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Optimal monetary policy with imperfect common knowledge ☆

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Abstract

This paper determines optimal nominal demand policy in a flexible price economy in which firms pay limited attention to aggregate variables. Firms' inattentiveness gives rise to idiosyncratic information errors and imperfect common knowledge about the shocks hitting the economy. This is shown to have strong implications for optimal nominal demand policy. In particular, if firms' prices are strategic complements and economic shocks display little persistence, monetary policy has strong real effects, making it optimal to stabilize the output gap. Weak complementarities or sufficient shock persistence, however, cause price level stabilization to become increasingly optimal. With persistent shocks, optimal monetary policy shifts from output gap stabilization in initial periods following the shock to price level stabilization in later periods, potentially rationalizing the medium-term approach to price stability adopted by some central banks.

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1. Introduction

The decentralized nature of economic activity suggests that not all economic decision-makers base their decisions on the same information set. Friedrich A. Hayek referred to information dispersion between decision-makers as the defining feature of economic policy problems:

> The peculiar character of the problem of rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. Hayek (1945).

Most economic models, however, derive policy recommendations on the assumption that private agents share a common information set. In the realm of monetary policy, for example, information asymmetries between private agents have not yet received much attention, and the literature has mainly focused on asymmetries between the private sector and the policy-maker.[1]

This paper determines optimal monetary policy when there is information dispersion between firms. Analyzing information dispersion between firms seems particularly important when studying monetary policy because the real effects of nominal demand policy ultimately depend on firms' pricing decisions. Moreover, it seems plausible to assume that firms do not pay attention to all aggregate developments, e.g., because of the limited number of managerial staff collecting and processing information and taking the corresponding decisions; see Radner (1992).

Following earlier work by Sims (2003), limited attention is modeled by assuming that firms have limited capacity to process information. Such processing limitations imply that firms cannot pay attention to all aggregate developments and have to *choose* what information to process and what information to neglect. Since each firm processes information in a slightly different way, information processing leads to idiosyncratic processing errors, whose size is inversely related to the firm's processing capacity.

Using a simple model with imperfectly competitive firms, flexible prices, and a policy-maker maximizing the welfare of the representative agent, it is shown that the presence of information processing constraints and differential information has stark consequences for the conduct of optimal nominal demand policy. In particular, if firms' prices are strategic complements and aggregate disturbances display little persistence, it tends to be optimal for monetary policy to stabilize the output gap. In the presence of either weak complementarities or sufficient shock persistence, however, monetary policy should increasingly emphasize price level stabilization.[2]

Intuitively, private information renders coordination among firms difficult because firms are uncertain about the decisions of other firms that base their price decisions on (slightly) different information sets. Strategic complementarities increase the relevance of other firms' pricing decisions, thereby increasing strategic uncertainty among firms, which causes each firm's price to react only weakly to own information. This amplifies the real effects of

[1] See, for example, Svensson and Woodford (2004).

[2] As shown in the paper, price level variability is a measure of the amount of information-based price dispersion in the economy.

monetary policy and makes it optimal to stabilize the output gap. However, when shocks display sufficient persistence, firms are able to observe shocks better over time, which leads to less information dispersion and weaker real effects of monetary policy. This makes it optimal to stabilize the price level. Optimal monetary policy thus shifts its emphasis from output stabilization during initial periods following the shock to price stabilization in later phases, potentially rationalizing the medium-term orientation to price stability adopted by a number of monetary authorities in industrialized economies.

Independent of whether monetary policy should stabilize the price level or the output gap, it is always optimal to nominally accommodate supply shocks, i.e., shocks that move the efficient output level. Firms then choose not to process any information about supply shocks. As a result, firms' prices do not react to these shocks and appropriate nominal demand adjustments then induce real output movements that cause output to follow its efficient level. Supply shocks, therefore, do not generate a trade-off between price and output gap stabilization.

The situation is different for real demand shocks, i.e., shocks to firms' desired mark-up. These shocks generate a trade-off between price and output gap stabilization because nominal accommodation of such shocks stabilizes the output gap but makes the price level more variable. Whether such shocks should be nominally accommodated or not thus depends on whether it is optimal to stabilize prices or the output gap.

As pointed out by Keynes (1936) and Phelps (1983), disparate information sets coupled with the assumption that agents hold rational expectations generates substantial technical difficulties: optimal decision-making requires that agents formulate 'higher order beliefs', i.e., beliefs about the beliefs of others and beliefs about what others believe about others, and so on ad infinitum.[3] This is the case because a firm's optimal price depends on the prices set by other firms, and thus on other firms' beliefs.[4]

Despite these difficulties, a number of recent papers successfully pioneer methods for determining rational expectations equilibria in imperfect common knowledge (ICK) environments, most notably Townsend (1983a,b), Pearlman (1986), Sargent (1991), Binder and Pesaran (1998), Kasa (2000), Woodford (2002), Sargent and Pearlman (2004), and the recent literature on global games, see Morris and Shin (2003a).

While the present setup in many respects is simpler than in these earlier contributions, it adds to the literature by solving an optimal policy problem for a private sector rational expectations equilibrium (REE) with ICK. Moreover, the paper endogenizes information imperfections by assuming limited information processing capacities. Limited processing capacity causes the information structure to be endogenous because firms choose what information to process, implying that the information structure reacts to the policy pursued by the central bank.

In related papers Morris and Shin (2003b) and Amato and Shin (2003, 2006) derive normative implications for ICK settings, but focus on the welfare effects of disclosing public information. Hellwig (2002) derives impulse responses to a large range of shocks for an economy with monopolistic competition and ICK. Nimark (2005) solves a forward-looking pricing model with nominal rigidities and ICK.[5]

[3] Morris and Shin (2003a) have shown that agents do not necessarily have to formulate such higher order beliefs. In binary action games, optimal decisions can be generated by holding simple uniform beliefs about other agents' *actions*.

[4] In the present model such dependencies arise from price competition among firms.

[5] Bacchetta and van Wincoop (2006) study the impact of public and private information in an asset-pricing context. Ui (2003) shows the non-neutrality of money in a Lucas island economy with private information.

Ball et al. (2005) analyze optimal monetary policy with disparate information by assuming that some agents set prices based on lagged information. While there are many similarities to the present paper, the assumed information lags do not generate ICK. Moreover, by contrast with the setting studied by Ball et al., price level targeting fails to be optimal in the current setup because the trade-off between price and output gap stabilization shifts over time when shocks are persistent.

The rest of this paper is organized as follows. Section 2 outlines a simple static economy with monopolistically competitive firms. As a benchmark, Section 3 derives optimal policy when there is common knowledge among firms. ICK and information processing constraints are introduced in Section 4, which builds on results from information theory. Section 5 determines the REE with ICK, and Section 6 characterizes optimal monetary policy. Results are extended to a dynamic setup in Section 7. A conclusion summarizes the main findings, and technical details are contained in an Appendix.

2. The model economy

This section introduces a representative agent economy with flexible prices, a continuum of monopolistically competitive firms, and a central bank. The central bank controls nominal demand and maximizes the utility of the representative agent. The economy is hit by stochastic shocks inducing variations in the efficient output level and variations in firms' desired mark-up.

Importantly, agents in the economy differ with respect to the attention they pay to aggregate variables. While the central bank pays perfect attention to aggregate shocks and processes information about them perfectly, this paper allows for the possibility that firms pay only limited attention to aggregate information. This is motivated by the observation that central banks tend to devote considerable resources to monitoring aggregate developments, while firms appear to be more concerned with microeconomic developments.[6] The assumption that the central bank can processes aggregate information perfectly will be relaxed later on.

2.1. Households

The household sector is described by a representative consumer choosing aggregate consumption Y and labor supply L so as to perform the following maximization

$$\max_{Y,L} \quad U(Y) - \nu V(L)$$

$$\text{s.t.}$$

$$0 = WL + \Pi - T - PY, \tag{1}$$

where W denotes a competitive wage rate, Π monopoly profits from firms, T nominal transfers from the monetary authority, and P the price index of the aggregate consumption good. The parameter $\nu > 0$ is a stochastic labor supply shifter with $\mathrm{E}[\nu] = 1$, which induces variations in the efficient level of output. For simplicity, the consumer observes all prices in

[6]Firms' trade-off of how much attention to allocate to aggregate versus idiosyncratic variables is analyzed in Mackowiak and Wiederholt (2005).

the economy. Furthermore, $U'>0$, $U''<0$, $\lim_{Y\to\infty} U'(Y)=0$, $V'>0$, $V''>0$ and $V'(0)<U'(0)$.

2.2. Firms

The production sector consists of a continuum of monopolistically competitive firms $i \in [0,1]$. Firm i produces an intermediate good Y^i with labor input L^i according to a linear production function of the form

$$Y^i = L^i,$$

and aggregate labor demand is $L = \int L^j \, \mathrm{d}j$. Intermediate goods enter into the aggregate output good Y according to a Dixit–Stiglitz aggregator

$$Y = \left(\int_{[0,1]} (Y^j)^{\theta-1/\theta} \, \mathrm{d}j \right)^{\theta/(\theta-1)}, \tag{2}$$

where $\theta>1$ is a stochastic shock, with mean $\mathrm{E}[\theta]=\overline{\theta}$, inducing variations in the price elasticity of firms' product demand.

Linearizing the first order condition of firm i delivers an expression for the profit-maximizing price:[7]

$$p(i) = \mathrm{E}[p + \xi(y - y^*) + u | I^i], \tag{3}$$

where lower case letters denote variables that are expressed as percentage deviations from deterministic steady state and I^i denotes the information set on which firm i bases its decision.

The profit-maximizing price $p(i)$ in Eq. (3) depends on the expected values of the average price level

$$p = \int p(j) \, \mathrm{d}j$$

and on the output gap $y - y^*$, where

$$y = \int y(j) \, \mathrm{d}j$$

denotes the average output across firms and y^* the efficient output level.[8] Fluctuations in the efficient output level y^* are induced by the labor supply shock v. The profit-maximizing price also depends on the expected value of u, which is a function of the demand shock θ,

$$u \sim -(\theta - \overline{\theta}).$$

Firms wish to charge a higher mark-up ($u>0$) whenever the price elasticity of demand θ falls below its mean $\overline{\theta}$. A positive value of u thus reflects the fact that product demand has become less price-sensitive.

For simplicity, it is assumed that $y^* \sim \mathrm{N}(0, \sigma^2_{y^*})$ and $u \sim N(0, \sigma^2_u)$ and these disturbances will be referred to as *supply shocks* and (*real*) *demand shocks*, respectively.[9]

[7]See Appendix A.1 for a derivation.

[8]Higher order approximation to the firm's first order condition would cause the optimal price to depend also on the expected level of price dispersion.

[9]Real demand shocks are called 'mark-up shocks' in Woodford (2003) and 'cost-push shocks' in Clarida et al. (1999).

