - T_{Y,t+k}) \left( \frac{P_{t+k} - 1}{P_{t-1}} \right)^{\gamma_w} \right] = 0 \quad (45)$$

Then by the law of large numbers the evolution of the price and wage indices are given

$$P_{t+1}^{1-\zeta} = \xi_p \left( P_t \left( \frac{P_t}{P_{t-1}} \right)^{\gamma_p} \right)^{1-\zeta} + (1 - \xi_p) (P_{t+1}^0(f))^{1-\zeta} \quad (46)$$

$$W_{t+1}^{1-\eta} = \xi_w \left( W_t \left( \frac{P_t}{P_{t-1}} \right)^{\gamma_w} \right)^{1-\eta} + (1 - \xi_w) (W_{t+1}^0(r))^{1-\eta} \quad (47)$$

3.5 Equilibrium and Interest Rate Rule

In equilibrium, goods markets, money markets and the bond market all clear. Equating the supply and demand of the consumer good we obtain

$$Y_t = A_t (Z_t^L) L_t^{1-\alpha} - F = C_t + G_t + I_t + \Psi (Z_t^L) K_{t-1} \quad (48)$$

We examine the dynamic behavior in the vicinity of a steady state in which the government budget constraint is in balance; i.e.,

$$TR_t + P_t G_t = (T_{Y,t} + T_{C,t}) P_t Y_t + T_{L,t} W_t L_t \quad (49)$$

As in Coenen et al. (2007) we further assume that changes in government spending are financed exclusively by changes in lump-sum taxes with tax rates $T_{Y,t}$, $T_{C,t}$ and $T_{L,t}$ held constant at their steady-state values.

Given the path of the interest rate, $\{R_t\}$ (expressed later in terms of an optimal or IFB rule) the money supply is fixed by the central banks to accommodate money demand. By
Walras’ Law we can dispense with the bond market equilibrium condition and therefore the household constraint. Then the equilibrium is defined at $t = 0$ by stochastic processes $C_t, B_t, I_t, P_t, L_t, K_t, Z_t, R_{K,t}, W_t, Y_t$, given past price indices and exogenous shocks and government spending processes.

The model is estimated in linearized form about a zero-inflation steady state set out in Appendix A. For estimation purposes only, the model is closed with a linear ‘empirical’ Taylor rule of the form

$$ r_t = \rho r_{t-1} + (1 - \rho)[\bar{\pi}_t + \theta \pi E_t(\pi_{t+j} - \bar{\pi}_{t+j}) + \theta_g(y_t - \bar{y}_t)] + \theta \Delta \pi_t \pi_{t-1} + \theta \Delta y_t (\Delta y_t - \Delta \bar{y}_t) + \epsilon_{R,t} $$

(50)

where $\bar{\pi}_{t+1} = \rho \pi_t + \epsilon_{\pi,t+1}$ is an inflation target shock process, and $\epsilon_{R,t}$ is an iid nominal interest rate shock. In the policy exercises, this rule is replaced with an optimal counterpart which may be subjected to a time consistency constraint (‘discretion’) or to a simplicity constraint (‘simple rules’). Appendix B and LMP provide details of the estimation.

4 Optimal Monetary Stabilization Policy

Now we tackle optimal monetary stabilization policy, discussing first the welfare-based welfare criterion, how we impose the lower bound constraint, the stabilization gains from the chosen simple rules, and then how to sustain commitment as an equilibrium.

4.1 Formulating the Policymaker’s Loss Function

Much of the optimal monetary policy literature has stayed with an ad hoc loss function (4) which, with an interest rate lower bound constraint, becomes

$$ \Omega_0 = E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left[ (y_t - \bar{y}_t - k)^2 + w_\pi \pi_t^2 + w_r r_t^2 \right] \right] $$

(51)

Indeed Clarida et al. (1999) provide a stout defense of a hybrid research strategy that combines a loss function based on the stated objectives of central banks with a micro-founded macro-model. A normative assessment of policy rules requires welfare analysis and for this, given our LQ framework, we require a quadratic approximation of the representative consumer’s utility function.

