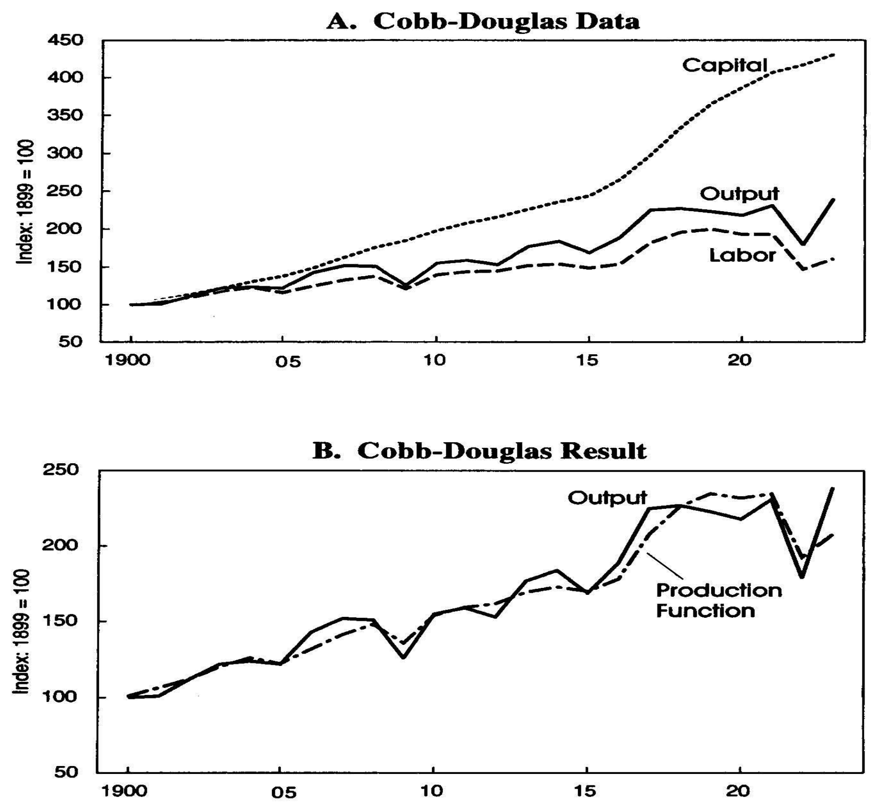
Lectures 11 & 12: Real business cycles

1. Solow and macroeconomic accounting
2. The real business cycle view: Kydland and Prescott, Long and Plosser
3. Prescott (86)
4. KPR (88)
5. Do productivity shocks need to be large?

1. The aggregate production function

- It is standard for us to think about economic activity at the level of the firm or industry using a production function
- But it once was not standard: it took work by economists like Paul Douglas to turn it from a theoretical device to one that was an a practical tool.
- Douglas noticed that shares paid to factors (like wn/y) were relatively constant as levels of inputs and outputs changed.
- Working with a UC mathematician, he determined a production function that had these properties and explored its fit

Figure 1


A. Cobb-Douglas Data
450
400
350
300
250
200
150
100
50
Index: 1899 = 100
Capital
Output
Labor
1900
05
10
15
20
B. Cobb-Douglas Result
250
200
150
100
50
Index: 1899 = 100
Output
Production
Function
1900
05
10
15
20