The parameter $\xi > 0$ in Eq. (3) indicates the sensitivity of firms' prices to the output gap and is given by

$$\xi = -\frac{U''(\overline{Y})\overline{Y}}{U'(\overline{Y})} + \frac{V''(\overline{Y})\overline{Y}}{V'(\overline{Y})},$$

where $\overline{Y}$ denotes the steady state output level. If the representative agent becomes risk neutral, then ξ approaches zero in the limit. Firms' prices are then virtually independent of the output gap because the real wage, i.e., the marginal rate of substitution between consumption and leisure, becomes independent of the output gap. Importantly, the value of ξ determines whether firms' prices are strategic complements or substitutes. This can be seen by defining nominal spending q as

$$q = y + p \tag{4}$$

and using it to substitute y in Eq. (3):

$$p(i) = \mathrm{E}[(1-\xi)p + \xi q - \xi y^* + u | I^i]. \tag{5}$$

For $\xi \leqslant 1$ prices are strategic complements, since the optimal price (weakly) increases in the average price level for a given level of nominal demand q. Note that this is the case whenever households are not too risk-averse. For $\xi > 1$ prices are strategic substitutes because the optimal price decreases in the average price level.[10] Throughout the paper, it is assumed that prices are strategic complements, i.e., $\xi \leqslant 1$, which appears to be the case of greatest economic interest. Analytical results, however, hold as long as $\xi < 2$.

2.3. Monetary policy

The central bank maximizes the utility of the representative agent by adjusting nominal demand. Appendix A.2 derives a second order approximation to the utility function of the representative agent. This allows an approximation of the monetary policy problem as

$$\begin{aligned} &\max_q -\mathrm{E}[(y-y^*)^2 + \lambda p^2] \\ &\quad \text{s.t.} \\ &p(i) = \mathrm{E}[(1-\xi)p + \xi q - \xi y^* + u | I^i], \\ &\quad p = \int p(j)\,\mathrm{d}j, \\ &\quad y = q - p \end{aligned} \tag{6}$$

for some $\lambda > 0$.[11] Output gap variability and variability of the aggregate price level both adversely affect the utility of the representative agent. While the first term is standard, the second term emerges because firms pay only limited attention to aggregate variables. As will be shown in Section 4, inattention gives rise to information dispersion between firms. The amount of dispersion thereby increases with the variance of aggregate variables, i.e.,

[10]Note that an increase in the price level reduces real demand. For $\xi > 1$ the demand shortfall reduces production costs by so much that a single firm finds it optimal to reduce prices.

[11]This requires steady state output to be first best, e.g., thanks to the existence of an output subsidy for firms; see Appendix A.2 for details.

the price level, because observing variables with greater variance is informationally more demanding. Price level variability is thus a measure of information dispersion. Such information dispersion is welfare-reducing because it leads to price dispersion and inefficient substitution between the goods of different firms.[12]

To determine first best policies, the central bank is assumed to have committed to its policy before the realization of shocks. This gives rise to the expectations operator in Eq. (6). For completeness, the paper also briefly addresses the implications of choosing policies after the realization of shocks.[13]

In the present model a trade-off between output gap stability and price stability can arise. This trade-off is generated by the pricing behavior of firms, as summarized by the constraints of problem (6). The goal of the paper is to show that the nature of this trade-off depends critically on the information sets I^i on which firms base their decisions. The next section explains how agents' information sets are generated.

2.4. Information processing

As is the case in standard rational expectations models, all agents in the economy observe all aggregate information perfectly. Unlike in standard models, however, it is assumed that observable information has to be processed before it can be incorporated into economic decisions, as in Sims (2003). Information processing may require, for example, the production of staff reports and memos which present information in such a way that it can be interpreted in the light of the particular decision that has to be taken.

The central bank is assumed to have sufficient staff, capable of processing all observable aggregate information. Firms, however, may devote only limited resources to the task of processing aggregate information, for reasons not modeled in this paper. As a result, the information sets I^i in Eq. (3) are not fully informative about aggregate variables and contain idiosyncratic noise elements. The presence of such noise implies that firms' decision-makers know neither the precise values of aggregate variables, nor exactly what other firms know. Information processing limitations thus reflect Hayek's view that information exists only in the form of dispersed bits of incomplete and frequently contradictory knowledge.

An important consequence of limited information processing is that firms have to *choose* what information to process and what information to neglect. Firms' information sets are thus endogenous and react, for example, to the policy pursued by the central bank, as will be explained in detail in later sections.

It is worth emphasizing that the present model is not a standard private information model. Although firms' decisions may not reflect all available information, this is not a consequence of the limited observability of information but rather the result of limited resources devoted to incorporating such information into decisions. The fact that firms' decisions do not reflect all available information thus cannot be eliminated by means of simple communication between decision-makers. As with other information, communicated information first has to be processed before it can be incorporated into decisions.

[12]See Appendix A.2 for details.

[13]This turns out to be less interesting because it does not generate a policy trade-off.

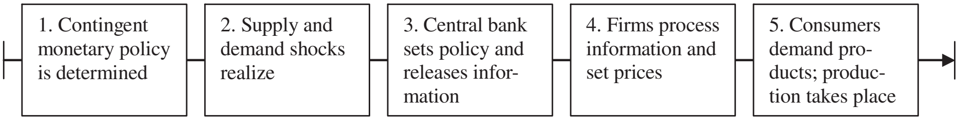

Fig. 1. Sequence of events.

2.5. *Timing of events*

The sequence of events taking place in the economy is illustrated in Fig. 1. After monetary policy is determined, the stochastic disturbances materialize. The central bank then processes the information about these disturbances, implements its policy, and releases information about the shocks and its policy decisions. Firms then optimally process information and simultaneously determine prices. Finally, consumers demand products for consumption, and production takes place.

3. Optimal policy in two benchmark settings

This section considers two settings with rather extreme informational assumptions. In the first, firms process information about aggregate variables perfectly, i.e., pay perfect attention to these variables; in the second, firms do not pay attention to aggregate variables at all, i.e., process no aggregate information. Both information settings give rise to common knowledge between firms and will serve as useful benchmarks when analyzing policy in ICK environments where firms pay positive but less than perfect attention to aggregate information.

3.1. *Benchmark I: perfect information processing*

With firms processing all information perfectly, all decisions reflect the same information and $p(i) = p$ in a symmetric equilibrium. Eq. (5) then simplifies to

$$p(i) = \mathrm{E}\left[q - y^* + \frac{1}{\xi}u|I\right]. \tag{7}$$

Since aggregate variables are contained in the information set I, the expectations operator in Eq. (7) can be omitted. This delivers:[14]

$$p = q - y^* + \frac{1}{\xi}u, \tag{8}$$

$$y - y^* = -\frac{1}{\xi}u. \tag{9}$$

Not surprisingly, nominal demand policy affects the price level only and has no effect on the output gap. In the absence of nominal rigidities and information asymmetries, monetary neutrality holds; see Lucas (1972). Optimal policy then stabilizes the price level. This is achieved by setting

$$q = y^* - \frac{1}{\xi}u, \tag{10}$$

[14] Eq. (9) follows from (8) and (4).

i.e., by accommodating supply shocks and by appropriately contracting in response to real demand shocks.

3.2. Benchmark II: no information processing

Now let us suppose that firms do not process information about aggregate variables at all. Firms' expectations in Eq. (7) are then determined by the unconditional mean values of the shocks and policy decision, which are all equal to zero. This delivers:[15]

$$p = 0, \tag{11}$$

$$y = q. \tag{12}$$

Nominal demand policy has real effects only and no effects on the price level, because nominal demand variations come as a 'surprise'. This is the reverse situation of the case with perfect information processing.

With nominal demand policy having real effects only, optimal policy stabilizes the output gap. This is achieved by nominally accommodating supply shocks, as in the case with perfect information, i.e.,

$$q^* = y^*.$$

Unlike in the case with perfect information processing, however, policy does not have to react to real demand shocks. Since firms do not observe these shocks, prices and the output gap both fail to react to them.

4. Modeling limited attention

The remainder of this paper considers the more realistic case of firms that pay positive but less than perfect attention to aggregate variables. Following Sims (2003), this is modeled by assuming that firms process information only at a finite rate.[16]

Firms' information processing capacity is described by a parameter $K \geqslant 0$. This parameter places an upper bound on the amount of information that firms can process per unit of time, as will be explained in detail below. For $K = \infty$ firms can process information perfectly, while for $K = 0$ firms fail to process any information. The setup thus nests the benchmark cases discussed in the previous section.

As mentioned above, firms can observe all existing information. Therefore, they will process the information released by the central bank only if the central bank announces firms' preferred information bundle, i.e., the one maximizing firms' profits.[17] Since the central bank is indifferent between making different announcements, the paper considers an equilibrium where the central bank announces firms' preferred information and firms process the information contained in central bank announcements.[18] Due to processing

[15] Eq. (12) follows from (11) and (4).

[16] Sims' approach is based on information theory, which was developed by Shannon (1948). See Cover and Thomas (1991) for a textbook treatment and Moscarini (2004) for a recent application in economics.

[17] In a setting where the central bank has monopoly power over information, it would announce the utility maximizing information bundle rather than that maximizing firms' profits.

[18] This assumption affects neither optimal policy nor the equilibrium outcome.

limitations, however, firms make idiosyncratic processing errors.[19] As a result, firms base their decisions on (slightly) different information about the state of the economy.

The next section introduces information processing constraints and derives the central bank announcement.

4.1. Processing constraints and central bank communication

Consider a firm that chooses a price $p \in R$ to maximize a quadratic profit function of the form:[20]

$$\max_{p} -\mathrm{E}[(p - \zeta' Z)^2 | I], \tag{13}$$

where $Z \sim \mathrm{N}(0, \Sigma_{zz})$ is a vector of uncorrelated fundamental shocks, i.e., $\Sigma_{zz} = diag(\sigma_1^2, \ldots, \sigma_I^2)$, and $\zeta' Z$ denotes the profit-maximizing price with perfect information about Z. The vector ζ' generally depends on the parameters of the underlying economic model, the policy pursued by the central bank, and other factors that the firm takes as given.

For a given information set I, the solution to problem (13) is trivially given by

$$p^* = \mathrm{E}[\zeta' Z | I] \tag{14}$$

and the expected loss equals

$$-Var(\zeta' Z | I). \tag{15}$$

Now suppose that the information set I is generated by observing a signal $s \in R$ given by

$$s = c' Z + \eta, \tag{16}$$

where $c' Z \in R$ denotes the central bank's communication about Z and $\eta \sim \mathrm{N}(0, \sigma_\eta^2)$ an idiosyncratic processing error independent of Z.[21]

The processing error in Eq. (16) arises because the firm has a finite capacity $K \in [0, \infty)$ to process the central bank announcement. This places an upper limit on the amount of information about Z that is contained in s. Formally, the processing constraint is given by

$$H(Z) - H(Z|s) \leqslant K, \tag{17}$$

where $H(Z)$ denotes the entropy of Z prior to observing the signal s, and $H(Z|s)$ the entropy after observing the signal.[22] Intuitively, entropy is a measure of the uncertainty about a random variable. Stated in these terms, Eq. (17) provides a bound for the uncertainty reduction about Z that can be achieved by observing the signal s; it thus constrains the information flow about Z.

