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# The magnitude–frequency distribution of earthquakes recorded with deep seismometers at Cajon Pass, southern California

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## Abstract

The cumulative $b$-value (the slope of the Gutenberg–Richter relationship between earthquake occurrence rate and magnitude) is commonly found to be constant (~1). Network catalogues, however, reveal a decrease at small magnitudes (<3). Some recent studies have suggested that this decrease in $b$-value is not just an artifact of catalogue incompleteness, but that small earthquakes are really not as numerous as a constant $b$-value extrapolated from larger events would predict. In the Cajon Pass area, southern California, the $b$-value of seismicity recorded by the local network (SCSN) appears to decrease below about $M_L$ 1.6. In order to investigate whether this decrease is real or simply represents the network detection threshold, we use seismicity recorded by the deep (1.5 and 2.5 km) seismometers deployed in the Cajon Pass Scientific Drillhole between April 1992 and October 1994. The maximum amplitudes recorded downhole are compared to SCSN magnitudes for events recorded by the network, to determine the relationship between amplitude and $M_L$ as a function of hypocentral distance from the borehole. Magnitudes are then calculated for 1300 earthquakes which occurred within 40 km of the borehole. Magnitude–frequency curves are calculated for those events within 18 km of the borehole, and a constant $b$-value is observed to $M_L$ 0.5.

## 1. Introduction

The empirical relationship between earthquake magnitude and frequency of occurrence proposed by Gutenberg and Richter (1944) is well known:

$$\log(N) = a - bM \tag{1}$$

where $N$ is the number of earthquakes which occur in a given area and time period, larger than magnitude $M$. The constant $a$ is a measure of the level of seismicity and the constant $b$ is typically close to 1. This relationship has been found to hold in regions throughout the world, over a wide range of magnitudes. Such a relationship is consistent with earthquake sources having a constant stress drop and thus being self-similar. Such a scale invariance of the earthquake rupture process was proposed by Aki (1967) and is in good agreement with observations that many geological processes are self-similar over a very wide range of scales (e.g., Turcotte, 1989). Magnitude–frequency statistics from earthquake catalogues, however, typically depart from the constant $b$-value at low magnitudes. This departure has traditionally been attributed to the detection threshold of the recording network, under the assumption that

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many small earthquakes occur undetected. Some recent studies, however, have proposed that this decrease in the number of small earthquakes, compared to that expected from a constant *b*-value, is not just an artifact of the recording system, but that there really are fewer small earthquakes than self-similarity predicts. Aki (1987) determined magnitudes for earthquakes recorded in a 1.5 km borehole in southern California. He found a decrease in the *b*-value below about *M* 2, but calculated that the catalogue was complete above *M* 0. Taylor et al. (1990) and Umino and Sacks (1993) analyzed network recorded seismicity in Hokkaido and northeastern Japan, respectively. They also find a departure from the constant *b*-value below about *M* 2.5 to 3.5 and estimate that the catalogues are complete above about *M* 2.

A decrease in the *b*-value could be explained by a breakdown at small magnitudes in the constant stress drop scaling relationship which governs larger earthquakes (Hanks, 1979; Aki, 1987; Rundle, 1993). Various studies, including Archuleta et al. (1982), observed an apparent decrease in earthquake stress drop below about *M* 3, with smaller events tending to a minimum source dimension of about 100 m. These observations were controversial, as such a change in scaling could also be interpreted as resulting from severe attenuation beneath the recording site (Hanks, 1982). Abercrombie and Leary (1993) and Abercrombie (1995) determine source parameters for earthquakes recorded at 2.5 km depth in granite, in the Cajon Pass borehole, southern California. They find no breakdown in constant stress drop scaling in the range $0 \leq M \leq 7$, and observe source dimensions ten times smaller than the proposed "minimum". By comparing surface and downhole recordings of the same events, they demonstrate that the apparent breakdown in scaling is simply an artifact of the limited spatial resolution of surface stations.