Production Function

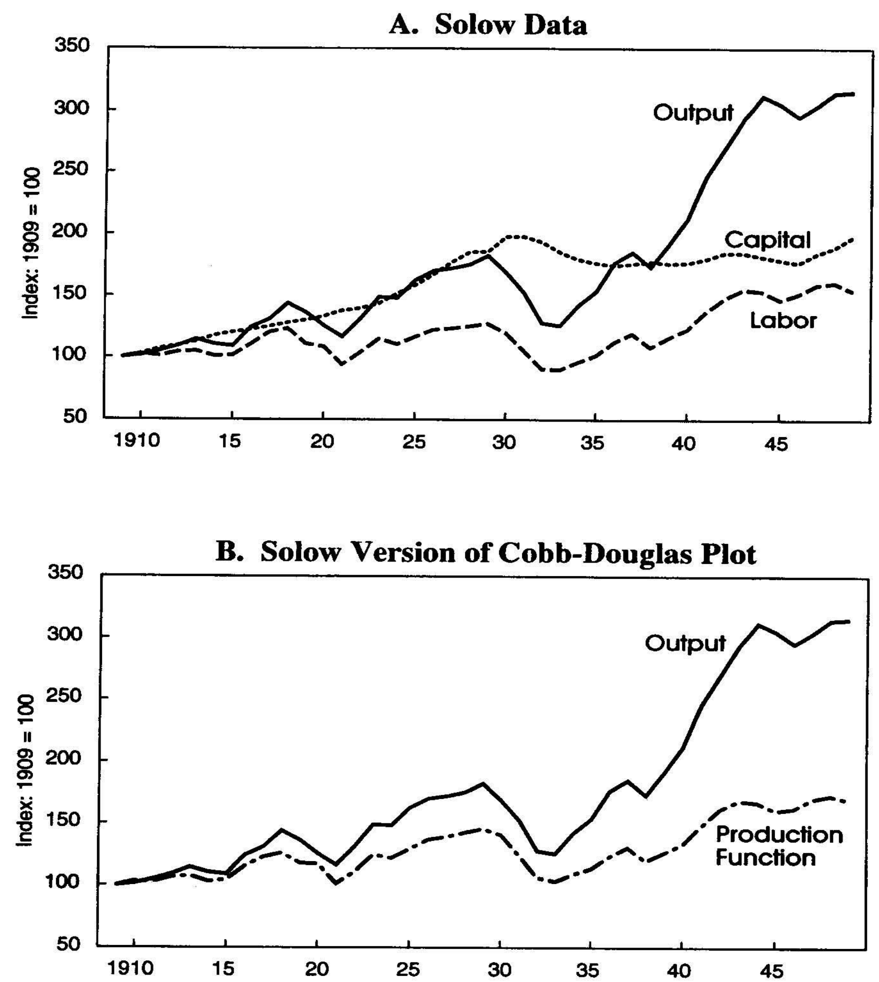
- CD production function

$$\log(Y_t) = \log(A) + \alpha \log(K_t) + (1-\alpha)\log(N_t) + e_t$$

Output = [Production Function] + error

- "Production Function" explains most of output

Figure 2


A. Solow Data
Index: 1909 = 100
350
300
250
200
150
100
50
1910
15
20
25
30
35
40
45
Output
Capital
Labor
B. Solow Version of Cobb-Douglas Plot
Index: 1909 = 100
350
300
250
200
150
100
50
1910
15
20
25
30
35
40
45
Output
Production Function

Solow

- Similar strategy, but very different findings
- [Some elements – such as using unemployment rate to correct for utilization are very important for year to year umbers, but not for longer-term evolution]
- Figure is based on equation below

$$\log(A_t) = \log(Y_t) - \alpha \log(K_t) - (1-\alpha)\log(N_t)$$

Residual (Produtivity) = Output - [Total Factor Input]

2. The Real Business Cycle hypothesis

F. Kydland and E.C. Prescott, "Time to Build and Aggregate Fluctuations," *Econometrica*, 1982.

J. Long and C.I. Plosser, "Real Business Cycles," *Journal of Political Economy*, 1983.

Background

Although real factors were suggested to be an important of sources by various authors (notably Hayek and Schumpeter in the 1920s and 1930s), the Keynesian revolution in macroeconomics led to a situation in which (a) changes in aggregate demand were viewed as the major source of business cycle impulses; and (b) various private market failures –involuntary unemployment– and various market frictions—including sticky prices and wages—were viewed as essential to understanding the positive nature and the normative consequences of economic fluctuations

Background

- Circa 1975, it was standard for macroeconomists to view “equilibrium activity” as a largely constant or slowly evolving value (a trend, perhaps) and to view cyclical variations as “disequilibrium phenomena”
- Circa 1975, the main debates among macroeconomists were mainly about whether monetary or fiscal impulses (or rules) had larger effects on economic activity and which should be used to stabilize fluctuations and to eliminate them if possible.

Background: The New Stimulae circa 1975

- Education of a generation of economists on the dynamic models of Solow and Cass that we just studied, focusing attention on economic dynamics in theory and the role of productivty in growth.
- Rise of quantitative (if static) models of general equilibrium in public finance and other areas.
- Lucas critique which suggested that existing policy models were inadequate and a new generation of models were necessary.

Basic RBC papers

- The initial set of dynamic general equilibrium models of the 1980s focused on productivity shocks as a source of economic fluctuations
- The Kydland-Prescott (henceforth KP) and Long-Plosser (henceforth LP) each described how a well-functioning market economy would respond to productivity shocks, concluding that it would display important characteristics of business cycles in the real world

Basic RBC papers

- Kydland and Prescott studied a variant of the one-sector neoclassical growth model, augmented to include variable labor supply, investment according to a “time to build” investment process and various other elements aimed at producing a modern business cycle model. Their model was driven by an aggregate productivity shock.
- Long and Plosser constructed a multisector neoclassical growth model, in which changes in sectoral productivity produced responses in other sectors and ultimately at the aggregate level in the economy.

Impact

- These papers launched a revolution in macroeconomics, which has resulted in methodology that is now used widely to study a wide range of problems (including in "New Keynesian" models that we will discuss later in the course).
- These papers stimulated much research into the influence of productivity on macroeconomic activity including work on its measurement, its origin and its consequences.

2A. Kydland-Prescott

- KP specified a version of the neoclassical growth model with several complications of preferences and the production function:
  - An investment technology designed to capture a range of phenomena described through the early section of the article, from which the paper takes its name.
  - Inventories (since inventories vary substantially over business cycles)
  - Variable labor (since labor varies over business cycles)
  - A utility function allowing for nonseparabilities in labor, designed to allow for different short-run and long-run labor supply elasticities.