The central bank chooses the communication policy c' that maximizes the firm's profits, anticipating that the firm will choose the profit-maximizing processing error σ_η^2 subject to

[19]This differs from Morris and Shin (2003b), who assume central bank announcements to be common knowledge to firms. Adam (2004) discusses the common knowledge case.

[20]The profit function (13) can be interpreted as a quadratic approximation to the firm's profit function. Such a quadratic approximation is convenient because it leads to linear decision rules that mimic the linearized first order conditions in Eq. (3).

[21]It is shown below that a univariate central bank announcement $c'Z$ is optimal.

[22]The entropy $H(X)$ of a continuous random variable X is defined as $H(X) = -\int \log(p(x))p(x)\,\mathrm{d}x$ where $p(x)$ is the probability density function of X, log is to the base 2, and the convention is to take $\log(p(x))p(x) = 0$ when $p(x) = 0$.

the information flow constraint (17). The communication policy and the variance of the processing errors must thus solve

$$\max_{c,\sigma_\eta^2} \quad -Var(\zeta' Z|s)$$
$$\text{s.t.} \quad \text{Eqs. (16) and (17).} \tag{18}$$

Appendix A.3 derives the solution to this problem, which is given by

$$c' = \zeta', \tag{19}$$

$$\sigma_\eta^2 = \frac{1}{2^{2K}-1}\, Var(\zeta' Z). \tag{20}$$

Eq. (19) shows that the central bank announces $\zeta' Z$, i.e., the optimal price chosen by a firm that has full information about Z. Since the vector ζ depends, inter alia, on the policy pursued by the central bank, communication depends on the monetary policy pursued. This reflects the fact that the relative importance firms attach to different fundamentals depends on how the central bank itself reacts to these fundamentals.[23] This feature will become important later on for understanding the optimal policy reaction to supply shocks.

Note that in Eq. (19) the central bank communication policy c' is independent of the noise statistic Σ_{zz} for the fundamental shocks. Intuitively, if the variability of a fundamental increases relative to that of others, the more variable fundamental will automatically contribute more to the overall variability of the central bank announcement $c'Z$, even without a change in the communication policy c'.

Eq. (20) shows that the variance of processing errors is proportional to the variance of the optimal price under full information. This is the case because it is informationally more demanding to process more variable central bank announcements.[24] Monetary policy thus also affects the size of agents' processing errors and thereby the amount of price dispersion in the economy.

From Eqs. (19) and (20) it follows that:

$$\mathrm{E}[\zeta' Z|s] = ks, \tag{21}$$

where the Kalman gain k is

$$k = \frac{Var(\zeta' Z)}{Var(\zeta' Z) + \sigma_\eta^2} = (1 - 2^{-2K}). \tag{22}$$

The Kalman gain k is a useful summary statistic indicating agents' ability to process information. For $k = 0$ firms cannot process any information since $\sigma_\eta^2 = \infty$. Conversely, with $k = 1$ firms process information perfectly since $\sigma_\eta^2 = 0$. These two cases thus generate the benchmark settings considered in Section 3. At intermediate values of k the variance of the observation noise is positive and decreases in k.

It should be noted that in the current setting it is optimal that the central bank announcement $c'Z \in R$ is univariate. This holds even though the fundamental shock Z is vector-valued. Appendix A.3 illustrates that for the case with two fundamental shocks, i.e.,

[23] Since the information flow constraint (17) only limits the overall flow of information and since the market for communication is competitive, firms essentially *choose* what information to process. Readers familiar with Kalman filtering may think of this situation as one where agents choose their observation equation.

[24] This follows from Eq. (17).

$Z = (Z_1, Z_2)'$, it would be suboptimal for firms to process information about Z_1 and Z_2 separately, even if the available processing capacity were allocated optimally across the two processing tasks. The intuition for this finding is simple: processing information about Z_1 and Z_2 separately generates superfluous information about Z whenever positive and negative elements in the sum $\zeta' Z$ cancel each other out.

5. Private sector equilibrium

In this section each firm is endowed with a given processing capacity and solve for a REE with ICK in which firms choose profit-maximizing prices and the central bank announcement maximizes firms' profits.

Solving for the REE is not trivial. Since there is 'information dispersion' in the economy, firms do not know what other firms have observed and must formulate beliefs about other agents' beliefs. This leads to a system of 'higher order beliefs' that affects the equilibrium outcome.

Section 5.1 determines how profit-maximizing prices depend on firms' higher order beliefs, and Section 5.2 derives the REE.

5.1. Price-setting with ICK

In order to be able to refer to firms' expectations of various order, we must first introduce some notation.

Let $x^{(m)}(i)$ denote firm i's mth order expectation of x and let

$$x^{(m)} = \int x^{(m)}(j)\,\mathrm{d}j$$

denote the average expectations of order m. Firms' expectations of order zero are given by the variable itself, i.e.,

$$x^{(0)}(i) = x.$$

Firms' expectations of order $m+1$ are their expectations of the average mth order expectation, i.e.,

$$x^{(m+1)}(i) = \mathrm{E}[x^{(m)}|I^i].$$

Therefore, $x^{(1)}(i)$ denotes the familiar (first order) expectation $\mathrm{E}[x_t|I^i]$; the second order expectations $x^{(2)}(i)$ denote i's expectations of the average (first order) expectations; the third order expectations $x^{(3)}(i)$ denote i's expectations of the average second order expectations, etc.

With this notation the price-setting (5) can be expressed as

$$p(i) = (1-\xi)p^{(1)}(i) + \xi q^{(1)}(i) - \xi y^{*(1)}(i) + u^{(1)}(i). \tag{23}$$

Iterating on Eq. (23) by repeatedly averaging across firms and taking conditional expectations obtains

$$p(i) = \mathrm{E}\left[\sum_{m=0}^{\infty}(1-\xi)^m(\xi q^{(m)} - \xi y^{*(m)} + u^{(m)})|I^i\right]. \tag{24}$$

Firms' optimal price depends on the first and higher order expectations of q, y^*, and u. For $\xi = 1$, i.e., without strategic interactions among firms, only first order expectations matter.

For smaller values of ξ, i.e., in the presence of strategic complementarities, higher order expectations increasingly influence the optimal price. Intuitively, this is due to the fact that strategic elements between firms cause them to attach greater weight to the beliefs about other firms' beliefs.

5.2. REE and central bank communication

This section determines the central bank announcement and characterizes the resulting REE with ICK.

Eq. (24) and the discussion in Section 4 imply that the central bank announces

$$\sum_{m=0}^{\infty}(1-\xi)^m(\xi q^{(m)} - \xi y^{*(m)} + u^{(m)}), \tag{25}$$

which is the profit-maximizing price chosen by a firm that has full information.[25]

Eq. (25) suggests that the central bank has to announce a combination of the fundamental shocks and agents' higher order expectations about these shocks. The latter implies that to construct a REE, a fixed point in the space of beliefs of infinite order has to be determined. A much simpler way to proceed, however, is to let the central bank announce the fundamentals

$$\xi q - \xi y^* + u. \tag{26}$$

Agents can then process information about (26) and construct the higher order beliefs in (25) using their (noisy) observation of these fundamentals. As shown in Appendix A.4, this leads to the same equilibrium outcome, but the equilibrium is much simpler to derive and easier to interpret.

The information available to firms is thus given by

$$s^i = (\xi q - \xi y^* + u) + \eta^i, \tag{27}$$

where η^i is an idiosyncratic Gaussian observation error. From Eq. (21) it then follows that:

$$\mathrm{E}[\xi q - \xi y^* + u | I^i] = ks^i, \tag{28}$$

where $k = 1 - 2^{-2K}$, see Eq. (22).

Integrating Eq. (28) over i, using Eq. (27) to substitute s^i, and taking the expectations $\mathrm{E}[\cdot|I^i]$ delivers

$$\mathrm{E}[\xi q^{(1)} - \xi y^{*(1)} + u^{(1)} | I^i] = k^2 s^i.$$

Applying the same operations m times obtains

$$\mathrm{E}[\xi q^{(m)} - \xi y^{*(m)} + u^{(m)} | I^i] = k^{m+1} s^i. \tag{29}$$

Expectations of higher order thus react less strongly to the signal s^i than expectations of lower order. This is rational because firms are increasingly uncertain about the expectations of higher order, which require them to repeatedly average (integrate) over information sets that differ from their own.

[25] More precisely, this is the price chosen by a firm with full information in an economy where other firms have ICK.

Substituting expression (29) into Eq. (24) delivers

$$p(i) = \frac{k}{1-(1-\xi)k}(\xi q - \xi y^* + u + \eta^i). \tag{30}$$

Averaging over firms yields an expression for the equilibrium price level

$$p = \frac{k}{1-(1-\xi)k}(\xi q - \xi y^* + u). \tag{31}$$

The equilibrium output gap follows from Eq. (4):

$$y - y^* = \frac{1-k}{1-(1-\xi)k}q - \frac{1-k}{1-(1-\xi)k}y^* - \frac{k}{1-(1-\xi)k}u. \tag{32}$$

As one would expect, nominal demand policy has real effects as long as firms have limited capacity to process information, i.e., $\partial y/\partial q > 0$ for $k < 1$. These real effects, however, are considerably amplified in the presence of strategic complementarities, i.e., when $\xi < 1$. In the limiting case $\xi \to 0$, for example, nominal demand policy has real effects only and no effect on prices.[26]

With strategic complementarities prices react more sluggishly to nominal demand variations because expectations of higher order receive more weight in the price-setting decisions; see Eq. (24). As shown above, however, expectations of higher order react more sluggishly to information than expectations of lower order. As a result, nominal demand variations increasingly affect output.

Finally, note that for the limiting cases $k = 1$ and $k = 0$, respectively, Eqs. (31) and (32) reduce to the benchmark expressions for the common knowledge settings derived in Section 3.

6. Optimal stabilization policy

This section determines optimal monetary policy when firms possess ICK about shocks. Using the results from Section 5, the policy problem (6) can be expressed as

$$\max_q \quad -\mathrm{E}[(y-y^*)^2 + \lambda p^2]$$

$$\text{s.t.} \tag{33a}$$

$$p = \frac{k}{1-(1-\xi)k}(\xi q - \xi y^* + u), \tag{33b}$$

$$y - y^* = \frac{1-k}{1-(1-\xi)k}q - \frac{1-k}{1-(1-\xi)k}y^* - \frac{k}{1-(1-\xi)k}u, \tag{33c}$$

which is a simple linear-quadratic maximization problem.[27] Since certainty equivalence holds, the results in this section do not depend on the assumption that the central bank observes shocks perfectly.[28]

[26] This holds for any value $k < 1$.

