Solutions to End-of-Chapter Exercises

Chapter 2: Theory of Consumer Behavior

1. (a) We know the tangency condition is

$$\frac{MU_G}{MU_M} = \frac{p_G}{p_M}.$$

Now

$$MU_G = \frac{\partial U}{\partial G} = \frac{0.4}{G} \qquad and \qquad MU_M = \frac{\partial U}{\partial M} = \frac{0.6}{M}.$$

Applying these to the tangency condition, we get

$$\frac{p_G}{p_M} = \frac{0.4M}{0.6G} \qquad or \qquad p_G G = \frac{2}{3} p_M M.$$

Substituting in the budget constraint and simplifying yields the demand functions:

$$G = \frac{2I}{5p_g}, \quad M = \frac{3I}{5p_M}.$$

(b) Substituting the data in the demand functions yields: $M = 10, \quad G = 40$.

(c) Again, after substitution of the new data in the demand functions, we get: $M = 10, \quad G = 80$.

2. This is a Leontief utility function, so we know Lily will always set $X = Y$ in order to maximize utility. Substituting for Y in the budget constraint and simplifying, we get the demand function for X:

$$X = \frac{I}{p_x + p_y}.$$

3. A necessity is a good whose income elasticity of demand satisfies:

$$0 < \eta < 1.$$

Now the expenditure share of a good is

$$\theta = \frac{px}{I}.$$

Then

$$\frac{\partial \theta}{\partial I} = \frac{I \cdot p\frac{\partial x}{\partial I} - px}{I^2} = \frac{px\left(\frac{\partial x}{\partial I} \cdot \frac{I}{x} - 1\right)}{I^2} = \frac{px(\eta - 1)}{I^2} < 0.$$

Thus the expenditure share falls as income rises, proving that the statement is true.

4. (a) An inferior good is one for which demand falls as income rises, that is, whose income elasticity of demand is negative.

   (b) Suppose X and Y are the two goods the consumer consumes. By the generalized Engel's Law, we know that

$$\theta_x \eta_x + \theta_y \eta_y = 1,$$

   where θ_i, η_i are the budget share and income elasticity of demand for good i respectively. But if both X and Y were inferior, we would have $\eta_x < 0$ and $\eta_y < 0$, which would imply

$$\theta_x \eta_x + \theta_y \eta_y < 0.$$

   This would violate the generalized Engel's Law. Therefore both goods cannot be inferior.

5. (a) The Lagrangian for this problem is

$$\mathcal{L} = xy + y + \lambda \left[I - p_x x - p_y y \right].$$

   The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x} = y - \lambda p_x = 0 \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial y} = x + 1 - \lambda p_y = 0 \qquad (2)$$

   Dividing (1) by (2) and simplifying, we get

$$p_y y = p_x x + p_x \qquad (3)$$

   Substituting this in the budget constraint gives us

$$p_x x + p_x x + p_x = I,$$

which simplifies to

$$x = \frac{I - p_x}{2p_x}.$$

This is the demand function for x.

Substituting this in (3) and simplifying gives us the demand function for y:

$$y = \frac{I + p_x}{2p_y}.$$

(b) Substituting the given data into the demand functions gives us the quantities consumed:

$$x = \frac{100 - 20}{40} = 2 \qquad \textit{and} \qquad y = \frac{100 + 20}{20} = 6.$$

(c) We know the elasticity of demand is given by

$$\epsilon_d = \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{x}.$$

From the demand function for x:

$$\frac{\partial x}{\partial p_x} = \frac{2p_x(-1) - (I - p_x) \cdot 2}{(2p_x)^2} = \frac{-2I}{4p_x^2}.$$

Then

$$\epsilon_d = \frac{-2I}{4p_x^2} \cdot \frac{p_x}{x} = \frac{-2(100)}{4\,(20)^2} \cdot \frac{20}{2} = -1.25.$$

6. (a) We know the tangency condition is

$$\frac{MU_A}{MU_B} = \frac{p_A}{p_B}.$$

Now

$$MU_A = \frac{\partial U}{\partial A} = \frac{1}{A - 10} \qquad \textit{and} \qquad MU_B = \frac{\partial U}{\partial MB} = \frac{1}{B}.$$

Applying these to the tangency condition, we get

$$\frac{p_A}{p_B} = \frac{B}{A - 10} \qquad \textit{or} \qquad p_B B = p_A A - 10p_A.$$

Substituting in the budget constraint and simplifying yields the demand functions:

$$A = \frac{I + 10p_A}{2p_A}, \quad B = \frac{I - 10p_A}{2p_B}.$$

(b) The own-price elasticity is

$$\epsilon_A = \frac{\partial A}{\partial p_A} \cdot \frac{p_A}{A}.$$

From the demand function, we have

$$\frac{\partial A}{\partial p_A} = \frac{2p_A(10) - (I + 10p_A)(2)}{4p_A^2} = -\frac{I}{2p_A^2}.$$

Then, substituting the demand function in the definition of the elasticity, we have

$$\epsilon_A = -\frac{I}{2p_A^2} \cdot \frac{p_A}{\frac{I+10p_A}{2p_A}} = -\frac{I}{I + 10p_A}$$

Since p_B does not appear in the demand function for A, we have

$$\frac{\partial A}{\partial p_B} = 0$$

and therefore the cross-price elasticity of demand for A is $\epsilon_{AB} = 0$.

(c) We need to calculate the income elasticity of demand for A. From the demand function, we have

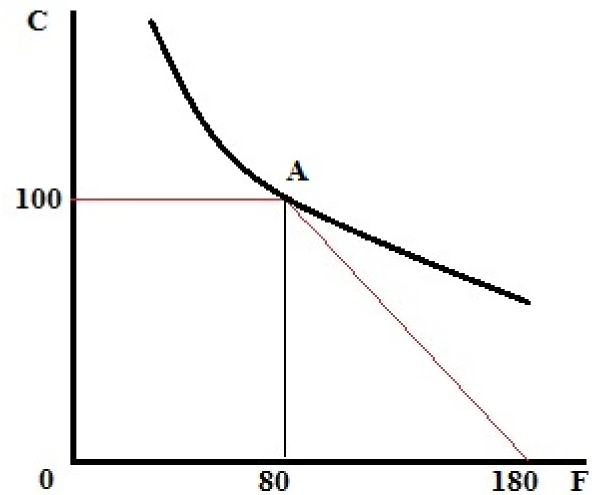
$$\frac{\partial A}{\partial I} = \frac{1}{2p_A}$$

and therefore

$$\eta_A = \frac{\partial A}{\partial I} \cdot \frac{I}{A} = \frac{1}{2p_A} \cdot \frac{I}{\frac{I+10p_A}{2p_A}} = \frac{I}{I + 10p_A}.$$

This is less than 1, so A is a necessity.

7. (a) Since the utility function is Cobb-Douglas, we know that Jane would spend one-third of her income on food and two-thirds on clothing. The diagram shows Jane's budget constraint (the red line).



If Jane had an income of \$180, she would consume the bundle $F = 60, C = 120$. This is outside her budget constraint, indicating that a tangency of an indifference curve with the budget constraint is not possible. Thus she will consume at the kink where $F = 80, C = 100$.

(b) Jane's utility level at A is

$$U_0 = (80)^{\frac{1}{3}}(100)^{\frac{2}{3}}.$$

Now her indirect utility function is given by

$$V(p, I) = \left(\frac{I}{3}\right)^{\frac{1}{3}} \left(\frac{2I}{3}\right)^{\frac{2}{3}},$$

where the terms within the parentheses are the demand functions when $p_F = p_C = 1$. Setting $U_0 = V(I)$ and solving for I gives us the income needed to attain U_0 at market prices, which is \$175.44. The needed cash subsidy is then 75.44.

8. (a) Since the utility function is Cobb-Douglas, we can readily write down the demand functions and then solve for the quantities with the given data:

$$x = \frac{I}{2p_x} = \frac{120}{2(4)} = 15 \qquad \textit{and} \qquad y = \frac{I}{2p_y} = \frac{120}{2(4)} = 15$$

(b) If the consumer is restricted to $y \leq 8$, she will consume $y = 8$ and spend the remainder of her income on x, giving

$$x^* = \frac{120 - 8(4)}{4} = 22.$$

(c) Without the quota, the choices are:

$$x = \frac{I}{2p_x} = \frac{120}{2(4)} = 15 \qquad and \qquad y = \frac{I}{2p_y} = \frac{120}{2(3)} = 20.$$

With the quota: $y = 8$ and

$$x^* = \frac{120 - 8(3)}{4} = 24.$$

(d) With no quota and a price of 4, the consumer's utility level is

$$U(15, 15) = 225.$$

With a quota q and a price of 3, the consumtion levels would be

$$x = \frac{120 - 3q}{4} \qquad and \qquad y = q,$$

yielding a utility level of

$$U_1 = \left(\frac{120 - 3q}{4}\right)(q) = 30q - \frac{3}{4}q^2.$$

The consumer will be indifferent between these two scenarios if

$$30q - \frac{3}{4}q^2 = 225,$$

which has the solutions $q = 10, 30$. However, we were told that the quota must be less than 15, thus the chosen quota will be $y = 10$, with which

$$x^* = \frac{120 - 10(3)}{4} = 22.5.$$

Chapter 3: Applications of Consumer Theory

1. (a) First let's calculate the income in each time period:

$$I_1 = (3 * 7) + (3 * 4) = 33$$

$$I_2 = (4 * 6) + (2 * 6) = 36$$

$$I_3 = (5 * 7) + (1 * 3) = 38.$$

Now let's see if he could have bought bundle 2 in year 1:

$$C(q_2, p_1) = (3 * 6) + (3 * 6) = 36 > I_1.$$

So bundle 2 was not available in year 1. Let's see if he could have bought bundle 1 in year 2:

$$C(q_1, p_2) = (4 * 7) + (2 * 4) = 36 = I_2.$$

Therefore he could have bought bundle 1 in year 2. By choosing bundle 2, he revealed a preference for bundle 2 over bundle 1; he is better of in year 2 compared to year 1.

(b) Was bundle 3 available in year 2?

$$C(q_3, p_2) = (4 * 7) + (2 * 3) = 34 < I_2.$$

So this reveals a preference for q_2 over q_3. Now let's look to see if bundle 2 was available in year 3:

$$C(q_2, p_3) = (5 * 6) + (1 * 6) = 36 < I_3.$$

This reveals a preference for q_3 over q_2. Thus we have an inconsistency with revealed preference.

(c) The Paasche index is:

$$PI_P = \frac{C(q_2, p_2)}{C(q_2, p_1)} = \frac{36}{36} = 1 \ \ (or\ 100).$$

The Laspeyre index is

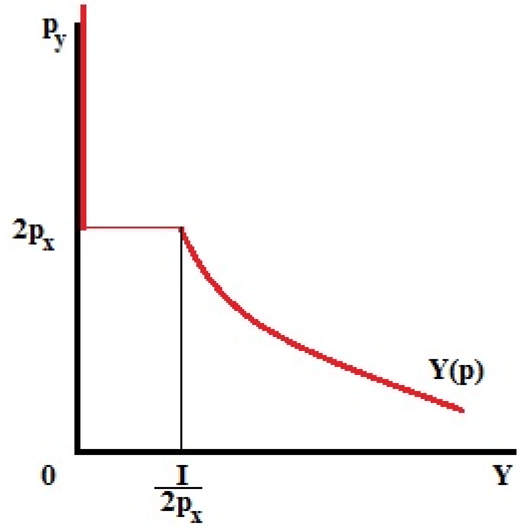
$$PI_L = \frac{C(q_1, p_2)}{C(q_1, p_1)} = \frac{36}{33} = 1.09 \ \ (or\ 109).$$

2. (a) Olivia's utility function is such that the goods are perfect substitutes and the indifference curves are straight lines with slope equal to $-\frac{1}{2}$. If the budget constraint is steeper than the indifference curves, Olivia will buy only Y and, if it is flatter, she will buy only X. Therefore, her demand function for Y is:

$$Y = \begin{cases} \frac{I}{p_y} & if \quad p_y < 2p_x \\ 0 & if \quad p_y > 2p_x \end{cases}$$

with Y being indeterminate between 0 and $\frac{I}{p_y}$ if $p_y = 2p_x$.

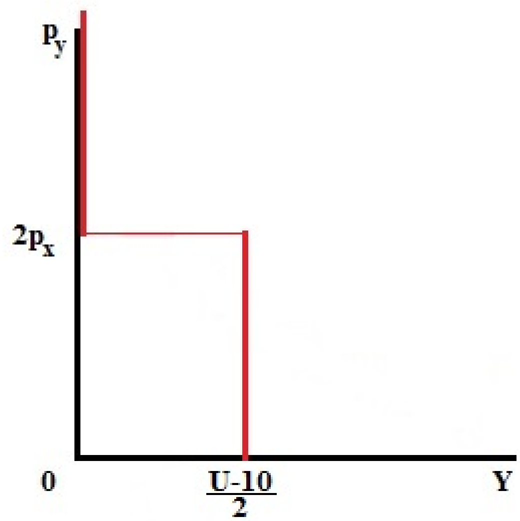
The demand curve is shown below as the red line. Demand is zero for $p_y > 2p_x$ and is a rectangular hyperbola for prices below that.



(b) For any desired utility level U, Olivia will buy only X if $p_y > 2p_x$ and only Y if the opposite is true. When she buys Y, $U = 10 + 2Y$. Therefore her compensated demand function is:

$$Y^h = \begin{cases} \frac{U-10}{2} & if \quad p_y < 2p_x \\ 0 & if \quad p_y > 2p_x \end{cases}$$

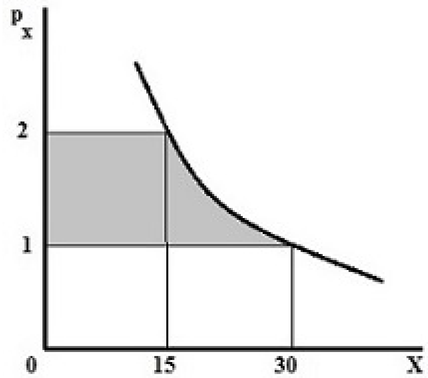
The demand curve is the red line below ... it is a step function.



3. (a) Since the utility function is Cobb-Douglas, we can write down the demand functions:

$$X = \frac{I}{3p_x} \qquad and \qquad Y = \frac{2I}{3p_y}.$$

Initially, $X_0 = 30$ and $Y_0 = 60$. After p_x rises, $X_1 = 15$ and $Y_1 = 60$.



The loss in consumer's surplus is the shaded area in the figure:

$$\Delta CS = \int_2^1 \frac{I}{3p_x} dp_x = \left[\frac{I}{3} ln p_x\right]_2^1 = -20.79.$$

If we had assumed the demand curve to be linear, we would have got

$$\Delta CS \approx -15 - \frac{1}{2}(15) = -22.5.$$

(b) Since only one price has changed, we can calculate CV as the area below the compensated demand curve between the price lines. To find the compensated demand curve, we need to minimize the expenditure needed to achieve the original utility level. That is, we need to satisfy the tangency condition and the utility function:

$$\frac{MU_x}{MU_y} = \frac{Y^2}{2XY} = \frac{p_x}{p_y} \qquad and \qquad U = XY^2.$$

This yields the compensated demand function for X:

$$X^h = \left(\frac{Up_y^2}{4p_x^2}\right)^{\frac{1}{3}}.$$

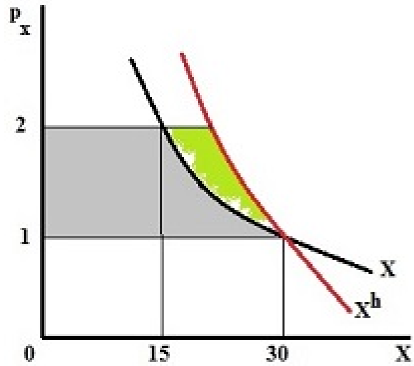
Since $U_0 = (30)(60)^2 = 108,000$ and $p_y = 1$, this reduces to

$$X^h = \left(\frac{27,000}{p_x^2}\right)^{\frac{1}{3}}.$$

Then the change in welfare using the compensated variation measure (equal to –CV) can be calculated as

$$\Delta W_{CV} = \int_2^1 \frac{30}{p_x^{\frac{2}{3}}} dp_x = 30\left[3p_x^{\frac{1}{3}}\right]_2^1 = -23.39.$$

(c) The graph below shows the grey shaded area as the change in consumer's surplus. The compensating variation measure adds the green shaded area to the grey one.



4. (a) Lizzie's problem is to

$$Maximize \quad U = C^{\frac{1}{2}} S^{\frac{1}{2}}$$

$$subject\ to \quad p_c C + p_s S = I.$$

The Lagrangian for this problem is

$$\mathcal{L} = C^{\frac{1}{2}} S^{\frac{1}{2}} + \lambda \left[I - p_c C - p_s S \right].$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{1}{2} C^{-\frac{1}{2}} S^{\frac{1}{2}} - \lambda p_c = 0, \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial S} = \frac{1}{2} C^{\frac{1}{2}} S^{-\frac{1}{2}} - \lambda p_s = 0, \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_c C - p_s S = 0. \tag{6}$$

Dividing (1) by (2), we get

$$\frac{S}{C} = \frac{p_c}{p_s} \quad \rightarrow \quad S = \frac{p_c}{p_s} C \tag{7}$$

and substituting (4) in (3) and simplifying gives us the demand function for soda:

$$S = \frac{I}{2p_s}. \tag{8}$$

Substituting (5) in (4) and simplifying, gives us the demand function for crackers:

$$C = \frac{I}{2p_c}. \tag{9}$$

(b) Substituting the values $p_c = 0.10, p_s = 0.25, I = 1$ in (5) and (6) gives us

$$C = 5 \quad and \quad S = 2.$$

So Lizzie would buy 5 crackers and 2 sodas every day.

(c) Lizzie's initial utility level is

$$U_0 = 5^{\frac{1}{2}} 2^{\frac{1}{2}} = \sqrt{10}.$$

Using the demand functions (5) and (6), we can write down her indirect utility function:

$$V(p_c, p_s, I) = \left(\frac{I}{2p_c}\right)^{\frac{1}{2}} \left(\frac{I}{2p_s}\right)^{\frac{1}{2}} = \frac{I}{2\sqrt{p_c p_s}}.$$

We need to find how much income she would need in order to have $U = \sqrt{10}$ when $p_c = 0.40, p_s = 0.25$. If we call this level of income I', it must satisfy the equation

$$\sqrt{10} = \frac{I^{'}}{2\sqrt{(0.4)(0.25)}}.$$

Solving, we find

$$I^{'} = 2.$$

Therefore Lizzie will need \$1 extra so she can achieve her original utility level.

5. The method for solving this problem is discussed in detail in the text. The ordinary demand function is:

$$X = \frac{3I}{4p_x}$$

and the compensated demand function is:

$$X^h = \left(\frac{3p_y}{p_x}\right)^{\frac{1}{4}} U.$$

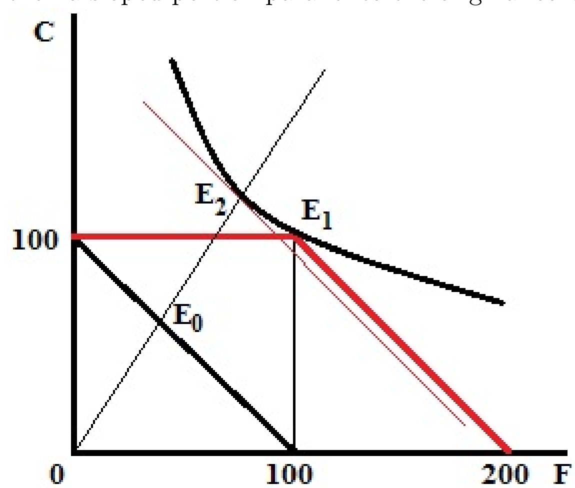
The elasticities and verification of the Slutsky equation proceed directly: $\epsilon_x = -1, \quad \eta_x = 1, \quad \alpha_x = 0.75, \quad \epsilon_x^h = -0.25, \quad so \quad \epsilon_x = \epsilon_x^h - \alpha_x \eta_x.$

6. (a) Since the utility function is Cobb-Douglas, we can write down the demand functions:

$$F = \frac{I}{4p_f} \qquad and \qquad C = \frac{3I}{4p_c}.$$

Substituting the data in these yields: $F = 25, \quad C = 75.$

(b) The initial consumption bundle is shown as the point E_0 in the graph. When the household is given 100 units of food, its constraint pushes out to the right by 100 units. The new constraint (the heavy red line in the graph) is kinked: it has a flat portion up to the point E_1 and then a sloped portion parallel to the original constraint.



If the household had \$200 of income, they would want to consume the bundle (50,150), but this point is outside the constraint and therefore is not available. Rather, they would consume the bundle E_1, which is at the kink in the constraint and where

$$F_1 = 100 \quad and \quad C_1 = 100.$$

Therefore they would buy the bundle: $F = 0, \quad C = 100$, since they have 100 units of food from the government.

(c) The utility level at the point E_1 is

$$U_1 = (100)^{\frac{1}{4}}(100)^{\frac{3}{4}} = 100.$$

Using the demand functions, we can write down the households's indirect utility function:

$$V(p, I) = \left(\frac{I}{4p_f}\right)^{\frac{1}{4}} \left(\frac{3I}{4p_c}\right)^{\frac{3}{4}}.$$

The income I_1 needed to achieve U_1 when $p_f = p_c = 1$ is then the solution to the equation

$$\left(\frac{I_1}{4}\right)^{\frac{1}{4}} \left(\frac{3I_1}{4}\right)^{\frac{3}{4}} = 100,$$

which yields $I_1 = 175.48$. Then both ΔW_{CV} and ΔW_{EV} equal 75.48, since prices have not changed.

7. (a) Since the utility function is Cobb-Douglas, we can write down the demand functions:

$$X = \frac{2I}{5p_x} \qquad \textit{and} \qquad Y = \frac{3I}{5p_y}.$$

Substituting the data in these, we find that the initial consumption bundle is $X_0 = 200$ and $Y_0 = 100$.

(b) Substituting the new data in the demand functions, we find $X_1 = 200, \quad Y_1 = 60$.

(c) The compensating variation in income is

$$CV = E(p_1, U_0) - E(p_1, U_1).$$

Now the initial utility level was

$$U_0 = (200)^2(100)^3.$$

Using the demand functions, we can write down Lily's indirect utility function:

$$V(p, I) = \left(\frac{2I}{5p_x}\right)^2 \left(\frac{3I}{5p_y}\right)^3.$$

Then, using the fact that $p_x = 2, p_y = 10$, $E(p_1, U_0)$ will be the income level that solves

$$\left(\frac{2I}{10}\right)^2 \left(\frac{3I}{50}\right)^3 = (200)^2(100)^3.$$

Solving, we find $E(p_1, U_0) = 1358.66$,which gives us

$$CV = 1358.66 - 1000 = 358.66.$$

8. (a) The goods are perfect substitutes. A simple function that could serve as his utility function is

$$U(x, y) = 0.75x + 2y.$$

(b) The rest of this problem is just like problem 2 above. Here, the demand function is

$$x = \begin{cases} \frac{I}{p_x} & if \quad p_x < \frac{3}{8}p_y \\ 0 & if \quad p_x > \frac{3}{8}p_y \end{cases}$$

(c) $x^h = \begin{cases} \frac{4U}{3} & if \quad p_x < \frac{3}{8}p_y \\ 0 & if \quad p_x > \frac{3}{8}p_y \end{cases}$

9. (a) A simple utility function that would capture these preferences would be

$$u(W, R) = W + R,$$

where *W, R* are the quantities of wheat and rice consumed by the Smith family. The slope of the typical indifference curve is *-1* and we know with these type of preferences the family will consume only one or the other good depending upon the prices. The demand functions are:

$$W = \begin{cases} \frac{I}{p_w} & if\ \frac{p_w}{p_r} < 1 \\ 0 & if\ \frac{p_w}{p_r} > 1 \end{cases}$$

$$R = \begin{cases} 0 & if\ \frac{p_w}{p_r} < 1 \\ \frac{I}{p_r} & if\ \frac{p_w}{p_r} > 1 \end{cases}$$

with demand indeterminate in the case where $\frac{p_w}{p_r} = 1$.

(b) Substituting the values of $I = 100, p_w = 4, p_r = 5$,we find that the quantities demanded are

$$W = 25 \qquad and \qquad R = 0.$$

With this bundle, the family's utility level is $u_0 = 25$.

(c) If $I = 100, p_w = 4, p_r = 2$, the family will switch to rice consumption and we will have

$$W = 0 \qquad and \qquad R = 50.$$

With this bundle, the family's utility level is $u_1 = 50$.

To achieve this utility level at the original prices, the Smiths would have consumed only wheat. The cost of achieving this utility level would then have been

$$E(u_1, p_0) = 50 * 4 = 200.$$

Thus the EV measure of welfare change would be

$$\triangle W_{EV} = E(u_1, p_0) - E(u_0, p_0) = 200 - 100 = 100.$$

(d) To provide the subsidy, the government must buy 50 lbs of rice at \$5 and sell it at \$2. Thus the cost of providing the subsidy is 3*50=\$150. The government could have achieved the same welfare gain for the Smiths more cheaply if it simply gave them \$100 (the EV), which would have allowed them to buy 50 lbs of wheat at the original prices.