Time to Build

- Let s_{jt} be the number of investment projects with j stages to go until they are completed and increment the capital stock.
- Suppose that there are J stages, so that

$$k_{t+1} - k_t = s_{1t} - \delta k_t$$

$$s_{j-1,t+1} = s_{j,t} \quad for \;\; j = 2, 3, 4, J$$

In this expression, s_{Jt} is the number of "starts", i.e., the number of new projects initiated in period t. Other projects simply move toward completion (they cannot be cancelled)

Investment

- Investment expenditures are distributed through time on any given project, with weights suggested by data on actual investment expenditure

$$i_t = \sum_{j=1}^{J} \varphi_j s_{jt} + (m_{t+1} - m_t)$$

where m_t is the predetermined stock of inventories at t so m_{t+1}-$m_t$is inventory investment and the other part is fixed investment (inventory notation differs from KP)

Production function

- Inventories are factor of production for KP, so that

$$y_t = f(a_t, n_t, k_t, m_t)$$

where we use our standard notation with "a" being productivity, "n" being labor input

- Further, the constraint on goods is then

$$c_t + i_t \le y_t$$

Utility Function

- KP posit a utility function of the form

$$u(c_t, l_t)$$

$$l_t = 1 - \gamma n_t - (1-\gamma)\eta \sum_{j=1}^{\infty} \eta^j n_{t-j}$$

$$= 1 - \gamma n_t - (1-\gamma)\eta z_t$$

$$z_{t+1} = (1-\eta) z_t + n_t$$

Utility Function (cont'd)

- This utility function imbeds the idea that supply of work is more costly if one has recently worked a lot.
- It therefore allows for a higher response of work to temporary variations in opportunities (temporary changes in wages) than permanent ones
- It is set up to allow for dynamic programming: z_t is a state variable at date t

Other features not emphasized in lecture

- Two stage decision structure, with some variables chosen before others
- Incomplete information about permanent or temporary nature of aggregate productivity situation (originally included to allow a potential source of monetary nonneutrality).

Computing Equilibrium

- KP set up a dynamic programming problem, essentially

$$v(k_t,[s_{jt}]_{j=1}^{J-1},m_t,z_t,\varsigma_t)=\max\{u(c_t,l_t)$$
$$+\beta E_t v(k_{t+1},[s_{j,t+1}]_{j=1}^{J-1},m_{t+1},z_{t+1},\varsigma_{t+1}\}$$

$$st \quad c_t+i_t=f(a(\varsigma_t),k_t,n_t,m_t)$$
$$i_t=\sum_{j=1}^{J}\varphi_j s_{jt}+(m_{t+1}-m_t)$$
$$k_{t+1}-k_t=s_{1t}-\delta k_t$$
$$s_{j-1,t+1}=s_{j,t} \quad for \;\; j=2,3,4,J$$
$$l_t=1-\gamma n_t-(1-\gamma)\eta z_t$$
$$z_{t+1}=(1-\eta)z_t+n_t$$

Computing equilibria (cont'd)

- KP designed their economy so that it was linear in the constraints, once they substituted out for consumption from the first constraint.
- They knew that optimization models with quadratic objectives and linear constraints (like those which we studied last term) could be rapidly and easily computed, resulting in linear decision rules, so that they sought a quadratic approximation to a reduced form objective (or indirect utility function).

Computing Equilibrium(cont'd)

"To determine the equilibrium process for this model, we exploit the well-known result that, in the absence of externalities, competitive equilibria are Pareto optima. With homogenous households, the relevant Pareto optimum is the one which maximizes the utility of the representative consumer subject to the technology constraints and the information structure" KP, pg. 1354.

Computing equilibria (cont'd)

"Approximations are necessary before equilibrium decision rules are computed. Our procedure is to determine the steady state for the model with no shocks to technology. Next, quadratic approximations (of an objective) are made in the neighborhood of the steady state. Equilibrium decision rules of the resulting approximate economy are then computed. These decision rules are linear, so … the approximated economy is generated by a system of stochastic difference equations for which covariances are easily determined." KP 1355.

Testing the theory

- KP propose a specific test of their theory, explained on pages 1359-1360.
- They begin by choosing most of the parameters of their model to match micro observations and steady state facts, a method of parameter selection which they call “calibration”.
- They then evaluate how well their model does at reproducing certain facts about business cycles (which they refer to as “the deviations”).

Example

- “The model is consistent with the high (percentage) variability in investment and the low variability in consumption and their high correlation with output.” KP, page 1364.

KP’s conclusion

- “The preference-technology environment was the simplest one that explained the quantitative comovements and serial correlation properties of output. The fit of the model was surprisingly good in light of the model’s simplicity.”
- “A crucial component of the model that contributed to persistence of output movements was the time-to-build requirement.”

KP's conclusions (cont'd)

- KP suggest that models of this class could be used to study design of policy rules, in a manner consistent with the Lucas critique, but that these analyses will require further developments of methods, as models with general policy rules are typically not models in which competitive equilibria are pareto optima.

2B. Long-Plosser

- Considered how two key economic ideas could lead to business cycle phenomena.

  1. The idea that consumers generally desire variety of goods and so will spread increments to wealth across many different commodities at many different dates.

  2. That producers generally require many different produced inputs to efficiently produce final goods.

LP's focus on particular aspects of business cycles

- Business cycles involve strong positive serial correlation of aggregates: *persistence*
- Over the course of business cycles expansions and contractions, many different sectors rise or fall together: *comovement*

LP's approach

- Describe general model in which ideas can be explored.
- Analyze in detail a particular analytical solution, obtained by making specific restrictions on preferences and technology (essentially loglinear preferences and Cobb-Douglas technology).
- We will discuss this model using notation which is compatible with the preceding presentation of KP and the later presentation of other models.