A common procedure for reducing optimal policy to a LQ problem is as follows. Linearize the model about a deterministic steady state as we have already done. Then expand the consumer’s utility function as a second-order Taylor series after imposing the economy’s resource constraint. In general this procedure is incorrect unless the steady state...
is close to the efficient outcome (see Woodford (2003), chapter 6, Benigno and Woodford (2004), Kim and Kim (2007) and Levine et al. (2007a)). This we have shown is indeed the case in our model with external habit and we show in Appendix C of LMP that a quadratic single-period loss function that approximates the utility then takes the form

\[
U_t = w_c(c_t - hc_{t-1})^2 + w_{\pi}(\pi_t - \gamma_p \pi_{t-1})^2 + w_{\Delta w}(\Delta w_t - \gamma_w \Delta w_{t-1})^2
+ w_{l}(l_t - k_{t-1} - z_t - \frac{1}{1-\alpha}a_t)^2 + w_{z}(z_t + \psi a_t)^2 - w_{\Delta a} l_{t} - w_i(i_t - i_{t-1})^2 \tag{52}
\]

where positive weights \(w_c\) etc are defined in LMP. Unlike the ad hoc formulations, these weights are functions of the fundamental parameters of the model. All variables are in log-deviation form about the steady state as in the linearization. The first four terms in (52) give the welfare loss from consumption, employment, price inflation and wage inflation variability respectively. The remaining terms are contributions that arise from the resource constraint in the quadratic approximation procedure.

### 4.2 Imposing the Interest Rate Zero Lower Bound

Incorporating the ZLB constraint, in the analysis that follows we now adopt a single period loss function of the form

\[
L_t = U_t + w_r(r_t - \pi^*)^2 \tag{53}
\]

where \(U_t\) is given by (52). As explained in Section 2.2.1, the policymaker’s optimization problem is to choose an unconditional distribution for \(r_t\) (i.e., the steady-state variance) shifted to the right about a new non-zero steady-state inflation rate and a higher nominal interest rate, such that the probability, \(p\), of hitting the lower bound is very low. This is implemented by calibrating the weight \(w_r\) for each of our policy rules so that \(z_0(p)\sigma_r < R\) where \(z_0(p)\) is the critical value of a standard normally distributed variable \(Z\) such that \(\text{prob}(Z \leq z_0) = p\), \(R = \frac{1}{\beta} - 1 + \pi^*\) is the steady-state nominal interest rate, \(\sigma_r\) is the unconditional variance and \(\pi^*\) is the new steady-state inflation rate. Given \(\sigma_r\), the steady-state positive inflation rate that will ensure \(r_t \geq 0\) with probability \(1 - p\) is given by\(^{14}\)

\[
\pi^* = \max[z_0(p)\sigma_r - \left(\frac{1}{\beta} - 1\right) \times 100, 0] \tag{54}
\]

\(^{14}\)If the inefficiency of the steady-state output is negligible, then \(\pi^* \geq 0\) is a credible new steady-state inflation rate. It contrasts with a transitional deflationary bias highlighted by Krugman (1998), Eggertsson (2006) and Adam and Billi (2007) which arises under discretion because the central bank cannot credibly lower the expected real interest rate, following a negative demand shock, by a promise to raise the inflation rate in the future. It must therefore rely on lowering the interest rate, hitting the zero lower bound more often. Reduced inflationary expectations, in turn, causes a temporary negative inflation bias. This effect is absent in the approximate approach to imposing the constraint in this paper.
In our LQ framework we can write the inter-temporal expected welfare loss at time $t = 0$ as the sum of stochastic and deterministic components, $\Omega_0 = \tilde{\Omega}_0 + \bar{\Omega}_0$. By increasing $w_r$ we can lower $\sigma_r$ thereby decreasing $\pi^*$ and reducing the deterministic component, but at the expense of increasing the stochastic component of the welfare loss. By exploiting this trade-off, we then arrive at the optimal policy that, in the vicinity of the steady state, imposes the zero lower bound constraint, $r_t \geq 0$ with probability $1 - p$.