10. (a) The Lagrangian for this problem is

$$\mathcal{L} = \sqrt{X} + \sqrt{Y} + \lambda \left[I - p_x X - p_y Y \right].$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial X} = \frac{1}{2}X^{-\frac{1}{2}} - \lambda p_x = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial Y} = \frac{1}{2}Y^{-\frac{1}{2}} - \lambda p_y = 0 \tag{11}$$

Dividing (10) by (11) and simplifying, we get

$$Y = X\left(\frac{p_x}{p_y}\right)^2 \tag{12}$$

Substituting this in the budget constraint gives us

$$p_x X + p_y X\left(\frac{p_x}{p_y}\right)^2 = I,$$

which simplifies to

$$X = \frac{p_y I}{p_x(p_x + p_y)}.$$

This is the demand function for X.
Substituting this in (12) and simplifying gives us the demand function for Y:

$$Y = \frac{p_x I}{p_y(p_x + p_y)}.$$

(b) Substituting the given data into the demand functions gives us the quantities consumed:

$$X = \frac{4 \cdot 100}{1 \cdot 5} = 80 \qquad \textit{and} \qquad Y = \frac{1 \cdot 100}{4 \cdot 5} = 5.$$

(c) To find the compensated demand functions, we need to minimize the expenditure needed to achieve any desired utility level. The Lagrangian for this problem is

$$\mathcal{L} = p_x X + p_y Y + \lambda\left[u - \sqrt{X} - \sqrt{Y}\right].$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial X} = p_x - \lambda\frac{1}{2}X^{-\frac{1}{2}} = 0 \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial Y} = p_y - \lambda\frac{1}{2}Y^{-\frac{1}{2}} = 0 \tag{14}$$

Dividing (13) by (14) and simplifying, we get

$$Y = X\left(\frac{p_x}{p_y}\right)^2, \tag{15}$$

which is the same tangency condition we had in the utility maximization case. Substituting (15) in the utility function and simplifying gives us the compensated demand function:

$$X^h = \left(\frac{p_y}{p_x + p_y} \cdot u\right)^2.$$

11. From the given data, we can calculate Lily's income in each time period:

$$Year\ 1:\ 1 \cdot 50 + 2 \cdot 25 = 100$$

$$Year\ 2:\ 3 \cdot 30 + 2 \cdot 40 = 170.$$

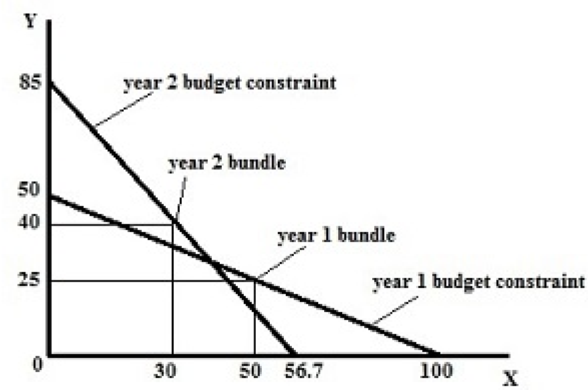
Now let's see if she could have bought year 2's bundle in year 1:

$$C(q_2, p_1) = 1 \cdot 30 + 2 \cdot 40 = 110 > 100$$

so she could not have bought year 2's bundle in year 1.
Let's see if she could have bought year 1's bundle in year 2:

$$C(q_1, p_2) = 3 \cdot 50 + 2 \cdot 25 = 200 > 170$$

so she could not have bought year 1's bundle in year 2.
Thus she has not revealed a preference for either bundle over the other, and we therefore cannot say which bundle makes her better off.
The situation is as pictured in the graph below:



12. (a) Since the utility function is Cobb-Douglas, we can write down the demand functions:

$$X = \frac{I}{3p_x} \qquad and \qquad Y = \frac{2I}{3p_y}.$$

Substituting the data in these functions, we find the chosen consumption bundle would be:

$$X_0 = 50, Y_0 = 50.$$

(b) Substituting the new data in the demand functions, we find the consumption bundle:

$$X_1 = 50, Y_1 = 40.$$

(c) We know

$$CV = E(p_1, U_0) - E(p_1, U_1).$$

From the utility function, we can find that

$$U_0 = (50)(50)^2 = 125,000.$$

From the demand functions, we can write down the indirect utility function:

$$V(p, I) = \left(\frac{I}{3p_x}\right)\left(\frac{2I}{3p_y}\right)^2.$$

Then $E(p_1, U_0)$ is the value of I that would solve

$$\left(\frac{I}{6}\right)\left(\frac{2I}{15}\right)^2 = 125,000,$$

which yields the solution $I = 348.12$. Then

$$CV = 348.12 - 300 = 48.12.$$

(d) We know

$$EV = E(p_0, U_1) - E(p_0, U_0).$$

From the utility function, we can find that

$$U_1 = (50)(40)^2 = 80,000.$$

Then $E(p_0, U_1)$is the value of I that would solve the equation

$$\left(\frac{I}{6}\right)\left(\frac{2I}{12}\right)^2 = 80,000,$$

which yields the solution $I = 258.53$. Then

$$EV = 258.53 - 300 = -41.47.$$

13. (a) The consumer's problem is to

$$Maximize \quad U = xy + y$$

$$subject\ to \quad p_x x + p_y y = I.$$

The Lagrangian for this problem is

$$\mathcal{L} = xy + y + \lambda\left[I - p_x x - p_y y\right].$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial x} = y - \lambda p_x = 0, \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial y} = x + 1 - \lambda p_y = 0, \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_x x - p_y y = 0. \tag{18}$$

Dividing (1) by (2), we get

$$\frac{y}{x+1} = \frac{p_x}{p_y} \quad \rightarrow \quad p_y y = p_x(x+1) \tag{19}$$

and substituting (4) in (3) and simplifying gives us the demand function for x:

$$x = \frac{I - p_x}{2p_x}. \tag{20}$$

Substituting (5) in (4) and simplifying, gives us the demand function for y:

$$y = \frac{I + p_x}{2p_y}. \tag{21}$$

(b) To find the compensated demand function, we can use the tangency condition (19) along with the utility function. (19) can be written

$$y = \frac{p_x}{p_y}(x+1). \tag{22}$$

Substituting this in the utility function yields

$$U = x \cdot \frac{p_x}{p_y}(x+1) + \frac{p_x}{p_y}(x+1).$$

Solving for x yields the compensated demand function:

$$x^h = \sqrt{\frac{Up_y}{p_x}} - 1.$$

Then, substituting (22) in this, we get the compnesated demand function for y:

$$y^h = \sqrt{\frac{Up_x}{p_y}}.$$

(c) Initially, when $p_x = 20$, we can see from the demand functions and utility function that

$$x_0 = 2, \quad y_0 = 6, \quad U_0 = 2(6) + 6 = 18.$$

We need to find $E(p_1, U_0)$, the minimum expenditure needed to achieve U_0 at the new prices. Now, from the demand functions, we can write down the indirect utility function:

$$V(p, I) = \left(\frac{I - p_x}{2p_x}\right)\left(\frac{I + p_x}{2p_y}\right) + \left(\frac{I + p_x}{2p_y}\right).$$

$E(p_1, U_0)$ will be the value of I that solves

$$\left(\frac{I - p_x}{2p_x}\right)\left(\frac{I + p_x}{2p_y}\right) + \left(\frac{I + p_x}{2p_y}\right) = 18,$$

with $p_x = 25, p_y = 10$. Solving for I, we find $I = 109.16$. The needed income change then, the compensating variation in income, is

$$CV = 109.16 - 100 = 9.16.$$

14. (a) The consumer's problem is to

$$Maximize \quad U = ln\ A + 2ln\ B$$

$$subject\ to \quad p_A A + p_B B = I.$$

The Lagrangian for this problem is

$$\mathcal{L} = ln\ A + 2ln\ B + \lambda\left[I - p_A A - p_B B\right].$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial A} = \frac{1}{A} - \lambda p_A = 0, \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{2}{B} - \lambda p_B = 0, \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_A A - p_B B = 0. \tag{25}$$

Dividing (1) by (2), we get

$$\frac{B}{2A} = \frac{p_A}{p_B} \quad \rightarrow \quad p_B B = 2p_A A \tag{26}$$

and substituting (4) in (3) and simplifying gives us the demand function for A:

$$A = \frac{I}{3p_A}. \tag{27}$$

Substituting (5) in (4) and simplifying, gives us the demand function for B:

$$B = \frac{2I}{2p_B}. \tag{28}$$

(b) Substituting the data into the demand functions yields the chosen consumption bundle:

$$A = 8, B = 32.$$

(c) We can find the compensated demand function by minimizing the expenditure needed to attain any desired level of utility. The solution is obtained by solving the tangency condition (26) and the utility function. From (26), we have

$$B = \frac{2p_A A}{p_B}.$$

Substituting this in the utility function gives us

$$U = ln\ A + 2ln\ \left(\frac{2p_A A}{p_B}\right)$$

Solving this for A gives us the compensated demand function:

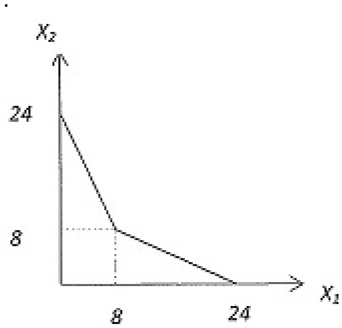
$$A^h = e^{\frac{U+2ln(p_b)-2ln(2p_a)}{3}}$$

(d) A normal good is one for which the income elasticity of demand is positive. To see whether bananas are normal, we can find its income elasticity of demand:

$$\eta_B = \frac{\partial B}{\partial I} \cdot \frac{I}{B} = \left(\frac{2}{2p_B}\right) \cdot \frac{I}{\frac{2I}{2p_B}} = 1.$$

Since this is positive, we can conclude that bananas are a normal good.

15. (a) .



(b) The bundle (10,7) yields a utility of 24, so it is on the indifference curve drawn in part (a). Clearly, the bundle is to the right of the kink, so it lies on a linear segment of the indifference curve. The slope of this segment is $-\frac{1}{2}$, so the $MRS = \frac{1}{2}$.

If Tim chose a bundle on this segment, his budget constraint must have coincided with the segment of the indifference curve. We can therefore conclude that

$$\frac{p_1}{p_2} = \frac{1}{2}.$$

Since $p_1 = 4$, we must have $p_2 = 8$. Then the cost of the bundle (10,7) would have been \$96, which must have been his income.

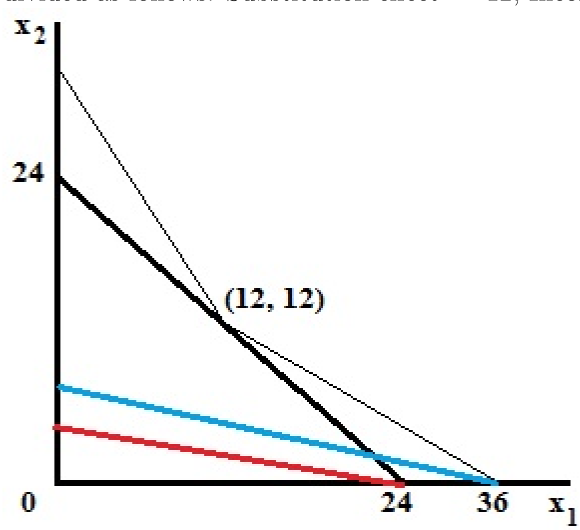
(c) The slope of the budget constraint is now -1, which lies in between the slopes of the two segments of the indifference curves. Therefore, the chosen bundle would be at the kink:

$$(x_1^*, x_2^*) = (12, 12).$$

(d) In this situation, the budget constraint is flatter than the indifference curve and so the chosen bundle would be at a corner:

$$(x_1^{**}, x_2^{**}) = (24, 0).$$

The new budget constraint is the red line in the diagram. The cheapest way to attain the original utility level at the new prices would be with the iso-expenditure line shown as the blue line; the chosen bundle would be (36,0). Therefore, the change in x_2 from 12 to 0 can be divided as follows: Substitution effect = -12, Income effect = 0.



(e) The slopes of the two segments of the indifference curve are -2 and $-\frac{1}{2}$. So, if the budget constraint is steeper than -2, the cheapest way to achieve any utility level would be to buy only x_2; if it is flatter than $-\frac{1}{2}$, only x_1 would be purchased; and if the slope of the budget constraint was in between the slopes of thsegments, the chosen bundle would be at the kink. Therefore, the compensated demand function is

$$x_1^h = \begin{cases} 24 \quad if \quad p_1 < 2 \\ 8 \quad if \quad 2 < p_1 < 8 \\ 0 \quad if \quad p_1 > 8 \end{cases}$$

16. (a) Since the utility function is Cobb-Douglas and the exponents are equal, we know Ms. Smith would spend equal amounts on the two goods. Therefore, she would spend \$50 on each good and so

$$F_0 = 25, C_0 = 10.$$

(b) Now when Ms. Smith spends \$50 on each good, her consumption will be

$$F_1 = 20, C_1 = 10.$$

The change in food consumption is -5 units. To find the substitution and income effects, we need to find the income level at which Ms. Smith would have been just as well off as she was before the price rose. Let this income level be I. If she faced this income level and p_f=2.5,p_c=5, her consumption pattern would be

$$F = \frac{I}{5}, C = \frac{I}{10}$$

and her utility level would be

$$U = \frac{I^2}{50}.$$

This is her indirect utility at the final prices. In the original situation, her utility level was

$$U_0 = (25)(10) = 250,$$

and we need her compensated utility to equal this. Therefore

$$\frac{I^2}{50} = 250 \qquad or \qquad I = 111.80.$$

With this income level and the final prices, Ms. Smith's consumption pattern would be

$$F_2 = 22.36, C_2 = 11.18.$$

Thus the substitution effect of the price change is

$$substitution\ effect = 22.36 - 25 = -2.64$$

and the income effect is

$$income\ effect = 20 - 22.36 = -2.36.$$

These add up to the total price effect of -5.

(c) We have already calculated the compensated income needed to restore Ms. Smith to her original utility level. The change in income needed is \$11.80.

Chapter 4: Extensions of Consumer Theory

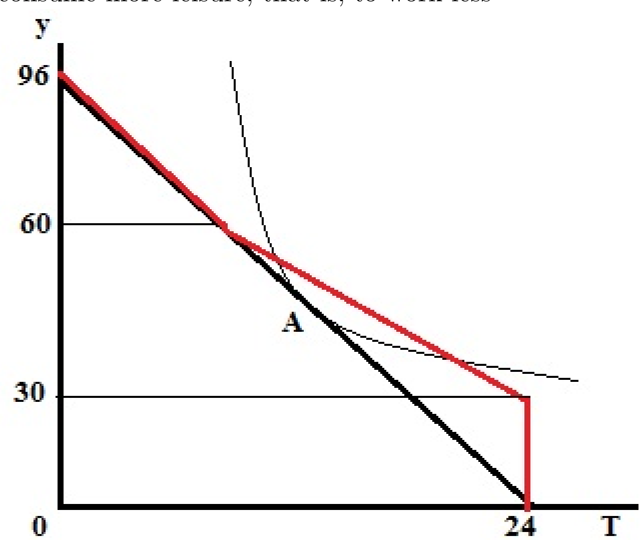
1. (a) Bill's problem is to

$$Maximize \quad U = yT$$
$$subject\ to \quad \frac{y}{4} + T = 24.$$

Since this utility function is Cobb-Douglas, we know he will "spend" half his "income" (of 24 hours) each on y and T. Therefore

$$T = 12, y = 48.$$

(b) Bill faced the solid black line originally as his constraint and chose point A. Under the welfare system, he faces the red line as his constraint. In his choice area, therefore, his constraint is flatter. On grounds of the substitution effect, he will choose a point on the original indifference curve somewhere to the right of A, and then, on grounds of income effect, he will choose even further to the right. Therefore, both substitution effect and income effect cause him to consume more leisure, that is, to work less



(c) Under the welfare system, Bill's constraint is

$$\frac{y-30}{2} + T = 24$$
$$i.e., \quad \frac{y}{2} + T = 39.$$

So now he will choose

$$T = \frac{39}{2} = 19.5.$$

He will therefore work 4.5 *hours*.
He will earn $4.5 * 4 = \$18$; and will receive welfare payments of $30 - 9 = \$21$. Therefore his income is $18 + 21 = \$39$.

2. (a) A simple way to think about this is to treat Joe's income as $p_a \cdot A_0$ and then find the demand functions in the usual way. Since the utility function is Cobb-Douglas, with equal coefficients for the two goods, we can write down the demand functions as

$$A = \frac{p_a \cdot A_0}{2p_a} \qquad \rightarrow \qquad A = \frac{A_0}{2}$$

$$B = \frac{p_a \cdot A_0}{2p_b}.$$

(b) The demand functions are of Cobb-Douglas form and can be written

$$A = \frac{1}{2}A_0 p_a^0 p_b^0 \qquad \textit{and} \qquad B = \frac{1}{2}A_0 p_a^1 p_b^{-1}.$$

Since we know that elasticities in Cobb-Douglas functions are given by the exponents on the respective variables, we can write down the elasticities by reading them from the demand functions:

$$\epsilon_a = 0, \epsilon_{ab} = 0, \qquad \textit{and} \qquad \epsilon_b = -1, \epsilon_{ba} = 1.$$

3. (a) Since Roger's utility function is Cobb-Douglas, we know his "expenditure shares" will be constant. He will consume 12 hours of leisure each day and therefore his labor supply function is simply

$$L^s = 12$$

regardless of the wage rate.

(b) Roger would work 12 hours regardless of the wage rate.

(c) If Roger receives an inheritance and starts to earn some non-labor income, his situation would be different; specifically, his constraint would no longer be

$$h + \frac{1}{w}c = 24.$$

Now

$$c = wL + 10$$

and so

$$L = \frac{1}{w}c - \frac{10}{w}$$

and his constraint would be

$$h + \frac{1}{w}c = 24 + \frac{10}{w}.$$

It is as if his "endowment" of time has gone up from 24 hours to $\left(24 + \frac{10}{w}\right)$. Since his "expenditure share" on leisure remains at 0.5, his consumption of leisure will be

$$h = \frac{1}{2} \cdot \left(24 + \frac{10}{w}\right) = 12 + \frac{5}{w}$$

and therefore his labor supply will be 24 minus his consumption of leisure:

$$L^s = 12 - \frac{5}{w}.$$

At *w=10*, he will supply 11.5 hours of labor.

4. Mary's constraint is

$$H + L = 24,$$

where L is the number of hours she works. But her income is $I = wL$, which means $L = \frac{I}{w}$. Thus her constraint can be written as

$$H + \frac{1}{w}I = 24.$$

Mary's problem is to then maximize her utility subject to this constraint, which looks much like a budget constraint where her "income" is 24, $p_H = 1$ and $p_I = \frac{1}{w}$.

Since the utility function is Cobb-Douglas, we can write down her "demand" for leisure:

$$H = \frac{b}{a+b}(24).$$

Then her labor supply, which is (24-H) is

$$L^s(w) = \frac{a}{a+b}(24).$$

5. By the fact that Mary chooses to work 4 hours per day when the wage rate is \$10, we know that the opportunity cost of her first four hours of work is less than or equal to \$10, while the opportunity cost of any additional hours over 4 is greater than \$10. Therefore, under no circumstances would Mary work more hours when she faces the alternative of being paid \$5 per hour for any hours beyond 2. In general, she would work fewer hours.

   Also, her surplus can never be higher under the \$10 scenario as compared to the \$15/\$5 scenario. Under the latter, she will certainly work 2 hours and enjoy an additional surplus of 5*2=\$10. If the opportunity cost of her third and fourth hours is greater than or equal to 5 (but of course less than or equal to 10), she would lose some surplus in the second scenario because she would not work beyond the 2 hours where she would get some surplus from working those hours if the wage were $10. But the maximum amount of surplus she could lose is $10, which would occur if the opportunity cost of the third and fourth hours was below $5. In general, this lost surplus would be smaller than $10 if any of this time has an opportunity cost greater than 5 but less than 10. Thus at worst she would be indifferent between the two jobs; in general, she would prefer the second one. Thus, she would accept it.

Chapter 5: Production, Cost and Supply

1. (a) Each firm's long run output will equal the output at which average cost is minimized. Now

$$AC(q) = \frac{C(q)}{q} = wq - 10 + \frac{100}{q}.$$

Then, using the fact that *w=1*, AC will be minimized where

$$\frac{dAC}{dq} = w - \frac{100}{q^2} = 0 \qquad \rightarrow \qquad q = 10.$$

To confirm we have a minimum:

$$\frac{d^2AC}{dq^2} = \frac{200}{q^3} > 0$$

so we do indeed have a minimum.

(b) At *q=10*,

$$AC = 10 - 10 + \frac{100}{10} = 10$$

and so $p = 10$ in the long run equilibrium. At this price, demand will be

$$Q_d = 40,000 - 10,000 = 30,000$$

and so, since each firm produces 10 units of output, the number of firms that will operate in the long run is

$$n = \frac{30,000}{10} = 3,000.$$

(c) If *w=4*, AC will be minimized where

$$\frac{100}{q^2} = 4 \qquad \rightarrow \qquad q = 5.$$

At this level of output,

$$AC = 20 - 10 + \frac{100}{5} = 30$$

and so $p = 30$ in the long run equilibrium. At this price, demand will be

$$Q_d = 40,000 - 30,000 = 10,000$$

and so, since each firm produces 5 units of output, the number of firms that will operate in the long run is

$$n = \frac{10,000}{5} = 2,000.$$

2. (a) First find each firm's marginal cost:

$$MC = \frac{dC}{dq} = 2q.$$

Next, find the average variable cost. From the cost function, we know that

$$VC = q^2,$$

so

$$AVC = \frac{VC}{q} = \frac{q^2}{q} = q.$$

We see that $MC > AVC$ for all q and so the entire MC curve is the firm's supply curve. Therefore, each firm's supply curve is

$$p = 2q \qquad or \qquad q = \frac{1}{2}p.$$

To find the short run price and quantity of widgets, we need to find the market supply curve and solve for the equilibrium. Now the market supply, Q_s, will simply be 12 times the individual firm supply. Therefore

$$Q_s = 12 \cdot \frac{1}{2}p = 6p.$$

At equilibrium, supply must equal demand, so

$$6p = 28 - p \qquad \rightarrow \qquad p = 4$$

is the equilibrium price and the quantity will be

$$Q = 6 * 4 = 24.$$

(b) In the long run, each firm would produce at the minimum point of the AC curve. Now,

$$AC = \frac{C(q)}{q} = \frac{q^2 + 1}{q} = q + \frac{1}{q}.$$

Then AC will be minimized where

$$\frac{dAC}{dq} = 1 - \frac{1}{q^2} = 0 \qquad \rightarrow \qquad q = 1.$$

To confirm this is a minimum, check

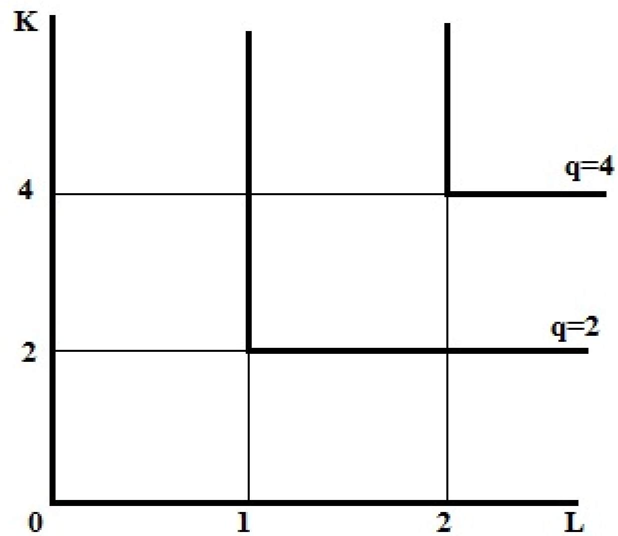
$$\frac{d^2AC}{dq^2} = \frac{2}{q^3} = 2 > 0,$$

so we do indeed have a minimum. Thus each firm will produce 1 unit of output. At *q=1*,

$$AC = q + \frac{1}{q} = 2$$

and so $p = 2$ in the long run equilibrium. At this price, demand will be $Q_d = 26$ and so 26 firms will operate in the long run.

3. (a) L-shaped isoquants:



(b) Since $\bar{K} = 20$, the firm's maximum output is $\bar{q} = 20$. For $q \leq 20$, L will be chosen such that

$$2L = q \qquad or \qquad L = \frac{q}{2}.$$

This is the conditional input demand function for labor. The short run cost function is then

$$C_{SR} = \begin{cases} 20r + \frac{w}{2}q \quad for \quad q \leq 20 \\ \infty \quad for \quad q > 20 \end{cases}$$

To find the supply curve, we need to look at the marginal cost curve:

$$MC = \frac{dC}{dq} = \frac{w}{2} \quad for \quad q \leq 20.$$

Then the supply curve is a reverse-L shaped curve:

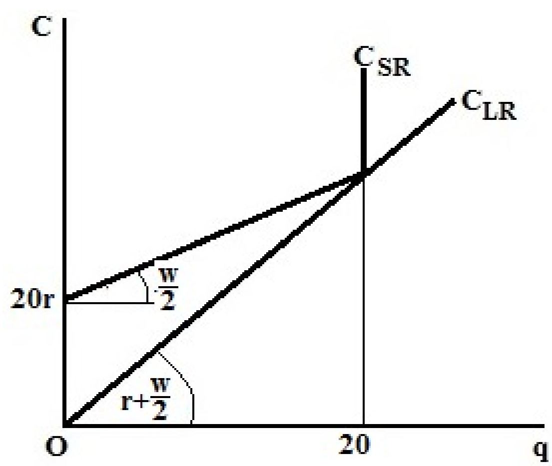
$$q^s(p) = \begin{cases} 20 & for \quad p \geq \frac{w}{2} \\ 0 & for \quad p < \frac{w}{2} \end{cases}$$

(c) In the long run, the input demand functions will be

$$K = q \qquad and \qquad L = \frac{q}{2}.$$

Therefore the long run cost function is

$$C_{LR} = \left(\frac{w}{2} + r\right) q.$$



4. (a) Since the production function is Leontief, we know that, when the firm chooses its inputs optimally,

$$q = 5K = 10L.$$

We can therefore write down the conditional input demand functions as

$$K = \frac{q}{5} \qquad and \qquad L = \frac{q}{10}.$$

The long run cost function is then

$$C_{LR} = r\frac{q}{5} + w\frac{q}{10} = \left(\frac{r}{5} + \frac{w}{10}\right) q.$$

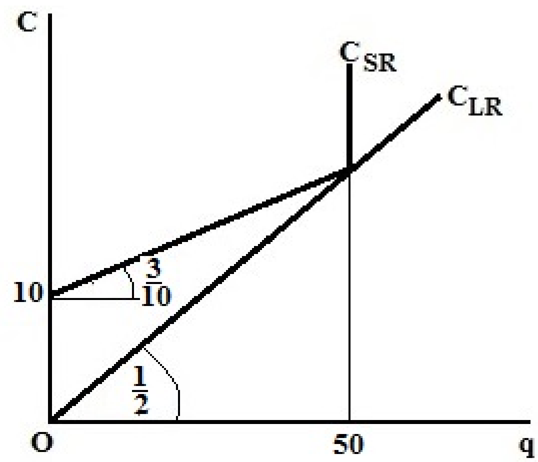
Given the input prices, this becomes

$$C_{LR} = \frac{1}{2}q.$$

(b) If $\bar{K} = 10$, output must be less than or equal to 50. The conditional input demand function for L remains as before as long as $q \leq 50$. the short run cost function is then

$$C_{SR} = \begin{cases} 10 + \frac{3}{10}q & for \quad q \leq 50 \\ \infty & for \quad q > 50 \end{cases}$$

(c) .