Since it has been shown that the scale invariance of the fundamental earthquake source param-

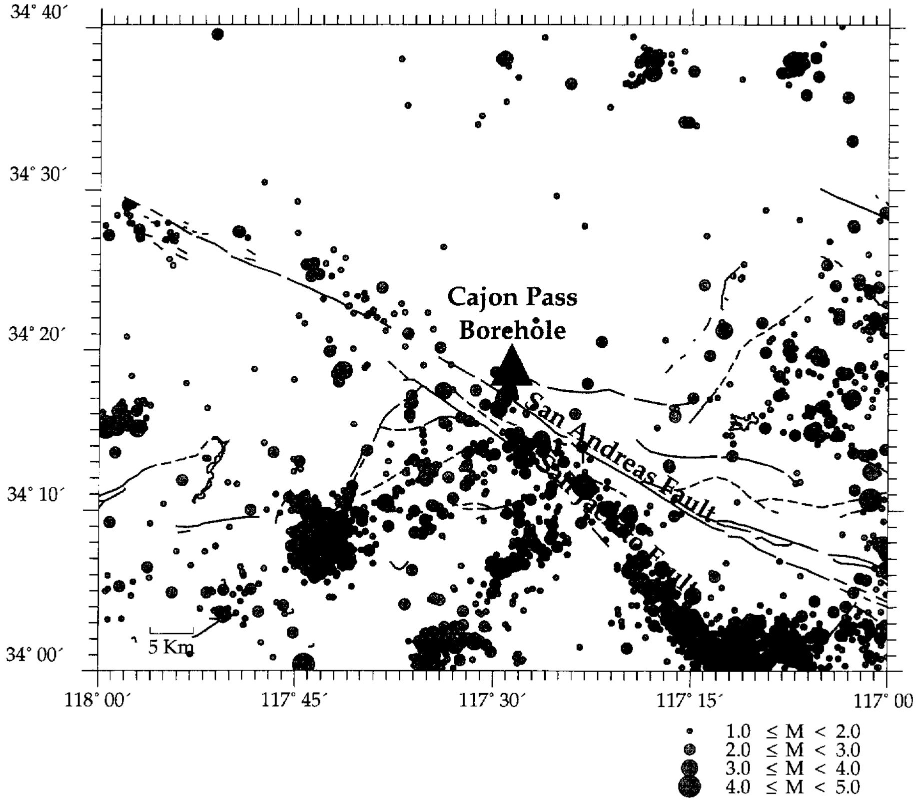

Fig. 1. Seismicity around Cajon Pass recorded by the Southern California Seismic Network between 1989 and 1994.

eters seismic moment and source dimension does not breakdown at $M$ 3 on two major fault zones in southern California (Abercrombie and Leary, 1993; Abercrombie, 1995) there is no clear reason why the $b$-value should decrease at small magnitudes. Also, two studies of microseismicity in California find the $b$-value to be constant above $M$ 0–0.5. Malin et al. (1989) calculate $b$-values for earthquakes recorded by the high resolution seismic network at Parkfield. This shallow borehole network has a lower detection threshold than surface networks and a constant $b$-value is observed above $M$ 0.5. More recently, Abercrombie and Brune (1994) combine the results of high gain microseismicity surveys with surface network recorded seismicity along three major fault zones in California. Their results imply a constant $b$-value of about 1 for earthquakes larger than $M$ 0.

Seismometers at depths of 1.5 to 3 km have been operating at Cajon Pass, southern California for over three years, recording local seismicity. In this study, magnitudes of most earthquakes recorded are calculated to determine how small earthquakes are occurring in this area around the San Andreas fault, and whether there is a minimum size, and whether the $b$-value is constant in this area.