Tables 1a and 1b show the results of this optimization procedure under discretion and commitment respectively. We choose $p = 0.025$. Given $w_r$, denote the expected inter-temporal loss (stochastic plus deterministic components) at time $t = 0$ by $\Omega_0(w_r)$. This includes a term penalizing the variance of the interest rate which does not contribute to utility loss as such, but rather represents the ZLB constraint. Actual utility, found by subtracting that interest rate term, is given by $\Omega_0(0)$. The steady-state inflation rate, $\pi^*$, that will ensure the lower bound is reached only with probability $p = 0.025$ is computed using (54). Given $\pi^*$, we can then evaluate the deterministic component of the welfare loss, $\bar{\Omega}_0$. Since in the new steady state the real interest rate is unchanged, the steady state involving real variables is also unchanged, so from (52) we can write

$$\tilde{\Omega}_0(0) = \left[w_\pi(1 - \gamma_p)^2 + w_{\Delta w}(1 - \gamma_w)^2\right] \pi^*^2$$

Both the ex-ante optimal and the optimal time-consistent deterministic welfare loss that guide the economy from a zero-inflation steady state to $\pi = \pi^*$ differ from $\tilde{\Omega}_0(0)$ (but not by much because the steady-state contributions by far outweighs the transitional one). From a timeless perspective (see Appendix A.1.2 of LMP), however, the policymaker will jump immediately to the new steady state further justifying the use of (55).

These results demonstrate the trade-off between reducing the stochastic component of policy at the expense of a higher steady-state inflation rate and therefore a higher deterministic component of policy. Under discretion in table 1a the optimal combination (i.e., the minimum of $\Omega_0^{TC}(0)$) is achieved at a quarterly rate inflation rate $\pi^* = 0.52$; i.e., at an annual rate around 2%. This pins down the parameter penalizing the variability of the interest rate at $w_r = 4$. The same exercise for optimal policy under commitment (Table 1b) sees $\pi^* = 0.26$ with $w_r = 20$, but in the case the loss function is very flat as $w_r$ falls from the value that results in $\pi^* = 0$, so there is little to gain from raising the steady-state inflation and interest rates.

Figure 1 further demonstrates the results in table 1a. The top-left figure shows the distribution for the nominal interest with zero steady-state inflation for the case $w_r^{TC} = 2$ where $\sigma_r = 1.00$. The probability of hitting the zero lower bound is now high, of the order $p = 0.30$. If in the top-right figure, the steady-state inflation increases to $\pi^* = 0.95,$
thus shifting the distribution by this amount to the right, the probability of \( r_t \leq 0 \) falls to \( p = 0.025 \). However this choice of \( \pi^* \) and \( \sigma_r \) is sub-optimal. In the bottom-left figure keeping \( p = 0.025 \), the total welfare loss falls if we set \( \sigma_r = 0.74 \) and \( \pi^* = 0.68 \), values obtained by tightening the variability constraint to \( w_r = 3 \). Finally, the bottom right figure illustrates the optimal combination of \( \sigma_r = 0.61 \) and \( \pi^* = 0.52 \) at \( p = 0.025 \), obtained at \( w_r = 4 \) and highlighted in Table 1a.

By reporting the expected inter-temporal utility loss at time \( t = 0 \) under both the time-consistent discretionary policy and optimal commitment, \( \Omega_{TC}^0(0) \) and \( \Omega_{OP}^0(0) \) respectively, we can now assess the stabilization gains from commitment as the interest rate lower bound takes greater effect. We compute these gains as equivalent permanent percentage increases in consumption and inflation, \( c_e^{gain} \) and \( \pi_e^{gain} \) given by\(^{15}\)

\[
c_e^{gain} = \frac{\Omega_{TC}^0(0) - \Omega_{OP}^0(0)}{1 - h} \times 10^{-2}; \quad \pi_e^{gain} = \frac{\sqrt{2(\Omega_{TC}^0(0) - \Omega_{OP}^0(0))}}{w_\pi} \tag{56}
\]

The measure \( c_e^{gain} \) (but not \( \pi_e^{gain} \)) usefully decomposes into a deterministic part that reflects the lower steady state inflation rate under commitment and a stochastic part that captures the superior stabilization performance of the commitment rule. We write this decomposition as \( c_e^{gain} = c_e^{gain} + \bar{c}_e^{gain} \) and report values in Table 1c. A further useful expression is the minimum cost of fluctuations\(^{16}\) in consumption and inflation equivalent terms obtained under the optimal commitment rule:

\[
c_e^{min} = \frac{\Omega_{OP}^0(0)}{1 - h} \times 10^{-2}; \quad \pi_e^{min} = \frac{\sqrt{2\Omega_{OP}^0(0)}}{w_\pi} \tag{57}
\]