5. (a) The long run cost function can be found by optimizing the short run cost function over K. Replacing the fixed K_0 by the variable K and differentiating the short run cost function with respect to K, we get:

$$\frac{dC}{dK} = r - \frac{q^2}{K^2} = 0 \qquad \rightarrow \qquad K = \frac{q}{\sqrt{r}}.$$

Taking the second derivative, we can confirm that this will give us a maximum, since

$$\frac{d^2C}{dK^2} = -\frac{2q^2}{K^3} < 0.$$

Substitute the found K in the short run cost function:

$$C = r \cdot \frac{q}{\sqrt{r}} + q^2 \cdot \frac{\sqrt{r}}{q} \qquad \rightarrow \qquad C = 2(\sqrt{r})q.$$

This is the long run cost function.

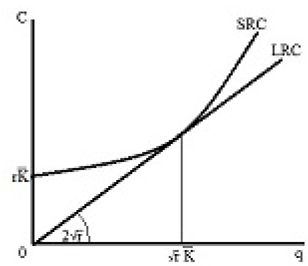
(b) Since the optimal K is given by

$$K = \frac{q}{\sqrt{r}}$$

K_0 would be the optimal K when

$$q = \sqrt{r}K_0.$$

The long run cost function is clearly a ray through the origin with slope equal to $2\sqrt{r}$. The short run cost function is convex and rising. The two are shown on the graph.



6. (a) To check for returns to scale, let Q_0 be the output level when K_0 is the level of capital input and L_0 is the level of labor input. Then

$$Q_0 = \frac{K_0 L_0}{K_0 + L_0}.$$

Now suppose the levels of input are increased to a multiple λ of the previous levels. Then the new level of output will be

$$Q_1 = \frac{(\lambda K_0)(\lambda L_0)}{\lambda K_0 + \lambda L_0} = \lambda \left(\frac{K_0 L_0}{K_0 + L_0} \right) = \lambda Q_0.$$

Thus output has also been multiplied by the same factor λ, indicating that the production function exhibits constant returns to scale

(b) To find the conditional demand functions, we must solve the firm's cost minimization problem:

$$\textit{Minimize} \qquad C = wL + rK$$

$$\textit{subject to} \qquad \frac{KL}{K+L} = Q.$$

The Lagrangian for the problem is

$$\mathcal{L} = wL + rK + \lambda \left[Q - \frac{KL}{K+L} \right].$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial L} = w + \lambda \frac{(K+L)K - KL}{(K+L)^2} = 0 \qquad \rightarrow \qquad w = -\lambda \frac{K^2}{(K+L)^2}$$

$$\frac{\partial \mathcal{L}}{\partial K} = r + \lambda \frac{(K+L)L - KL}{(K+L)^2} = 0 \qquad \rightarrow \qquad r = -\lambda \frac{L^2}{(K+L)^2}$$

Dividing one equation by the other, we get

$$\frac{w}{r} = \frac{K^2}{L^2} \qquad \rightarrow \qquad K = \sqrt{\frac{w}{r}} \cdot L.$$

Substituting this in the production function, we find

$$Q = \frac{\sqrt{\frac{w}{r}} \cdot L \cdot L}{\sqrt{\frac{w}{r}} \cdot L + L} = \frac{\sqrt{\frac{w}{r}} \cdot L}{\sqrt{\frac{w}{r}} + 1}.$$

Rearranging,

$$L = \left(1 + \sqrt{\frac{r}{w}}\right) Q.$$

This is the conditional demand function for L. Substituting in the expression we found earlier for K and simplifying, we get the conditional demand function for K:

$$K = \left(1 + \sqrt{\frac{w}{r}}\right) Q.$$

(c) If $w=0$, we see from the conditional demand function for K that $K=Q$. So K is not set equal to zero.

7. (a) To find the long run cost function, we must minimize the cost of producing any desired output level. Now the production function is of a Leontief type, so we know that, to minimize cost, Widget must produce at the corners of its isoquants. In other words,

$$q^2 = K \qquad \textit{and} \qquad q^2 = \frac{L}{4}.$$

Thus the conditional input demand functions are

$$K = q^2 \qquad \textit{and} \qquad L = 4q^2.$$

Therefore the long run cost function is

$$C(q, w, r) = rq^2 + 4wq^2 = (r + 4w)q^2.$$

(b) If Widget's capital input is fixed at $K = 100$, the maximum output it can produce is 10. The demand for labor will be the same as we saw in part (a). Therefore the short run cost function is

$$C(q, w, r, \bar{K}) = \begin{cases} 100r + 4wq^2 & for \quad q \leq 10 \\ \infty & for \quad q > 10 \end{cases}$$

The average cost function is

$$AC(q, w, r, \bar{K}) = \begin{cases} \frac{100r}{q} + 4wq & for \quad q \leq 10 \\ \infty & for \quad q > 10 \end{cases}$$

and the marginal cost function is

$$MC(q, w, r, \bar{K}) = \begin{cases} 8wq & for \quad q \leq 10 \\ \infty & for \quad q > 10 \end{cases}$$

(c) Let's consider the AVC and the MC curves first. Clearly, MC is a ray through the origin with slope equal to *8w*, for values of $q \leq 10$, shown as the red line in each of the two figures. The AVC is the term in AC that does not include $\bar{K}$. Therefore

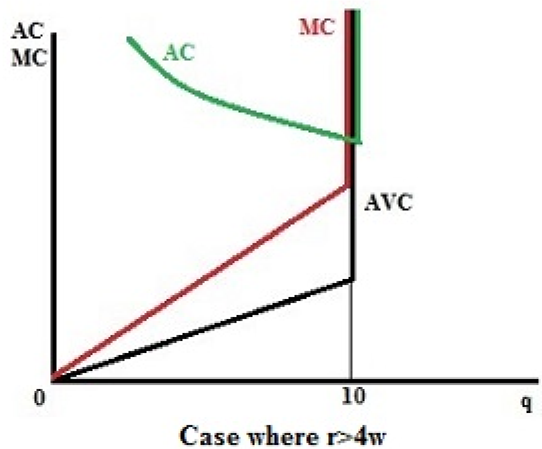
$$AVC = \begin{cases} 4wq & for \quad q \leq 10 \\ \infty & for \quad q > 10 \end{cases}$$

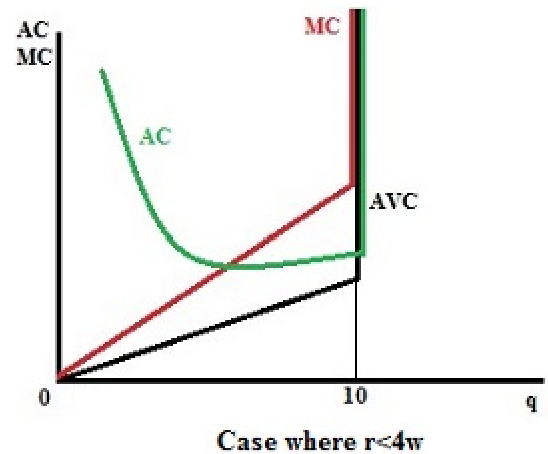
Thus AVC is a ray through the origin with slope equal to *4w* for values of $q \leq 10$, shown as the black line in each of the two figures.

The AC curve is a bit trickier to draw, because it includes the average fixed cost, which is a declining function of *q*. The AC curve will therefore be U-shaped, crossing the MC curve at its minimum point. What is not clear, however, is whether this crossover point (or the minimum point of the AC curve) will be at values of $q \leq 10$ or for a value of q greater than 10. The minimum point of the AC curve will be where

$$\frac{dAC}{dq} = -\frac{100r}{q^2} + 4w = 0 \qquad i.e., \ where \qquad q = 5\sqrt{\frac{r}{w}}.$$

Thus, when $r > 4w$, the minimum point of the AC curve will be at a value of q greater than 10; this case is shown in the panel at left; the AC curve is the green line and we see that it does not cross the MC curve in the area where q<10. If, on the other hand, $r < 4w$, the minimum point of the AC curve will be at a value of q greater than 10; this case is shown in the panel at left. [If $r = 4w$, the minimum point of the AC curve will be at a value of $q = 10$; this case is not shown but would imply that the AC curve meets the MC curve at precisely $q = 10$.]



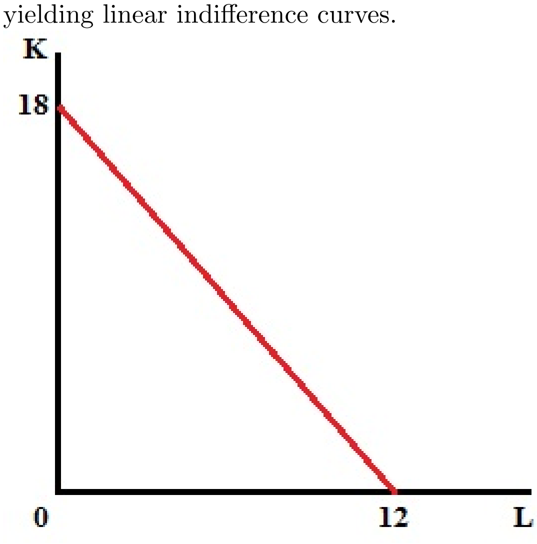


(d) Widget Corp's short run supply curve is simply its MC curve, since its MC curve lies everywhere above the AVC curve. Of course the maximum amount it can supply is 10 units, where its $MC=80w$. Therefore the supply curve is

$$q_s = \begin{cases} \frac{p}{8w} & for\ p \leq 80w \\ 10 & for\ p > 80w \end{cases}$$

If $w = 1$ and $p = 1000$, we see that $p > 80w$ and so, from the supply curve, we see that Widget Corp will supply 10 units to the market.

8. (a) Capital and labor are perfect substitutes in this production function, yielding linear indifference curves.



$MRTS = \frac{2}{3}$.

(b) To check for returns to scale, let

$$q_0 = (2K_0 + 3L_0)^{\frac{1}{2}}$$

and now consider

$$q_1 = (2\{\lambda K_0\} + 3\{\lambda L_0\})^{\frac{1}{2}} = \lambda^{\frac{1}{2}} q_0.$$

Since $q_1 < \lambda q_0$ (for $\lambda > 1$), we can conclude that the production function exhibits decreasing returns to scale.

(c) If capital is fixed at K_0 in the short run, the firm can produce $(2K_0)^{\frac{1}{2}}$ units of output using capital alone. Therefore, it will use only capital for all output levels less than or equal to $(2K_0)^{\frac{1}{2}}$ and will then use labor for any additional output it wants to produce. The short run cost function is then

$$C_{SR}(q, K_0) = \begin{cases} 3K_0 \quad for \quad q \leq (2K_0)^{\frac{1}{2}} \\ 2q^2 - K_0 \quad for \quad q \geq (2K_0)^{\frac{1}{2}} \end{cases}$$

(d) Given the input prices, the firm will want to use only capital in the long run. Therefore the conditional demand functions will be

$$K = \frac{1}{2}q^2 \qquad and \qquad L = 0.$$

The long run cost function is then

$$C_{LR}(q) = \frac{3}{2}q^2.$$

Chapter 6: Markets in Partial Equilibrium

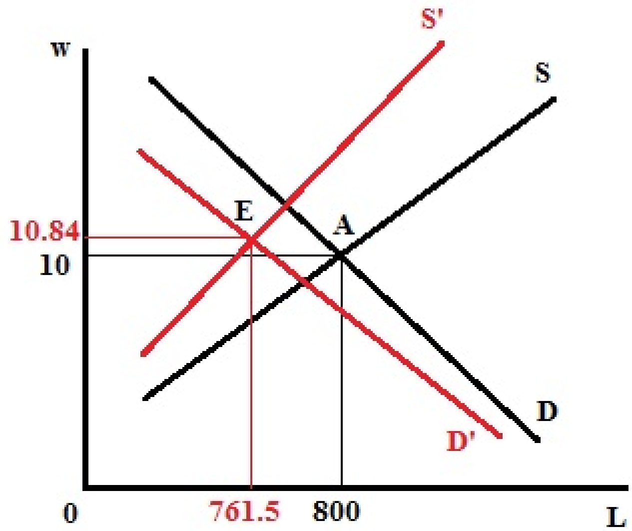
1. (a) If $w_t = w_c = w$, the equilibrium of $L_s = L_d$ will be where

$$160w - 800 = 1000 - 20w \qquad or \qquad w = 10.$$

Then

$$L_s = L_d = 800.$$

(b) The effect of the tax policy is illustrated in the graph. *D'* and *S'* represent the new demand and supply curves that take into account the new tax rules. *S* is 10% lower than *S'* and *D* is 10% higher than *D'*.



D was

$$w_c = 50 - \frac{1}{20}L_d.$$

So D' will be

$$w_c^{'} = \frac{1}{1.1}\left(50 - \frac{1}{20}L_d\right).$$

Similarly, S was

$$w_t = 5 + \frac{1}{160}L_s.$$

So S' will be

$$w_t^{'} = \frac{1}{0.9}\left(5 + \frac{1}{160}L_s\right).$$

At the equilibrium E, $w_c^{'} = w_t^{'}$ and $L_d = L_s = L$. Therefore

$$\frac{1}{1.1}\left(50 - \frac{1}{20}L\right) = \frac{1}{0.9}\left(5 + \frac{1}{160}L\right).$$

Solving, we find $L^* = 761.5, w^* = 10.84$.

(c) Workers' take-home pay is

$$w_t = 0.9 * w_t^{'} = 9.76,$$

and the total cost per worker to the firm is

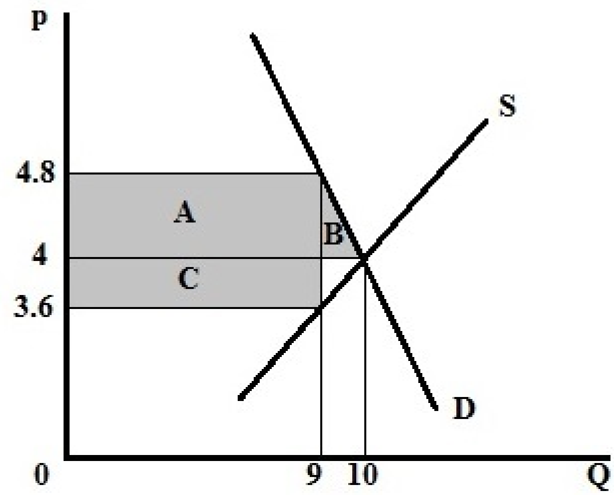
$$w_c = 1.1 * w_c^{'} = 11.93.$$

Thus the burden of the tax is much greater on firms than on workers. This is because demand is less elastic than supply. We can easily calculate the demand and supply elasticities at the original equilibrium:

$$\varepsilon_s = \frac{dL_s}{dw} \cdot \frac{w_0}{L_0} = 160 \cdot \frac{10}{800} = 2,$$

$$\varepsilon_d = \frac{dL_d}{dw} \cdot \frac{w_0}{L_0} = (-20) \cdot \frac{10}{800} = -\frac{1}{4}.$$

2. (a) By distributing coupons for only 9 million gallons of gasoline, government ensures that only 9 million gallons are traded. The demand price p_d will have to rise to clear the demand at that quantity. At the same time, the supply price p_s will have to fall so that only 9 million gallons are supplied. The gap between p_d and p_s will be the value of each coupon. The figure shows the situation.



To calculate p_d and p_s, we can use the discrete versions of the definitions of elasticity of demand and supply respectively.

$$\epsilon_d = \frac{\triangle Q_d}{\triangle p_d} \cdot \frac{p_0}{Q_0} \quad \rightarrow \quad -0.5 = \frac{-1}{\triangle p_d} \cdot \frac{4}{10} \quad \rightarrow \quad \triangle p_d = 0.8.$$

Therefore $p_d = 4.8$. This is the effective price buyers will pay after imputing the value of the coupons they have to surrender.

$$\epsilon_s = \frac{\triangle Q_s}{\triangle p_s} \cdot \frac{p_0}{Q_0} \quad \rightarrow \quad 1 = \frac{-1}{\triangle p_s} \cdot \frac{4}{10} \quad \rightarrow \quad \triangle p_s = -0.4.$$

Therefore $p_s = 3.6$. This is the price sellers will receive and will be the announced price of gasoline.

(b) The price of the coupons will be the gap between p_d and p_s:

$$p_{coupon} = p_d - p_s = 1.2.$$

(c) The calculation of the welfare effect on consumers is complicated by the fact that they not only buy gasoline but are also the owners of the coupons. The loss in consumers' surplus is given by the area A+B in the figure, but they have a gain of A+C in the value of the coupons. Thus, on balance, consumers are better off by C-B:

$$Gain\ in\ Welfare = (9 * 0.4) - (\frac{1}{2} * 1 * 0.8) = \$3.2\ million.$$

3. (a) If MC falls by \$4 per unit, the supply curve will shift downwards by \$4 everywhere. The original supply curve was

$$p = \frac{1}{50}Q_s + 12,$$

so the new supply curve will be

$$p = \frac{1}{50}Q_s^{'} + 8$$

which can be written as

$$Q_s^{'} = 50p - 400.$$

Equilibrium is where $Q_s^{'} = Q_d$, that is where

$$50p - 400 = \frac{1000}{p}.$$

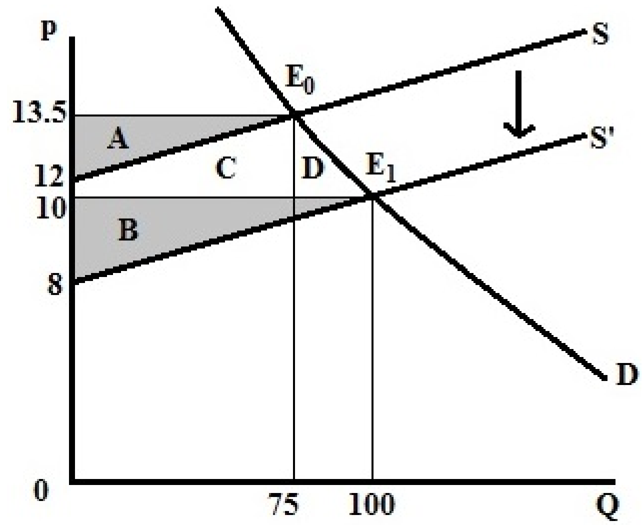
Solving for p:

$$50p^2 - 400p - 1000 = 0$$
$$p^2 - 8p - 20 = 0$$
$$(p + 2)(p - 10) = 0$$
$$p = 10$$

Then

$$Q = \frac{1000}{10} = 100.$$

(b) The situation is illustrated in the figure. The initial equilibrium is at E_0, and the final equilibrium is at E_1. Producers' surplus has gone up by area B minus area A, while consumers' surplus has gone up by area A+B+C.



To calculate these:

$$\triangle PS = \frac{1}{2}(100)(2) - \frac{1}{2}(75)(1.5) = 43.75.$$

$$\triangle CS = \int_{10}^{13.5} \frac{1000}{p} dp = 1000\, ln\, p \,|_{10}^{13.5} = 300.10.$$

Thus the total gain from this cost reduction is 43.75 + 300.10 = 343.85, which is less than the cost of dissemination of 400. The government should not distribute the information.

If we treated the line segment E_0E_1 as linear, we could measure the change in consumers' surplus as follows:

$$\triangle CS = (75)(3.5) + \frac{1}{2}(25)(3.5) = 306.25.$$

This would not change our answer.

4. (a) Equilibrium is where $Q_d = Q_s$, that is, where

$$100 - 2P = 20 + 6P.$$

Solving, we find $P^* = 10, Q^* = 80$.

(b) The new supply curve will be everywhere vertically higher by \$4. The original supply curve was

$$P = \frac{1}{6}\left(Q_s - 20\right).$$

Then the new supply curve will be

$$P = \frac{1}{6}\left(Q_s - 20\right) + 4 \qquad or \qquad Q_s = 6P - 4.$$

(c) The new equilibrium is where

$$100 - 2P = 6P - 4.$$

Solving, we find $Q^* = 74, \quad P_d = 13, \quad P_s = 9.$

(d) The Tax Revenue collected is

$$R = 4 * 74 = 296.$$

Of this,

$$Buyers'\ burden = 3 * 74 = 222,$$

$$Sellers'\ burden = 1 * 74 = 74.$$

$$Deadweight\ Loss = \frac{1}{2} \cdot 6 \cdot 4 = 12.$$

5. (a) Using the formulae for the effects of taxes in competitive markets, we find

$$dp_d = \frac{\varepsilon_s}{\varepsilon_s - \varepsilon_d} dT = \frac{2}{2.5} \cdot 1 = 0.8.$$

$$dp_s = \frac{\varepsilon_d}{\varepsilon_s - \varepsilon_d} dT = \frac{-0.5}{2.5} \cdot 1 = -0.2.$$

$$dQ_s = \varepsilon_s \cdot \frac{Q_0}{p_0} \cdot dp_s = 2 \cdot \frac{100}{10} \cdot (-0.2) = -4.$$

Therefore, the new equilibrium is: $Q^* = 96, \quad P_d = 10.80, \quad P_s = 9.80.$

(b) Tax Revenue collected is

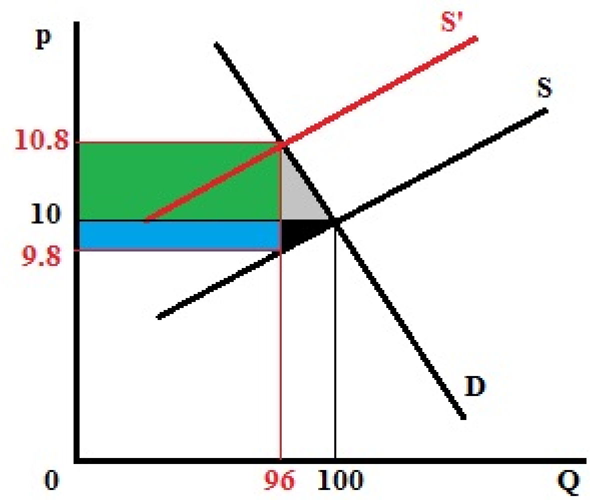
$$TR = 1(96) = 96.$$

(c) The graph shows the effects of the tax. The change in consumers' surplus is the loss of the green plus grey areas. The change in producers' surplus is the loss of the blue plus black areas. The excess burden is the sum of the grey and black areas. So

$$\triangle CS = -\left\{96(0.8) + \frac{1}{2}(4)(0.8)\right\} = -78.4.$$

$$\triangle PS = -\left\{96(0.2) + \frac{1}{2}(4)(0.82)\right\} = -19.6.$$

$$Excess\ Burden = \Delta W = \Delta CS + \Delta PS + TR = -78.4 - 19.6 + 96 = -2.$$



6. (a) Each firm's supply curve is its MC curve above the AVC curve. To find MC:

$$MC(q) = \frac{dC(q)}{dq} = q + w,$$

and, noting that there is no fixed cost in the cost function, AVC=AC:

$$AVC = \frac{1}{2}q + w.$$

Clearly MC>AVC throughout, so the entire MC curve is the firm's supply curve. Thus the supply curve is

$$p = q_s + w \qquad or \qquad q_s = p - w.$$

Now the industry supply $Q_s = 10q_s$, so

$$Q_s = 10p - 10w.$$

But we know that $w = 0.9\,Q_s$. Substituting, we get

$$Q_s = 10p - 10\,(0.9Q_s) \qquad or \qquad Q_s = p.$$

This is the industry supply curve.

(b) At equilibrium, $Q_s = Q_d$, so

$$p = 6 - p \qquad \rightarrow \qquad p = 3.$$

This is the equilibrium price. The equilibrium quantity will be

$$Q_s = Q_d = 3.$$

Then $w = 0.9Q_s = 2.7$ and so, from the firm supply curve, we find each firm's output is

$$q_s = 3 - 2.7 = 0.3.$$

We can confirm that, with each firm producing 0.3, the 10 firms together will produce the total needed output of 3.

(c) If government imposed a \$1 excise tax, the industry supply curve would everywhere shift up by \$1. That is, instead of being $p = Q_s$, the supply curve would be

$$p = Q_s + 1 \qquad \to \qquad Q_s = p - 1.$$

Then, at equilibrium:

$$p - 1 = 6 - p \qquad \to \qquad p = 3.5.$$

This is the demand price p_d, with the supply price $p_s = 2.5$. From the demand curve, we find the equilibrium quantity to be

$$Q = 6 - 3.5 = 2.5.$$

7. (a) Equilibrium will occur where $Q_d = Q_s$, that is, where

$$5000 - 100p = 150p \qquad \to \qquad p = 20.$$

Then $Q_d = Q_s = 3000$.

(b) If gadgets can be imported and sold at a price of \$10, the price would have to be \$10. At this price, the demand would be

$$Q_d = 5000 - 1000 = 4000,$$

and the domestic supply will be

$$Q_s = 150 \cdot 10 = 1500.$$

The remainder $4000 - 1500 = 2500$ would be imported.