LP model elements

- Preferences
  - Depend on a J element vector of consumer goods, with c_t being the date t vector and c_{jt} the quantity of the jth good (jth element of vector).
  - Restricted so demand for each good increases with wealth (positive wealth elasticities)
  - Depend on the quantity of leisure (do not introduce dynamic elements as in KP)

$$U = \sum_{t=o}^{\infty} \beta^t u(c_t, l_t)$$

LP model elements

- Sectoral production $y_{i,t+1}$ depends on
  - labor allocated to the sector n_{it}
  - the amount of intermediate inputs from other sectors m_{ijt}, with i being the sector of use and and j being the sector of origin
  - a sector-specific shock $a_{i,t+1}$

$$y_{j,t+1} = f_j(a_{j,t+1}, n_{jt}, [m_{ijt}]_{j=1}^{J})$$

LP elements
(indicated by notation on prior page)

- f_j indicates that production function in sectors need not be same
- Dating of outputs (t+1) and inputs (t) indicates that length of period is the amount of time that production requires, which is assumed constant across sectors

LP elements

- Write production function compactly as

$$y_{t+1} = F(a_{t+1}, n_t, m_t)$$

where
  - y is the output vector (J by 1)
  - a is the productivity vector (J by 1)
  - n is the labor input vector (J by 1)
  - m is the intermediate input matrix (J by J)

LP model elements

- There are constraints

$$c_{jt} + \sum_{i=1}^{J} m_{ijt} = y_{jt}$$

$$1 = l_t + \sum_{i=1}^{J} n_{it}$$

Equilibrium computation: Solving Robinson Crusoe's problem

$$v(y_t, \varsigma_t) = \max\{u(c_t, l_t) + \beta E_t v(y_{t+1}, \varsigma_{t+1})\}$$

$s.t.$

$$c_t + (\phi m_t)' = y_t$$

$$y_{t+1} = F(a_{t+1}, n_t, m_t)$$

$$l_t + \phi n_t = 1$$

with ϕ being a J element row vector of 1s so that this sums elements of labor vector and $(\phi m_t)'$ is the column vector of total input use.

Example

- Preferences

$$u(c_t, l_t) = \theta_0 \log(l_t) + \sum_{j=1}^{J} \theta_j \log(c_{jt})$$

- Production

$$\log(y_{i,t+1}) = \log(a_{i,t+1}) + \alpha_i^n \log(n_{it}) + \sum_{j=1}^{J} \alpha_{ij}^x \log(m_{ijt})$$

LP solution

- Remarkable (positively) in that they are able to actually generate an exact loglinear difference system as the solution for the "policy functions". Generalizes solution we explored in earlier "computational dp lecture" to many goods
- Remarkable (negatively) that labor in the economy as a whole and to each sector is constant in this solution, reflecting a complicated general equilibrium pattern of offsetting wealth and substitution effects.

LP solution

- Takes the form

$$\log(y_{t+1}) = \kappa + \alpha^x \log(y_t) + \log(a_{t+1})$$

$$where\ \alpha^x = [\alpha^x_{ij}]$$

- Depends centrally on "input output" information from matrix of cost-shares α^x
- Is invariant to nature of driving process (analogous to Sandmo's offsetting effects of return level and uncertainty)

Calibration

- Choice of sectors (degree of disaggregation)

- Input-output information

Analysis

- Response of system to transitory shocks: how does output shock in sector i respond to events in sector j through time? (depends on input-output structure)
- How much persistence – measured by serial correlation – do mechanisms produce? (some)
- How much comovement – measured by correlation—do mechanisms produce? (some)
- How does system respond to permanent productivity shocks, e.g., in a simulation? (looks more like macro data)

LP conclusions

- Real shocks and real mechanisms may be important for business cycle (enduring)
- Sectoral structure and sectoral shocks are important (largely unexplored, but tantalizing research problem)
- "Technical progress is possible in obtaining explicit solutions to models with alternative specifications of preferences and technology": 20 years later, we don't have many additional ones
- "Empirical research": active with highly aggregated models as in KP but relatively little with models of sectoral interactions and/or shocks

Questions raised by initial RBC studies

- How should we best measure productivity change and to what extent is it exogenous to business cycle?
- How important are various mechanisms stressed in individual studies? (e.g., time-to-build)
- How should we select parameters for and evaluate the performance (fit) of simple dynamic models?
- How should we solve models with suboptimal equilibria?