Table 1c sets out the gains from commitment under four scenarios: the first where the ZLB constraint is ignored (\( w_{TC}^r = w_{OP}^r = \pi^* = 0 \)); second, where it is taken into account but only the steady state inflation rates are adjusted to accommodate the constraint; third, under optimal combinations of \( \sigma_r \) and \( \pi^* \) highlighted in Tables 1a and 1b (\( w_{TC}^r = 4, w_{OP}^r = 20 \)); fourth, under an added constraint that the steady-state inflation rate remains at zero (\( w_{TC}^r = 60, w_{OP}^r = 45 \)) and the stabilization bears the full brunt of the constraint. The deterministic component of \( c_e^{gain} \) and by implication its decomposition into deterministic and stochastic components is also provided.

A number of interesting points emerge from these tables. First using (57) the minimal cost of consumption fluctuations is given by \( c_e^{min} = 0.55\% \), a value much larger than the welfare cost reported by Lucas (1987) which was of the order 0.05\%. The reason why they

\(^{15}\)See LMP, Appendix C.

\(^{16}\)But it should be noted that our quadratic approximation to the utility function omits terms independent of policy so the cost of fluctuations is under-estimated.
are much larger is down to the welfare costs of price and wage inflation not included in the Lucas calculations and to the estimated variances of shocks to preferences, government spending, investment, price and wage mark-ups, equity prices and technology.\textsuperscript{17} Moreover, our figure rises when we impose the ZLB constraint and is further inflated by the existence of external habit which reduces the utility improvement from a increase in consumption.

Second, the most important point from these tables endorses the conclusion reached by Adam and Billi (2007) discussed in the Introduction, namely that the lower bound constraint on the nominal interest rate increases the gains from commitment several fold. In terms of the consumption equivalent for the welfare-based case $c_e^{\text{gain}}$ we can see that the stabilization gain from commitment rises and at the optimal combination of $\sigma_r$ and $\pi^*$ reaches $c_e^{\text{gain}} = 0.42\%$ and $\pi_e^{\text{gain}} = 0.62\%$. The deterministic component of this arising from a lower steady-state inflation rate is $\bar{c}_e^{\text{gain}} = 0.09\%$. These gains are much higher than those in Adam and Billi (2007). The reason for this is that the stabilization gains are very low in their simple New Keynesian model with no capital, habit or wage stickiness and only shocks to the price mark-up and government spending. Wage stickiness in particular adds a large welfare cost from wage inflation in (52). It is true that by imposing a symmetric interest rate distribution the LQ approach to the ZLB constraint exaggerates its cost. However in imposing a zero steady-state inflation rate Adam and Billi (2007) also exaggerate the cost of the constraint within their non-linear approach. They report the gains in terms of a percentage increase in welfare loss as one proceeds from commitment to discretion. Our results indicate an increase of 76\% with is remarkably close to their 65\% increase reported for the baseline calibration. If, as in their results, we require that there is no long-run inflation under discretion, from the last row of Table 1c we see that the commitment gain increases dramatically to $c_e^{\text{gain}} = 10.8\%$ and $\pi_e^{\text{gain}} = 3.20\%$.

The conclusion that emerges from these results is that the stabilization gains from commitment are significantly greater than those previously reported in the literature. We find these gains to be a 0.42\% equivalent permanent increase in consumption corresponding to a 0.62\% permanent increase in quarterly inflation. The latter, for instance, compares with a range of 0.04 – 0.4\% found in the comprehensive study of Dennis and Söderström (2006) across several models.\textsuperscript{18} We have carried out robustness checks on these results using two variants of the model: a no-indexation one which performed best empirically

\textsuperscript{17}The Lucas calculation is based on $sd(c_t) = 1.5\%$ which is somewhat lower than the standard deviation we found under optimal commitment of $sd(c_t) = 2.7\%$. Reworking the Lucas calculation would then give a consumption equivalent loss from fluctuations of 0.16\%, still a low figure and far below the loss reported here.