[27] This is the case because the Kalman gain k in Eqs. (33b) and (33c) is independent of policy. If the variance of observation noise was specified exogenously, this property would be lost and closed form solutions would be unavailable, even for the relatively simple policy problem at hand.

[28] Firms, however, will then also process information about shocks directly rather than processing only the central bank announcement, since the latter is contaminated by the central bank's processing noise.

The solution to (33a) is readily calculated to be

$$q = au + y^*, \tag{34}$$

where

$$a = \frac{(1-k)k - \lambda\xi k^2}{(1-k)^2 + \lambda\xi^2 k^2}. \tag{35}$$

The equilibrium behavior of prices and output under the optimal monetary policy is given by

$$p = \frac{(1-k)k}{(1-k)^2 + k^2\lambda\xi^2} u, \tag{36a}$$

$$y - y^* = \frac{-\xi\lambda k^2}{(1-k)^2 + k^2\lambda\xi^2} u, \tag{36b}$$

and does not depend on the supply shock y^*. The economic forces shaping the optimal monetary policy function (34) and the resulting behavior of output and inflation are discussed below.

6.1. *Policy reaction to supply shocks*

It is optimal to nominally accommodate supply shocks y^*. This holds independently of the degree of strategic complementarity, the extent to which firms can process information, and the relative weight given to price stabilization in the policy objective. The response thus remains unchanged if compared with the common knowledge benchmarks analyzed in Section 3.

Accommodating supply shocks is optimal because firms then *choose* not to observe any information about these shocks and the resulting policy reaction; see Eq. (27). As a result, they do not make any processing errors, which would result in inefficient price dispersion. Moreover, their prices will not react and the nominal demand adjustment comes as a (deliberate) surprise, affecting real output only and causing output to follow its efficient level. Therefore, supply shocks do not induce a trade-off between output gap stabilization and price level stabilization, i.e., the minimization of price dispersion.

6.2. *Policy reaction to demand shocks*

The situation differs notably when considering real demand shocks. The optimal reaction coefficient a then depends on the degree of strategic complementarity, the extent to which firms can process information, and the relative weight given to the price level objective.

Demand shocks also generate a trade-off between output and price stabilization. Consider the case of a central bank pursuing output gap stabilization only ($\lambda = 0$). The optimal reaction coefficient (35) is then given by

$$a_y = \frac{k}{1-k} > 0. \tag{37}$$

Shocks are nominally accommodated and the degree of accommodation increases in the processing index k. Higher values of k imply that firms receive more information, which reduces the real effects of monetary policy and requires stronger policy reactions.[29] Note that the reaction coefficient is independent of the degree of strategic complementarity. While lower values of ξ increase the real effects of monetary policy, they also increase the real effects of the demand shocks u; see Eq. (33c).

Next, consider the case of pure price level stabilization ($\lambda \to \infty$). The optimal reaction coefficient (35) is then given by

$$a_p = -\frac{1}{\xi} < 0. \tag{38}$$

Nominal contraction in response to demand shocks is required in this case, indicating that a trade-off exists between price level and output gap stabilization. Moreover, stronger complementarities (lower values of ξ) require a stronger policy reaction: strategic complementarities increase the importance of higher order beliefs, causing firms' prices to respond more sluggishly to policy.

For $0 < \lambda < \infty$ the optimal reaction coefficient (35) is a convex combination of the optimal reaction coefficients for the single objectives and can be written as

$$a = \omega a_y + (1 - \omega) a_p,$$

where the weight on the output coefficient is

$$\omega = \frac{(1-k)^2}{1 - 2k + k^2 + \lambda \xi^2 k^2}.$$

As is easily seen, ω increases as strategic complementarities become stronger and in the limit $\omega \to 1$ as $\xi \to 0$.[30] Strategic complementarities thus shift the trade-off between output and prices in favor of output gap stabilization. Strategic complementarities imply that price stabilization is rather costly in terms of its output implications: with prices responding sluggishly, policies aiming at price stabilization have to be rather aggressive and would generate large output gaps.

This fact is illustrated in Fig. 2. Depicted is the optimal reaction coefficient (35) as a function of the processing index k for various degrees of strategic complementarity.[31] As strategic complementarities become stronger, the policy reaction at intermediate values of k is increasingly characterized by nominal accommodation, as suggested by Eq. (37).

6.3. *Simple description of optimal policy*

Optimal policy in the previous section is described as a function of the fundamental shocks u and y^*. This section presents a simpler way to describe optimal policy using endogenous variables only.

From Eqs. (36a) follows the targeting rule:

$$p = -\frac{1-k}{k\xi}\frac{1}{\lambda}(y - y^*), \tag{39}$$

which is a relationship between endogenous variables that has to hold in equilibrium under optimal policy. It shows that optimal monetary policy should target the price level and the

[29] Policy successfully stabilizes output at the target as long as $k < 1$.

[30] This holds for all $k < 1$.

[31] The figure assumes $\lambda = 1$, i.e., equal weights for the output gap and the price level target. The bottom panel considers the case $\xi = 0.15$, which Woodford (2001) suggests as a plausible parameter value for the U.S. economy.

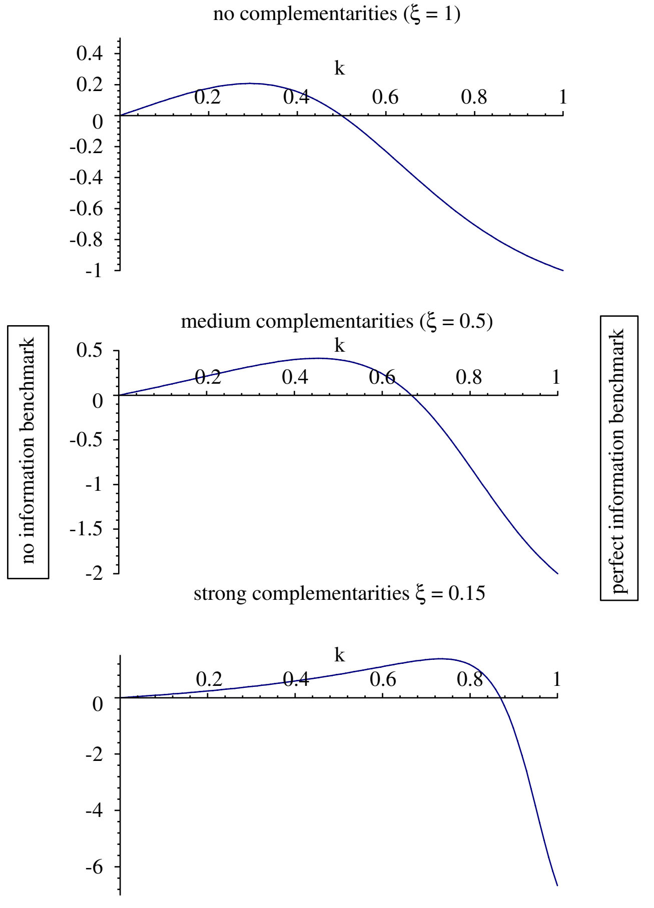

Fig. 2. Optimal reaction coefficient (real demand shocks).

output gap. In particular, prices should deviate from target only if output deviates from its efficient level.

Interestingly, the optimal targeting rule (39) is very similar to the one derived by Ball et al. (2005), who study a monopolistic price-setting model where firms update their information sets only infrequently. Using the notation of the current paper, their targeting rule can be expressed as

$$\mathrm{E}_{t-1}p_t = -\frac{1}{\xi\gamma}\mathrm{E}_{t-1}(y - y^*), \tag{40}$$

where the expectations operator appears because monetary policy is determined one period in advance.[32] Abstracting from the expectations operator in Eq. (40) and using the expressions for γ and λ derived in Appendix A.2, targeting rules (39) and (40) can both be

[32]The targeting rule derived in Ball et al. (2005) is slightly more general than the one stated above because the authors allow for a deterministically varying steady state price level. This feature could easily be introduced in the present model. Targeting rule (39) would then generalize correspondingly.

expressed as

$$p = -\frac{1}{\overline{\theta}}(y - y^*), \tag{41}$$

where $\overline{\theta}$ is the steady state elasticity of demand. This result shows that in the current model, as well as in the one studied by Ball et al. (2005), the price level should react less strongly to the output gap the more competitive the economy (the larger $\overline{\theta}$). Intuitively, price level movements lead to information and price dispersion. In competitive economies, such price dispersion is particularly harmful to welfare because demand reacts strongly to price differences.

Unlike in Ball et al. (2005), however, the optimality of price level targeting does not generalize to the case with persistent shocks, as is shown in Section 7.

6.4. Discretionary policy

For the sake of completeness this section briefly considers a discretionary policy-maker determining policy after or simultaneously with firms, i.e., after the shocks have materialized. Such a policy-maker takes firms' pricing decisions and thus the aggregate price level as given and the output gap remains the only policy objective. Optimal discretionary policy is thus given by

$$q = y^* + a_y u,$$

where a_y is defined in Eq. (37). Clearly, discretionary policy generates inefficiently high price level volatility, i.e., price dispersion.

7. Optimal policy in a dynamic economy

This section extends the static setting considered so far to an infinite horizon economy. A dynamic setting is of interest because it allows an analysis of persistent shock disturbances, which generate additional policy incentives. In particular, monetary policy decisions then not only affect current economic outcomes but also the prior beliefs with which firms enter the next period. This intertemporal aspect of policy is absent in a static economy or when shocks are white noise.

Time is discrete, indexed by $t = 0, 1, 2, \ldots$, and the policy-maker commits to a policy rule at the beginning of period zero. In each period, steps 2–5 depicted in Fig. 1 take place, i.e., the central bank sets its policy and releases information that maximizes firms' current profits, and firms process the information released by the central bank and set profit-maximizing prices.[33] To simplify on notation, let us abstract from supply shocks and consider real demand shocks only.[34] The demand shock now evolves according to

$$u_t = \rho u_{t-1} + v_t, \tag{42}$$

where $v_t \sim ii\mathrm{N}(0, \sigma_v^2)$ and $\rho \in (-1, 1)$. For $\rho = 0$ the economy reduces to a sequence of independent static economies.

[33] It is an open question whether information processing that maximizes period-by-period profits also maximizes the present discounted value of profits.

[34] It is not difficult to show that supply shocks still do not generate a policy trade-off and should be nominally accommodated in the same way as in the static setup.

The policy-maker is assumed to maximize

$$-\mathrm{E}[y^2 + \lambda p^2], \tag{43}$$

where E[·] denotes the unconditional expectations operator.[35] The efficient level of output has been normalized to zero.