(c) If the government imposes an import tax of \$5, the price of the imported gadgets will go up to \$15, and this would now be the market price. Now the demand would be

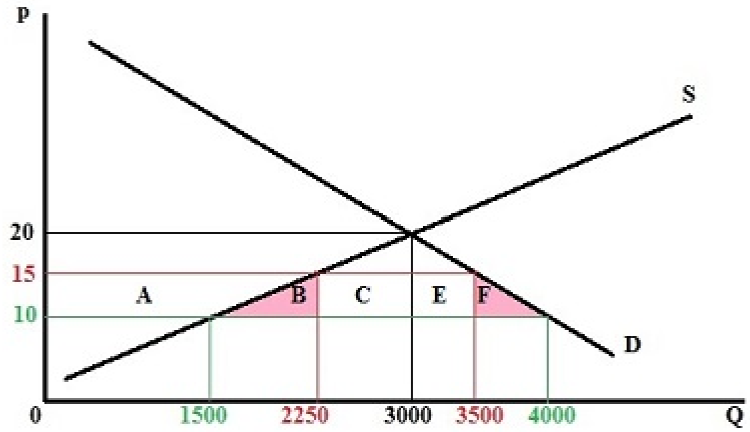
$$Q_d = 5000 - 1500 = 3500,$$

and the domestic supply will be

$$Q_s = 150 \cdot 15 = 2250.$$

The remainder $3500 - 2250 = 1250$ would be imported.

(d) .



The diagram shows the various equilibria. With the tax, consumers lose, since the price they face rises from \$10 to \$15. The net welfare effect on consumers is the negative of the area A+B+C+E+F in the diagram:

$$\Delta CS = -[5 * 3500 + \frac{1}{2} * 5 * 500] = -18,750.$$

Producers are better off, since the price they receive rises from \$10 to \$15. The net welfare effect on domestic producers is the area A:

$$\Delta PS = 5 * 1500 + \frac{1}{2} * 5 * 750 = 9,375.$$

Government also benefits, since it collects tax revenue of the area C+E:

$$Tax = 5 * 1250 = 6,250.$$

The net welfare effect then is

$$\Delta W = -18,750 + 9,375 + 6,250 = -3,125.$$

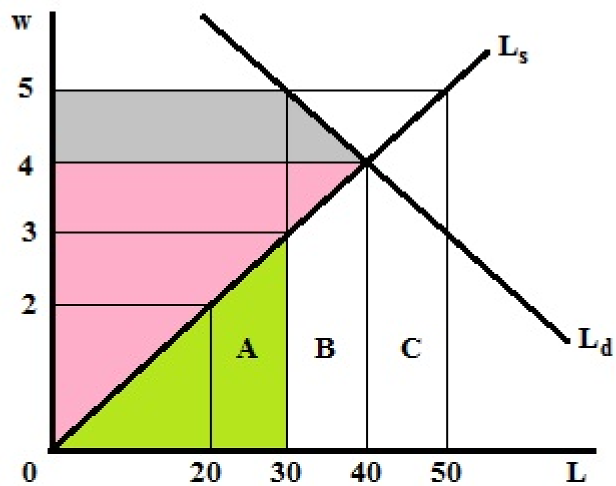
This is equal to the sum of the two shaded areas in the diagram (B+F), which show the quantity distortions caused by the tax and the resulting deadweight losses associated with them.

8. (a) Market equilibrium will occur where $L_d = L_s$, that is, where

$$80 - 10w = 10w \qquad or \qquad w^* = 4.$$

Then $L^* = 40$.

(b) If $w = 5$, $L_d = 30$ and $L_s = 50$. Thus there will be excess supply of labor. The level of employment will be $L^* = 30$ and 20 units of labor will be unemployed.



Calculating the net welfare impact of this policy is difficult, because we do not know how the scarce jobe will be rationed amongst the willing workers. The welfare loss to employers is clear; it is the grey-shaded area in the graph (lost consumers' surplus) and this can easily be calculated as

$$\triangle W_{employers} = -\left(1 \cdot 30 + \frac{1}{2} \cdot 1 \cdot 10\right) = -35$$

But the welfare gain to workers is problematic. Prior to the implementation of the minimum wage policy, workers as a group enjoyed a surplus equal to the pink shaded area in the graph, which can be calculated as

$$S_0 = \frac{1}{2} \cdot 4 \cdot 40 = 80$$

The surplus after the implementation of the minimum wage policy depends upon who gets the jobs. If the jobs go to the 30 workers without the lowest opportunity cost, which is the green-shaded area in the graph, the surplus can be calculated as

$$S_1 = 5 \cdot 30 - \frac{1}{2} \cdot 3 \cdot 30 = 105.$$

Thus workers would have gained surplus of 25 and the net welfare impact would be

$$\Delta W_1 = -35 + 25 = -10.$$

At the other extreme, if the jobs went to the workers with the highest opportunity costs (the area A+B+C), the surplus accruing to workers would be their wages minus the opportunity cost:

$$S_2 = 5 \cdot 30 - \left(\frac{1}{2} \cdot 5.50 - \frac{1}{2} \cdot 2 \cdot 20\right) = 45.$$

In this case, the workers would have suffered a loss in surplus of 35 and so the net welfare effect of the policy would be

$$\Delta W_2 = -35 - 35 = -70.$$

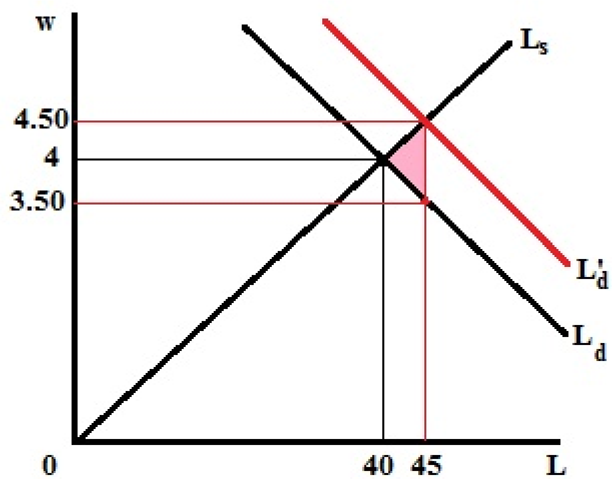
Thus $\triangle W$ is indeterminate in the range $-10 \leq \triangle W \leq -70$ because it depends upon how the scarce jobs are rationed.

(c) The effect of a wage subsidy would be to shift the labor demand curve vertically higher by \$1 from L_d to $L_d^{'}$. The original labor demand curve was

$$w = 8 - \frac{1}{10} L_d.$$

The new labor demand will be

$$w = 9 - \frac{1}{10} L_d^{'}.$$



The equilibrium is therefore

$$w^* = 4.50, L^* = 45.$$

The welfare loss is the shaded area in the graph. It can be broken down as follows:

$$\triangle W_{employers} = \Delta CS = 0.5(42.5) = 21.25.$$

$$\triangle W_{workers} = \Delta PS = 0.5(42.5) = 21.25.$$

$$\triangle W_{taxpayers} = -1(45) = -45.$$

$$Net \triangle W = 21.25 + 21.25 - 45 = -2.50.$$

9. (a) The Lagrangian for this problem is

$$\mathcal{L} = rK + wL + \lambda\left[q - K^{\frac{1}{4}}L^{\frac{1}{4}}\right].$$

The first-order tangency condition is

$$\frac{w}{r} = \frac{F_L}{F_K} = \frac{-\frac{1}{4}K^{\frac{1}{4}}L^{-\frac{3}{4}}}{-\frac{1}{4}K^{-\frac{3}{4}}L^{\frac{1}{4}}} = \frac{K}{L}.$$

Since $w = r = 1$, this yields $K = L$. Then the input demand functions are

$$K = q^2 \qquad and \qquad L = q^2.$$

Then total cost is

$$C(q) = rq^2 + wq^2 = 2q^2.$$

(b) Each firm's marginal cost is

$$MC = \frac{dC}{dq} = 4q.$$

There are no fixed costs; the average variable cost is

$$AVC = \frac{C(q)}{q} = 2q.$$

Thus we see that $MC > AVC$ everywhere and so the MC curve is the firm's supply curve. Thus each firm's supply curve is

$$q_s = \frac{1}{4}p.$$

Since there are 100 firms, the market supply curve is $100q_s$, which is

$$Q_s = 25p.$$

Setting $Q_s = D(p)$ and solving, we get the equilibrium:

$$p_1 = 8, Q = 200, q_1 = 2, K_1 = 4.$$

(c) If K is fixed at 4, the conditional demand function for labor will be the solution to

$$q = 4^{\frac{1}{4}}L^{\frac{1}{4}} \qquad or \qquad L = \frac{1}{4}q^4.$$

Then, remembering that $w = r = 1$, the short-run cost function is

$$C_{SR}(q) = 4 + \frac{1}{4}q^4.$$

(d) In the short run, each firm's marginal cost is

$$MC = \frac{dC_{SR}(q)}{dq} = q^3.$$

At the same time, the average variable cost is

$$AVC = \frac{1}{4}q^3.$$

Thus we see that $MC > AVC$ everywhere and so the MC curve is the firm's supply curve. Thus each firm's short run supply curve is

$$p = q^3 \qquad or \qquad q_{SR} = p^{\frac{1}{3}}.$$

Then the market supply curve is

$$Q_{SR} = 100p^{\frac{1}{3}}.$$

Setting this equal to $D^*(p)$ and solving, we get the short run equilibrium:

$$p_2 = 64, Q_2 = 400, q_2 = 4.$$

(e) In the intermediate run, the market supply curve will be $Q_s = 25p$, as we found in (b). Setting this equal to $D^*(p)$ and solving, we get the intermediate run equilibrium:

$$p_3 = (256)^{0.6} \approx 27.858, q_3 \approx 6.9644.$$

Chapter 7: General Equilibrium and Welfare

1. (a) From the production functions, we can write

$$L_w = \frac{W}{5} \qquad and \qquad L_g = \frac{G^2}{100}.$$

Substituting in the labor availability constraint $L_w + L_g = 400$, we get

$$\frac{W}{5} + \frac{G^2}{100} = 400$$

which can be rewritten as

$$W = 2000 - \frac{G^2}{20}.$$

This is the production possibilities frontier (ppf).

(b) We need

$$\frac{W}{G} = 10 \qquad or \qquad W = 10G.$$

Substituting this in the ppf and solving for G, we get

$$G^* = 123.6, W^* = 1236.$$

We know the competitive equilibrium requires the slope of the ppf to equal the price ratio:

$$\frac{dW}{dG} = -\frac{G}{10} = -\frac{p_g}{p_w}.$$

Since $G = 123.6$ and we are setting $p_w = 1$, this yields

$$p_g = 12.36.$$

(c) If trade can take place at a price ratio $\frac{p_g}{p_w} = 10$, production should take place at a point where that is the slope of the ppf. Thus we can find the optimal production bundle:

$$\frac{G}{10} = 10 \qquad \rightarrow \qquad G_p^* = 100, W_p^* = 1500.$$

The value of this production bundle (think of it as income) is 2500. To optimize consumption, this income needs to be spent in such a way that $\frac{W_c}{G_c} = 10$. Thus we must have

$$10G_c + W_c = 2500 \qquad \rightarrow \qquad 10G_c + 10G_c = 2500.$$

This gives us the optimal consumption pattern:

$$G_c^* = 125, W_c^* = 1250.$$

2. (a) Both Smith and Jones desire "balanced" consumption bundles, so it would be desirable that both goods are produced. We know that the marginal rate of transformation must equal the price ratio in the competitive equilibrium. But

$$MRT = \frac{MPL_x}{MPL_y} = \frac{2}{2} = 1.$$

Therefore the equilibrium price ratio must be

$$\frac{p_x}{p_y} = 1.$$

(b) Since $w = 1$, both individuals earn \$10 per day. Now, we know that

$$w = p_k MPL_k \qquad for\ k = x, y.$$

Since $MPL_x = MPL_y = 2$, this means we must have $p_x = p_y = \frac{1}{2}$. Given that the utility functions are Cobb-Douglas, we can write down the demand functions for each consumer and find their consumption levels:

$$X_s = \frac{3}{10} \cdot \frac{I_s}{p_x} = 6 \qquad and \qquad Y_s = \frac{7}{10} \cdot \frac{I_s}{p_y} = 14.$$

$$X_j = \frac{1}{2} \cdot \frac{I_j}{p_x} = 10 \qquad and \qquad Y_j = \frac{1}{2} \cdot \frac{I_j}{p_y} = 10.$$

(c) Production levels are

$$X = 16 \qquad and \qquad Y = 24.$$

The labor allocations follow from the production functions:

$$L_x = 8, \quad L_y = 12.$$

3. (a) From the production functions, we can write

$$L_c = \frac{C}{2} \qquad and \qquad L_f = F^2.$$

Substituting in the labor availability constraint $L_c + L_f = 9$, we get

$$\frac{C}{2} + F^2 = 9,$$

which can be rewritten as

$$C = 18 - 2F^2.$$

This is the production possibilities frontier (ppf).
The MRT is the absolute value of the slope of the ppf. Therefore

$$MRT = 4F.$$

(b) To maximize his utility, Crusoe will maximize his utility subject to the ppf as a constraint. The Lagrangian for this problem is

$$\mathcal{L} = CF + \lambda \left[9 - \frac{C}{2} - F^2 \right].$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial C} = F - \frac{\lambda}{2} = 0$$

$$\frac{\partial \mathcal{L}}{\partial F} = C - 2\lambda F = 0.$$

Combining these equations, we get

$$\frac{C}{F} = 4F \qquad or \qquad F^2 = \frac{C}{4}.$$

Substituting this in the ppf and solving, we get the optimal production levels:

$$C^* = 12, F^* = \sqrt{3}.$$

The labor allocations follow from the production functions:

$$L_c^* = 6, L_f^* = 3.$$

(c) We know

$$MRS = \frac{U_f}{U_c} = \frac{C}{F} = \frac{12}{\sqrt{3}} = 4\sqrt{3}.$$

In part (a), we found

$$MRT = 4F = 4\sqrt{3}.$$

Thus we have$MRT = MRS = 4\sqrt{3}$.
In competitive equilibrium, we have

$$\frac{p_f}{p_c} = MRS = MRT.$$

Therfore the equilibrium price ratio would be

$$\left(\frac{p_f}{p_c}\right)^* = 4\sqrt{3}.$$

4. (a) In competitive equilibrium,

$$w = p_x \cdot MPL_x \qquad and \qquad w = p_y \cdot MPL_y.$$

Therefore

$$\left(\frac{p_x}{p_y}\right)^* = \frac{MPL_y}{MPL_x} = \frac{3}{2}.$$

(b) Since the productivities of the two individuals are identical, there are no gains from trade possible in this situation. Each individual will maximize his utility subject to the production constraint. This can be derived as follows for each individual:

$$L_x + L_y = 10 \qquad \rightarrow \qquad \frac{x}{2} + \frac{y}{3} = 10 \qquad \rightarrow \qquad 3x + 2y = 60.$$

This looks just like a budget constraint where $p_x = 3, p_y = 2, I = 60$. Since the utility functions are Cobb-Douglas, we can write down the demand functions and then find the equilibrium prodiction and consumption levels for each good and for each individual:

$$x_s^* = \frac{3}{10} \cdot \frac{I}{p_x} = 6 \qquad and \qquad y_s^* = \frac{7}{10} \cdot \frac{I}{p_y} = 21.$$

$$x_j^* = \frac{1}{2} \cdot \frac{I}{p_x} = 10 \qquad and \qquad y_j^* = \frac{1}{2} \cdot \frac{I}{p_y} = 15.$$

Total production of the two goods is then

$$Production: \ x^* = 16, y^* = 36.$$

5. (a) To find the ppf, we need to solve the following problem:

$$Maximize \qquad B = L_b^{\frac{1}{2}} T_b^{\frac{1}{2}}$$

$$subject\ to \qquad C = 4(100 - L_b) + (400 - T_b).$$

The Lagrangian for the problem is

$$\mathcal{L} = L_b^{\frac{1}{2}} T_b^{\frac{1}{2}} + \lambda \left[C - 4(100 - L_b) - (400 - T_b) \right].$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial L_b} = \frac{1}{2} L_b^{-\frac{1}{2}} T_b^{\frac{1}{2}} + \lambda(4) = 0 \qquad \rightarrow \qquad \frac{1}{2} \frac{T_b^{\frac{1}{2}}}{L_b^{\frac{1}{2}}} = -4\lambda$$

$$\frac{\partial \mathcal{L}}{\partial T_b} = \frac{1}{2} L_b^{\frac{1}{2}} T_b^{-\frac{1}{2}} + \lambda(1) = 0 \qquad \rightarrow \qquad \frac{1}{2} \frac{L_b^{\frac{1}{2}}}{T_b^{\frac{1}{2}}} = -\lambda$$

Dividing one equation by the other, we get

$$\frac{T_b}{L_b} = 4 \qquad \rightarrow \qquad T_b = 4L_b.$$

Substituting this in the production function, we find

$$B = L_b^{\frac{1}{2}} \left(4L_b\right)^{\frac{1}{2}} = 2L_b.$$

Therefore

$$L_b = \frac{B}{2} \qquad and \qquad T_b = 4L_b = 2B.$$

Substituting this in the constraint of the optimization exercise (the production function for C, we find

$$C = 4\left(100 - \frac{B}{2}\right) + (400 - 2B) \qquad \rightarrow \qquad C = 800 - 4B.$$

This is the ppf.

(b) Since all the utility functions are identical, we can take the given utility function as the typical or social utility function. Then, to find the efficient levels of production, we need to solve the problem:

$$\begin{aligned} &Maximize \qquad U = lnB + lnC \\ &subject\ to \qquad C = 800 - 4B. \end{aligned}$$

Substituting for C in the utility function, we can convert this to the unconstrained problem to

$$Maximize \qquad U = lnB + ln\,(800 - 4B)\,.$$

Then the first-order condition is

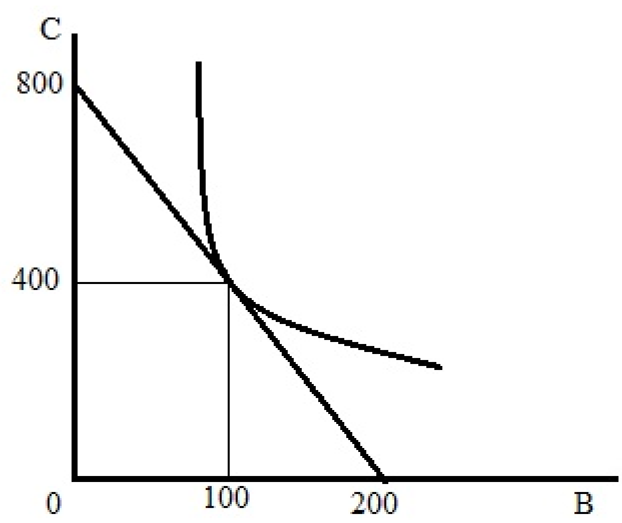
$$\frac{dU}{dB} = \frac{1}{B} - \frac{4}{800 - 4B} = 0 \qquad \rightarrow \qquad B = 100.$$

Then

$$C = 800 - 4(100) = 400.$$

These are the efficient levels of output. Since the ppf is linear, for both goods to be produced, the price ratio must equal the absolute value of the slope of the ppf. Therefore

$$\frac{p_b}{p_c} = 4.$$



We know that, under perfect competition, factors must be paid their marginal revenue products. Looking at the production function for C, we see that

$$MPL_c = 4 \qquad and \qquad MPT_c = 1.$$

and to find the MRP for each of these, we just need to multiply each by the price of corn, p_c. Then

$$w = 4p_c \qquad \textit{and} \qquad r = p_c.$$

If the rental rate on land, r, is set equal to 1, then $p_c = 1$ and w=*4*.

We have already seen that $p_c = 1$ and $\frac{p_b}{p_c} = 4$. Therefore $p_b = 4$.

6. Since the labor market is competitive, the marginal revenue product of labor (MRP) must be equal in the two markets. Now

$$MPL_c = \frac{2000}{L_c} \qquad \textit{and so} \qquad MRP_c = 20 \cdot \frac{2000}{L_c} = \frac{40,000}{L_c}.$$

Similarly

$$MPL_b = \frac{100}{L_b} \qquad \textit{and so} \qquad MRP_c = 100 \cdot \frac{100}{L_b} = \frac{10,000}{L_b}.$$

Setting these equal, using the fact that $L_c + L_b = 10,000$, we get

$$\frac{40,000}{L_c} = \frac{10,000}{(10,000 - L_c)} \qquad \rightarrow \qquad L_c = 8,000 \quad \textit{and so}\, L_b = 2,000.$$

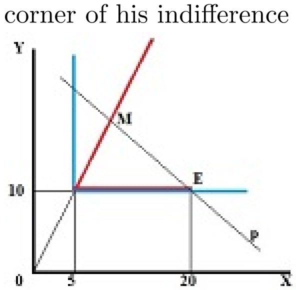
The equilibrium wage rate will be the MRP in each of the markets (which are equal):

$$w = \frac{40,000}{8,000} \qquad \rightarrow \qquad w = \$5.$$

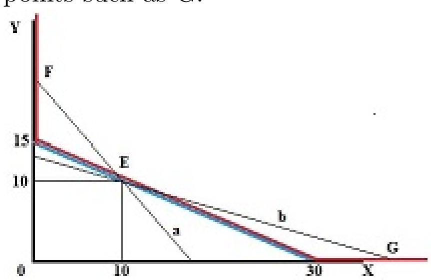
7. (a) Adam's utility function is of the Leontief type, with Adam always wanting to consume X and Y in the fixed ratio that should satisfy

$$x_A = \frac{y_A}{2}, \quad \textit{that is} \quad \frac{x_A}{y_A} = \frac{1}{2}.$$

A typical indifference curve looks like the blue L-shaped line in the figure. In that case, Adam's offer curve will be the red line in the figure, since, facing a price ratio such as p, he would want to consume at points along the ray through the origin (so that he can be on the corner of his indifference curve) such as M.



(b) Becky's utility function shows that she regards X and Y as perfect substitutes. Her indifference curves will be linear, like the blue line in the figure below, with slope equal to $-(1/2)$. Her offer curve will then be the red line. When the price ratio is steeper than her indifference curve (such as line a), she will want to consume only Y at a point such as F. And when the price ratio is flatter than her indifference curve (such as line b), she will want to consume nothing but X at points such as G.



(c) If Adam and Becky form a pure exchange economy, their Edgeworth box will look like the one shown below. Here the red line is Adam's offer curve and the green line is Becky's offer curve. The two offer curves intersect at C and this would be the competitive (Walrasian) equilibrium. The price ratio would be the slope of the green line:

$$\left(\frac{p_x}{p_y}\right)^* = \frac{1}{2} \tag{29}$$

which is also the slope of Becky's indifference curve. Becky is indifferent between her endowment at E and any point on this line. Thus the extent of trade will depend on Adam's choice. Now Adam's endowment is 20 units of X and 10 units of Y, and he wants his consumption pattern to conform to:

$$\frac{x_A}{y_A} = \frac{1}{2}, \quad \textit{that is} \quad y_A = 2x_A. \tag{30}$$

Further, the value of his consumption must equal the value of his endowment, therefore we must have

$$p_x x_A + p_y y_A = 20p_x + 10p_y. \tag{31}$$

But we know from (29) that

$$\frac{p_x}{p_y} = \frac{1}{2}.$$

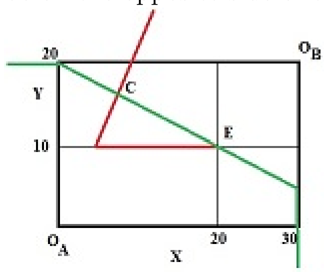
If we let $p_x = 1$ and $p_y = 2$, (31) becomes

$$x_A + 2y_A = 40.$$

And, using (30) then allows us to find

$$x_A = 8 \qquad and \qquad y_A = 16.$$

Thus Adam will sell 12 units of X and buy 6 units of Y; Becky will be on the opposite side of these trades.



8. (a) Jack's budget constraint is

$$p_x x_j + p_y y_j = 8p_x,$$

since his endowment is 8 xylophones. His utility function is of the Leontief type, so we know that, at his optimum,

$$2x_j = y_j.$$

Substituting this in his budget constraint and solving, we get his demand functions:

$$x_j = \frac{8p_x}{p_x + 2p_y} \qquad and \qquad y_j = \frac{16p_x}{p_x + 2p_y}.$$

Bernice's budget constraint is

$$p_x x_b + p_y y_b = 12p_y.$$

Her utility function is Cobb-Douglas, and she will spend half her income on each good. Therefore her demand functions are

$$x_b = \frac{6p_y}{p_x} \qquad and \qquad y_b = 6.$$

(b) Since Bernice's demand for yogurt is $y_b = 6$ and the total supply of xylophones is 12, we must have $y_j = 6$. That is, in equilibrium,

$$\frac{16p_x}{p_x + 2p_y} = 6 \qquad \rightarrow \qquad \left(\frac{p_x}{p_y}\right)^* = \frac{6}{5}.$$

If we then for convenience set $p_x = 6, p_y = 5$, we can use the demand functions to find the consumption bundles:

$$x_j^* = 3, y_j^* = 6; \quad x_b^* = 5, y_b^* = 6.$$

(c) If the endowments were switched, Jack's budget constraint would be

$$p_x x_j + p_y y_j = 12p_y$$

and so his demand functions would now be

$$x_j = \frac{12p_y}{p_x + 2p_y} \qquad and \qquad y_j = \frac{24p_y}{p_x + 2p_y}.$$

Similarly, we can find Bernice's demand functions which are now

$$x_b = 4 \qquad and \qquad y_b = \frac{4p_x}{p_y}.$$

Since $x_b = 4$, we must have

$$x_j = \frac{12p_y}{p_x + 2p_y} = 4 \qquad \rightarrow \qquad \left(\frac{p_x}{p_y}\right)^* = 1.$$

As before, we can then solve for the consumption bundles:

$$x_j^* = 4, y_j^* = 8; \quad x_b^* = 4, y_b^* = 4.$$

Jack is better off now, with $U_j = 8 > 6$.