\textsuperscript{18}We have adjusted their reported annual inflation rate equivalents. Note that they examine models without explicit micro-foundations and an ad hoc loss function.
in achieving the highest marginal likelihood, and a variant with a lower degree of price stickiness corresponding to our prior \( \xi_p = 0.75 \), but which was massively outperformed by the others. Full details of these results are reported in Appendix C and LMP. Gains from commitment for the first of these models are slightly greater than those of the core model, whilst for the low price stickiness variant substantially higher gains of over 2% consumption equivalent are found.

4.3 Stabilization Gains with Simple Rules

We now turn to results for simple commitment rules of the general form:

\[
rt = \rho rt-1 + \Theta_\pi E_t \pi_{t+j} + \Theta_y (y_t - \hat{y}_t) + \Theta_{\Delta w} \Delta w_t + \Theta_{wr} wr_t
\]  

(58)

where \( \rho \in [0,1] \), \( \Theta_\pi, \Theta_y, \Theta_{\Delta w}, \Theta_{wr} > 0, j \geq 0 \). Putting \( \Theta_{\Delta w} = \Theta_{wr} = 0 \) gives the standard Taylor rule where the interest rate only to current price inflation and the output gap, \( \Theta_{\Delta w} = \Theta_{wr} = \Theta_y = 0 \) gives a price inflation rule, \( \Theta_\pi = \Theta_{wr} = \Theta_y = 0 \) gives a wage inflation rule and \( j = \Theta_{\Delta w} = \Theta_y = 0 \) gives a current price inflation and real wage rule.

Results for these rules are summarized in Table 2. Since the welfare gains from increasing the steady-state inflation rate and widening the interest rate distribution consistent with \( p = 0.025 \) is very small, we confine ourselves to \( \pi^* = 0 \). Two notable results emerge: First, we assess the effect of using an arbitrary rather than an optimized simple commitment rule by examining the outcome when a minimal rule \( i_i = 1.001 \pi_t \) that just produces saddle-path stability. This is the worst case and we see that the costs are substantial: \( c_e^{gain} = 7.03\% \). Interestingly, this outcome is still better than that under discretion if the same constraint on the variance of the interest rate as for optimal commitment is imposed. Second, simple price inflation and wage inflation rules perform reasonably well in that they achieve over 80% of the commitment gains achieved by the optimal rule when \( \pi^* = 0 \) is imposed. The simple rule that closely mimics optimal commitment for the welfare-based case is the price inflation plus real wage rule.

From table 2 almost all the gains from commitment are achieved by this rule though simplicity per se still leaves a small cost of \( c_e^{gain} = 0.02\% \) and \( \pi_e^{gain} = 0.15\% \approx 0.60\% \) annually.

The ability of the simple rule to mimic the optimal one is clearly demonstrated in Figure 2 which provides comparisons of the responses under the optimal commitment,
discretion and the optimized simple price inflation/real wage rule following an unanticipated productivity shock \((a_0 = 1)\), with and without interest rate ZLB considerations.\(^{20}\)

In all cases, a positive technology shocks increase potential output \((\hat{y}_t)\), given sluggish adjustment on the demand side, a positive output gap (defined as \(\hat{y}_t - y_t\)) therefore opens up. This, coupled with the drop in inflation (reflecting lower unit labor costs from higher labor productivity), prompts a real and nominal monetary expansion. Under discretion, the inability to commit to future rises in the interest rate results in a higher immediate reduction in the nominal interest rate and a bigger drop in inflation. In a stochastic environment with repeated technology shocks of either sign this results in higher variances for both the interest rate and inflation, the former having been seen in Table 1a.

When we relax the ZLB constraint in the right-hand-side graphs we witness two features. First there is a substantial drop in the nominal interest rate for both optimal commitment and discretion of 1.4 – 1.6% below the steady state interest rate which is about 1%. In other words the ZLB constraint is violated. Second, the optimal commitment and discretionary paths now almost coincide and the time-inconsistency problem becomes minor, as the low gains reported in Table 1c have already indicated. This is because in the absence of a ZLB constraint we are much closer to a world of ‘divine coincidence’ of simple New Keynesian models where both inflation and the output gap can be perfectly stabilized. The bottom line is that ZLB considerations really do matter.

### 4.4 Sustaining Commitment as an Equilibrium

We now examine numerically the no-deviation condition for commitment to be a perfect Bayesian equilibrium (PBE). We confine ourselves to reporting results for the condition (28) which assumes an instantaneous loss of reputation following deviation. Experimentation revealed this to give very similar results to those using (26).