7.1. Simple policy rules

To facilitate the comparison with the static case, this section first considers a policy rule that conditions on the fundamental shock only, i.e.,

$$q_t = au_t, \tag{44}$$

where the reaction coefficient a remains to be determined. General policy rules will be considered in Section 7.2.

Following Woodford (2002), an equilibrium law of motion is conjectured of the form

$$X_t = MX_{t-1} + mv_t, \tag{45}$$

where

$$X_t = \begin{pmatrix} u_t \\ f_t \end{pmatrix} \quad \text{with } f_t = \sum_{m=0}^{\infty} (1-\xi)^m u_{t|t}^{(m+1)},$$

$$M = \begin{pmatrix} \rho & 0 \\ M_{21} & M_{22} \end{pmatrix}, \quad m = \begin{pmatrix} 1 \\ m_2 \end{pmatrix},$$

and where M_{21}, M_{22}, and m_2 are unknown parameters. Using (24), (44), and (4), the equilibrium price level and output level can be expressed as linear functions of X_t:

$$p_t = (\xi a + 1) f_t, \tag{46}$$

$$y_t = au_t - (\xi a + 1) f_t. \tag{47}$$

Thus, once the equilibrium law (45) has been determined, the objective function (43) is readily evaluated.

Appendix A.5 derives analytical expressions for the equilibrium values of M and m in Eq. (45). The equilibrium values must satisfy the following fixed point property: given the law of motion (45), optimal belief updating by firms must exactly generate (45) again.

The Appendix shows that the equilibrium law (45) depends on the parameters ξ, k, and ρ, but is independent of the policy parameter a.[36] This is the case because the optimal Kalman gains in agents' filtering equations happen to be independent of the policy function, as in the simple static case. The optimal policy problem, therefore, retains its simple linear quadratic structure and results do not depend on the variance of the innovations σ_v^2.

The upper panel of Fig. 3 illustrates the effects of persistent demand shocks on the optimal reaction coefficient, assuming $\lambda = \xi = 1$. It shows that shock persistence causes the optimal reaction coefficient to be on average closer to its perfect observability

[35] Maximizing (43) is identical to maximizing $-\mathrm{E}_0[\sum_{t=0}^{\infty} \beta^t (y_t^2 + \lambda p_t^2)]$ for the limiting case $\beta \to 1$.

[36] This does not imply that firms' beliefs about p_t are independent of policy; see Eq. (46).

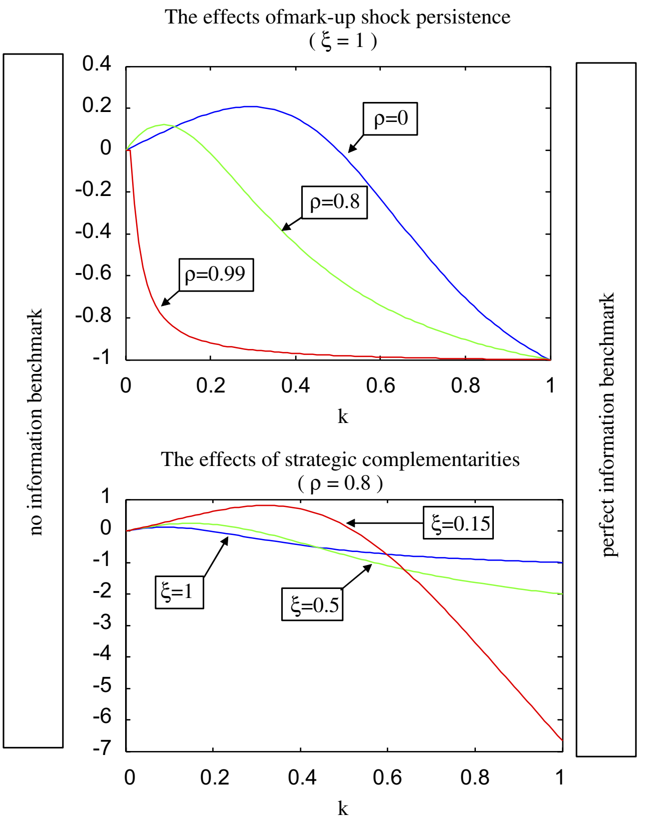

Fig. 3. Optimal policy reaction coefficients (real demand shocks).

benchmark $a = -1/\xi$. Persistence thus induces a policy shift towards more aggressive price stabilization.

This shift is optimal because persistent demand shocks imply that f_t follows (on average) more closely the pattern of u_t. In particular, simulations shows that for the limiting case $\rho \to 1$, the results $var(f_t)/var(u_t) \to (1/\xi)^2$ and $cov(f_t, u_t)/\sqrt{var(f_t)var(u_t)} \to 1$ are obtained. This suggests that the approximation $f_t \approx (1/\xi)u_t$ can be taken in Eqs. (46) and (47), i.e.,

$$p_t \approx \left(a + \frac{1}{\xi}\right)u_t,$$

$$y_t \approx -\frac{1}{\xi}u_t,$$

which implies that $a = -(1/\xi)$ is optimal.[37] Note that this is the optimal reaction coefficient with a pure price level objective in the static economy.

[37]This holds independently of the value of λ in (43).

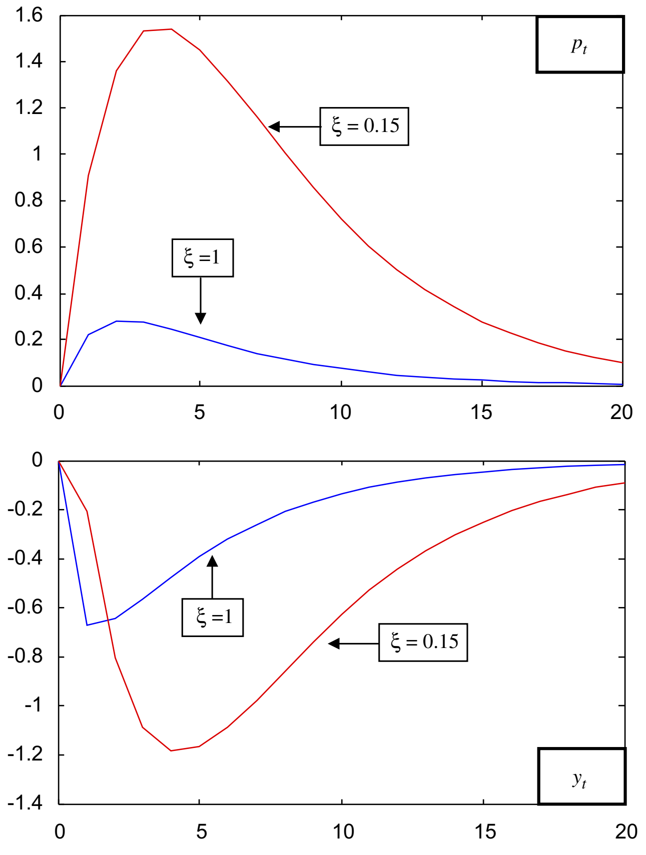

Fig. 4. Impulse responses ($\rho = 0.8$, $k = 0.4$).

The lower panel of Fig. 3 illustrates the effects of strategic complementarities on the optimal reaction coefficient when demand shocks are persistent.[38] The findings closely resemble those for the static economy. In particular, strategic complementarities make it optimal to accommodate demand shocks over a wider range of values for the processing index k. As before, strategic complementarities amplify the real effects of monetary policy and entail a more accommodative policy stance.

Fig. 4 depicts the response of the price level and output to a persistent demand shock under optimal policy.[39] Without strategic complementarities ($\xi = 1$) output drops immediately in response to the shock while prices display a hump-shaped pattern.[40] With strategic complementarities ($\xi = 0.15$) output and inflation are more sluggish and the peak

[38]The lower panel assumes equal weights to price and output targets ($\lambda = 1$) and persistent mark-up shocks ($\rho = 0.8$).

[39]The figure assumes equal weights to price and output targets ($\lambda = 1$), persistent mark-up shocks ($\rho = 0.8$), and $k = 0.4$, which implies that firms' Kalman gain for estimating ε_t is 0.4. The shock hits the economy in period 1.

[40]For other parameterizations the maximum drop in output is not necessarily immediate but may also occur with some delay.

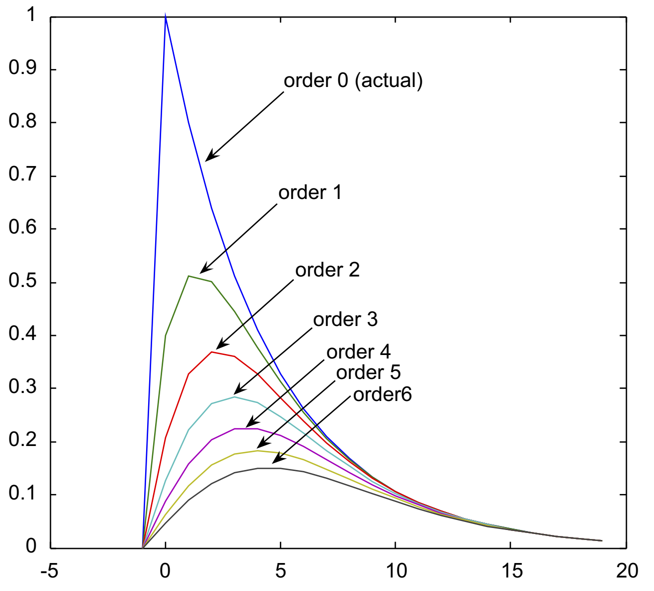

Fig. 5. Dynamics of higher order beliefs $u_{t|t}^{(j)}$ ($\rho = 0.8, k = 0.4, \xi = 0.15$).

reaction occurs with additional delay.[41] Importantly, for both parameterizations the peak of output and prices occurs at different points in time, suggesting that it is optimal to deviate from the price level targeting rule (39). The next section shows that this feature is independent of the form of the assumed policy rule.

7.2. General policy rules

This section considers policy rules of the form

$$q_t = a_0 u_{t|t}^{(0)} + a_1 u_{t|t}^{(1)} + a_2 u_{t|t}^{(2)} + \cdots + a_n u_{t|t}^{(n)}, \tag{48}$$

where n is an arbitrary positive integer. For $n > 0$ monetary policy conditions not only on the fundamental u_t but also on the average higher order beliefs of the fundamental. This is potentially important because—unlike in the static setup— there fails to be a simple linear relationship between u_t and $u_{t|t}^{(j)}$ ($j \geqslant 1$) in a dynamic setting. This is shown in the upper panel of Fig. 5, which depicts the evolution of beliefs of various order in response to a persistent real demand shock.[42] Beliefs of higher order react more sluggishly and reach their peak later than beliefs of lower order. Allowing monetary policy to condition on higher order beliefs thus adds linearly independent conditioning variables.

The upper panel of Fig. 6 shows the impulse responses of output and prices to a real demand shock under optimal monetary policy for various values of n.[43] Notable differences exist between the cases $n = 0$ and 6. In particular, for $n = 6$ the output response

[41]The absolute size of the peak response is more pronounced because complementarities amplify the effects of shocks.