3. Prescott and "Theory Ahead of Business Cycle Measurement"

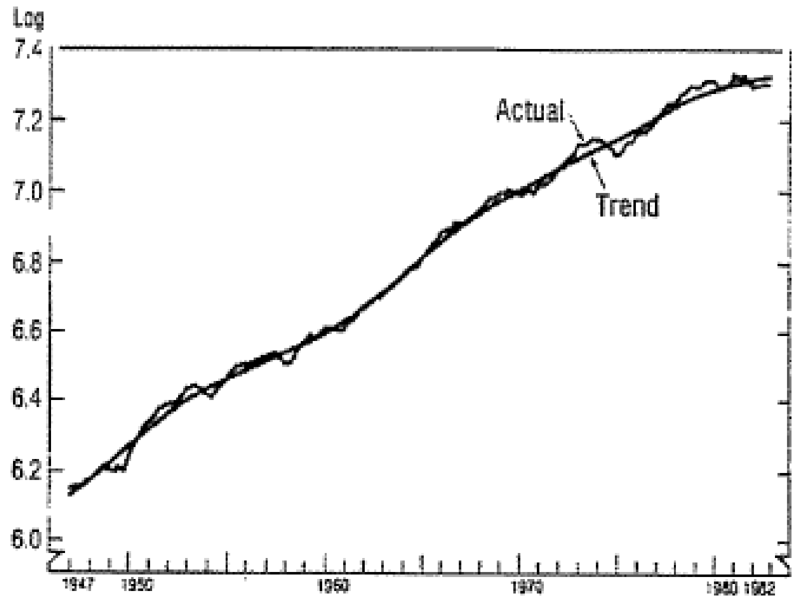
- Detrending of data via "HP filter"
- Computation of moments "stylized facts"
- Exploration of simple model via moments and stochastic simulations
- Role of highly elastic labor supply

Detrending of data via Hodrick-Prescott filter

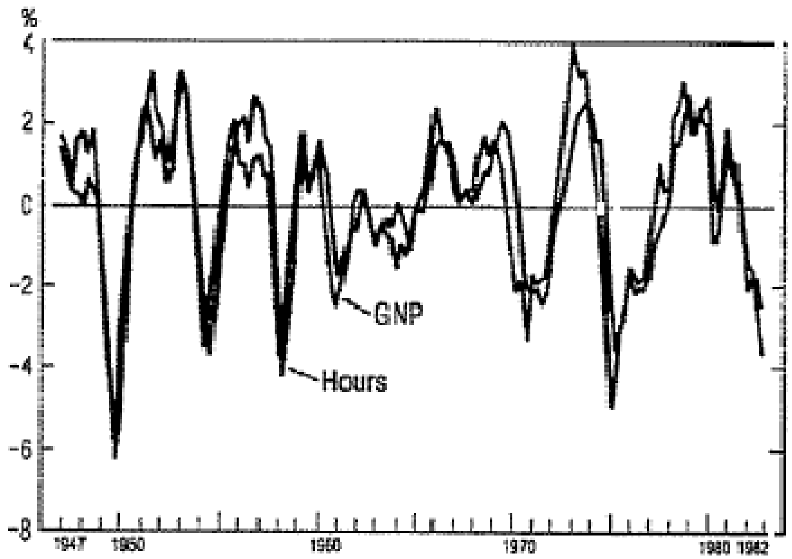
- Filtered outcomes are solutions to a minimization problem
- “best estimate” of unobserved component if there is a particular probability model
- More generally, a procedure that eliminates low frequency elements from the data

$$d_t = \sum_{j=-J}^{J} m_j y_{t-j} \quad \text{with } m_j = m_{-j} \text{ and } \sum_{j=-J}^{J} m_j = 0$$

Figure 1
Actual and Trend Logs of U.S. Gross National Product
Quarterly, 1947–82



Source of basic data: Citicorp's Citibase data bank



Source of basic data: Citicorp's Citibase data bank

Stylized Facts (Moments)

Table 1

Cyclical Behavior of the U.S. Economy

Deviations From Trend of Key Variables, 1954:1–1982:4

Variable x	Standard Deviation	Cross Correlation of GNP With		
		$x(t-1)$	$x(t)$	$x(t+1)$
Gross National Product	1.8%	.82	1.00	.82
Personal Consumption Expenditures				
Services	.6	.66	.72	.61
Nondurable Goods	1.2	.71	.76	.59
Fixed Investment Expenditures	5.3	.78	.89	.78
Nonresidential Investment	5.2	.54	.79	.86
Structures	4.6	.42	.62	.70
Equipment	6.0	.56	.82	.87
Capital Stocks				
Total Nonfarm Inventories	1.7	.15	.48	.68
Nonresidential Structures	.4	-.20	-.03	.16
Nonresidential Equipment	1.0	.03	.23	.41
Labor Input				
Nonfarm Hours	1.7	.57	.85	.89
Average Weekly Hours in Mfg.	1.0	.76	.85	.61
Productivity (GNP/Hours)	1.0	.51	.34	-.04

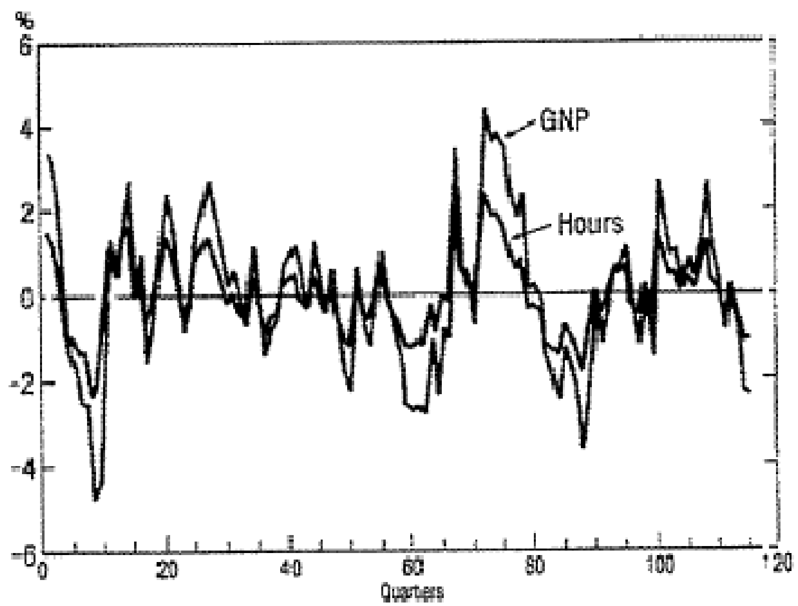
Source of basic data: Citicorp's Citibase data bank

