EC 501: Problem Set 11, Solutions

1. (a) Firm 1's profit is

$$\pi_1 = q_1 \cdot (300 - q_1 - q_2).$$

This will be maximized where

$$\frac{\partial \pi_1}{\partial q_1} = 300 - 2q_1 - q_2 = 0 \qquad \rightarrow \qquad q_1 = 150 - \frac{1}{2}q_2.$$

This is firm 1's best-response function. Firm 2's best-response function will be similar and the equilibrium will be symmetric. Solving the two best-response functions simultaneously yields the Cournot equilibrium:

$$q_{1C} = q_{2C} = 100, p_C = 100, \pi_{1C} = \pi_{2C} = 10,000.$$

(b) If firm 1 plays Stackelberg to firm 2's Cournot, firm 1 will incorporate firm 2's best-response function for q_2 in its objective function:

$$\pi_{1S} = q_{1S} \cdot \left(300 - q_{1S} - \left\{150 - \frac{1}{2}q_{1S}\right\}\right).$$

This will be maximized where

$$\frac{\partial \pi_{1S}}{\partial q_{1S}} = 150 - q_{1S} = 0 \qquad \rightarrow \qquad q_{1S} = 150.$$

Then the rest of the Stackelberg equilibrium will be

$$q_{2S} = 75, p_S = 75, \pi_{1S} = 11250, \pi_{2S} = 5625.$$

(c) If firm 2 has not yet entered, firm 1 might consider deterring entry. If it accomodated entry, the best it could do would be the Stackelberg solution in part (b). To deter entry, it would need to set q_1 high enough so as to make firm 2's entry unprofitable, by reducing firm 2's profit to its entry cost of 900. Now firm 2's profit, as a function of q_1, taking into account its best-response to firm 1's choice of q_1, is

$$\pi_2 = \left(150 - \frac{1}{2}q_1\right) \cdot \left(300 - q_1 - 150 + \frac{1}{2}q_1\right) = \left(150 - \frac{1}{2}q_1\right)^2.$$

Setting $\pi_2 = 900$ and solving for q_1, we find $q_1^* = 240$. So, if firm 1 set its output at 240, it would deter firm 2's entry ($q_2^* = 0$) and so $p^* = 60$. Then firm 1's profit would be

$$\pi_1^* = (240)(60) = 14,400.$$

This is higher than its profit in the Stackelberg equilibrium, so firm 1 would deter firm 2's entry.

2. (a) Firm 1's profits are given by

$$\pi_1 = p_1(120 - 2p_1 + p_2) - 1000$$

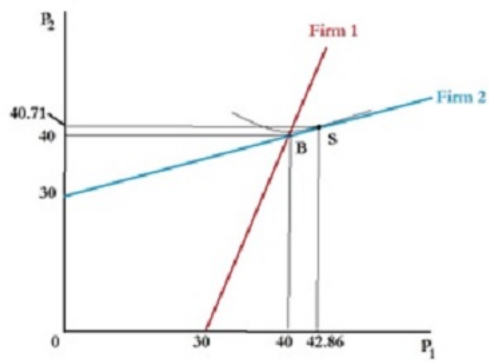
Profits are maximized when

$$\frac{\partial \pi_1}{\partial p_1} = 120 - 4p_1 + p_2 = 0 \;\rightarrow\; p_1 = 30 + \frac{1}{4}p_2.$$

This is firm 1's best response function. By the symmetry of the problem, firm 2's best response function will be

$$p_2 = 30 + \frac{1}{4}p_1.$$

In the figure, the red line is firm 1's best response function and the blue line is Firm 2's best response function. The Bertrand equilibrium is at B, where the two best response functions intersect.



Solving the two best response functions simultaneously, we find $p_1 = p_2 = 40$. Then, substituting in the profit functions, we find$\pi_1 = \pi_2 = 2200$.

(b) If firm 1 moves first, it will incorporate firm 2's best response function into its calculation. Thus its profits will be

$$\pi_1 = p_1(120 - 2p_1 + 30 + \frac{1}{4}p_1) - 1000.$$

Differentiating and solving, we get

$$p_1 = \frac{300}{7} = 42.86.$$

Substituting this into Firm 2's best response function, we find

$$p_2 = \frac{285}{7} = 40.71.$$

The new equilibrium is at S in the figure, where a firm 1 iso-profit line is tangent to firm 2's best response function.

Substituting in the expressions for profits, we find

$$\pi_1 = 2214.29, \quad \pi_2 = 2315.31.$$

We see that both firms' profits have gone up, illustrating the notion that, in Bertrand competition, the firms' prices are strategic complements. Further, note that firm 2's profits are higher, illustrating the fact that, in Bertrand competition, it is advantageous to be the follower rather than the leader.