[42]Figure 5 uses $k = 0.4$, $\xi = 0.15$, and $\rho = 0.8$.

[43]Figure 6 uses $k = 0.4$, $\xi = 0.15$, and $\rho = 0.8$. Appendix A.6 shows how to compute the impulse responses.

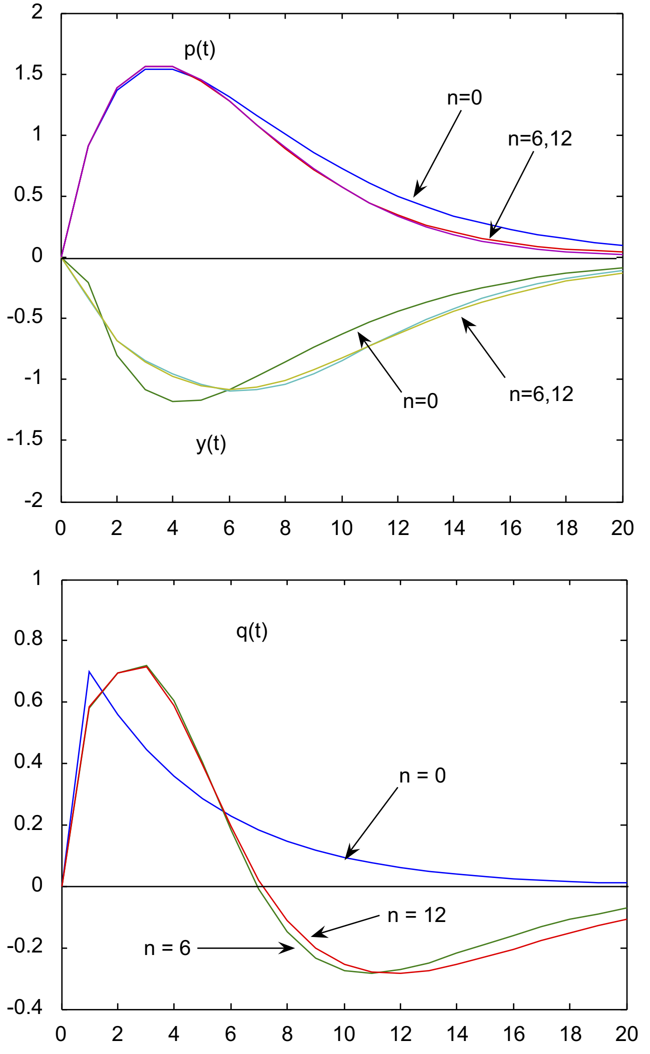

Fig. 6. Impulse responses for general policy rules ($\rho = 0.8$, $k = 0.4$, $\xi = 0.15$).

is more protracted while prices return faster to their steady-state value. The impulse responses for the cases $n = 6$ and 12, however, are virtually identical, indicating that beliefs of order higher than 6 have a negligible influence on the impulse response. Importantly, the responses for $n = 6, 12$ show that the peaks for output and inflation occur at different points in time. Therefore, unlike in Ball et al. (2005), the optimality of the price level targeting rule (39) does not generalize to a dynamic setting. The reasons for this result are explored below.

The lower panel of Fig. 6 depicts the optimal response of nominal demand. Again, the response for $n = 0$ differs substantially from that for $n = 6, 12$, with the difference between the latter being fairly small. The responses for $n = 6, 12$ show that, initially, as long as firms are still uncertain about the value of the real demand shock, it is optimal to nominally accommodate. Over time, however, firms become increasingly informed and information dispersion is reduced correspondingly. This weakens the real effects of monetary policy and makes it optimal to nominally contract, so as to stabilize the price level. The optimal trade-off between output and price stabilization thus shifts over time and this explains why the simple price level targeting rule (39) fails to be optimal. Numerical simulations show that the speed at which this trade-off shifts increases with firms' processing capacity k, the complementarity parameter ξ, and the persistence of shocks ρ, as suggested by the results derived for the simple static economy.

The fact that optimal monetary policy insures that price stability will hold only at a certain horizon in the future may potentially rationalize the medium-term approach to price stability adopted by some central banks. The current results suggest, however, that the horizon at which price stability is (approximately) achieved should depend on the persistence of the shocks to which monetary policy reacts, a feature that is usually not incorporated in monetary policy strategies. It seems important to explore this issue further in future work.

8. Conclusions

It has been shown by Phelps (1970) and Lucas (1973) that monetary policy has real effects when firms are only imperfectly informed about the shocks hitting the economy.

Considering firms that can process information only at a finite rate, this paper shows that information dispersion, i.e., ICK about shocks, may significantly enhance the real effects of nominal demand policy in a flexible price economy.

When firms' prices are strategic complements, prices respond rather sluggishly to shocks and policy decisions. This gives rise to substantial real effects of monetary policy and makes it optimal to stabilize the output gap. However, when mark-up shocks are persistent, the ability to affect the output gap is reduced over time and optimal policy is again increasingly characterized by price level stabilization.

Whether it is optimal to stabilize the output gap or the price level thus depends on the importance of strategic complementarities, on the degree of information frictions, and on the persistence of the shocks that hit the economy. Empirical work seeking to quantify the relative importance of these three determinants of optimal monetary policy would thus be of interest.

Appendix A

A.1. Derivation of the price setting equation

Consider the nonlinear economy outlined in Section 2. The product demand function associated with the Dixit–Stiglitz aggregator (2) is

$$Y^i(P^i) = (P^i/P)^{-\theta} Y, \tag{49}$$

where P^i is the price charged by firm i and

$$P = \left(\int (P^i)^{1-\theta} \, \mathrm{d}i \right)^{1/(1-\theta)}. \tag{50}$$

The profit maximization problem of firm i is given by

$$\max_{P^i} \mathrm{E}[(1+\tau)P^i Y^i(P^i) - WY^i(P^i)|I^i], \tag{51}$$

where τ denotes an output subsidy, and I^i firm i's information set, containing information about the labor supply shock v, the demand shock θ, and monetary policy decisions. Using (49) the first order condition is

$$\mathrm{E}\left[(1+\tau)(1-\theta)\left(\frac{P^i}{P}\right)^{-\theta} + \theta W\left(\frac{P^i}{P}\right)^{-\theta-1}|I^i\right] = 0. \tag{52}$$

Since θ is stochastic one cannot solve this expression for P^i as in the standard case but has to linearize before solving for P^i.[44] Using the household's first order condition

$$W = \frac{vV'(\widehat{Y})}{U'(Y)}P, \tag{53}$$

where

$$\widehat{Y} = \int Y^j \,\mathrm{d}j \tag{54}$$

and assuming

$$\tau = \frac{1}{\overline{\theta} - 1},$$

there exists a symmetric deterministic steady state with $P^i = \overline{P}$, $Y^i = \overline{Y}$, $\theta = \overline{\theta}$, and $v = 1$ where $\overline{Y}$ solves

$$\frac{V'(\overline{Y})}{U'(\overline{Y})} = 1 \tag{55}$$

and $\overline{P}$ is any value chosen by the central bank. Linearizing (52) around this steady state delivers

$$\frac{P^i - \overline{P}}{\overline{P}} = \mathrm{E}\left[\frac{P - \overline{P}}{\overline{P}} + \frac{V''(\overline{Y})U'(\overline{Y}) - V'(\overline{Y})U''(\overline{Y})}{(U'(\overline{Y}))^2}\overline{Y}\frac{Y - \overline{Y}}{\overline{Y}}\right. \tag{56}$$

$$\left. +(v-1) + \frac{1}{1-\overline{\theta}}\frac{\theta - \overline{\theta}}{\overline{\theta}} + \mathrm{O}(1)|I^i\right], \tag{57}$$

where O(1) denotes an approximation error of order smaller than one and where the fact that $Y = \widehat{Y} + \mathrm{O}(1)$ has been used. The first best output level Y^* solves

$$\frac{vV'(Y^*)}{U'(Y^*)} = 1. \tag{58}$$

Linearizing (58) around the steady-state delivers

$$v - 1 = -\frac{V''(\overline{Y})U'(\overline{Y}) - V'(\overline{Y})U''(\overline{Y})}{(U'(\overline{Y}))^2}\overline{Y}\left(\frac{Y^* - \overline{Y}}{\overline{Y}}\right). \tag{59}$$

[44] Thanks go to Christian Hellwig for pointing this out to me.

Substituting (59) into (56) delivers (3) where:

$$
\begin{aligned}
u_t &= \frac{1}{1-\overline{\theta}}\frac{(\theta-\overline{\theta})}{\overline{\theta}},\\
\xi &= \frac{V''(\overline{Y})U'(\overline{Y})-V'(\overline{Y})U''(\overline{Y})}{(U'(\overline{Y}))^2}\overline{Y}\\
&= \frac{V''(\overline{Y})\overline{Y}}{V'(\overline{Y})}-\frac{U''(\overline{Y})\overline{Y}}{U'(\overline{Y})}.
\end{aligned}
$$

A.2. A welfare-based monetary policy objective

Consider the nonlinear economy outlined in Section 2. A second-order expansion of the utility Ω of the representative agent around the steady state level $\overline{\Omega}$ is given by

$$
\begin{aligned}
\Omega-\overline{\Omega} &= U'(\overline{Y})(Y-\overline{Y})-V'(\overline{Y})(\widehat{Y}-\overline{Y})\\
&\quad+\frac{1}{2}U''(\overline{Y})(Y-\overline{Y})^2-\frac{1}{2}V''(\overline{Y})(\widehat{Y}-\overline{Y})^2\\
&\quad-V'(\overline{Y})(\widehat{Y}-\overline{Y})(v-1)+\mathrm{O}(2)+t.i.p.,
\end{aligned}
\tag{60}
$$

where $\widehat{Y}$ is defined in (54), *t.i.p.* denotes (first and higher order) terms that are independent of policy, and O(2) summarizes endogenous terms of order larger than two.