Simple model

- Basic neoclassical model (Ramsey, Cass, Koopmans)
- With productivity shock (as in Solow’s empirics)
- With u(c,l): endogenous labor and leisure

Implications

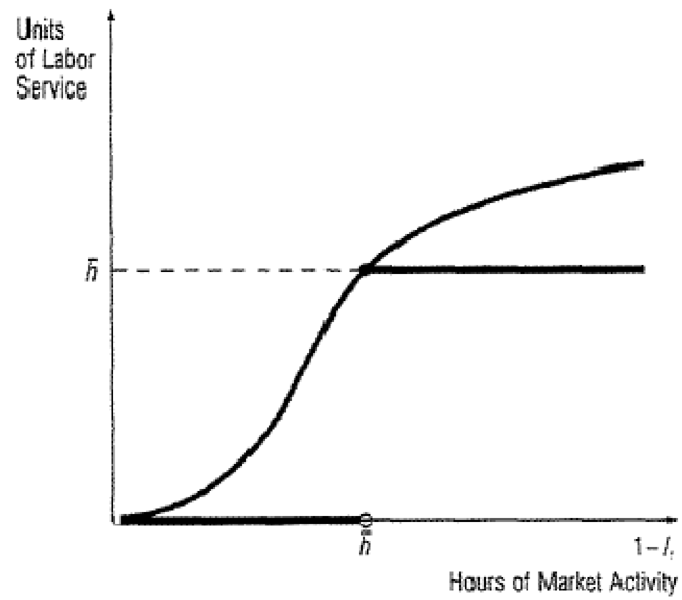
Figure 3
Deviations From Trend of GNP and Hours Worked in the Basic Growth Economy



Labor supply on the extensive margin

Figure 4

Relation Between Time Allocated to Market Activity and Labor Service



Labor supply on the intensive margin

Standard model:

$$\max \quad u(c,l) \;\; \text{s.t.} \;\; c + \ldots \leq w(1-l) + \ldots$$

Discrete model

$$\max \quad (1-\pi)u(c_1,1) + \pi u(c_2, 1-\bar{n})$$

$$s.t. \quad (1-\pi)c_1 + \pi c_2 \ldots \leq w\pi + \bar{n} \ldots$$

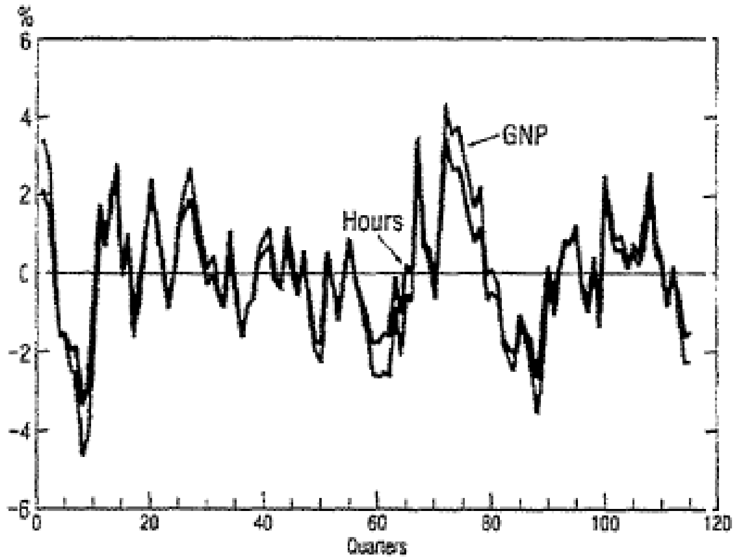
Key points

- Lotteries
- Efficient consumption equated across individuals
- Labor supply elasticity is effectively infinite (holding fixed marginal utility of consumption)
- Works, but what does this correspond to in real world?

Implications for labor-output comovement in simulation

Figure 5

Deviations From Trend of GNP and Hours Worked in Hansen's Indivisible Labor Economy



Source. Gary D. Hansen, Department of Economics, University of California, Santa Barbara

Conclusions of Prescott

- Basic theory “works” in sense of explaining a great deal of cyclical fluctuations
- Theory should be used to guide future measurement (e.g., aggregation of workers with diverse skills)
- Stabilization policy is unnecessary or undesirable, as outcomes are approximately optimal

4. KPR and the mechanics of Real Business Cycles:

R.G. King, C.I. Plosser and S.T. Rebelo, "Production, Growth and Business Cycles," 2 papers in *Journal of Monetary Economics*, 1988.