## 2. Data

The earthquake seismograms used in this study were recorded at depth in the Cajon Pass Scientific Drill hole, 4 km from the San Andreas fault (see Fig. 1) in two successive deployments. Initially a seismometer was installed at 2.5 km depth and reliable recording began in April 1992. This continued until July 1993, just before this seismometer was retrieved. In November 1993 two deep seismometers were installed, at 1.5 and 3 km depth, and recording continued to November 1994. During both installations there were periods of down time, amounting to about 10% of the total time. During the second time period, seismograms recorded by the 1.5 km instrument are used as this seismometer experienced less down time than the one at 3 km. All three deep seismometers were triaxial 10 Hz high-temperature geophones, and recording was carried out with RefTeks in triggered mode ($\sim$5 × background noise level) at 500 or 1000 samples/second, and constant gain. A pre-trigger window of 10 s was used throughout ensuring that all events within about 75 km which triggered on the S wave (being too small to trigger on the P arrival) would have the P-wave onset recorded.

## 3. Analysis

### 3.1. The catalogue

The first stage of the analysis was to produce a catalogue of the events recorded. As the vast majority of events recorded downhole were not detected by the Southern California Seismic Network (SCSN) it was necessary to determine for each earthquake, its distance from the borehole seismometer and some measure of its size. A specially adapted version of the program XQR (A. Martin, pers. commun.) was therefore used to pick P- and S-wave arrival times ($t_P$ and $t_S$, respectively), and maximum amplitudes on all three components from the raw velocity seismograms. It was necessary to pick the amplitude on all three components as the incoming ray paths to the downhole instruments have not been refracted to vertical incidence as is typically the case for surface recordings.

Two time periods were thus catalogued, 15 April to 28 June 1992 (day of the Landers earthquake, $M_w$ 7.3) during which 1634 earthquakes were recorded, and a further 2004 for which the P wave was missing—principally aftershocks of the Joshua Tree earthquake (23 April 1992, $M_w$ 6.1). As $t_S - t_P$ is used to determine the hypocentral distance, these latter events are not included in the magnitude analysis. The second time period catalogued was 25 November 1993 to 13 October 1994, in which 1089 events, and 619 with P waves missing, were recorded. The first time period was recorded at 2.5 km depth and the second at 1.5 km and these recording levels are used to distinguish them in the following.

### 3.2. Calculation of magnitudes

The magnitudes of the earthquakes recorded at Cajon Pass are calculated by determining the relationship between amplitude recorded and the SCSN $M_L$, as a function of distance from the borehole, for the events which the SCSN detected and located. An important point to consider in this calculation is that although we measure magnitude, we

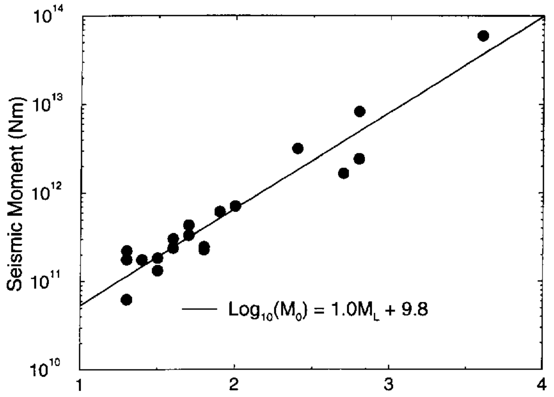

Fig. 2. The relationship between seismic moment (from pulse areas and spectra of Cajon Pass borehole recordings, Abercrombie and Leary, 1993; Abercrombie, 1995), and $M_L$ for earthquakes $<M_L$ 4.

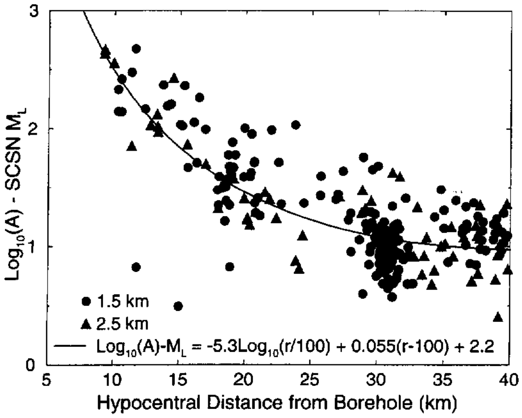

Fig. 3. Determination of relationship between maximum amplitude recorded in the borehole and SCSN $M_L$ as a function of hypocentral distance.