Substituting (59) into (60), using (55) and the fact that $Y=\widehat{Y}+\mathrm{O}(1)+t.i.p.$, one obtains

$$
\begin{aligned}
\Omega-\overline{\Omega} &= U'(\overline{Y})(Y-\overline{Y})-V'(\overline{Y})(\widehat{Y}-\overline{Y})\\
&\quad+\tfrac{1}{2}(U''(\overline{Y})-V''(\overline{Y}))(\widehat{Y}-\overline{Y})^2\\
&\quad-(U''(\overline{Y})-V''(\overline{Y}))(\widehat{Y}-\overline{Y})(Y^*-\overline{Y})+\mathrm{O}(2)+t.i.p.
\end{aligned}
$$

Adding $\frac{1}{2}(U''(\overline{Y})-V''(\overline{Y}))(Y^*-\overline{Y})^2$, which is a term independent of policy, one obtains

$$
\begin{aligned}
\Omega-\overline{\Omega} &= U'(\overline{Y})(Y-\overline{Y})-V'(\overline{Y})(\widehat{Y}-\overline{Y})\\
&\quad-\tfrac{1}{2}(V''(\overline{Y})-U''(\overline{Y}))((\widehat{Y}-\overline{Y})-(Y^*-\overline{Y}))^2+\mathrm{O}(2)+t.i.p.
\end{aligned}
\tag{61}
$$

A second order expansion of (2) yields after some tedious but straightforward calculations

$$
Y-\overline{Y}=(\widehat{Y}-\overline{Y})-\frac{1}{2}\frac{1}{\overline{\theta}\overline{Y}}\int(Y^j-\widehat{Y})^2\,\mathrm{d}j+\mathrm{O}(2)+t.i.p.
\tag{62}
$$

A first order approximation to (49) delivers

$$
Y^i=Y-\frac{\overline{\theta}\overline{Y}}{\overline{P}}(P^i-P)+\mathrm{O}(1)+t.i.p.
\tag{63}
$$

Integrating (63) over all firms, subtracting the result from (63), and taking squares delivers

$$
(Y^i-\widehat{Y})^2=\frac{\overline{\theta}\overline{Y}}{\overline{P}}(P^i-\widehat{P})+\mathrm{O}(2)+t.i.p.,
\tag{64}
$$

where

$$\widehat{P} = \int P^j \, \mathrm{d}j.$$

Using (62) and (64) one can express (61) as

$$\begin{aligned}\Omega - \overline{\Omega} = & -\frac{1}{2} U'(\overline{Y})\overline{\theta}\,\overline{Y} \int \left(\frac{P^j - \overline{P}}{\overline{P}} - \frac{\widehat{P} - \overline{P}}{\overline{P}} \right)^2 \mathrm{d}j \\ & -\frac{1}{2}(V''(\overline{Y}) - U''(\overline{Y}))\overline{Y}^2 \left(\frac{(\widehat{Y} - \overline{Y})}{\overline{Y}} - \frac{(Y^* - \overline{Y})}{\overline{Y}} \right)^2 + \mathrm{O}(2) + t.i.p. \qquad (65)\end{aligned}$$

Maximizing (65) is, thus, equivalent to maximizing

$$-(y - y^*)^2 - \gamma \int (p(j) - p)^2 \, \mathrm{d}j \qquad (66)$$

for

$$\gamma = \frac{\overline{\theta}}{\dfrac{V''(\overline{Y})\overline{Y}}{V'(\overline{Y})} - \dfrac{U''(\overline{Y})\overline{Y}}{U'(\overline{Y})}} > 0.$$

Eq. (30) implies

$$\int (p(j) - p)^2 \, \mathrm{d}j = \left(\frac{k}{1 - (1 - \xi)k} \right)^2 Var\,(\eta^i), \qquad (67)$$

where

$$VAR(\eta^i) = \frac{1 - k}{k} \mathrm{E}[(\xi q - \xi y^* + u)^2] \qquad (68)$$

by Eq. (20). Using (67), (68), and (31) the welfare-based objective (66) can be written as

$$\begin{aligned}& -(y - y^*)^2 - \gamma \frac{k(1 - k)}{(1 - (1 - \xi)k)^2} \mathrm{E}[(\xi q - \xi y^* + u)^2] \\ & \quad = -(y - y^*)^2 - \lambda p^2 \qquad (69)\end{aligned}$$

for $\lambda = \gamma(1 - k/k)$. Eq. (69) is the central bank's objective assumed in the text.

A.3. The optimal allocation of attention

The entropy of a m-dimensional normal random vector $X \sim \mathrm{N}(0, \Sigma_{xx})$ is given by

$$H(X) = \tfrac{1}{2}\log_2 [(2\Pi e)^m \det \Sigma_{xx}]. \qquad (70)$$

Constraint (17) can then be written as

$$2^{-2K} \leqslant \det(I_n - \Sigma_{zz}^{-1} \Sigma_{zs} \Sigma_{ss}^{-1} \Sigma_{sz}), \qquad (71)$$

where Σ_{zs} denotes the covariance matrix between Z and s (correspondingly for the other matrices). Using

$$\Sigma_{zs} = \Sigma_{zz}c,$$
$$\Sigma_{sz} = c'\Sigma_{zz},$$
$$\Sigma_{ss} = c'\Sigma_{zz}c + \sigma_\eta^2,$$

the determinant on the r.h.s. of (71) can be expressed as

$$\begin{aligned}\det(I_n - \Sigma_{zz}^{-1}\Sigma_{zs}\Sigma_{ss}^{-1}\Sigma_{sz}) &= \det\left(I_n - \frac{cc'\Sigma_{zz}}{c'\Sigma_{zz}c + \sigma_\eta^2}\right)\\ &= \det\left(\Sigma_{zz}^{1/2}\left(I_n - \frac{cc'\Sigma_{zz}}{c'\Sigma_{zz}c + \sigma_\eta^2}\right)\Sigma_{zz}^{-1/2}\right)\\ &= \det\left(I_n - \frac{\Sigma_{zz}^{1/2}cc'\Sigma_{zz}^{1/2}}{c'\Sigma_{zz}c + \sigma_\eta^2}\right)\\ &= 1 - \frac{c'\Sigma_{zz}c}{c'\Sigma_{zz}c + \sigma_\eta^2}\end{aligned}$$

and constraint (71) can be written as

$$1 - \frac{c'\Sigma_{zz}c}{c'\Sigma_{zz}c + \sigma_\eta^2} \geqslant 2^{-2K}. \tag{72}$$

Using the above notation, the objective function (18) can be expressed as

$$-Var(\zeta'Z|s) = -\left[\zeta'\Sigma_{zz}\zeta - \frac{(\zeta'\Sigma_{zz}c)^2}{c'\Sigma_{zz}c + \sigma_\eta^2}\right]. \tag{73}$$

The previous equation implies that for any c the value of $-Var(\zeta'Z|s)$ is maximized by choosing σ_η^2 as small as possible. As a result, (72) must hold with equality and can be solved for σ_η^2:

$$\sigma_\eta^2 = \frac{1}{(2^{2K} - 1)}c'\Sigma_{zz}c. \tag{74}$$

Substituting this expression into (73), maximization problem (18) can be expressed as

$$\max_c -\left[\zeta'\Sigma_{zz}\zeta - (1 - 2^{-2K})\frac{(\zeta'\Sigma_{zz}c)^2}{c'\Sigma_{zz}c}\right], \tag{75}$$

where the optimal noise follows from Eq. (74). The FOC's of (75) are given by

$$(c'\Sigma_{zz}c)\Sigma_{zz}\zeta - (\zeta'\Sigma_{zz}c)\Sigma_{zz}c = 0$$

and are solved by

$$c = \lambda\zeta,$$

where $\lambda \neq 0$ is some constant normalizing the signal. Choosing $\lambda = 1$, one obtains

$$c = \zeta,$$
$$\sigma_\eta^2 = \frac{1}{(2^{2K} - 1)} \zeta' \Sigma_{zz} \zeta,$$

as claimed in the text. The maximized objective is

$$-2^{-2K} \zeta' \Sigma_{zz} \zeta. \tag{76}$$

Now consider the case $Z \in R^2$ and suppose the central bank announces Z_1 and Z_2 separately. Below it is illustrated that this results in lower expected utility, even if firms allocate the available processing capacity optimally between the two announcements. To economize on notation it is assumed that $\Sigma_{zz} = I$.

With separate signals the maximization problem is given by

$$\max_{a_1, a_2, \sigma_1^2, \sigma_2^2} \quad - Var(\zeta_1 Z_1 + \zeta_2 Z_2 | s_1, s_2)$$
$$\text{s.t.} \tag{77}$$
$$s_1 = c_1 Z_1 + \eta_1, \tag{78}$$
$$s_2 = c_2 Z_2 + \eta_2, \tag{79}$$
$$K \geqslant H(Z_1, Z_2) - H(Z_1, Z_2 | s_1, s_2), \tag{80}$$

where $\eta_1 \sim \mathrm{N}(0, \sigma_1^2)$ and $\eta_2 \sim \mathrm{N}(0, \sigma_2^2)$ denote the observation noise for Z_1 and Z_2, respectively. It is assumed that η_1 and η_2 are mutually independent and independent of (Z_1, Z_2). Furthermore, $\zeta_1 \neq 0$ and $\zeta_2 \neq 0$, which implies that both fundamentals affect the firm's optimal decision. Without loss of generality one can label shocks such that $|\zeta_1| \geqslant |\zeta_2|$ and normalize the signals by choosing

$$c_1 = \zeta_1,$$
$$c_2 = \zeta_2.$$

The independence assumptions imply that (80) can be written as

$$K \geqslant H(Z_1) - H(Z_1 | s_1) + H(Z_2) - H(Z_2 | s_2), \tag{81}$$

where $H(Z_1) - H(Z_1 | s_1) \geqslant 0$ can be interpreted as the capacity allocated to observing Z_1 and $H(Z_2) - H(Z_2 | s_2) \geqslant 0$ as the capacity allocated to Z_2. Using (71), Eq. (81) simplifies further to

$$(1 - \rho_1)(1 - \rho_2) \geqslant 2^{-2K}, \tag{82}$$

where

$$\rho_1 = \frac{\zeta_1^2}{\zeta_1^2 + \sigma_1^2} \quad \text{and} \quad \rho_2 = \frac{\zeta_2^2}{\zeta_2^2 + \sigma_2^2}$$

are the signal-to-noise ratios for Z_1 and Z_2, respectively. For signals of the form (78) and (79), objective function (77) can be expressed as

$$-[\zeta_1^2(1 - \rho_1) + \zeta_2^2(1 - \rho_2)] \tag{83}$$

and the policy problem now consists of maximizing (83) over ρ_1 and ρ_2 subject to (82) and the constraint that $\rho_i \in [0,1]$ for $i = 1,2$. The first order conditions deliver:

$$\rho_1^* = \begin{cases} 1 - 2^{-K}\sqrt{\frac{\zeta_2^2}{\zeta_1^2}} & \text{for } \sqrt{\frac{\zeta_2^2}{\zeta_1^2}} > 2^{-K}, \\ 1 - 2^{-2K} & \text{otherwise,} \end{cases}$$

$$\rho_2^* = \begin{cases} 1 - 2^{-K}\sqrt{\frac{\zeta_1^2}{\zeta_2^2}} & \text{for } \sqrt{\frac{\zeta_2^2}{\zeta_1^2}} > 2^{-K}, \\ 0 & \text{otherwise.} \end{cases}$$

Firms will thus not pay attention to Z_2 unless it affects the optimal decision with a sufficiently high weight. As is easily verified,

$$-[\zeta_1^2(1-\rho_1^*) + \zeta_2(1-\rho_2^*)] < -2^{-2K}(\zeta_1^2 + \zeta_2^2), \tag{84}$$

where the expression on the left-hand side is the utility achieved with separate announcements and the expression on the right-hand side the utility achieved with a one-dimensional central bank announcement, which follows from (76) for $\Sigma_{zz} = I$.