Figure 3 plots a histogram from 10,000 draws of the sector \([z_t^T p_{21}^T]^T\) in the vicinity of the steady state of the economy under the optimal commitment rule. The probability of the weak government deviating from the optimal rule, \(q_t\), is then the proportion of these draws for which (28) does not hold; i.e., \(\Phi = \text{tr}((1 - \beta)(N_{11} + S)(Z_t + \beta \Sigma) + \text{tr}((1 - \beta)N_{22}p_{21}^T \Sigma p_{21}^T) < 0\). For our model and sample of 10,000 draws we see that in fact \(q_t = 0\) so that optimal commitment for a weak government turns out to be a perfect Bayesian equilibrium.

As discussed in Section 2.3, the no-deviation condition compares the temporary stabilization gains from reneging (‘temptation’) with the long-run stabilization loss from losing reputation (the ‘penalty’). The latter depends crucially on the policymaker’s rate of discount \(\beta\). In all our welfare-based results we set \(\beta = 0.99\) on a quarterly basis for both the

\(^{20}\)Responses to an unanticipated government spending shock \((g_0 = 1)\) are also reported in LMP.
policymaker and the private sector. But suppose that the policymaker was more myopic than the private sector. For all $\beta \geq 0.75$ we find that $q_t = 0$. In Figure 4 we set $\beta = 0.5$ which could be appropriate for a non-independent central bank in which optimal monetary policy depends on the probability of the survival (re-election) of government was very low, in fact $0.5^4 = 0.0625$ per year. We find that there is now a very small probability of a break-down in the no-deviation condition, namely $q_t = 0.002$. Thus our result that commitment can be sustained as a PBE is robust to variations in the policymaker’s discount factor for all conceivable institutional arrangements in the euro area.

5 Conclusions

This paper re-examined the welfare gains from monetary commitment in the context of an empirical micro-founded DSGE model of the euro area. We utilized a quadratic approximation of the representative household’s utility as the welfare criterion and with the interest rate ZLB constraint imposed, we found these gains to be relatively large. Given this, we then examined the question of how such commitment (to an optimal rule) can be sustained as an equilibrium. Finally, we sought a simple interest rate rule that closely approximates the optimal one.

Our findings are as follows: first, we use a ‘small distortions’ quadratic approximation to the consumer’s utility which is accurate if the steady state is close to the social optimum. In assessing whether this condition holds, we highlight a neglected aspect of NK models: external habit in consumption tends to make labor supply and the natural rate of output too high compared with the social optimum. If the habit effect is sufficiently high and labor market, product market and tax distortions are not too big, then the natural rate can actually be above the social optimum. This would then render the long-run ‘inflationary bias’ negative. Given the high tax wedge in the euro-area model, however, the inflationary bias turns out to be positive in the model.

Second, whilst the validity of an inflationary bias arising from the pursuit of an ambitious output target above its natural rate has been criticized (notably in Blinder (1998)), our analysis suggests a rather different form of bias arising from the interest rate zero lower bound. We find that the optimal steady state inflation rate necessary to avoid the lower bound is far lower under commitment than under discretion, so there is a new sense in which there is a long-run inflationary bias which is really an integral part of the stabilization bias.

Third, in terms of an equivalent permanent increase in consumption, $c^\text{gain}_t$ for the welfare-based loss function and a permanent decrease in inflation $\pi^\text{gain}_t$, the stabilization
gains from commitment rise considerably if the lower bound is taken into account. The reason is that following a shock which diverts the economy from its steady state and given expectations of inflation, the opportunist discretionary policy-maker can increase or decrease output by reducing or increasing the interest rate which increases or decreases inflation. This results in a higher variability of inflation and the nominal interest rate under discretion. The latter means that the ZLB constraint is tighter under discretion and its presence increases the stabilization gains from commitment. Using empirical estimates from the core model, we find an average consumption and inflation-equivalent gains of $c^\text{gain}_e = 0.42\%$ and $\pi^\text{gain}_e = 0.62\%$ respectively, the latter on a quarterly basis. For a variant of the model with lower price stickiness, these rise considerably to $c^\text{gain}_e = 2.35\%$ and $\pi^\text{gain}_e = 4.39\%$

Fourth, given these large gains from commitment, the incentive for central banks to avoid a loss of reputation for commitment is quite substantial. Consequently, unless the policymaker is implausibly myopic, a commitment rule can be sustained as a perfect Bayesian equilibrium in which deviation from commitment hardly ever happens.