(d) No, because competitive equilibrium is Pareto efficient. Therefore, as long as the total supply of the goods remains unchanged, no reallocation could be a Pareto improvement.

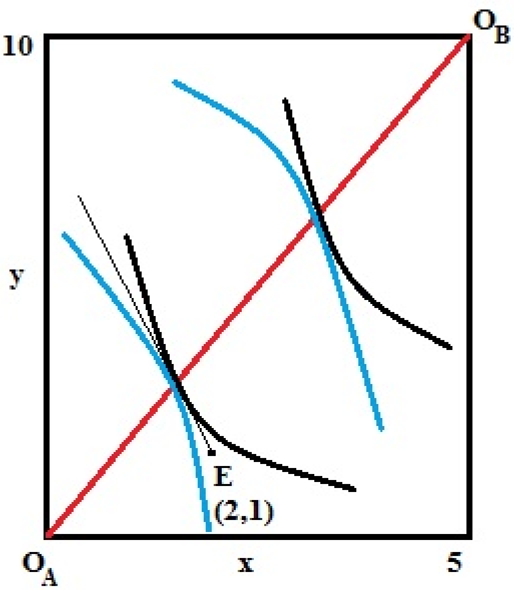
9. (a) We know that the condition for Pareto efficiency is

$$MRS_A = MRS_B \qquad that\ is \qquad \frac{y_A}{x_A} = \frac{y_B}{x_B}.$$

But $(x_A + x_B) = 5$ and $(y_A + y_B) = 10$. Substituting in the efficiency condition, we find

$$\frac{y_A}{x_A} = 2 \qquad or \qquad y_A = 2x_A.$$

The set of Pareto efficient allocations then (the contract curve) will be the red line in the figure.



(b) If the Walrasian auctioneer announces prices p_x and p_y, the income levels for A and B will be respectively

$$I_A = 2p_x + p_y \qquad and \qquad I_B = 3p_x + 9p_y.$$

Given their Cobb-Douglas utility functions, each consumer will wish to spend half their income on each good. Therefore, their demands for x will be

$$x_A = \frac{2p_x + p_y}{2p_x} \qquad and \qquad x_B = \frac{3p_x + 9p_y}{2p_x}.$$

But, in equilibrium, we must have $x_A + x_B = 5$. Combining this restriction with the demand functions yields the equilibrium price ratio in competitive equilibrium:

$$\left(\frac{p_x}{p_y}\right)^* = 2.$$

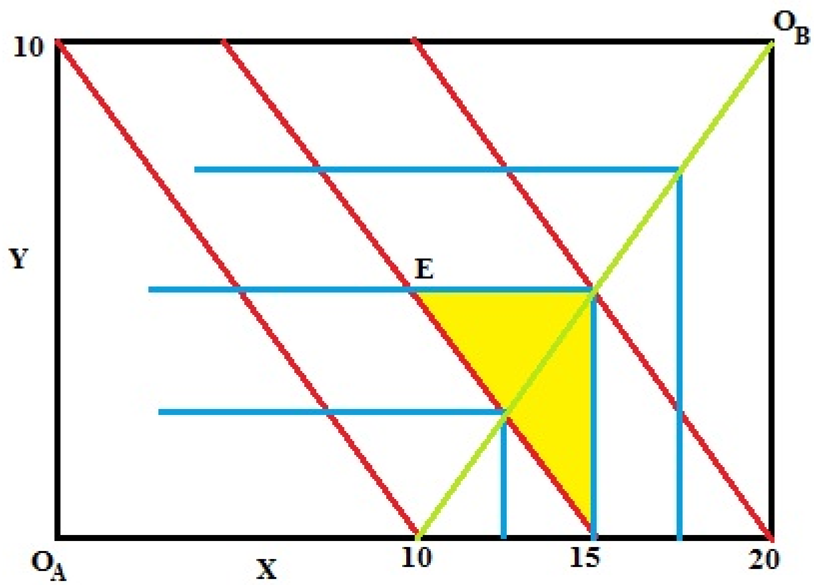
Setting $p_x = 2, p_y = 1$, we can solve each consumer's utility maximization problem to find the consumption bundles:

$$x_A^* = \frac{5}{4}, y_A^* = \frac{5}{2}; \quad x_B^* = \frac{15}{4}, y_B^* = \frac{15}{2}.$$

Since each consumer is setting *MRS* equal to the price ratio, their *MRS's* must be equal and so the equilibrium is efficient.

10. (a) A's utility function is linear, yielding the linear indifference curves seen as the red lines in the graph. B's utility function is Leontief,

yielding the blue L-shaped indifference curves.The Pareto efficient allocations have $x_B = y_B$ and $x_B, y_B \leq 10$, seen as the green line in the graph, which is the contract curve. E is the endowment point. The Pareto improving allocations over E are represented by the yellow shaded area, whose allocations simultaneously have $(x_A + y_A) \geq 15$ $and\ min[x_B, y_B] \geq 5$.



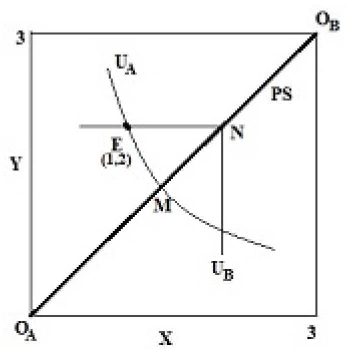
(b) The Walrasian equilibrium must have a price ratio equal to the slope of A's indifference curves, since otherwise A will want to consume at a corner, while B will want to be in the interior of the Edgeworth box. Therefore, in equilibrium,

$$\left(\frac{p_x}{p_y}\right)^* = 1.$$

Given $p_x = p_y = 1$, B's income will be 15 and so he will consume 7.5 units of each good. A's consumption can be calculated as the residual. the bundles therefore are:

$$x_A^* = 12.5, y_A^* = 2.5; \quad x_B^* = 7.5, y_B^* = 7.5.$$

11. (a) A's indifference curves have the usual convex shape, but B's indifference curves are L-shaped. The contract curve PS is the diagonal of the box.



(b) Pareto improvements over the endowment E that are also efficient are on the line segment MN where A's allocations are, at M: $(\sqrt{2}, \sqrt{2})$, and at N: $(2, 2)$.

(c) If the Walrasian auctioneer announces prices p_x and p_y, the income levels for A and B will be respectively

$$I_A = p_x + 2p_y \qquad \textit{and} \qquad I_B = 2p_x + p_y.$$

Now, given his Cobb-Douglas utility function, A will want to spend half his income on x. His demand function for x is therefore

$$x_A = \frac{p_x + 2p_y}{2p_x}.$$

B has a Leontief utility function and, in equilibrium, would want to set $x_B = y_B$. Therefore, his demand function for x is

$$x_B = \frac{2p_x + p_y}{p_x + p_y}.$$

In Walrasian equilibrium, we must have

$$x_A + x_B = 3.$$

Substituting the demand functions in this equation and rearranging gives us the equilibrium price ratio:

$$\left(\frac{p_x}{p_y}\right)^* = 1.$$

Setting $p_x = p_y = 1$, we can use the demand functions to solve for the chosen consumption bundles:

$$x_A^* = 1.5, y_A^* = 1.5; \quad x_B^* = 1.5, y_B^* = 1.5.$$

Chapter 8: Uncertainty and Information

1. (a) The expected value of the ticket is

$$EV = (0.5)(5) + (0.5)(0) = 2.50.$$

(b) With the lottery ticket, Jerry's expected utility is

$$EU_0 = (0.5)\left(1 - \frac{1}{65}\right) + (0.5)\left(1 - \frac{1}{60}\right) = 0.9839742.$$

Suppose Jerry sold the ticket for a price of p. Then his expected utility would be

$$EU_1 = 1 - \frac{1}{60 + p}.$$

The minimum price for which Jerry would sell the ticket would be that value of p for which $EU_1 = EU_0$. Solving yields:

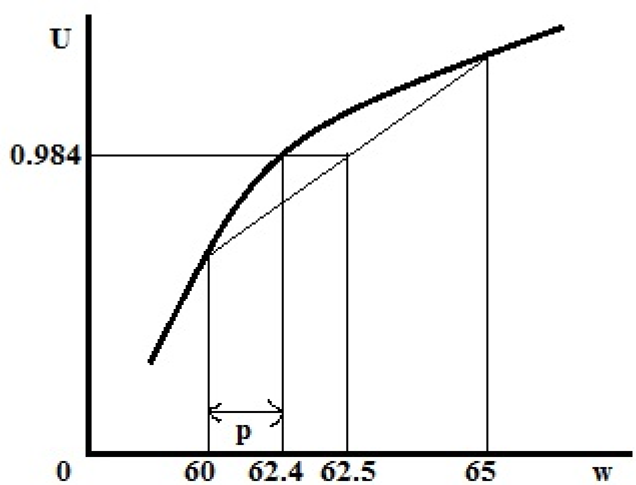
$$p_{min} = 2.40.$$

(c) The answer to (a) is bigger because of risk aversion. We can check whether Jerry is risk-averse by checking:

$$U^{'}(w) = \frac{1}{w^2} > 0,$$

$$U^{"}(w) = -\frac{2}{w^3} < 0.$$

Thus *U(w)* is a rising concave function, confirming that Jerry is risk-averse. The minimum price for the ticket is indicated in the graph.



(d) The cost of risk is the difference between the expected wealth (62.50) and the certainty-equivalent wealth (62.40). Thus

$$C = 62.50 - 62.40 = 0.10.$$

2. (a) The expected utility from the trip is

$$EU_0 = 0.75(ln\ 10{,}000) + 0.25(ln\ 9{,}000) = 9.184.$$

(b) The actuarially fair premium would be

$$\pi = E(Loss) = 0.25(1{,}000) = 250.$$

With insurance, Will's expected utility would be

$$EU_1 = ln\ 9{,}750 \approx 9.185 > 9.184,$$

He is better off with insurance, so he will buy it. This is to be expected, because he is risk-averse.

(c) The maximum willingness to pay would be the solution to the equation

$$ln\ (10{,}000 - \pi_{max}) = 9.184.$$

Solving, we find

$$\pi_{max} = 260.$$

(d) The actuarially fair insurance under this moral hazard scenario would be

$$\pi_{mh} = E(Loss) = 0.3(1{,}000) = 300.$$

With this insurance, his expected utility would be

$$EU_{mh} = ln\ 9{,}700 \approx 9.18.$$

He will not buy this insurance, since his expected utility is higher without insurance (remembering that there is no moral hazard if he does not have insurance): $9.184 > 9.18$.

3. (a) The cost of risk is the difference between the expected wealth and the certainty equivalent wealth. Bob's expected wealth is

$$E(w) = 0.5(2500) + 0.5(1600) = 2050.$$

The certainty equivalent wealth is that level of wealth that has the same utility as the expected utility under the risky situation. Now the latter is

$$EU(w) = 0.5(2500)^{\frac{1}{2}} + 0.5(1600)^{\frac{1}{2}} = 45,$$

so the certainty equivalent wealth satisfies

$$U(w_{CE}) = 45 \qquad \rightarrow \qquad w_{CE} = 45^2 = 2025.$$

Then the cost of risk is

$$C = 2050 - 2025 = \$25.$$

(b) Bob's expected utility without insurance is

$$EU_{no\ insurance} = \frac{1}{2}(50) + \frac{1}{2}(40) = 45,$$

while his utility with insurance is

$$EU_{insurance} = \sqrt{2000} = 44.72.$$

The latter is lower, so he will not buy insurance.

(c) The maximum willingness to pay for insurance would reduce Bob to the same level of expected utility as he has without insurance. That is, it would reduce his wealth to the certainty equivalent wealth. Then

$$max\ WTP = 2500 - 2025 = \$475.$$

4. (a) If Widget doesn't invest in the new product, its expected utility is

$$u_0 = 100.$$

If it does invest in the new product, its expected utility is

$$E(u_1) = 0.8(100 - 36) + 0.2(100 - 36 + 192) = 102.4.$$

Its expected utility is higher with the investment, so it will undertake the project.

(b) Now if Widget doesn't invest in the new product, its expected utility is

$$u_0 = 100^{\frac{1}{2}} = 10$$

and if it does invest in the new product, its expected utility is

$$E(u_1) = 0.8(100 - 36)^{\frac{1}{2}} + 0.2(100 - 36 + 192)^{\frac{1}{2}} = 9.6.$$

So now its expected utility is higher without the investment, so it will not undertake the project.

(c) We know that, if Widget does not conduct the survey, its optimal action is to undertake the project and its expected utility will be $E(u_1) = 102.4$.

If they conduct the survey, they will know for sure if they have a market for the new product, so they would spend the 36 for the project only if they know that it will be successful. So in this scenario their expected utility is

$$E(u_2) = 0.8(100 - 20) + 0.2(100 - 20 - 36 + 192) = 111.2.$$

Since this expected utility is higher than 102.4, Widget will undertake the survey.

If the utility function is $U(y) = \sqrt{y}$, we know from part (b) that Widget would not undertake the project without the survey and so would have an expected utility of 10. If they conducted the survey, their expexted utility would be

$$E(u_2) = 0.8(100-20)^{\frac{1}{2}} + 0.2(100-20-36+192)^{\frac{1}{2}} = 10.228.$$

Their expected utility is higher with the survey, so they would conduct it.

(d) Their maximum willingness to pay for the survey would be the solution to the equation

$$0.8(100-x)^{\frac{1}{2}} + 0.2(100-x-36+192)^{\frac{1}{2}} = 10.$$

Solving this equation numerically yields the maximum willingness to pay:

$$x_{max} = 24.39.$$

5. (a) The expected utility under each of the crops is as follows:

$$EU_{wheat} = (0.5)\sqrt{30,000} + (0.5)\sqrt{10,000} = 136.6025.$$

$$EU_{corn} = (0.5)\sqrt{23,000} + (0.5)\sqrt{15,000} = 137.0660.$$

Since $EU_{corn} > EU_{wheat}$, he will plant corn.

(b) Under this scenario, his expected utility is

$$EU_{mixed} = (0.5)\sqrt{26,500} + (0.5)\sqrt{12,500} = 137.2958.$$

Since $EU_{mixed} > EU_{corn}$, he will plant the mixed crops.

(c) Suppose he plants a fraction θ of his field with wheat and the remainder with corn. His expected utility would now be

$$EU_1 = (0.5)\sqrt{\theta(30,000) + (1-\theta)(23,000} + (0.5)\sqrt{\theta(10,000) + (1-\theta)(15,000)}$$

$$= (0.5)\sqrt{7,000\theta + 23,000} + (0.5)\sqrt{15,000 - 5,000\theta}.$$

The first order condition to find the maximum of EU_1 is

$$\frac{dEU_1}{d\theta} = \frac{(0.5)^2}{\sqrt{7,000\theta + 23,000}} \cdot 7,000 + \frac{(0.5)^2}{\sqrt{15,000 - 5,000\theta}} \cdot (-5,000) = 0.$$

Solving this equation, we find $\theta = 0.38095$. Therefore he will plant his field with 38.1% wheat and 61.9% corn.

6. (a) The well costs \$200 and returns nothing if the rain is normal but a gain of \$600 if there is a drought. The net expected value is therefore

$$EV = -200 + (0.3)(600) = -20.$$

(b) Pablo's expected utility under the alternative scenarios is as follows:

$$EU_{no\ well} = (0.7)(ln\ 1000) + (0.3)(ln\ 200) = 6.425.$$

$$EU_{well} = (0.7)(ln\ 800) + (0.3)(ln\ 600) = 6.598.$$

Since $EU_{well} > EU_{no\ well}$, he will dig the well.

7. (a) The expected income in the two scenarios is as follows:

$$E_{wheat}(Y) = (0.5)(64) + (0.5)(81) = 72.5.$$

$$E_{rice}(Y) = (0.5)(100) + (0.5)(49) = 74.5.$$

(b) The expected utility with each of the crops is as follows:

$$EU_{wheat} = (0.5)(\sqrt{64}) + (0.5)(\sqrt{81}) = 8.5.$$

$$EU_{rice} = (0.5)(\sqrt{100}) + (0.5)(\sqrt{49}) = 8.5.$$

Since $EU_{wheat} = EU_{rice}$, he is indifferent between the two crops.

(c) With the mixed cropping pattern, his expected utility would be

$$EU_{mixed} = (0.5)(\sqrt{64\theta + 100(1-\theta)}) + (0.5)(\sqrt{81\theta + 49(1-\theta)})$$
$$= (0.5)(\sqrt{100 - 36\theta}) + (0.5)(\sqrt{49 + 32\theta}).$$

The first order condition to find the maximum of EU_1 is

$$\frac{dEU_{mixed}}{d\theta} = \frac{(0.5)^2}{\sqrt{100 - 36\theta}} \cdot (-36) + \frac{(0.5)^2}{\sqrt{49 + 32\theta}} \cdot (32) = 0.$$

Solving this equation, we find $\theta = 0.4965$. Therefore he will plant his field with 49.65% wheat and 50.35% rice.

(d) The expected utility in the scenario with mixed cropping is

$$EU_{mixed} = (0.5)(\sqrt{82.126}) + (0.5)(\sqrt{64.888}) = 8.5588.$$

With actuarially fair insurance, his income under each cop would be certain; these income levels would be the average income under each crop:

$$Y_{wheat} = \frac{64 + 81}{2} = 72.5 \qquad and \qquad Y_{rice} = \frac{100 + 49}{2} = 74.5.$$

Of these, his income is higher under rice and his utility level would then be

$$EU_{rice} = \sqrt{74.5} = 8.6313.$$

Since $EU_{rice} > EU_{mixed}$, he will plant rice and buy insurance.

8. (a) Calculate the expected utility under each of the options:

$$EU_i = \sqrt{(1.05)(250,000)} = 512.35.$$

$$EU_{ii} = (0.7)\sqrt{500,000 + 52,500} + (0.3)\sqrt{50,000 + 52,500} = 616.36.$$

$$EU_{iii} = (0.7)\sqrt{400,000} + (0.3)\sqrt{350,000} = 620.20.$$

Therefore, he will choose option (iii) which yields the highest expected utility.

(b) The actuarially fair premium would be the expected loss:

$$\pi = (0.3)(450,000) = 135,000.$$

The maximum WTP will be π_{max} which would result in an expected utility equal to 620.20, the best he can do without the insurance. So π_{max} will be the solution to the equation

$$\sqrt{552,500 - \pi_{max}} = 620.20 \qquad \rightarrow \qquad \pi_{max} = 167,851.96.$$

9. (a) $W_{fire} = 60,000 + (1 - \pi)x, \quad W_{no\ fire} = 100,000 - \pi x.$

(b) If firms make zero expected profits, they must be charging the actuarially fair premium for insurance. Then π would equal the probability of a loss, i.e., $\pi = 0.10$.

(c) Since Al is risk-averse (which we can tell from his utility function) he would buy full insurance (40,000) if it was available at an actuarially fair premium. The total premium would be 4,000 and so his expected utility will be

$$EU_{insured} = ln(96,000) \approx 11.47.$$

(d) If the per-dollar premium is 0.20 and Al buys x units of insurance, his expected utility would be

$$EU_1 = (0.9)ln(100,000 - 0.2x) + (0.1)ln(60,000 + 0.8x).$$

To maximize expected utility, Al would want to set

$$\frac{dEU_1}{dx} = (0.9) \cdot \frac{(-0.2)}{(100,000 - 0.2x)} + (0.1) \cdot \frac{(0.8)}{(60,000 + 0.8x)} = 0.But$$

this would yield a negative value of x, which is not possible. Therefore, Al will buy zero insurance. His expected utility will be

$$EU_0 = (0.9)ln(100,000) + (0.1)ln(60,000) \approx 11.46.$$

Chapter 9: Monopoly and Market Power

1. (a) Under perfect competition in the long run, firms will produce at the minimum average cost. The average cost is

$$AC(q) = \frac{20}{q} + 2 + 0.05q.$$

To find the minimum:

$$\frac{dAC}{dq} = -\frac{20}{q^2} + 0.05 = 0 \qquad \rightarrow \qquad q = 20.$$

At that level of output, $AC = 4$; therefore the long-run equilibrium will be:

$$p_{LR} = 4, Q_{LR} = 10,000, number\ of\ firms = 500.$$

(b) The monopolist can build multiple plants, each one producing 20 units at an $AC = 4$. Therefore, we can treat the monpolist's AC curve as flat at \$4. Then the monopolist's profits are

$$\pi_m = \frac{400}{\sqrt{Q}} \cdot Q - 4Q.$$

To maximize profits, the monopolist will set

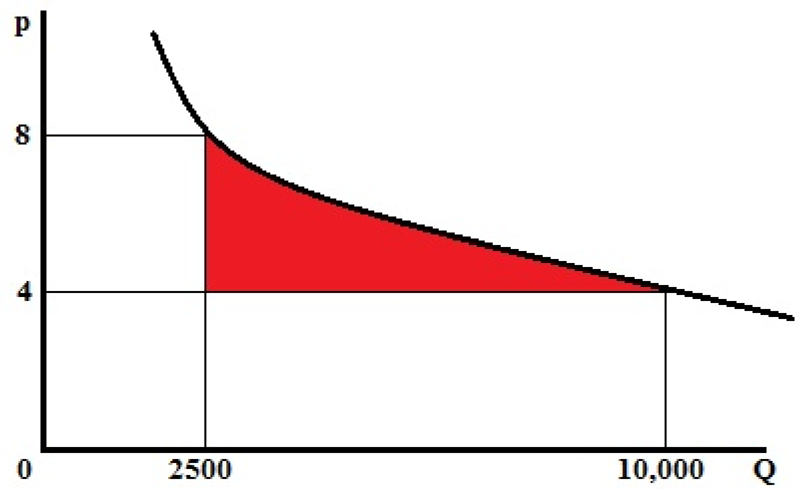
$$\frac{d\pi_m}{dQ} = \frac{200}{\sqrt{Q}} - 4 = 0 \rightarrow Q^* = 2500.$$

The monopoly equilibrium is then

$$p_m = 8, Q_m = 2,500, number\ of\ plants = 125.$$

(c) The Deadweight Loss is the red shaded area in the graph. We can calculate it as follows:

$$DWL = \int_4^8 \frac{160,000}{p^2} dp - (8-4) \cdot (2500) = 10,000.$$



If we assumed linearity of the demand curve, we would find

$$DWL \approx \frac{1}{2} \cdot (8-4) \cdot (7500) = 15,000,$$

which is not a very good approximation in this case.

2. (a) If the firm acts as a price-taker, it will set $w = MRP_L$. That is,

$$\frac{L_c}{80} = 10 - \frac{L_c}{40} \qquad \rightarrow \qquad L_c = \frac{800}{3}.$$

Then$w_c = 3.33$.

(b) As a monopsonist, the firm will set $ME_L = MRP_L$. Now

$$E_L = wL = \frac{L^2}{80}, \qquad so \qquad ME_L = \frac{L}{40}.$$

Set this equal to MRP_L:

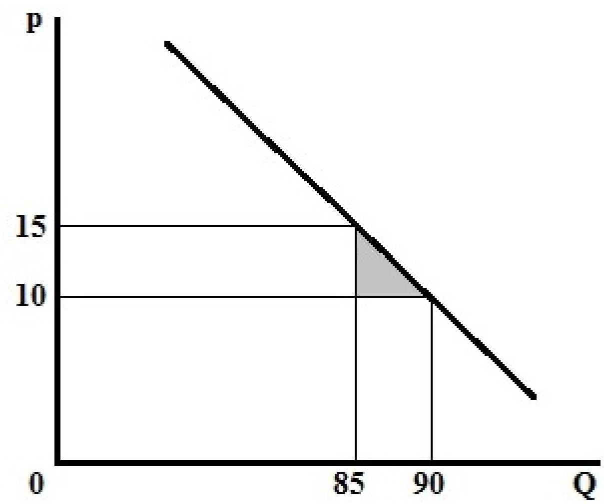
$$\frac{L_m}{40} = 10 - \frac{L_m}{40} \qquad \rightarrow \qquad L_m = 200.$$

Then $w_m = 2.5$.

(c) In both cases, whether the firm is a price-taker or a monopsonist, $w = 4, L = 240$. So employment will fall in the competitive case but rise in the monopsony case.

3. (a) Under competition, the price will simply be the constant average and marginal cost of production, \$10. At that price, demand is 90. After imposition of the excise tax of \$5, price will rise to \$15 and demand will fall to 85. The deadweight loss is the shaded area in the graph (graph is not to scale):

$$DWL_c = \frac{1}{2} * 5 * 5 = \$12.5.$$



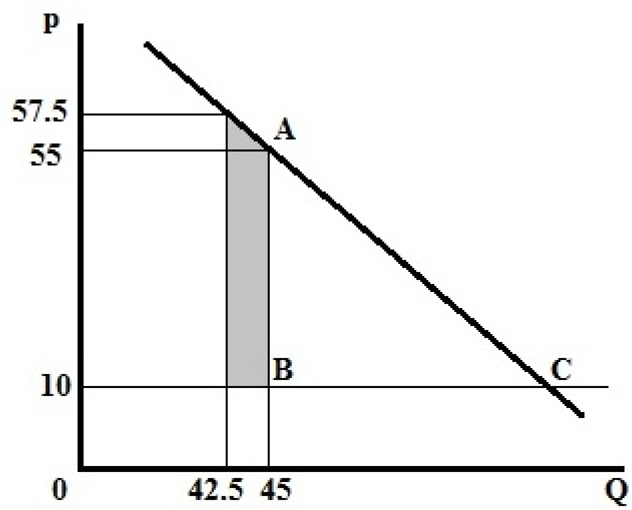
(b) Since demand is linear and AC is constant, we know the monopoly output will be half the competitive output level. Therefore:

$$before\, tax: \; Q_m^0 = 45, p_m^0 = \$55$$

$$after\, tax: \; Q_m^1 = 42.5, p_m^1 = \$57.5$$

and therefore the deadweight loss *due to the tax* is the shaded area in the next graph. Prior to the imposition of the tax, there was a deadweight loss due to monopoly equal to the area ABC. Once the tax is imposed, the deadweight loss *rises* by the shaded area. It is the loss of surplus on the units of output that are now eliminated. This area is:

$$DWL_m = \frac{1}{2} * 2.5 * 2.5 + 2.5 * 45 = \$115.625.$$



The deadweight loss of the tax is much higher under monopoly than under competition, even though the size of the quantity distortion is smaller (2.5 instead of 5). This is because the quantity is already distorted under monopoly *(p>MC)*, so the per-unit welfare loss from further distortion is a lot higher. Under competition, the per-unit losses vary from zero to 5, but, under monopoly, the per-unit losses vary from 45 to 47.5.