A.4. The central bank announcement

The text assumes that the central bank announces (26). Below it is shown that the REE derived under this assumption is unaffected when the central bank announces (25) instead.

Let f denote the infinite sum of Eq. (25). Eq. (29) implies that in the REE where the central bank announces (26):

$$f = \sum_{m=0}^{\infty} ((1-\xi)k)^m (\xi q - \xi y^* + u) \tag{85}$$

and

$$\begin{aligned} \mathrm{E}[f|s^i] &= k\sum_{m=0}^{\infty} ((1-\xi)k)^m (\xi q - \xi y^* + u + \eta^i) \\ &= k\sum_{m=0}^{\infty} ((1-\xi)k)^m (\xi q - \xi y^* + u) + \frac{k}{1-(1-\xi)k}\eta^i. \end{aligned} \tag{86}$$

Next, suppose instead that the central bank announces (85) and firms observe

$$\tilde{s}^i = f + \tilde{\eta}^i.$$

Expectations are then given by

$$\begin{aligned} \mathrm{E}[f|\tilde{s}^i] &= k\tilde{s}^i \\ &= kf + k\tilde{\eta}^i \\ &= k\sum_{m=0}^{\infty} ((1-\xi)k)^m (\xi q - \xi y^* + u) + k\tilde{\eta}^i. \end{aligned} \tag{87}$$

The expectations in (87) are identical to ones in (86) if

$$\widetilde{\eta}^i = \frac{1}{1-(1-\xi)k}\eta^i,$$

which follows from the fact that the firm faces the same processing constraint, independently of which object is observed. Eq. (24) together with $\mathrm{E}[f|\widetilde{s}^i] = \mathrm{E}[f|s^i]$ then implies that agents set the same profit-maximizing price, independently of whether the central bank announces (25) or (26).

A.5. Simple policy in a dynamic economy

With just one shock, central bank communication consists of announcing the value of the shock value. Firms' signals are thus given by

$$s_t^i = h' X_t + \eta_t^i,$$

where $h' = (1, 0)$. Let $X_{t|t}$ denote agents' average believe about X_t based on information up to t. The Kalman filter equations then imply

$$X_{t|t} = (I - gh')MX_{t-1|t-1} + gh'(MX_{t-1} + mv_t), \tag{88}$$

where

$$g = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}$$

is a vector of Kalman gains that remains to be determined.

Now note that

$$f_t = \overline{\xi} X_{t|t}, \tag{89}$$

where $\overline{\xi} = (1, 1-\xi)$. Using (88), Eq. (89) can be expressed as

$$\begin{aligned} f_t = {} & ((1-z)\rho + (1-\xi)M_{21})u_{t-1|t-1} \\ & + (1-\xi)M_{22} f_{t-1|t-1} \\ & + z\rho u_{t-1} + zv_t, \end{aligned} \tag{90}$$

where

$$z = g_1 + (1-\xi)g_2. \tag{91}$$

Using Eq. (89) for $t-1$ to substitute $f_{t-1|t-1}$ in (90) delivers

$$\begin{aligned} f_t = {} & ((1-z)\rho + (1-\xi)M_{21} - M_{22})u_{t-1|t-1} \\ & + M_{22} f_{t-1} + z\rho u_{t-1} + zv_t. \end{aligned} \tag{92}$$

Eq. (92) is consistent with the second line of the conjectured equilibrium law (45) when

$$M_{21} = \rho z, \tag{93}$$

$$M_{22} = \rho(1 - \xi z), \tag{94}$$

$$m_2 = z. \tag{95}$$

The previous equations determine M and m in (45) up to z, which depends on the Kalman gains, see (91). To determine the Kalman gain note that the Kalman filter updating equations imply that

$$g = P_{t|t-1}h(h'P_{t|t-1}h + \sigma_\eta^2)^{-1}, \tag{96}$$

where σ_η^2 is the variance of the private sector observation error and $P_{t|t-1}$ denotes the prior uncertainty about X_t. By Eq. (20)

$$\sigma_\eta^2 = \frac{1-k}{k}h'P_{t|t-1}h, \tag{97}$$

where k is defined in (22). Substituting this result into (96) delivers

$$\begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = \begin{pmatrix} k \\ k\dfrac{P^{21}_{t|t-1}}{P^{11}_{t|t-1}} \end{pmatrix}, \tag{98}$$

where $P^{ij}_{t|t-1}$ denotes the (i,j)th element of $P_{t|t-1}$. Note that (98) already determines g_1. To find g_2 one has to compute the steady state values of $P_{t|t-1}$. The Kalman filter updating equations for P are

$$\begin{aligned} P_{t|t} &= P_{t|t-1} - P_{t|t-1}h(h'P_{t|t-1}h + \sigma_\eta^2)^{-1}h'P_{t|t-1} \\ &= \begin{pmatrix} (1-k)P^{11}_{t|t-1} & (1-k)P^{11}_{t|t-1} \\ (1-k)P^{11}_{t|t-1} & P^{22}_{t|t-1} - k\dfrac{P^{21}_{t|t-1}P^{12}_{t|t-1}}{P^{11}_{t|t-1}} \end{pmatrix}. \end{aligned} \tag{99}$$

Using (99) and

$$P_{t+1|t} = MP_{t|t}M' + mm'\sigma_v^2,$$

one can solve for the steady state values of $P^{11}_{t+1|t}$ and $P^{21}_{t+1|t}$. These are given by

$$P^{11}_{t+1|t} = \frac{\sigma_v^2}{1-\rho^2,(1-k)}, \tag{100}$$

$$P^{21}_{t+1|t} = \frac{z\sigma_v^2}{1-\rho^2(1-k)(1-z\xi)}\left(1+\rho^2\frac{1-k}{1-\rho^2(1-k)}\right). \tag{101}$$

Substituting (100) and (101) into the second line of (98) and using (91) one can analytically determine z. For $\rho = 0$ or $k = 1$:

$$z = \frac{k}{1-k+k\xi},$$

otherwise:

$$z = \frac{(1+k\xi-k+\rho^2(-1+k-k\xi+k^2\xi)-\sqrt{A})}{2\rho^2\xi(k-1)},$$

where

$$\begin{aligned} A = {} & 1 + k^2 + 2k\xi - 2k^2\rho^2 + 2k^2\rho^2\xi + 4k\rho^2 - 2\rho^2 - 2k - 2k^2\xi \\ & - 2k^3\xi\rho^2 + 4k^2\rho^4\xi - 2k^3\rho^4\xi - 2\rho^4 k + \rho^4 k^2 + k^2\xi^2 - 2k^2\xi^2\rho^2 \\ & + 2k^3\xi^2\rho^2 - 2\rho^4 k\xi + k^2\rho^4\xi^2 - 2k^3\rho^4\xi^2 + k^4\rho^4\xi^2 + \rho^4. \end{aligned}$$

This together with (93)–(95) completes the determination of the equilibrium law.

A.6. General policy rule in a dynamic setting

The dynamics of higher order beliefs can be computed as follows. In Appendix A.5 is has been shown that

$$\begin{pmatrix} u^{(0)}_{t|t} \\ u^{(1)}_{t|t} \end{pmatrix} = \begin{pmatrix} \rho & 0 \\ k\rho & (1-k)\rho \end{pmatrix} \begin{pmatrix} u^{(0)}_{t-1|t-1} \\ u^{(1)}_{t-1|t-1} \end{pmatrix} + \begin{pmatrix} 1 \\ k \end{pmatrix} v_t. \tag{102}$$

Applying the Kalman filter with observation equation

$$s = u^{(0)}_{t|t} + \eta^i_t,$$

to Eq. (102) delivers, after averaging across agents, the law of motion for $(u^{(1)}_{t|t}, u^{(2)}_{t|t})'$, i.e., the equilibrium law for beliefs of order 2. Applying the Kalman filter to the larger system $(u^{(0)}_{t|t}, u^{(1)}_{t|t}, u^{(2)}_{t|t})'$ then delivers the equilibrium law for $(u^{(1)}_{t|t}, u^{(2)}_{t|t}, u^{(3)}_{t|t})'$, i.e., beliefs of order 3. Iterating in this manner one can compute the dynamics for any order of beliefs.

From Eq. (24), averaged across firms, the policy rule (48), and

$$f^{(m)}_{t|t} = \frac{f^{(m-1)}_{t|t} - u^{(m)}_{t|t}}{1-\xi},$$

one obtains

$$\begin{aligned} p_t = {} & \left(1 + \xi a_0 + \frac{\xi a_1}{1-\xi} + \frac{\xi a_2}{(1-\xi)^2} + \cdots + \frac{\xi a_n}{(1-\xi)^n}\right) f_t \\ & - \left(\frac{\xi a_1}{1-\xi} + \frac{\xi a_2}{(1-\xi)^2} + \cdots + \frac{\xi a_n}{(1-\xi)^n}\right) u^{(1)}_{t|t} \\ & - \left(\frac{\xi a_2}{(1-\xi)} + \frac{\xi a_3}{(1-\xi)^2} + \cdots + \frac{\xi a_n}{(1-\xi)^{n-1}}\right) u^{(2)}_{t|t} \\ & - \cdots - \frac{\xi a_n}{(1-\xi)} u^{(n)}_{t|t}. \end{aligned}$$

The corresponding expression for output is given by

$$y_t = a_0 u^{(0)}_{t|t} + a_1 u^{(1)}_{t|t} + a_2 u^{(2)}_{t|t} + \cdots + a_n u^{(n)}_{t|t} - p_t.$$

The previous results show that the equilibrium price and output levels can be written as

$$p_t = \alpha'_p Z_t \tag{103}$$

$$y_t = \alpha'_y Z_t,, \tag{104}$$

where

$$Z_t = (u^{(0)}_{t|t}, u^{(1)}_{t|t}, \ldots, u^{(n)}_{t|t}, f_t)$$

and (α_p, α_y) depend on the coefficients a_n in the policy rule. The equilibrium law of motion for Z_t follows from the equilibrium law for the $u^{(n)}_{t|t}$ (determined above) and the law of motion for f_t derived in Appendix A.5. Once can thus compute the asymptotic covariance matrix Σ_{ZZ} of Z_t, where Σ_{ZZ} it is independent of policy. Optimal policy is then readily determined by maximizing over a_n

$$-(Var(y_t^2) + \lambda Var(p_t^2)) = -(\alpha_y' \Sigma_{zz} \alpha_y + \lambda \alpha_p' \Sigma_{zz} \alpha_p).$$

The impulse responses then follow from the law of motion for Z_t and Eqs. (103) and (104).

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