are really interested in the seismic moment ($M_0$). Magnitude scales are empirical and affected by instrument saturation and the relationship between the instrument frequency and the corner frequency (inversely proportional to the rupture dimension) of the earthquake (e.g., Hanks and Boore (1984). To avoid these problems here the analysis is limited to the magnitude range over which $M_L$ is proportional to $M_0$. This should be true when the corner frequency of the earthquake ($f_c$) is greater than the frequency of the Wood–Anderson torsion seismograph (1.25 Hz), i.e., for events smaller than about $M_L$ 4.5 (assuming stress drop approx. 100 bar, Brune, 1970). This is confirmed by Hanks and Boore (1984) for Californian earthquakes, and also for the earthquakes recorded at Cajon Pass (Fig. 2). The downhole recordings, however, are made on 10 Hz geophones and therefore only earthquakes with $f_c > 10$ Hz should be included in the magnitude calculation. Abercrombie (1995) calculates source parameters for over 100 earthquakes recorded at Cajon Pass and finds that events $<M_L$ 2.0 have S-wave corner frequencies of at least 10 Hz. Earthquakes recorded by the SCSN, less than $M_L$ 2.0 are therefore used. The selection of SCSN recorded events is further restricted to include only those occurring within 40 km hypocentral distance of the borehole instrument. The smallest events are clearly recorded close to the borehole, and therefore this is the region of chief interest. The distance of 40 km is selected as that is the approximate radius within which all SCSN recorded events are recorded downhole (see Fig. 4). Including more distant events could bias the correction towards attenuation effects only occurring at distance from the borehole.

The amplitude ($A$) of each event recorded downhole at Cajon Pass, in counts, is determined by summing over the three components ($z$, $h1$ and $h2$):

$$A = \sqrt{z^2 + h1^2 + h2^2}$$

Fig. 3 is a plot of $\log_{10}(A)$ minus the SCSN magnitude, against hypocentral distance ($R$) for the 292 events which met the above selection criteria. $R$ is calculated from $t_S - t_P$ assuming that the P-wave velocity ($V_P$) = 6 km/s, and the S-wave velocity ($V_S$) = $V_P/\sqrt{3}$. The data are modeled with the best fitting attenuation relationship of the form used by Hutton and Boore (1987) to calculate $M_L$ from the Cajon Pass recordings for the SCSN recorded events. The 1.5 and 2.5 km recorded earthquakes are fit separately and combined. The difference between the fits to the two separate data sets is less than 0.15 magnitude units, less than the scatter in the data, and so the best fit to the combined data sets was used to calculate the magnitude of all the events recorded within 40 km of Cajon Pass from measured $A$ and $R$. It is worth noting that the lack of any data points constraining the rapid rise in the model fit at hypocentral distances less than about 7 km is unimportant as almost no earthquakes are detected this close to the borehole (Fig. 4).

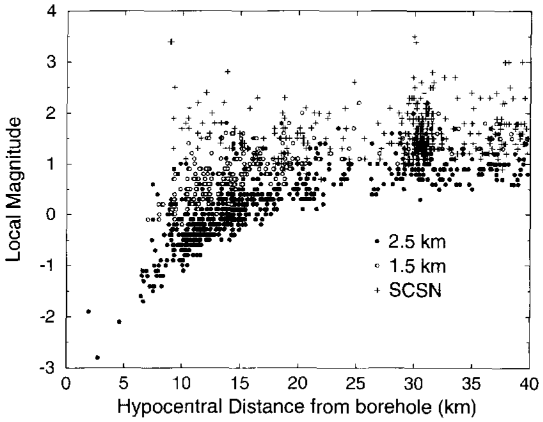

Fig. 4. Magnitudes of earthquakes recorded within 40 km of Cajon Pass by the deep seismometers. The magnitudes calculated using the relationship in Fig. 3 for earthquakes not recorded by the SCSN are shown as circles. The earthquakes recorded by both the SCSN and one of the borehole instruments are plotted as crosses at the SCSN magnitudes.