Another related paper: R.G. King and S.T. Rebelo, "Resuscitating Real Business Cycles," in *Handbook of Macroeconomics* 1999

Questions posed by KPR

- What are the business cycle properties of the basic neoclassical model of capital accumulation, when it is augmented by productivity shocks?
- What are desirable strategies for computing linear approximation (or loglinear approximation) solutions to dynamic equilibrium models?
- Do these methods continue to work if there is a stochastic trend in productivity, i.e., there are permanent variations in the level of productivity
- If government policy or private market failure leads competitive equilibrium to be suboptimal, are entirely new methods necessary?

Recipe: Part A

1. Start with model which includes technical progress and describes a growing economy (depends on t via deterministic trend growth)
2. Restrict preferences and technology so that steady state growth is feasible
3. As in growth models, find a transformed economy which is stationary (i.e, in which the equations do not depend on t except as subscript)
4. Write down a discrete time Lagrangian
5. Find the FOCs and TC for this economy
6. Find the stationary point of these FOCS (which surely satisfies the TC)

Recipe: Part B

1. Linear/loglinear approximation
2. Solve the resulting linear RE model in general: {Y} depends on {X}
3. Calculate the solution under a particular driving process, obtaining a solution in state space form
4. Use this solution to calculate
   a. Comparative dynamics (impulse responses)
   b. Population moments
   c. Stochastic simulations
5. Use economic analysis to interpret solution: e.g., permanent income theory including Fisher's rule for consumption growth and real interest rate

Extensions of Recipe

- KPR88b: To suboptimal equilibria
- KPR88b: To permanent variations in productivity (stochastic trend productivity) motivated by unit root findings
- Later: to all sorts of real and monetary models, including New Keynesian models
- Now routine:
  - Write down FOCs and other conditions of equilibrium
  - Linearize around appropriately defined stationary point
  - Get dynamic outcomes using linear RE techniques

Recipe: Part A

- The original economy: production with deterministic technical progress expressible in labor augmenting form

$$Y_t = A_t F(K_t, X_t N_t) \quad and \quad X_{t+1} = \gamma X_t$$

$$\Rightarrow y_t = A_t F(k_t, N_t) \;\; with\, y_t = \frac{Y_t}{X_t}\, and\; k_t = \frac{K_t}{X_t}$$

[form necessary for ss growth]

Consumption, Investment and Capital Accumulation

$$C_t + I_t = Y_t \ \ and \ \ K_{t+1} = (1-\delta)K_t + I_t$$

$$\Rightarrow c_t + i_t = y_t \ \ and \ \ \gamma k_{t+1} = (1-\delta)k_t + i_t$$

$$with\, c_t = \frac{C_t}{X_t} and\, i_t = \frac{I_t}{X_t}$$

$$------------------------$$

$$(1-\delta)\frac{K_t}{X_t} + \frac{I_t}{X_t} = \frac{K_{t+1}}{X_t} = \frac{K_{t+1}}{X_{t+1}}\frac{X_{t+1}}{X_t} = \gamma k_{t+1}$$

Labor and leisure

- Per-capita hours of work and leisure must be constant in steady state
- This comes despite fact that wage rate will grow due to technical progress

$$W_t = wX_t = wX_0\gamma^t$$

Preference restrictions necessary for ss growth to be optimal

- Need: constant consumption growth in face of constant ss real interest rate (implied by above)
- Need: constant work level in face of real wage growth
- Utility (general): u(C,L)

Preference restrictions
(sometimes called KPR utility)

- Intertemporal MRS: must be constant elasticity (as in permanent income model)
- MRS between work and leisure, must display invariance to growing consumption and wage rate (offsetting income and substitution effects)

$$u(c,L)=\frac{1}{1-\sigma}c^{1-\sigma}v(L)\ or\ u(c,L)=\log(c)+v(L)$$

Original and modified utility

For first utility function above

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) = (X_0)^{1-\sigma} \sum_{t=0}^{\infty} (\beta\gamma^{1-\sigma})^t u(c_t, L_t)$$

For both utility functions:

(i) X_0 affects welfare but not preferences (over c,L), so set $X_0=1$ for convenience and abstract from it

(ii) modification of discount factor

Solving optimization problem

- Via Lagrangian

$$L = \sum_{t=0}^{\infty} (\beta^*)^t u(c_t, L_t)$$
$$+ \sum_{t=0}^{\infty} (\beta^*)^t \lambda_t [A_t f(k_t, N_t) + (1-\delta)k_t - \gamma k_{t+1} - c_t]$$
$$+ \sum_{t=0}^{\infty} (\beta^*)^t \omega_t [1 - N_t - L_t]$$

$$Concepts: (\beta^*)^t \lambda_t \; is \; shadow \; price \; of \; c, y$$
$$(\beta^*)^t \omega_t \; is \; shadow \; price \; of \; N, L$$