4. (a) Let q_h, q_x represent the quantities of widgets sold at home and exported, respectively. Then Widget Corp's profits are:

$$\pi = q_h(50 - q_h) + 10q_x - \frac{(q_h + q_x)^2}{6}.$$

The first-order conditions to maximize profits are:

$$\frac{\partial \pi}{\partial q_h} = 50 - 2q_h - \frac{1}{3}(q_h + q_x) = 0 \tag{32}$$

$$\frac{\partial \pi}{\partial q_x} = 10 - \frac{1}{3}(q_h + q_x) = 0. \tag{33}$$

Substituting (33) in (32), we can solve for q_h and then find p_h from the demand curve:

$$q_h = 20, p_h = 30.$$

From (33), we can then find

$$q_x = 10.$$

Of course, $p_x = 10$. And total output is $q = 30$.

(b) The imposition of the excise tax adds 5 to the per-unit cost. Then Widget Corp's profits are:

$$\pi = q_h(50 - q_h) + 10q_x - \frac{(q_h + q_x)^2}{6} - 5(q_h + q_x).$$

The first-order conditions to maximize profits are:

$$\frac{\partial \pi}{\partial q_h} = 50 - 2q_h - \frac{1}{3}(q_h + q_x) - 5 = 0.$$

$$\frac{\partial \pi}{\partial q_x} = 10 - \frac{1}{3}(q_h + q_x) - 5 = 0.$$

Substituting as before, we find

$$q_h = 20.$$

Solving for q_x, we find

$$q_x = -5!$$

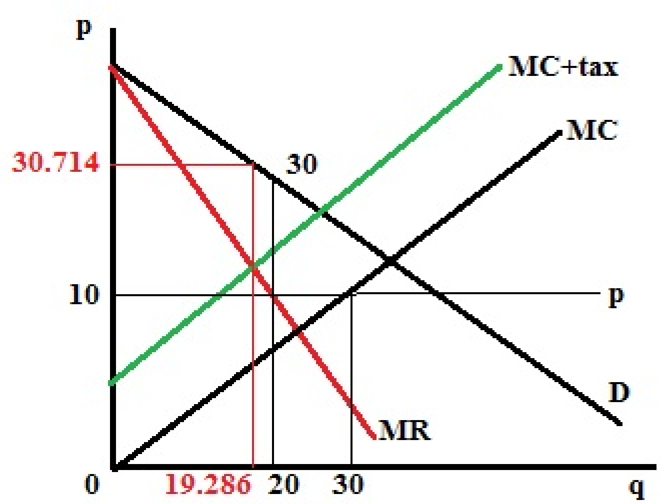
Obviously, exports cannot be negative; therefore, we can set $q_x = 0$ and solve again:

$$\pi = q_h(50 - q_h) - \frac{q_h^2}{6} - 5q_h.$$

$$\frac{\partial \pi}{\partial q_h} = 50 - 2q_h - \frac{1}{3}q_h - 5 = 0 \qquad \rightarrow \qquad q_h = 19.286.$$

Then $p_h = 30.714$.

(c) .



5. (a) Suppose Foxhunt buys and re-sells Q furs. Then the revenue from their sale will be

$$R(q) = (210 - Q) \cdot Q.$$

To find the cost, rearrange the supply curve of the untreated furs:

$$2P_s = Q_s + 100 \qquad \rightarrow \qquad P_s = \frac{Q_s}{2} + 50.$$

This will be the per-unit price Foxhunt will have to pay for untreated furs. Remembering that Foxhunt has to spend an extra \$10 to treat the furs, the total cost will be

$$C(Q) = \frac{Q^2}{2} + 50Q + 10Q.$$

Then Foxhunt's profits are

$$\pi = 210Q - Q^2 - \frac{Q^2}{2} - 60Q = 150Q - \frac{3}{2}Q^2.$$

Profits are maximized when

$$\frac{d\pi}{dQ} = 150 - 3Q = 0 \qquad \rightarrow \qquad Q = 50.$$

The price of untreated furs can be found from the supply curve:

$$P_s = \frac{50}{2} + 50 = \$75$$

and the price of treated furs can be derived from the demand curve:

$$P_d = 210 - 50 = \$160.$$

(b) For efficiency, the gap between P_s and P_d should just be the \$10 cost of treatment. So we should have

$$P_d = P_s + 10 \qquad \rightarrow \qquad 210 - Q = \frac{Q}{2} + 60 \qquad \rightarrow \qquad Q = 100.$$

Then

$$P_s = \frac{100}{2} + 50 = \$100 \qquad and \qquad P_d = 210 - 100 = \$110.$$

6. (a) AFC's demand for skins will be its marginal revenue product from the skins. Now the marginal product of the skins is

$$MP_x = 240 - 4x$$

and so the MRP is

$$MRP_x = 5(240 - 4x) = 1200 - 20x.$$

If TJC acts as a monopolist, this will be its AR curve. The cost of the skins is

$$wL = 10x^2.$$

Therefore, TJC's profit can be written as

$$\pi_T = 1200x - 20x^2 - 10x^2 = 1200x - 30x^2.$$

Profit is maximized where

$$\frac{d\pi_T}{dx} = 1200 - 60x = 0 \qquad \rightarrow \qquad x = 20.$$

Then the price of skins will be

$$p = 1200 - 20(20) = \$800.$$

(b) If TJC acts as a price-taker, its supply curve will be its MC curve, which is found by differentiating the "cost of skins" found in part (a):

$$MC = 20x$$

so the supply curve is

$$p = 20x.$$

For AFC, the total expense on skins will then be

$$E = p \cdot x = 20x^2$$

and so the marginal expense on skins will be

$$ME = 40x.$$

To maximize its profit, AFC will set its MRP equal to the ME:

$$1200 - 20x = 40x \qquad \rightarrow \qquad x = 20.$$

So x is still 20. But now the price of skins will come from the supply curve of skins:

$$p = 20(20) = \$400.$$

(c) From the social point of view, the MC of skins should be set equal to the MRP from skins:

$$20x = 1200 - 20x \qquad \rightarrow \qquad x = 30.$$

Then price can be calculated either from the supply curve or the demand curve:

$$p = 20(30) = \$600$$

$$p = 1200 - 20(30) = \$600.$$

So we get the same answer either way.

7. (a) NW's profit can be written as follows:

$$\pi = PQ - wL = (100 - 2L) \cdot 2L - (20 + 2L) \cdot L = 180L - 6L^2.$$

Profits are maximized where

$$\frac{d\pi}{dL} = 180 - 12L = 0 \qquad \rightarrow \qquad L_m = 15.$$

The profit maximizing equilibrium is therefore

$$p_m = 70, Q_m = 30, L_m = 15, w_m = 50.$$

(b) The socially optimal equilibrium would be one where NW acted as a price taker and set $p \cdot MP_L = w$, that is, where

$$(100 - 2L) \cdot 2 = 2L + 20 \rightarrow L^* = 30.$$

The socially optimal equilibrium is therefore

$$p^* = 40, Q^* = 60, L^* = 30, w^* = 80.$$

8. (a) Since there are 100 munchkins, the total demand for rides from munchkins is 100 times the per-munchkin demand:

$$Q_m = 1200 - 100p.$$

Similarly, the total demand from smurfs is

$$Q_s = 800 - 100p.$$

Then total market demand is

$$Q = Q_m + Q_s = 2000 - 200p.$$

(b) The firm's profits are

$$\pi = Q \cdot \left(10 - \frac{Q}{200}\right),$$

so profits are maximized where

$$\frac{d\pi}{dQ} = 10 - \frac{Q}{100} = 0 \qquad \rightarrow \qquad Q_1 = 1000.$$

Then

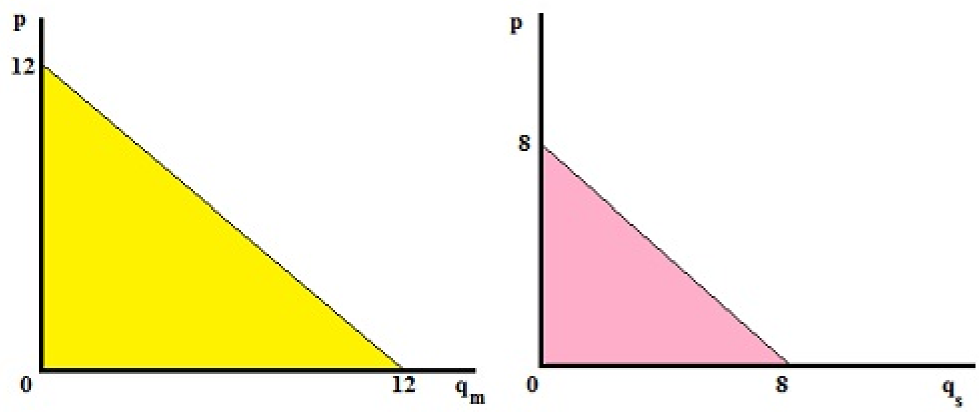
$$p_1 = 5, \pi_1 = 5000.$$

(c) By charging a fixed fee and offering rides for free, JRCo can potentially extract all of the surplus of the consumer. The yellow area is what JRCo could extract from munchkins and the pink area is what can be extracted from smurfs. These are, respectively:

$$F_m = \frac{1}{2}(12)(12) = 72 \qquad and \qquad F_s = \frac{1}{2}(8)(8) = 32.$$

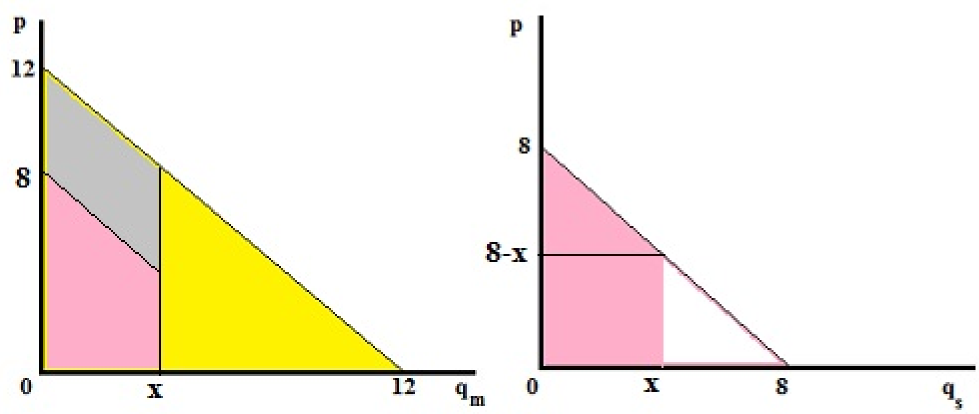
So JRCo could charge 32 and get 200 customers, or 72 and get 100 customers. The second option yields a higher profit so the choice will be

$$F_2 = 72, \pi_2 = 7200.$$



(d) With second-degree price discrimination, JRCo can offer small bundles to try to entice the smurfs and then bundles of 12 rides to sell to the munchkins. Suppose the small bundles consist of x rides. Then the smurfs can be charged the pink area, which is

$$F_3 = x(8-x) + \frac{1}{2}x^2 = 8x - \frac{1}{2}x^2.$$



The munchkins would get a surplus of the grey area if they bought the small bundle. Therefore, the maximum they can be charged for the large bundles of 12 rides is the sum of the pink and yellow areas. This is

$$F_4 = \frac{1}{2}(12)(12) - 4x = 72 - 4x.$$

JRCo's profits then are

$$\pi = 100\left(8x - \frac{1}{2}x^2\right) + 100(72 - 4x) = 7200 + 400x - 50x^2.$$

This is maximized when

$$\frac{d\pi}{dx} = 400 - 100x = 0 \qquad \rightarrow \qquad x = 4.$$

From the formulae for F_3 and F_4, we can calculate the prices of the two bundles. The final choices are to sell two bundles: 4 rides at 24, and 12 rides at 56. Then $\pi^* = 8000$.

9. (a) Find the typical firm's MC and AVC curves:

$$MC = \frac{dC}{dq} = q + 10 \qquad \textit{and} \qquad AVC = 0.5q + 10.$$

So $MC > AVC$ for all q, and so the MC curve is the firm's supply curve. Therefore the firm's supply curve is

$$p = q_s + 10 \qquad \textit{or} \qquad q_s = p - 10.$$

The industry supply curve is just 50 times this, or

$$Q_s = 50p - 500.$$

(b) Setting $Q_s = Q_d$ and solving yields the short run equilibrium:

$$p_{SR} = 20.5, Q_{SR} = 525, q_{SR} = 10.5.$$

(c) In the long run, firms will produce at the minimum point of their AC curves. Now

$$AC = \frac{C(q)}{q} = 0.5q + 10 + \frac{18}{q}.$$

This is minimized where

$$\frac{dAC}{dq} = 0.5 - \frac{18}{q^2} = 0 \qquad \rightarrow \qquad q = 6.$$

At this quantity, AC=*16*, and so the market supply curve in the long run will be perfectly elastic at p=*16*. The long run equilibrium is then

$$p_{LR} = 16, Q_{LR} = 1200, q_{LR} = 6, n_{LR} = 200.$$

(d) The multi-plant monopoly will set MC equal in all its plants. The monopolist's combined *MC* curve will then be the same curve that was the competitive supply curve:

$$MC_m = \frac{Q_m}{50} + 10.$$

From the demand curve, we can find the monopolist's total revenue:

$$R = 24Q - \frac{Q^2}{150}$$

and so marginal revenue will be

$$MR = \frac{dR}{dQ} = 24 - \frac{Q}{75}.$$

Setting *MR=MC* and solving yields the monopolist's short run profit maximizing equilibrium:

$$p_{mSR} = 21.2, Q_{mSR} = 420, q_{mSR} = 8.4.$$

(e) The monoplist's long run average and marginal cost will essentially be constant at 16, since he can build multiple plants, each operating at the minimum point of its AC curve. Setting *MR=16* and solving yields the monopolist's long run equilibrium:

$$p_{mLR} = 20, Q_{mLR} = 600, q_{mLR} = 6, n_{LR} = 100.$$

10. (a) Find the typical firm's *MC* and AVC curves:

$$MC = \frac{dC}{dq} = 100q \qquad and \qquad AVC = 50q.$$

So $MC > AVC$ for all q, and so the MC curve is the firm's supply curve. Therefore the firm's supply curve is

$$p = 100q_s \qquad or \qquad q_s = \frac{p}{100}.$$

The industry supply curve is just 100 times this, or

$$Q_s = p.$$

Setting $Q_s = Q_d$ and solving yields the short run equilibrium:

$$p_{SR} = 80, Q_{SR} = 80, q_{SR} = 0.8, \pi_{SR} = -18\ (or{+}32\ if\ fixed\ cost\ is\ ignored).$$

(b) In the long run, firms will produce at the minimum point of their AC curves. Now

$$AC = \frac{C(q)}{q} = \frac{50}{q} + 50q.$$

This is minimized where

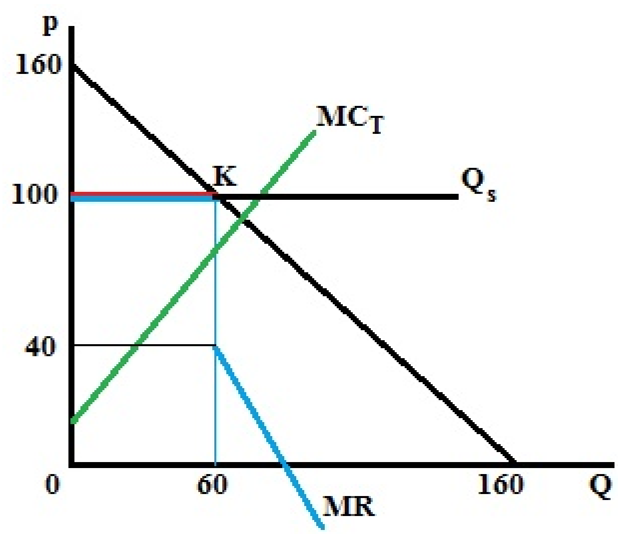
$$\frac{dAC}{dq} = -\frac{50}{q^2} + 50 = 0 \qquad \rightarrow \qquad q = 1.$$

At this quantity, AC=100, and so the market supply curve in the long run will be perfectly elastic at p=100. The long run equilibrium is then

$$p_{LR} = 100, Q_{LR} = 60, q_{LR} = 1, n_{LR} = 60.$$

(c) Tech Corp's residual demand curve is flat at p=100 up to the market demand curve and is the whole market demand for prices below 100. The marginal revenue curve is the blue line in the figure, with the lower segment having the equation

$$MR = 160 - 2Q.$$



Thus, at $Q = 60$, $MR = 40$. Their marginal cost curve is

$$MC_T = 10 + q_T$$

which is represented by the green line in the figure. At $Q_T = 60$, $MC = 70$. Therefore, we see that the MC_T line crosses the residual MR in the vertical segment, indicating that Tech's choice will be to produce at K, limiting entry from the fringe firms. The equilibrium is therefore:

$$p_T = 100, Q_T = 60, q_T = 60, q_{fringe} = 0, n_{fringe} = 0.$$

(d) Tech's entry does not change the price or quantity, so consumers are unaffected by the entry. Social welfare will therefore hinge on whether the costs incurred in the production of widgets have gone up or down. Costs under competition are

$$C_C = 60 \cdot 100 = 6000.$$

Costs under Tech production are

$$C_T = 100 + 10(60) + \frac{1}{2}(60)^2 = 2500.$$

Therefore

$$\triangle W = 6000 - 2500 = +3500.$$

We could have got the same result by looking at profits. Profits in the competitive equilibrium are 0, but Tech's profits are 3500, which is the welfare gain.

11. (a) Rewrite the demand curves as

$$p_1 = 55 - Q_1 \qquad \textit{and} \qquad p_2 = \frac{55}{2} - \frac{1}{2}Q_2.$$

Then the monopolist's profits are

$$\pi = (55 - Q_1) \cdot Q_1 + \left(\frac{55}{2} - \frac{1}{2}Q_2\right) \cdot Q_2 - 5\left(Q_1 + Q_2\right).$$

The first-order conditions for a maximum are

$$\frac{\partial \pi}{\partial Q_1} = 55 - 2Q_1 - 5 = 0 \qquad \rightarrow \qquad Q_1 = 25.$$

$$\frac{\partial \pi}{\partial Q_2} = \frac{55}{2} - Q_2 - 5 = 0 \qquad \rightarrow \qquad Q_2 = 22.5.$$

The profit-maximizing equilibrium is therefore

$$p_1 = 30, Q_1 = 25, p_2 = 16.25, Q_2 = 22.5.$$

(b) If it only costs \$5 to transport widgets between markets, the firm can have a gap of only \$5 between the two prices. Therefore the new equilibrium will involve

$$p_1^{'} = p_2^{'} + 5.$$

It will now be more convenient to solve this problem by choosing a price rather than quantities. We can write the firm's profits as

$$\pi^{'} = \left(p_2^{'} + 5\right) \cdot Q_1^{'} + p_2^{'}Q_2^{'} - 5\left(Q_1^{'} + Q_2^{'}\right).$$

Substituting the demand curves and simplifying, this can be written as

$$\pi^{'} = -275 + 115p_2^{'} - 3\left(p_2^{'}\right)^2.$$

Profits are maximized where

$$\frac{d\pi^{'}}{dp_2^{'}} = 115 - 6p_2^{'} = 0 \qquad p_2^{'} = \frac{115}{6} = 19.17.$$

The profit-maximizing equilibrium is then

$$p_1^{'} = 24.17, Q_1^{'} = 30.83, p_2^{'} = 19.17, Q_2^{'} = 16.67.$$

12. (a) First find the firm's MC and AVC curves:

$$MC(q) = 10q - 10 \qquad and \qquad AVC = 5q - 10.$$

So $MC > AVC$ for all q, and so the MC curve is the firm's supply curve. Therefore the firm's supply curve is

$$p = 10q_s - 10 \qquad or \qquad q_s = 1 + \frac{p}{10}.$$

The industry supply curve is just 90 times this, or

$$Q_s = 90 + 9p.$$

With competition, no price discrimination will be possible; we therefore need to find the total market demand curve. Assuming that $p < 20$ so that both demands are positive, and given that $p_a = p_b$, the market demand curve will be

$$Q_d = 200 - 10p + 250 - 5p = 450 - 15p.$$

Setting $Q_s = Q_d$ and solving yields the short run equilibrium:

$$p_{aSR} = p_{bSR} = 15, Q_{SR} = 225, q_{SR} = 2.5, Q_{aSR} = 50, Q_{bSR} = 175.$$

(b) In the long run, firms will produce at the minimum points of their AC curves:

$$AC(q) = 5q - 10 + \frac{20}{q}.$$

$$\frac{dAC}{dq} = 5 - \frac{20}{q^2} = 0 \qquad \rightarrow \qquad q = 2.$$

At this quantity, AC=*1*0, and so the market supply curve in the long run will be perfectly elastic at p=*1*0. The long run equilibrium is then

$$Q_{LR} = 300, q_{LR} = 2, Q_{aLR} = 100, Q_{bLR} = 200, p_{aLR} = p_{bLR} = 10, n = 150.$$

(c) The monopolist will operate each plant at $q = 2$, so effectively his $AC = MC = 10$. He will set this equal to MR in each market.

$$TR_a = 20Q_a - \frac{1}{10}Q_a^2.$$

$$MR_a = 20 - \frac{1}{5}Q_a = 10 \qquad \rightarrow \qquad Q_{am} = 50, p_{am} = 15.$$

$$TR_b = 50Q_b - \frac{1}{5}Q_b^2.$$

$$MR_b = 50 - \frac{2}{5}Q_b = 10 \qquad \rightarrow \qquad Q_{bm} = 100, p_{bm} = 30.$$

Total production is $Q_m = 150$ and $n = 75$ is the number of plants.

(d) With no price discrimination, the monopoly will work with the combined demand curve $Q_d = 450 - 15p$. Then

$$TR = 30Q - \frac{1}{15}Q^2.$$

$$MR = 30 - \frac{2}{15}Q = 10 \qquad \rightarrow \qquad Q_m^{'} = 150, p_m^{'} = 20.$$

The final equilibrium may be characterized as

$$Q_m^{'} = 150, q_m^{'} = 2, Q_{am}^{'} = 0, Q_{bm}^{'} = 150, p_{am}^{'} = p_{bm}^{'} = 20, n^{'} = 75.$$

13. (a) In long run competitive equilibrium, $p^* = MC = 6$. From the demand curve, we see $Q^* = 4$. Consumer Surplus will be

$$CS^* = \frac{1}{2}(10 - 6)(4) = 8.$$

(b) The subsidy will reduce the final price to $p_s = 4$. This raises the quantity sold to 6. Consumer surplus is now

$$CS_s = \frac{1}{2}(6)(6) = 18 \qquad so \qquad \triangle CS = 10.$$

$\triangle\pi = 0$ since profits are always zero. The subsidy cost to the government is $(6)(2) = 12$. Thus

$$Subsidy > \triangle CS + \triangle\pi.$$

(c) For part (a): The monopolist's profit is

$$\pi_m = 10Q_m - Q_m^2 - 6Q_m.$$

Profit is maximized where

$$\frac{d\pi_m}{dQ_m} = 4 - 2Q_m = 0 \qquad \rightarrow \qquad Q_m = 2, p_m = 8, \pi_m = 4.$$

Consumer Surplus is now

$$CS_m = \frac{1}{2}(2)(2) = 2.$$

For part (b): The monopolist's profit is now

$$\pi_{ms} = 10Q_{ms} - Q_{ms}^2 - 4Q_{ms}.$$

This leads to the profit maximizing equilibrium

$$Q_{ms} = 3, p_{ms} = 7, \pi_{ms} = 9.$$

Consumer Surplus is now

$$CS_{ms} = \frac{1}{2}(3)(3) = 4.5.$$

So $\triangle CS_m = 2.5, \triangle\pi_m = 5$, and therefore

$$Subsidy_m = (3)(2) = 6 < \triangle CS_m + \triangle\pi_m = 6.5.$$

14. (a) Note that marginal cost of production is the same (1) for sale in either market. To maximize profits, the firm would set $MR_1 = MR_2 = 1$. Now

$$TR_1 = 3y_1 - \frac{y_1^2}{2} \quad so \quad MR_1 = 3 - y_1 = 1 \quad \rightarrow \quad y_1^* = 2, p_1^* = 2.$$

$$TR_2 = 2y_2 - \frac{y_2^2}{2} \quad so \quad MR_2 = 2 - y_2 = 1 \quad \rightarrow \quad y_2^* = 1, p_2^* = \frac{3}{2}.$$

Note that, at this equilibrium,

$$\pi = TR_1 + TR_2 - c(y_1, y_2) = 4 + \frac{3}{2} - \frac{16}{3} = \frac{1}{6}.$$

The demand elasticities at the equilibrium are

$$\epsilon_1 = \frac{\partial y_1}{\partial p_1} \cdot \frac{p_1}{y_1} = (-2) \cdot \frac{2}{2} = -2.$$

$$\epsilon_2 = \frac{\partial y_2}{\partial p_2} \cdot \frac{p_2}{y_2} = (-2) \cdot \frac{1.5}{1} = -3.$$

(b) If the firm cannot price discriminate, we need to find the combined demand curve. The individual demand curves are

$$y_1 = 6 - 2p_1 \quad and \quad y_2 = 4 - 2p_2.$$

Knowing that $p_1 = p_2 = p$, we can add these to yield the combined demand curve

$$y = 10 - 4p \quad or \quad p = \frac{10}{4} - \frac{y}{4}.$$

The MR curve is then

$$MR = \frac{10}{4} - \frac{y}{2}.$$

Setting this equal to the MC of 1 and simplifying yields the equilibrium

$$y = 3, p = \frac{7}{4}, \pi = -\frac{1}{12}.$$

Since the firm is making losses, it would rather set output to zero and the firm would shut down.