The resulting magnitudes of all 1300 earthquakes recorded within 40 km of Cajon Pass are plotted in Fig. 4. It is immediately clear that at the quiet, low attenuation deep borehole sites, many small earthquakes are recorded which are not detected by the SCSN. It is also interesting to note that the 2.5 km instrument was significantly more sensitive, recording smaller events (SCSN recorded about 9% of total recorded downhole), than the 1.5 km instrument (SCSN recorded 58%). This finding is not unexpected as the background noise at 2.5 km was typically about five counts, whereas at 1.5 km it was about 30 counts, at the same gain. At these sites, the seismic noise is not detectable above the system noise, so the increased level during the second installation is most likely to result from the use of older (prototype) RefTeks for recording, or more leakage in the cable which was the same as that previously used at 2.5 km, redeployed, than from it being seismically more noisy at 1.5 km depth than at 2.5 km.

## 4. Frequency–magnitude relationship

To investigate the frequency–magnitude relationship within the vicinity of Cajon Pass it is necessary to select a volume centred at the borehole, small enough such that small-magnitude events are recorded throughout, but large enough that sufficient earthquakes are recorded by the borehole instruments and the SCSN to calculate reliable curves. A radius of 18 km is chosen as best matching these requirements. This volume includes earthquakes from both the San Jacinto and San Andreas fault zones. Cumulative *b*-value curves are calculated following Eq. 1 for events within this volume and plotted in Fig. 5. The two periods recorded downhole are considered separately and combined, and two longer periods of SCSN recorded data are included to show that there is little long-term temporal variation in seismicity rate. The curves are all normalized in time to allow direct comparison, and all cover the same crustal volume. A total of 483 events are included recorded at 2.5 km and 175 at 1.5 km.

The SCSN curves exhibit a *b*-value of 1 above about $M_L$ 1.6 at which the slope of the curve decreases. The curves from the borehole recordings, however, exhibit good agreement with those of the SCSN in the $M_L$ range 1.6–2.5, but continue with a constant *b*-value of about 1 to $M_L$ 1 for 1.5 km and $M_L$ 0.5 for 2.5 km recordings. It is clear, therefore, that the decrease in *b*-value in the SCSN data below about $M_L$ 1.6 is purely an artifact of incomplete detection of smaller earthquakes. It is most probable that the decrease in *b*-value at small magnitudes in the borehole data is also simply reflecting the detection threshold, as the decrease starts at higher $M_L$ at 1.5 km, the recording period with significantly higher background noise. Also it is clear from Fig. 4 that the magnitude of completeness is strongly distance dependent, again implying a detection threshold origin for the apparent decrease in *b*-value. Above the detection thresholds, both the borehole and SCSN recorded magnitude–frequency curves are quite straight. No significant variation from a constant *b*-value, of the type observed at smaller magnitudes in mining induced seismicity (Trifu et al., 1993), is seen.

The 1.5 km recording period and the combined borehole data both exhibit levels of seismicity consistent with those observed over the last 10 years in the area. The 2.5 km time period, corresponding to the 3 months prior to the Landers earthquake, shows a higher level of seismicity. The seismicity recorded by the SCSN in this general area remained essentially stationary during the six years before the Landers earthquake (Qian et al., 1992) and so this increase is unlikely to be anomalous.