FOCs+TC

$$c_t : (\beta^*)^t [D_1 u(c_t, L_t) - \lambda_t] = 0$$
$$L_t : (\beta^*)^t [D_2 u(c_t, L_t) - \omega_t] = 0$$
$$N_t : (\beta^*)^t [-\omega_t + \lambda_t A_t D_2 f(k_t, N_t)] = 0$$
$$k_{t+1} : (\beta^*)^t [-\lambda_t + \beta^* \lambda_{t+1} (A_{t+1} D_1 f(k_{t+1}, N_{t+1}) + (1-\delta))] = 0$$
$$(\beta^*)^t \lambda_t : [A_t f(k_t, N_t) + (1-\delta) k_t - \gamma k_{t+1} - c_t] = 0$$
$$(\beta^*)^t \omega_t : [1 - N_t - L_t] = 0$$

$$TVC : \lim_{t \to \infty} (\beta^*)^t \lambda_t k_{t+1} = 0$$

Stationary point

- Constrained by

$$c:[D_1u(c,L)-\lambda]=0$$
$$L:[D_2u(c,L)-\omega]=0$$
$$N:-\omega+\lambda AD_2f(k,N)=0$$
$$k:[-\lambda+\beta^*\lambda(AD_1f(k,N)+(1-\delta))]=0$$
$$\lambda:[Af(k,N)+(1-\delta)k-\gamma k-c]=0$$

Recipe: Part B

- Linearization/loglinearization

$$\xi_{cc}\hat{c}_t + \xi_{cL}\hat{L}_t - \hat{\lambda}_t = 0$$

$$\xi_{Lc}\hat{c}_t + \xi_{LL}\hat{L}_t - \hat{\omega}_t = 0$$

$$-\hat{\omega}_t + \hat{A}_t + \xi_{nk}\hat{k}_t + \xi_{nk}\hat{k}_t = 0$$

$$N^*\hat{N}_t + L^*\hat{L}_t = 0$$

$$-\hat{\lambda}_t + \hat{\lambda}_{t+1} + \eta_A\hat{A}_{t+1} + \eta_k\hat{k}_{t+1} + \eta_N\hat{N}_{t+1} = 0$$

$$-[\hat{A}_t + s_N\hat{N}_t + s_k\hat{k}_{t+1}] + [s_c\hat{c}_t + s_i\phi\hat{k}_{t+1} - s_i(\phi-1)\hat{k}_t] = 0$$

Solution of Linear Model

- Stage 1: Relate endogenous variables to sequences of exogenous variables (as in Blanchard-Kahn)

$$\hat{k}_{t+1} = \mu_1 \hat{k}_t + \psi_1 \hat{A}_t + \psi_2 \sum_{j=0}^{\infty} \mu_2^{-j} E_t \hat{A}_{t+j+1}$$

- Stage 2: Assuming driving process and evaluate discounted sums

$$\hat{A}_t = \rho \hat{A}_{t-1} + e_t$$

$$\hat{k}_{t+1} = \mu_1 \hat{k}_t + [\psi_1 + \psi_2 \frac{\rho}{1-(\rho / \mu_2)}] \hat{A}_t = \mu_1 \hat{k}_t + \pi_{kA} \hat{A}_t$$

$$Y_t = \Pi s_t \qquad\qquad s_t = M s_{t-1} + G e_t$$

- Linear difference system (Bellman)
- State space model (Chow/Hamilton)
- RE solution (from BK or KW)
- Used for calculations of responses to one-time shocks (impulse responses); stochastic simulations; calculations of moments…

$$\begin{bmatrix} y_t \\ c_t \\ i_t \\ n_t \\ \ldots \end{bmatrix} = \begin{bmatrix} \pi_{yk} & \pi_{ya} \\ & \\ & \\ & \\ & \end{bmatrix} \begin{bmatrix} k_t \\ a_t \end{bmatrix}$$

$$\begin{bmatrix} k_t \\ a_t \end{bmatrix} = \begin{bmatrix} \mu & \pi_{ka} \\ 0 & \rho \end{bmatrix} \begin{bmatrix} k_{t-1} \\ a_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e_t$$

Substantive conclusions from KPR

- Productivity shocks (A) must be persistent to generate business cycle phenomena: these give rise to important wealth effects on consumption.
- Although labor is invariant to trend growth in productivity (X), it varies sharply in response to productivity shocks (A)
- Large productivity variations are necessary to produce substantial volatility in output [not stressed in paper, but implicit in graphs].

Do we need large productivity shocks?

- KR in *Handbook of Macroeconomics* review developments from 1988 to 1999 when there was:
  - A huge number of RBC studies
  - A rising concern that productivity shocks measured via Solow residual were “too big” and “to important” to RBC modeler conclusions

Switch in model presentation

- Recursive optimization

$$V(k_t A_t) = \max\{u(c_t, L_t) + \beta^* E_t V(k_t, A_{t+1})\}$$

$$A_t f(k_t, N_t) + (1-\delta)k_t - \gamma k_{t+1} - c_t] \geq 0$$

$$[1 - N_t - L_t] \geq 0$$

FOCs + ET

FOCs + ET

KR “volatility enhancing” strategy

- Modified basic 1988 model to introduce several key features that had previously been studied separately
  - Labor choice on extensive margin (to work or not) rather than intensive margin (how many hours to work)
  - Varying utilization of capital

Implications

- Model economy:
  - Solow residual as poor proxy for actual productivity shock (utilization not observed)
  - High response to actual productivity shocks (so the implied shocks could be smaller)
  - Alternative method of extracting shocks