(c) In this situation, price discrimination would be desirable because otherwise the firm would make losses and shut down. By staying open and price discriminating, it generates profit and consumer surplus.

Chapter 10: Theory of Games

1. (a) (compete, compete) is the only NE; in fact it is a dominant strategy equilibrium. Each player is better off playing that strategy regardless of what the other player does.

   (b) Suppose there is a NE in mixed startegies with AIR playing *(p, 1-p)* and RMS playing *(q, 1-q)*. Then AIR's expected payoff is

   $$E\pi_{AIR} = p\left[40q + 10(1-q)\right] + (1-p)\left[80q + 30(1-q)\right].$$

   Now note that

   $$\frac{\partial E\pi_{AIR}}{\partial p} = 40q + 10(1-q) - 80q - 30(1-q) = -20 - 20q < 0.$$

   Thus the optimal value of p is zero, the pure strategy of "compete." The same will be true for RMS, because "compete" is a dominant strategy for both players.

   (c) If the firms merged and could coordinate their behaviour, they would play (compete, cooperate) to get a total payoff of 90. Without the merger, AIR's payoff is 30 so its maximum willingness to pay for RMS is 60. RMS's payoff without the merger is 20; this would be the minimum price at which it would be willing to be acquired.

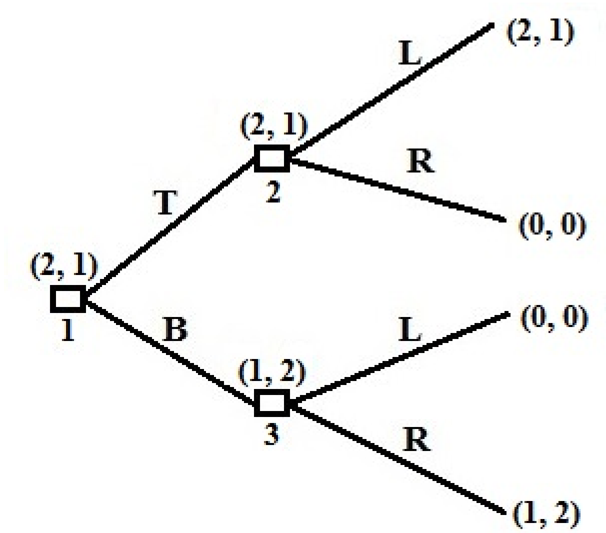
2. (a) The normal form game is

	L L	L R	R L	R R
Top	2, 1	2, 1	0, 0	0, 0
Bottom	0, 0	1, 2	0, 0	1, 2

   The Nash Equilibria of this game are: (T, LL), (T, LR), (B,RR). In all cases, neither player has any incentive to deviate from their strategy, given what the other player is doing.

The subgame perfect equilibrium of this game is (T, LR) as it does not involve any non-credible threat. The other two Nash equilibria involve non-credible threats. (T, LL) involves the wife playing L if the husband plays B, which is sub-optimal. (B, RR) involves the wife playing R if the husband plays T, which is also sub-optimal.

(b) The extensive form of the game is as follows:



The solution is for the husband to play T, with the wife following with L, which is her best response, yielding the payoffs (2, 1). Had the husband played B, her best response would have been to play R, yielding payoffs (1, 2). As the husband's payoff is lower here, he will play T.

3. (a) Maximize joint profits:

$$\pi = (44 - 4Q)\,Q - 4Q.$$

$$\frac{d\pi}{dQ} = 44 - 8Q - 4 = 0 \qquad \rightarrow \qquad Q = 5.$$

Based on the agreed 80%-20% market shares: $q_G = 4, q_L = 1$.

(b) Under the cartel, $Q = 5$ and so $p = 24$. Then

$$\pi_G = (24-4)(4) = 80 \qquad \textit{and} \qquad \pi_L = (24-4)(1) = 20.$$

There are three other possibilities under cheating:
(1) Only Little cheats: Then $Q_G = 4, Q_L = 2$, so $Q = 6, p = 20$. Profits will be

$$\pi_G = (20-4)(4) = 64 \qquad \textit{and} \qquad \pi_L = (20-4)(2) = 32.$$

(2) Only Grand cheats: Then $Q_G = 5, Q_L = 1$, so $Q = 6, p = 20$. Profits will be

$$\pi_G = (20-4)(5) = 80 \qquad and \qquad \pi_L = (20-4)(1) = 16.$$

(3) Both cheat: Then $Q_G = 5, Q_L = 2$, so $Q = 7, p = 16$. Profits will be

$$\pi_G = (16-4)(5) = 60 \qquad and \qquad \pi_L = (16-4)(2) = 24.$$

The payoff matrix is then

		Q_L	
		1	2
Q_G	4	80, 20	64, 32
	5	80, 16	60, 24

(c) $Q_G = 4, Q_L = 2$ are both dominant strategies; we would therefore expect this outcome. Total profit is, as expected, lower than in the cartel solution (96<100), with Grand worse off and Little better off.

4. (a) The Nash equilibria in pure strategies are (T, L) and (B, R). For (T, L), the row player has no incentive to deviate since 10>5 and the column player has no incentive to deviate since 5>3. For (B, R), the row player has no incentive to deviate since 20>5 and the column player has no incentive to deviate since 2>0.

(b) Suppose the row player plays T with probability p and B with probability *(1-p)* and the column player plays L with probability q and R with probability *(1-q)*. Then the row player's expected payoff is

$$E(\pi_r) = p\left\{10q + 5(1-q)\right\} + (1-p)\left\{5q + 20(1-q)\right\}.$$

Setting the derivative of this with respect to p equal to zero gives us

$$\frac{\partial E(\pi_r)}{\partial p} = 10q + 5(1-q) - 5q - 20(1-q) = 0 \qquad \rightarrow \qquad q = \frac{3}{4}.$$

Similarly, the column player's expected payoff is

$$E(\pi_c) = q\left\{5p\right\} + (1-q)\left\{3p + 2(1-p)\right\}.$$

Setting the derivative of this with respect to q equal to zero gives us

$$\frac{\partial E(\pi_c)}{\partial q} = 5p - 3p - 2(1-p) = 0 \qquad \rightarrow \qquad p = \frac{1}{2}.$$

Thus there is a Nash equilibrium in mixed strategies with the row player playing (T, B) with probabilities $\left(\frac{1}{2}, \frac{1}{2}\right)$ and the column player playing (L, R) with probabilities $\left(\frac{3}{4}, \frac{1}{4}\right)$.

(c) The normal form of the sequential game is as follows:

			Column	Player	
		LL	LR	RL	RR
Row	T	10,5	10,5	5,3	5,3
Player	B	5,0	20,2	5,0	20,2

(d) There are three Nash equilibria in pure strategies for this game: (T, LL), (B, LR) and (B, RR):
(T, LL) because 10>5 and 5>3
(B, LR) because 20>10 and 2>0
(B, RR) because 20>5 and 2>0.

Of these, (B, LR) is the subgame perfect equilibrium, because L is the best response to T (5>3) and R is the best response to B (2>0). (T, LL) involves the non-credible threat of L in response to B, while (B, RR) involves the non-credible threat of R in response to T; thus these two strategies are not subgame perfect equilibria.

Chapter 11: Market Structures Between Competition and Monopoly

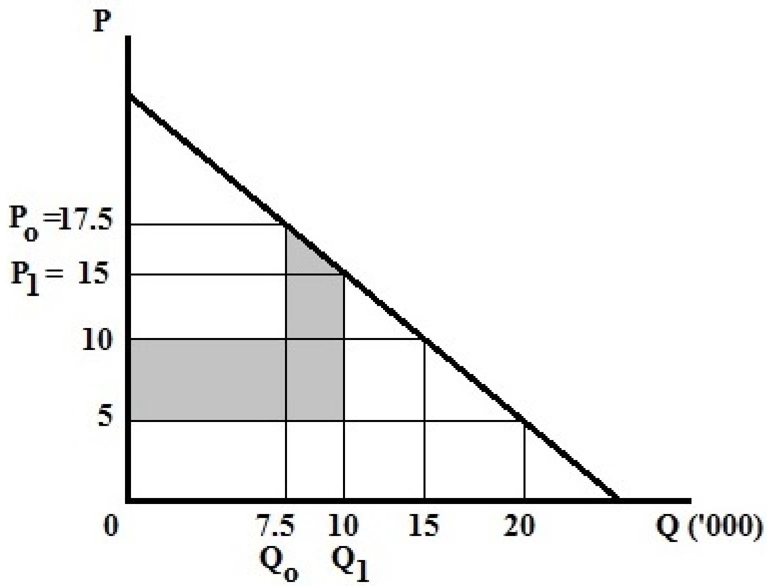
1. (a) Calculate Incumbent's profits without and with the investment:
Without the investment:

$$Q_0 = 7,500, p_0 = 17.5 \qquad \rightarrow \qquad \pi_0 = (7500)(17.5) = 56,250.$$

With the investment:

$$Q_1 = 10,000, p_1 = 15 \qquad \rightarrow \qquad \pi_1 = (10,000)(10) = 100,000.$$

The gain in profit from the investment is 100,000-56,250 = 43,750, which is less than the needed investment of $60,000, so Incumbent would NOT invest.



The social gain from Incumbent's investment would be the shaded area in the graph. Thus

$$\triangle W = (5 * 10,000) + (5 * 2,500) + \frac{1}{2}(2.5)(2,500) = 65,625.$$

This is greater than the needed investment of \$60,000, so the investment is desirable from the social point of view.

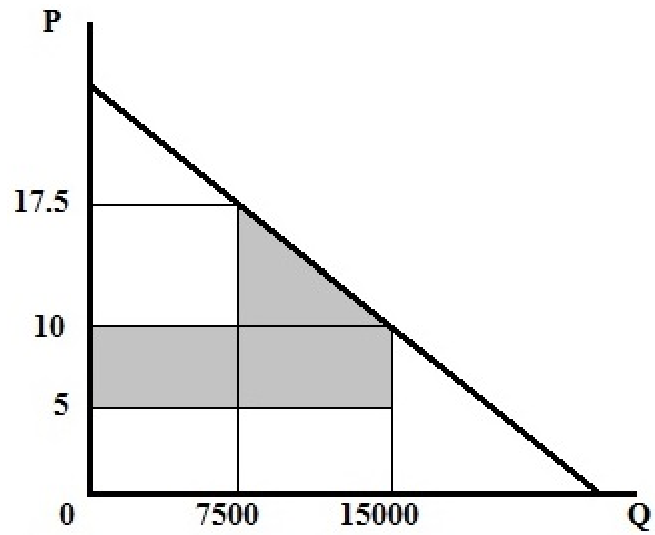
(b) The maximum amount Incumbent would be willing to pay for the cost reduction is the potential gain in profits:

$$Max\ WTP = \triangle\pi_I = 43,750.$$

(c) If Rival entered as a low cost producer, it could charge epsilon below \$10 and drive Incumbent from the market. At that price, it would sell 15,000 units, so its profit would be

$$\pi_R = 5 * 15,000 = 75,000.$$

This is greater than the cost of entry (60,000) so Rival would enter.



The social gain from Rival's entry is that price falls from 17.5 to 10 and cost falls from 10 to 5 ... see the shaded area in the new graph. Then the gain in welfare is:

$$\triangle W_R = (5 * 15,000) + \frac{1}{2}(7.5)(7,500) = 103,125.$$

So, yes, it is socially desirable because $\triangle W_R = 103,125 > 60,000$.

(d) With Rival's entry, Incumbent's profits go to zero. If it took over Rival, it would be a monopoly again, and now its cost would be \$5 per unit. It would therefore be able to make a profit of \$100,000 (see the answer to part (a) above). Note that it makes more profit than Rival would because Rival was not a monopolist and had to set a limit price to keep Incumbent out. Thus Incumbent would be willing to pay \$100,000 to take over Rival. This is more than it was willing to pay in part (b) because there the alternative was for it to be a monopolist with a per-unit cost of \$10, while now the alternative is to be out of business.

2. (a) Polaroid's profit is

$$\pi_P = 25Q - \frac{Q^2}{1000} - 5Q.$$

Profit is maximized where

$$\frac{d\pi_P}{dQ} = 20 - \frac{Q}{500} = 0 \qquad \rightarrow \qquad Q = 10,000, p = 15, \pi = 100,000.$$

(b) Polaroid's profit is now

$$\pi_P = q_P \cdot \left[25 - \frac{q_P + q_K}{1000}\right] - 5q_P.$$

This is maximized where

$$\frac{d\pi_P}{dq_P} = 20 - \frac{q_P + q_K}{1000} - \frac{q_P}{1000} = 0 \qquad \to \qquad 20 - \frac{q_P}{500} - \frac{q_K}{1000} = 0.$$

This is Polaroid's best-response function.

Kodak's profit is

$$\pi_K = q_K \cdot \left[25 - \frac{q_P + q_K}{1000}\right] - 10q_K.$$

This is maximized where

$$\frac{d\pi_K}{dq_K} = 15 - \frac{q_P + q_K}{1000} - \frac{q_K}{1000} = 0 \qquad \to \qquad 15 - \frac{q_P}{1000} - \frac{q_K}{500} = 0.$$

This is Kodak's best-response function.

Solving the two best-response functions simultaneously, we get the Cournot equilibrium:

$$q_P = \frac{25,000}{3}, q_K = \frac{10,000}{3}, p = \frac{40}{3}, \pi_P = \frac{625,000}{9}, \pi_K = \frac{100,000}{9}.$$

(c) CRC could ask up to $\frac{100,000}{9} = 11,111$. Polaroid would pay

$$100,000 - \frac{625,000}{9} = \frac{275,000}{9} = 30,556,$$

which is more than CRC would ask, because it would like to preserve its monopoly.

3. (a) The firm's profit is

$$\pi_m = \left(\frac{360}{Q}\right)^{\frac{1}{2}} \cdot Q - 3Q.$$

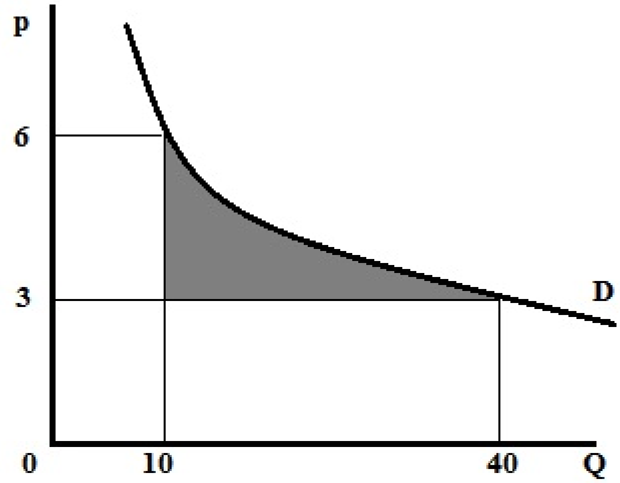
Profit is maximized where

$$\frac{d\pi_m}{dQ} = \frac{3\sqrt{10}}{Q^{\frac{1}{2}}} - 3 = 0 \qquad \to \qquad Q_m = 10, p_m = 6.$$

(b) Social welfare would be maximized by setting $p = MC$. The social optimum therefore is at

$$Q^* = 40, p^* = 3.$$

(c) The deadweight loss is the shaded area in the graph.



$$\triangle W = \int_3^6 \frac{360}{p^2} dp - 30 = \left[-\frac{360}{p} \right]_3^6 - 30 = 30.$$

(d) If there were two firms, the first firm's profit could be written as

$$\pi_1 = \frac{6\sqrt{10}}{(q_1 + q_2)^{\frac{1}{2}}} \cdot q_1 - 3q_1.$$

Profit would be maximized where

$$\frac{\partial \pi_1}{\partial q_1} = \frac{6\sqrt{10}}{(q_1 + q_2)^{\frac{3}{2}}} \cdot \left(-\frac{1}{2} \right) \cdot q_1 + \frac{6\sqrt{10}}{(q_1 + q_2)^{\frac{1}{2}}} - 3 = 0.$$

This is form 1's best-response function. Firm 2's best-response function will look just like this one, and we know that, in the final equilibrium, $q_1 = q_2$ because of the symmetry of the problem. Imposing this condition into Firm 1's best-response function, we can solve for the final equilibrium. It will be:

$$Q_c = 22.5, p_c = 4, q_1 = q_2 = 11.25.$$

4. (a) Let q_s represent Select's output and q_e represent Exclusive's output. Now the demand curve can be rewritten as

$$P = 1000 - 2Q_d.$$

Then Select's profit can be written as

$$\pi_s = [1000 - 2(q_s + q_e)] \cdot q_s - 100q_s.$$

To maximize profits, Select will set

$$\frac{\partial \pi_s}{\partial q_s} = 1000 - 2q_e - 4q_s - 100 = 0 \qquad \rightarrow \qquad q_s = 225 - \frac{1}{2}q_e.$$

This is Select's best-response function. Similarly, we can find Exclusive's best-response function by the same procedure:

$$\pi_e = [1000 - 2(q_s + q_e)] \cdot q_e - 200q_e.$$

$$\frac{\partial \pi_e}{\partial q_e} = 1000 - 2q_s - 4q_e - 200 = 0 \qquad \rightarrow \qquad q_e = 200 - \frac{1}{2}q_s.$$

The Cournot-Nash equilibrium is at the intersection of the two reaction functions. Solving them simultaneously gives us the equilibrium levels of output:

$$q_s = \frac{500}{3} = 166\frac{2}{3} \qquad \textit{and} \qquad q_e = \frac{700}{6} = 116\frac{2}{3}.$$

Then total output is:

$$Q = \frac{500}{3} + \frac{700}{6} = \frac{1700}{6}$$

and so the equilibrium price will be:

$$P = 1000 - 2\left(\frac{1700}{6}\right) = \frac{1300}{3} = 433\frac{1}{3}.$$

(b) If each of the firms thought the other would match its quantity changes, we would have

$$\frac{\partial q_e}{\partial q_s} = 1 \qquad \textit{and} \qquad \frac{\partial q_s}{\partial q_e} = 1.$$

So Select's first-order condition would be

$$\frac{\partial \pi_s}{\partial q_s} = 1000 - 2q_e - 4q_s - 2q_s - 100 = 0 \qquad \rightarrow \qquad q_s = \frac{450 - q_e}{3}.$$

Exclusive's first-order condition would be

$$\frac{\partial \pi_e}{\partial q_e} = 1000 - 2q_s - 4q_e - 2q_e - 200 = 0 \qquad \rightarrow \qquad q_e = \frac{400 - q_e}{3}.$$

Solving for the intersection of the two best-response functions, we find

$$q_s = 118.75 \qquad \textit{and} \qquad q_e = 93.75.$$

Total output is $Q = 212.5$ and so the price will be $P = 575$.

(c) If Select plays the game as a Stackelberg leader, it will take Exclusive's best-response function into account when making its own optimizing decision. Therefore we now have

$$\pi_s = \left[1000 - 2(q_s + 200 - \frac{1}{2}q_s)\right] \cdot q_s - 100q_s.$$

and this will be maximized when

$$\frac{\partial \pi_s}{\partial q_s} = 1000 - 400 - 2q_s - 100 = 0 \qquad \rightarrow \qquad q_s = 250.$$

Then

$$q_e = 200 - \frac{1}{2}(250) = 75, \qquad Q = 325 \qquad \textit{and} \qquad P = 1000 - 2(325) = 350.$$

5. (a) We can look at Acme's profits:

$$\pi_a = (300 - q_a) * q_a - 60q_a.$$

To maximize profits, Acme will set

$$\frac{\partial \pi_a}{\partial q_a} = 300 - 2q_a - 60 = 0 \qquad \rightarrow \qquad q_a = 120.$$

Then

$$p = 180 \qquad \textit{and} \qquad \pi_a = (120)(120) = 14,400.$$

(b) If Bingo enters, Acme's profits would be:

$$\pi_a = (300 - q_a - q_b) * q_a - 60q_a.$$

Setting the derivative of this with respect to q_a equal to zero gives us

$$\frac{\partial \pi_a}{\partial q_a} = 300 - 2q_a - q_b - 60 = 0 \qquad \rightarrow \qquad q_a = 120 - \frac{1}{2}q_b.$$

This is Acme's reaction function, By symmetry, we can say that Bingo's reaction function will be

$$q_b = 120 - \frac{1}{2}q_a.$$

Solving these two reaction functions simultaneously gives us the Cournot equilibrium:

$$q_a = 120 - \frac{1}{2}\left(120 - \frac{1}{2}q_a\right) \qquad \rightarrow \qquad q_a = 80.$$

Then

$$q_b = 80, p = 140 \;\; \textit{and} \;\; \pi_a = \pi_b = (80)(80) = 6,400.$$

(c) If Acme and Bingo collude, and since they each have constant AC and MC of 60, they should adopt the monopoly solution of part (a) and simply share the market equally. Thus, in this solution:

$$q_a = q_b = 60, p = 180, \pi_a = \pi_b = 7,200.$$

(d) If Acme sets $q_a = 60$ as required by the collusion agreement, Bingo effectively faces the residual demand curve

$$q_b = 240 - p.$$

Then we can find Bingo's profit-maximizing output level by looking at its profit:

$$\pi_b = (240 - q_b) * q_b - 60q_b.$$

Setting the derivative of this with respect to q_b equal to zero gives us

$$\frac{\partial \pi_b}{\partial q_b} = 240 - 2q_b - 60 = 0 \qquad \rightarrow \qquad q_b = 90.$$

Now

$$Q = q_a + q_b = 150, \qquad so \;\; p = 150,$$

$$\pi_a = (90)(60) = 5,400 \qquad and \qquad \pi_b = (90)(90) = 8,100.$$

6. (a) Able's profit, as a function of its level of production, is

$$\pi_a = (14 - W) \cdot W - 2W.$$

Differentiating with respect to W and setting equal to zero yields the profit-maximizing output level: $W_a = 8. \qquad Then \qquad p_a = 6, \pi_a = 36.$

(b) Let's assume Baker finds it profitable to enter and see what the Cournot equilibrium would be. Let W_a and W_b represent the output levels of Able and Baker respectively. Baker's profit would be

$$\pi_b = (14 - W_a - W_b) \cdot W_b - 2W_b.$$

Differentiating with respect to W_b, setting equal to zero, and simplifying, yields Baker's best-response function:

$$W_b^* = \frac{12 - W_a}{2}.$$

Able's best-response function will be symmetrical to Baker's. Solving the two functions simultaneously yields the Cournot equilibrium:

$$W_a = W_b = 4.$$

Then $p_2 = 6$ and profits are $\pi_a = 16, \pi_b = 12$ (remember that Baker has the fixed entry cost of 4).

(c) If Able can pre-commit to an output level, it needs to decide whether to accommodate or deter Baker's entry. If it accommodates entry, it will play as a Stackelberg leader, taking Baker's best-response function into account when maximizing its profit:

$$\pi_a = \left\{14 - W_a - \frac{12 - W_a}{2}\right\} \cdot W_a - 2W_a.$$

Optimizing over W_a and then completing all the calculations, we get:

$$W_a = 6, W_b = 3, p = 5, \pi_a = 18, \pi_b = 5.$$

Since $\pi_b > 0$, Baker would find it profitable to enter and this equilibrium would emerge.

In order to deter entry, Able would have to pre-commit to an output level high enough so as to reduce Baker's operating profit to 4, thereby making its entry unprofitable. Suppose this output level is W_a^*. Then, using Baker's best-response function, we can infer that Baker would produce

$$W_b = \frac{12 - W_a^*}{2}.$$

Then Baker's profit will be

$$\pi_b = \left\{14 - W_a^* - \frac{12 - W_a^*}{2}\right\} \cdot \left\{\frac{12 - W_a^*}{2}\right\} - 2\left\{\frac{12 - W_a^*}{2}\right\}.$$

Setting this equal to 4 and solving, we find

$$W_a^* = 8.$$

If Able pre-committed to this output level, Baker would not enter and therefore:

$$p = 6, \pi_a = 32, \pi_b = 0.$$

Able is therefore better off deterring entry and this will be the equilibrium.