Finally we find that the optimal commitment rule can be closely approximated in terms of its good stabilization properties by an interest rate rule that responds positively to current inflation and to the current real wage. Policies which target the real wage as opposed to nominal wage growth seem not only more plausible, but also theoretically more coherent.

There are a number of possible directions for future research. First, the robustness of our finding, that gains from commitment may be far higher than previously thought, needs to be investigated further for other DSGE models, including, for example, the SW model fitted to US data and small open economy models such as Adolfson et al. (2007). Second, whereas for our estimated models with a high degree of habit we find that the steady-state inefficiency is small, justifying the small distortions approximation, robustness considerations may demand a more accurate quadratic approximation of the household utility as in the ‘large distortions’ procedure of Benigno and Woodford (2004) and Levine et al. (2007a). Then using posterior estimates of the densities of the parameters and model probabilities, a consistently Bayesian approach to the design of robust interest rate rules can be employed as in Batini et al. (2006).

References


$\pi^* = \max\{z_0(p)\sigma^{TC}_r - (\frac{1}{2} - 1) \times 100, 0\} = \max\{1.96\sigma^{TC}_r - 1.01, 0\}$ with $p = 2.5\%$ probability of hitting the zero-lower bound and $\beta = 0.99$. $\tilde{\Omega}^{TC}_0(0) = \frac{1}{2}[w_\pi(1 - \gamma_\pi)^2 + w_\Delta w(1 - \gamma_w)^2] \pi^* = 19.81 \pi^*$. $\Omega^{TC}_0(0) = \tilde{\Omega}^{TC}_0(0) + \bar{\Omega}^{TC}_0(0)$.

<table>
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<tr>
<th>Weight $w^{TC}_r$</th>
<th>$(\sigma^{TC}_r)^2$</th>
<th>$\Omega^{TC}_0(w_r)$</th>
<th>$\Omega^{TC}_0(0)$</th>
<th>$\pi^*$</th>
<th>$\Omega^{TC}_0(0)$</th>
<th>$\Omega^{TC}_0(0)$</th>
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Table 1a. Optimal Discretion.

$\pi^*, \Omega^{OP}_0(0)$ defined as for discretion above.

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<th>Weight $w^{OP}_r$</th>
<th>$(\sigma^{OP}_r)^2$</th>
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<td>17.1</td>
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Table 1b. Optimal Commitment.

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<th>$(\sigma^{TC}_r, \sigma^{OP}_r)$</th>
<th>$((\pi^<em>)^{TC}, (\pi^</em>)^{OP})$</th>
<th>$c^{gain}_e$</th>
<th>$\pi^{gain}_e$</th>
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<td>Imposed</td>
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<td>0.09</td>
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<td>Imposed</td>
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<td>(0, 0)</td>
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<td>3.20</td>
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Table 1c. Stabilization Gains From Commitment

% Consumption Equivalent ($c^{gain}_e$) and % Inflation Equivalent ($\pi^{gain}_e$)
<table>
<thead>
<tr>
<th>Rule</th>
<th>$\rho$</th>
<th>$\Theta_\pi$</th>
<th>$\Theta_{\Delta w}$</th>
<th>$\Theta_{wr}$</th>
<th>$w_r$</th>
<th>$\Omega_0(w_r)$</th>
<th>$\Omega_0(0)$</th>
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<th>$\sigma_{c}^{gain}$</th>
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Table 2. Comparison of Optimal and Simple Commitment Rules.

Figure 1: Imposing the Interest Rate Zero Lower Bound under Discretion.
Figure 2: Responses to a Positive Technology Shock with and without a ZLB Constraint.
Figure 3: **The No-Deviation Condition:** \( \Phi = \text{tr}((1 - \beta)(N_{11} + S)(Z_t + \beta \Sigma) + \text{tr}((1 - \beta)N_{22}p_2T_{2t}^T) \). \( \beta = 0.99 \)
Figure 4: The No-Deviation Condition: $\Phi$ defined as above. $\beta = 0.5$