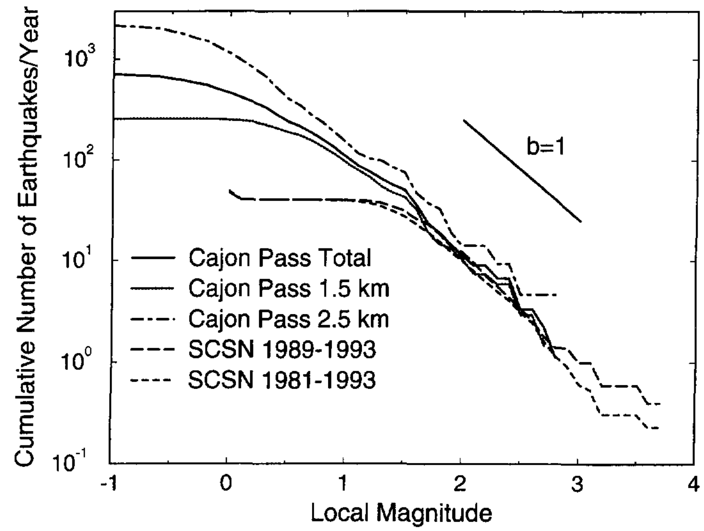

Fig. 5. Magnitude–frequency curves for earthquakes recorded within 18 km hypocentral distance of the borehole seismometers.

## 5. Discussion

The results presented here of constant *b*-value to about $M_L$ 0.5 are in good agreement with previous microseismicity surveys. Malin et al. (1989) find a constant *b*-value to *M* 0.6 at Parkfield, using the High Resolution Seismic Network, and Abercrombie and Brune (1994) find constant *b*-value to $M \sim 0$ on the San Andreas, San Jacinto and San Miguel faults and in the Mammoth Lakes area. These results all suggest that earthquake self-similarity extends to such small magnitudes and this is in excellent agreement with the source parameter results obtained from the deep Cajon Pass borehole recordings (Abercrombie and Leary, 1993). Abercrombie and Leary (1993) and Abercrombie (1995) find that when observations are made in the absence of near surface attenuation, earthquakes have a constant stress drop of ~10 to 100 bar to $M \sim 0$.

A few studies have suggested that the breakdown in *b*-value observed at small magnitudes in network catalogues may be real, e.g., Taylor et al. (1990) and Umino and Sacks (1993). In such studies, the decrease in *b*-values reported is typically within less than one magnitude unit of the completeness threshold. Microseismicity studies in these areas, such as those of Malin et al. (1989), Abercrombie and Brune (1994) and this one, would be useful to determine whether the reported decrease is really significant.

Attempts have been made to relate observations of apparent breakdown in earthquake self-similarity to the width of mature faults zones (e.g., Aki, 1987), typically of the order of a few hundred meters. This corresponded to the minimum rupture dimension calculated for small earthquakes from band limited data, uncorrected for near-surface attenuation. Recent source parameter studies, in which near-surface attenuation is correctly compensated for, or eliminated (e.g., Frankel and Wennerberg, 1989; Abercrombie and Leary, 1993), and microseismicity studies such as this one show that there is no breakdown in self-similarity above $M_L$ 0–0.5, corresponding to a source size of ~10 m (Abercrombie and Leary, 1993; Abercrombie, 1995). There is little evidence that this size represents anything other than the detection threshold with current seismometer installations. Recent studies using trapped waves (Li et al., 1990, 1994) suggest that fault zones are typically a few hundred meters thick, an order of magnitude larger than the smallest detected earthquake sources. This discrepancy implies that the width of the fault zone does not play any part in controlling the size and scaling of earthquake sources.

## 6. Conclusions

Magnitudes are calculated for 1300 earthquakes recorded at depths of 1.5 and 2.5 km in the Cajon

Pass borehole, southern California during a total time period of 326 days. The resulting magnitude–frequency statistics within a volume of radius 18 km, centred at the borehole exhibit a constant $b$-value to $\sim M_L$ 0.5 or less, and any decrease at lower magnitudes almost certainly results from catalogue incompleteness. These results imply no breakdown in earthquake self-similarity above $\sim M_L$ 0.5 and that the thickness of mature fault zones such as the San Andreas and San Jacinto faults does not affect earthquake source sizes in any way.

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