7. (a) Firm 1's profit is

$$\pi_1 = q_1 (100 - q_1 - q_2) - 20q_1.$$

This is maximized where

$$\frac{\partial \pi_1}{\partial q_1} = 100 - q_1 - q_2 - q_1 - 20 = 0 \qquad \rightarrow \qquad q_1 = 40 - \frac{1}{2}q_2.$$

This is firm 1's reaction function.
Firm 2's profit is

$$\pi_2 = q_2 (100 - q_1 - q_2).$$

This is maximized where

$$\frac{\partial \pi_2}{\partial q_2} = 100 - q_1 - q_2 - q_2 = 0 \qquad \to \qquad q_2 = 50 - \frac{1}{2}q_1.$$

This is firm 2's reaction function.
Solving the two reaction functions simultaneously, we find the Cournot equilibrium:

$$q_1^* = 20, q_2^* = 40, p^* = 40, \pi_1^* = 400, \pi_2^* = 1600.$$

(b) Firm 1's reaction function remains as before. However, firm 2's profit will be written taking into account firm 1's reaction function. Thus

$$\pi_2 = q_2\left(100 - 40 + \frac{1}{2}q_2 - q_2\right) = 60q_2 - \frac{1}{2}q_2^2.$$

This is maximized where

$$\frac{\partial \pi_2}{\partial q_2} = 60 - q_2 = 0 \qquad \to \qquad q_2 = 60.$$

The Stackelberg equilibrium is therefore

$$q_1^{**} = 10, q_2^{**} = 60, p^{**} = 30, \pi_1^{**} = 100, \pi_2^{**} = 1800.$$

(c) Under the Stackelberg equilibrium, price is lower and quantity sold is higher.

$$\triangle CS = 10 * 60 + \frac{1}{2}(10)(10) = 650.$$

In addition,

$$\triangle \pi_1 = -300 \qquad and \qquad \triangle \pi_2 = +200.$$

Therefore the net welfare change is

$$\triangle W = 650 - 300 + 200 = +550.$$

8. (a) Firm 1's profit is

$$\pi_1 = q_1\left(15 - \frac{q_1 + q_2}{2}\right) - 6q_1.$$

This is maximized where

$$\frac{\partial \pi_1}{\partial q_1} = 15 - \frac{q_1 + q_2}{2} - \frac{q_1}{2} - 6 = 0 \qquad \to \qquad q_1 = 9 - \frac{1}{2}q_2.$$

This is firm 1's best-response function. By a similar process, we can find firm 2's best-response function:

$$q_2 = 6 - \frac{1}{2}q_1.$$

Solving the two best-response functions, we get the Cournot equilibrium:

$$q_1^* = 8, q_2^* = 2, p^* = 10, \pi_1^* = 32, \pi_2^* = 2.$$

(b) Under a cartel equilibrium, firm 2 would not produce anything, since its costs are higher than those of firm 1. Firm 1 would then find the monopoly equilibrium:

$$q_1^{**} = 9, q_2^{**} = 0, p^{**} = 10.5, \pi_1^{**} = 40.5, \pi_2^{**} = 0.$$

(c) Firm 1 would pay 8.5 (= 40.5 - 32) for firm 2 and would require 32 to be bought out.
Firm 2 would pay 38.5 (= 40.5 - 2) for firm 1 and would require 2 to be bought out.

9. (a) For joint profits to be maximized, all production should be by Acme, as its MC is lower:

$$MC_A = 10 \qquad and \qquad MC_B = 12.$$

Then total profits will be

$$\pi = \pi_A + \pi_B = 50Q - 5Q^2 - 20 - 10Q - 10$$

(remember, B's fixed costs must still be incurred). Profits are maximized where

$$\frac{d\pi}{dQ} = 50 - 10Q - 10 = 0 \qquad \rightarrow \qquad Q^* = 4.$$

Individual production levels are: $q_A^* = 4, q_B^* = 0$. In this equilibrium, $\pi = 50$.

(b) If entry has not already taken place, we need to think of what would happen if each firm entered separately. If firm A alone entered, we would have the outcome from part (a), except that $\pi = 60$, since there will be no fixed cost from firm B. If firm B alone entered,

$$\pi_B = 50q_B - 5q_B^2 - 10 - 12q_B.$$

This is maximized where

$$\frac{d\pi_B}{dq_B} = 50 - 10q_B - 12 = 0 \qquad \rightarrow \qquad Q^* = 3.8 \qquad \rightarrow \qquad \pi_B = 62.2.$$

Thus profits are higher if B alone enters and the choice would be: $q_A^{**} = 0, q_B^{**} = 3.8$.

(c) Under Cournot competition, Acme's profit would be

$$\pi_A = 50q_A - 5\left(q_A + q_B\right)q_A - 20 - 10q_A.$$

This is maximized where

$$\frac{\partial \pi_A}{\partial q_A} = 50 - 10q_A - 5q_B - 10 = 0 \qquad \rightarrow \qquad q_A = \frac{40 - 5q_B}{10}.$$

This is Acme's best-response function. By a similar process, we can find Best's best-response function:

$$q_B = \frac{38 - 5q_A}{10}.$$

Solving the two best-response functions simultaneously, we find the Cournot-Nash equilibrium:

$$q_A^C = 2.8, q_B^C = 2.4, \pi_A^C = 19.2, \pi_B^C = 18.8.$$

(d) Under the takeover scenarios, we would end up with the solution in part (a), where the joint profits of the firms are maximized at $\pi = 50$. Each firm would compare this to the profits it makes under the Cournot equilibrium. Then:
Acme would pay $50 - 19.2 = 30.8$ to take over Best.
Best would pay $50 - 18.8 = 31.2$ to take over Acme.

10. (a) Under competition, each firm's supply curve will be that part of its marginal cost curve that lies above its average variable cost curve. Now

$$MC_i = 10 + q_i \qquad and \qquad AVC_i = 10 + \frac{1}{2}q_i.$$

Therefore the entire MC curve lies above the AVC curve and will be each firm's supply curve, which can be written

$$q_i = p - 10.$$

Then total supply will be

$$Q_s = 2p - 20.$$

Setting $Q_s = Q_d$ and solving, we find the short-run equilibrium:

$$p^* = 40, Q^* = 60, q_i^* = 30.$$

(b) The cartel would want to maximize joint profits:

$$\pi = \pi_1 + \pi_2 = (100 - q_1 - q_2)(q_1 + q_2) - 10(q_1 + q_2) - \frac{1}{2}q_1^2 - \frac{1}{2}q_2^2.$$

Profits would be maximized where

$$\frac{\partial \pi}{\partial q_1} = 100 - q_1 - q_2 - (q_1 + q_2) - 10 - q_1 = 0$$

and a similar equation for $\frac{\partial \pi}{\partial q_2}$. By the symmetry of the problem, we know that $q_1 = q_2$ at the optimum. Imposing this in the first-order condition above, we can find the equilibrium:

$$q_i^C = 18, Q^C = 36, p^C = 64.$$

(c) From part (a):

$$CS^* = \frac{1}{2}(60)(60) = 1800.$$

$$\pi^* = 2\left[(30)(40) - 10(30) - \frac{1}{2}(30)^2\right] = 900.$$

From part (b):

$$CS^C = \frac{1}{2}(36)(36) = 648.$$

$$\pi^C = (64)(36) - 2\left[180 + \frac{1}{2}(18)^2\right] = 1620.$$

Therefore consumers lose and firms gain:

$$\triangle CS = 648 - 1800 = -1152.$$

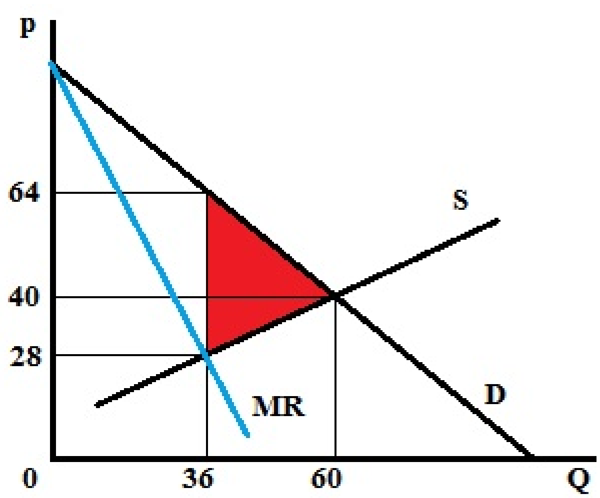
$$\triangle \pi = 1620 - 900 = +720.$$

(d) The net loss to society is

$$\triangle W = -1152 + 720 = -432.$$

This is seen as the red area in the graph, whose area can be computed graphically as:

$$\frac{1}{2}(64 - 28)(60 - 36) = 432.$$



11. (a) Joint profits are

$$\pi = \pi_A + \pi_B = 2\sqrt{x_A} + 2\sqrt{x_B} - x_A + 2\sqrt{x_B} - x_B.$$

These are maximized when

$$\frac{\partial \pi}{\partial x_A} = \frac{1}{\sqrt{x_A}} - 1 = 0 \qquad or \qquad x_A^* = 1,$$

$$\frac{\partial \pi}{\partial x_B} = \frac{2}{\sqrt{x_B}} - 1 = 0 \qquad or \qquad x_B^* = 4.$$

(b) To maximize π_A, firm A will set

$$\frac{\partial \pi_A}{\partial x_A} = \frac{1}{\sqrt{x_A}} - 1 = 0 \qquad or \qquad x_A^e = 1.$$

To maximize π_B, firm B will set

$$\frac{\partial \pi_B}{\partial x_B} = \frac{1}{\sqrt{x_B}} - 1 = 0 \qquad or \qquad x_B^* = 1.$$

(c) With the subsidy, firm A's problem becomes to maximize

$$\pi_A = 2\sqrt{x_A} + 2\sqrt{x_B} - x_A + s_A x_A.$$

It will set

$$\frac{\partial \pi_A}{\partial x_A} = \frac{1}{\sqrt{x_A}} - 1 + s_A = 0 \qquad or \qquad x_A = \left(\frac{1}{1 - s_A}\right)^2.$$

To get $x_A^* = 1$, we will need to set $s_A = 0$.
With the subsidy, firm B's problem becomes to maximize

$$\pi_B = 2\sqrt{x_B} - x_B + s_B x_B.$$

It will set

$$\frac{\partial \pi_B}{\partial x_B} = \frac{1}{\sqrt{x_B}} - 1 + s_B = 0 \qquad or \qquad x_B = \left(\frac{1}{1 - s_B}\right)^2.$$

To get $x_B^* = 4$, we will need to set $s_B = \frac{1}{2}$.

(d) The total subsidy paid will be

$$S = \left(\frac{1}{2}\right)(4) = 2.$$

Any combination of T_A, T_B such that $T_A + T_B = 2$ will achieve the desired end.

12. (a) Toyota's profit is

$$\pi_T = \frac{32,000 w_T}{w_T + w_H} - 2,000 w_T.$$

To maximize this, the company would set

$$\frac{\partial \pi_T}{\partial w_T} = \frac{(w_T + w_H) \cdot 32,000 - 32,000 w_T}{(w_T + w_H)^2} - 2,000 = 0 \qquad \rightarrow \qquad 16 w_H = (w_T + w_H)^2 .$$

This is Toyota's best-response function. By a similar process, we can find Honda's bast-response function:

$$16 w_T = (w_T + w_H)^2 .$$

Solving the two best-response functions simultaneously, we get the Cournot-Nash solution:

$$w_T^C = 4, w_H^C = 4, \pi_T^C = 8000, \pi_H^C = 8000.$$

(b) If Honda moves first, it will take Toyota's best-response function into account when choosing its warranty level. So it will be trying to maximize

$$\pi_H = \frac{32,000 w_H}{4\sqrt{w_H}} - 2,000 w_H .$$

To maximize this, the company would set

$$\frac{\partial \pi_H}{\partial w_H} = \frac{4,000}{\sqrt{w_H}} - 2,000 = 0 \qquad \rightarrow \qquad w_H = 4.$$

Toyota's best response to this is to set $w_T = 4$. Therefore, we get the same solution as in (a).

(c) If the companies decide to collude, they could agree to set $w_T = w_H$. Then Toyota's profit function will be

$$\pi_T = \frac{32,000 w_T}{2 w_T} - 2,000 w_T = 16,000 - 2,000 w_T .$$

Now note that

$$\frac{\partial \pi_T}{\partial w_T} = -2,000 < 0.$$

Therefore, the optimal solution is to set $w_T^* = 0, w_H^* = 0$.

13. (a) The firms' strategy sets are their output levels, which can take on any real non-negative values. Under the Cournot ssumption, firm 1's profit will be

$$\pi_1 = q_1 (10 - q_1 - q_2) - 2 q_1 .$$

To maximize this, the firm would set

$$\frac{\partial \pi_1}{\partial q_1} = 10 - 2q_1 - q_2 - 2 = 0 \qquad \rightarrow \qquad q_1 = 4 - \frac{1}{2} q_2 .$$

This is firm 1's best-response function. Firm 2's best-response function will be symmetric to this:

$$q_2 = 4 - \frac{1}{2}q_1.$$

Solving the two best-response functions simultaneously, we get the Cournot equilibrium:

$$q_1^C = q_2^C = \frac{8}{3}, Q^C = \frac{16}{3}.$$

(b) If the firms collude, they will maximize joint profits:

$$\pi = Q(10-Q)-2Q \qquad \rightarrow \qquad \frac{d\pi}{dQ} = 8-2Q = 0 \qquad \rightarrow \qquad Q = 4.$$

So the equilibrium will be $q_1^* = q_2^* = 2, Q^* = 4, p^* = 6$.

(c) If firm 1 moves first, its strategy set is, as before, any non-negative output level. Firm 2 will have the same set of strategies for each possible choice that firm 1 may make. Firm 1 will maximize its profit taking into account firm 2's best response to its choice. Therefore, firm 1's profit can be written

$$\pi_1 = q_1\left(10 - q_1 - 4 + \frac{1}{2}q_1\right) - 2q_1 = 4q_1 - \frac{1}{2}q_1^2.$$

To maximize this, the firm will set

$$\frac{\partial \pi_1}{\partial q_1} = 4 - q_1 = 0 \qquad \rightarrow \qquad q_1^S = 4.$$

From firm 2's best-response function, we see that it will set $q_2^S = 2$. The final equilibrium is then

$$Q^S = 6, p^S = 4.$$

(d) From part (a) we have

$$\pi_1^C = \pi_2^C = \left(\frac{14}{3} - 2\right)\left(\frac{8}{3}\right) = \frac{64}{9}.$$

From part (b) we have

$$\pi_1^* = \pi_2^* = \frac{1}{2}(6-2)(4) = 8.$$

From part (c) we have

$$\pi_1^S = (4-2)(4) = 8 \qquad and \qquad \pi_2^S = (4-2)(2) = 4.$$

Joint profits are highest in the collusive equilibrium, as would be expected. Firm 1 does just as well in the Stackelberg case, but firm 2 does worse. The lowest profits are in the most competitive situation, the Cournot case.

Chapter 12: Externalities and Public Goods

1. (a) In the competitive equilibrium, average catch will be equalized on the two lakes:

$$10 - \frac{1}{2}L_x = 5 \qquad \rightarrow \qquad L_x = 10 \qquad \rightarrow \qquad Q_x = 50.$$

Then

$$L_y = 10 \qquad \rightarrow \qquad Q_y = 50.$$

The total catch will be 100.

(b) In the efficient allocation, the marginal catch would be equalized across the two lakes:

$$10 - L_x = 5 \qquad \rightarrow \qquad L_x = 5 \qquad \rightarrow \qquad Q_x = 37.5$$

Then

$$L_y = 15 \qquad \rightarrow \qquad Q_y = 75.$$

The total catch will be 112.5.

(c) Since we want to reduce the number of fisherman on lake X, we need to levy a fee on those who choose that lake. The fee F should be such that

$$AP_x - F = AP_y \qquad \rightarrow \qquad 10 - \frac{1}{2}L_x - F = 5.$$

Since we want $L_x = 5$, this becomes:

$$10 - \frac{1}{2}(5) - F = 5 \qquad \rightarrow \qquad F = 2.5.$$

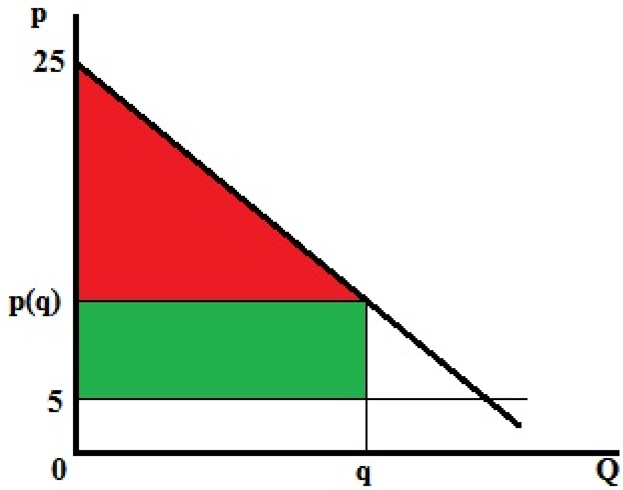
This is the required fee.

2. (a) Since the MC of producing lead is constant at 5, social welfare would be maximized if the price were set equal to MC and the level of production chosen accordingly. The socially optimal equilibrium is therefore

$$Q^* = 20,000, p^* = 5.$$

(b) We know that the inverse demand curve is

$$p(Q) = 25 - \frac{Q}{1,000}.$$



For any value of Q, say q, the graph shows the Consumer Surplus (CS) as the red area and the firm profits (π) as the green area. If the environmental damage caused by the toxic waste is D, we can write social welfare as

$$W(Q) = CS + \pi - D = \frac{Q^2}{2,000} + \left(20Q - \frac{Q^2}{1,000}\right) - \frac{Q^2}{8,000} = 20Q - \frac{5Q^2}{8,000}.$$

This is maximized where

$$\frac{dW}{dQ} = 20 - \frac{Q}{800} = 0 \qquad \rightarrow \qquad Q^e = 16,000.$$

Then $p^e = 9$.

(c) Social welfare can now be written as

$$W = \frac{Q^2}{2,000} + \left(20Q - \frac{Q^2}{1,000}\right) - \frac{(Q-A)^2}{8,000} - \frac{A^2}{2,000}.$$

This is maximized where

$$\frac{\partial W}{\partial Q} = 20 - \frac{Q}{1,000} - \frac{(Q-A)}{4,000} = 0,$$

$$and \qquad \frac{\partial W}{\partial A} = -\frac{(Q-A)}{4,000}(-1) - \frac{A}{1,000} = 0.$$

Solving these two first-order conditions simultaneously, we get the optimal solution:

$$Q^a = \frac{100,000}{6}, p^a = \frac{50}{6}.$$

3. (a) The equilibrium number of people would be that number at which the benefit per person is \$6. Thus, at equilibrium,

$$20 - \frac{n^e}{200} = 6 \qquad or \qquad n^e = 2800.$$

(b) Since the cost of providing beach access is zero, the socially optimal number of visitors would be the number at which the marginal benefit of coming to the beach is zero. Now B is the average benefit per person, so the total benefit is

$$TB = nB = 20n - \frac{n^2}{200}.$$

Differentiating with respect to n and setting equal to zero would yield the optimal number of visitors:

$$20 - \frac{n^*}{100} = 0 \qquad or \qquad n^* = 2000.$$

In order to induce this number of visitors, we must set an entrance fee f such that the average benefit per person is f when the number of visitors is 2000:

$$f = 20 - \frac{2000}{200} = 10.$$

4. (a) If it abates a, CCC's profit will be

$$\pi = 100 - a^2 - 14(10 - a),$$

since it pays \$14 per ton not abated. Profit is maximized where

$$\frac{d\pi}{da} = -2a + 14 = 0 \qquad \rightarrow \qquad a_1 = 7.$$

Then $\pi_1 = 9$.

(b) Under the subsidy,

$$\pi = 100 - a^2 + 14a.$$

This is maximized where

$$\frac{d\pi}{da} = -2a + 14 = 0 \qquad \rightarrow \qquad a_2 = 7.$$

Then $\pi_2 = 149$.

(c) Either is optimal if marginal damage is 14. (a) is preferable because it raises revenue, in contrast to (b) which involves subsidy costs.

5. (a) The supply curve is the aggregation of the individual firm MC curves; in other words, its the private marginal cost curve (PMC). Now the supply curve can be written as

$$P = 5 + \frac{1}{2000}Q_s \qquad so \qquad PMC = 5 + \frac{1}{2000}Q_s.$$

The marginal external cost (MEC) of blodget production is \$6 per unit (=2 units of gunk produced x \$3 damage per unit of gunk). So the social marginal cost (SMC) is \$6 more than the PMC:

$$SMC = 11 + \frac{1}{2000}Q_s.$$

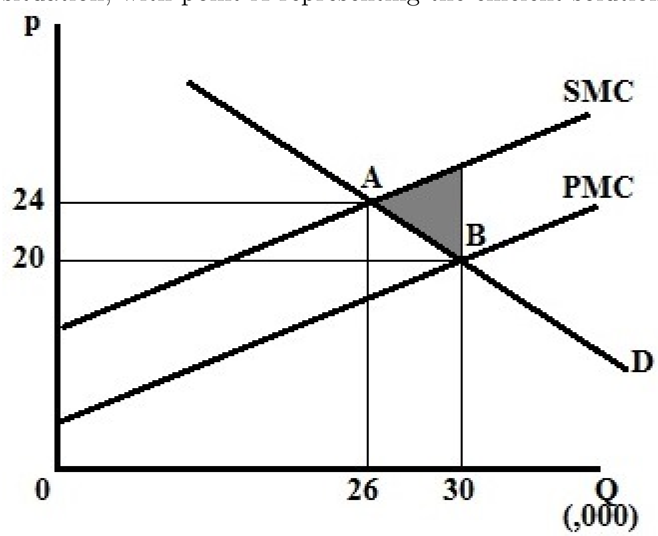
At the efficient solution, $p = SMC$, so

$$Q_s = 2000P - 22,000.$$

Equate this to Q_d to get the equilibrium:

$$2,000P - 22,000 = 50,000 - 1,000P \qquad \rightarrow \qquad P = \$24 \;\; and\, so\, Q = 26,000.$$

This is the welfare-maximizing solution. The graph illustrates the situation, with point A representing the efficient solution.



(b) The competitive equilibrium would occur where demand and supply intersect:

$$2,000P - 10,000 = 50,000 - 1,000P \qquad \rightarrow \qquad P = \$20 \;\; and\, so\, Q = 30,000.$$

This equilibrium is point B in the graph. The deadweight loss in this equilibrium is the shaded area in the graph, since there is excessive production and this represents the amount by which social marginal cost exceeds the social marginal benefit over these excessive units of output. We can calculate this area (remember the vertical difference between PMC and SMC is $6):

$$DWL = \frac{1}{2} \cdot 6 \cdot 4000 = \$12,000.$$

(c) The Pigouvian tax necessary to force the efficient solution is the MEC at the optimum, which is $6. There are four classes of economic agents affected by this tax:
Consumers of blodgets lose because price rises from $20 to $24:

$$\triangle CS = -4 \cdot 26,000 - \frac{1}{2} \cdot 4 \cdot 4,000 = -\$112,000.$$

Producers of blodgets lose because their net price falls from $20 to $18:

$$\triangle PS = -2 \cdot 26,000 - \frac{1}{2} \cdot 2 \cdot 4,000 = -\$56,000.$$

Government gains tax revenue:

$$\triangle Tax\,Revenue = 6 \cdot 26,000 = \$156,000.$$

Society at large benefits from the reduced gunk pollution. Since output of blodgets falls by 4,000, the amount of gunk produced falls by 8,000, and, at a marginal damage cost of $3 per unit of gunk, that gives us:

$$\triangle External\,Cost = 3 \cdot 8,000 = \$24,000.$$

If we add up all these welfare changes, we get

$$\triangle W = -112,000 - 56,000 + 156,000 + 24,000 = \$12,000,$$

exactly the DWL we calculated in part (b).

(d) The marginal benefit of any gunk abatement is $3 per unit, which is the damage that is prevented. Now the MC of abatement is

$$\frac{dA}{da} = \frac{a}{10,000},$$

so the optimal level of abatement (where MC=MB) will be where

$$\frac{a}{10,000} = 3 \qquad \rightarrow \qquad a = 30,000.$$

The optimal level of blodget production is still 26,000, since the MEC has remained at $6.

The gain to society from this solution compared to the solution with the Pigouvian tax is the reduced external cost from the 30,000 units of gunk abated minus the cost of abatement:

$$\triangle W = 3 * 30,000 - \frac{(30,000)^2}{20,000} = \$45,000.$$

6. (a) The equilibrium number of boats will be the number at which the average catch per boat equals the cost of running a boat:

$$100(200 - n) = 1,000 \qquad \rightarrow \qquad n^e = 190.$$

Then $Q^e = 1900$.

(b) The socially efficient number of boats will be the number at which the *marginal* catch per boat equals the cost of running a boat:

$$100(200 - 2n) = 1,000 \qquad \rightarrow \qquad n^* = 95.$$

Then $Q^* = 9975$.

(c) At the optimum, the average catch per boat is

$$ARP = 100(200 - 95) = 10,500.$$

To achieve the optimum, we need to make the average cost of running a boat equal to this average revenue product, which means we need to charge a licence fee of

$$F^* = 10,500 - 1,000 = 9,500.$$

The budgetary impact of this policy would be a gain in tax revenue of

$$T = (9,500)(95) = +902,500.$$

7. (a) First find the MC and AVC curves for each type of firm:

$$MC_A = q_A \qquad and \qquad AVC_A = \frac{1}{2}q_A.$$

$$MC_B = \frac{4}{5}q_B \qquad and \qquad AVC_B = \frac{2}{5}q_B.$$

Both MC curves are entirely above the AVC curves and so will constitute the firms' supply curves. The supply curves are

$$q_A = p \qquad and \qquad q_B = \frac{5}{4}p.$$

The market supply curve then is (remembering that there are 40 firms of each type)

$$Q_s = 90p.$$

Setting this equal to Q_d and solving, we get the competitive equilibrium:

$$p^c = 5, Q^c = 450, q_A^c = 5, q_B^c = \frac{25}{4}, \pi_A^c = \frac{25}{2}, \pi_B^c = \frac{125}{8}.$$

(b) With the negative externality, the SMC curve is everywhere \$1 higher than the PMC curve (the supply curve). Thus

$$SMC = \frac{Q}{90} + 1 \qquad or \qquad Q_s^{'} = 90p + 90.$$

Equating this to Q_d and solving, we get the optimal equilibrium under externality:

$$p^* = \frac{11}{2}, Q^* = 405, q_A^* = \frac{9}{2}, q_B^* = \frac{45}{8}.$$

(c) Since the marginal damage caused by a unit of carbon is \$1, we know that the optimal carbon tax would be \$1. Each firm should abate carbon to the point where the marginal cost of abatement is also \$1. But

$$\frac{dy_A}{dx_A} = \frac{1}{2}x_A \qquad and \qquad \frac{dy_B}{dx_B} = \frac{2}{5}x_B.$$

Setting each equal to \$1 gives us the optimal levels of abatement:

$$x_A^a = 2, x_B^a = \frac{5}{2}.$$