EC 501: Problem Set 10, Solutions

1. (a) The Nash equilibria in pure strategies are (T, L) and (B, R). For (T, L), the row player has no incentive to deviate since 10>5 and the column player has no incentive to deviate since 5>3. For (B, R), the row player has no incentive to deviate since 20>5 and the column player has no incentive to deviate since 2>0.

   (b) Suppose the row player plays T with probability p and B with probability *(1-p)* and the column player plays L with probability q and R with probability *(1-q)*. Then the row player's expected payoff is

$$E(\pi_r) = p\left\{10q + 5(1-q)\right\} + (1-p)\left\{5q + 20(1-q)\right\}.$$

   Setting the derivative of this with respect to p equal to zero gives us

$$\frac{\partial E(\pi_r)}{\partial p} = 10q + 5(1-q) - 5q - 20(1-q) = 20q - 15.$$

   Then

$$p^* = \begin{cases} 0 & if \quad q < \frac{3}{4} \\ 1 & if \quad q > \frac{3}{4} \end{cases}$$

   and p^* can take any value in the interval [0,1] if $q = \frac{3}{4}$.
   Similarly, the column player's expected payoff is

$$E(\pi_c) = q\left\{5p\right\} + (1-q)\left\{3p + 2(1-p)\right\}.$$

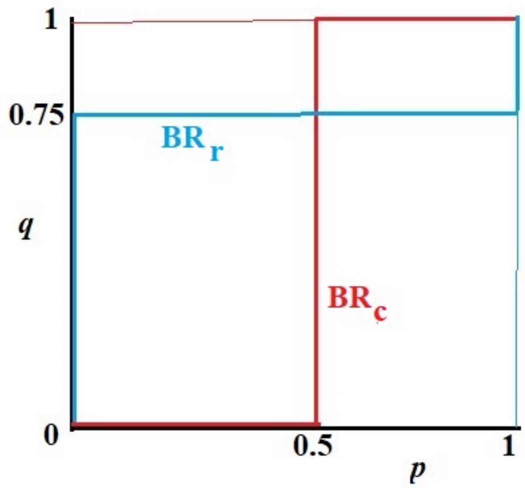
   Setting the derivative of this with respect to q equal to zero gives us

$$\frac{\partial E(\pi_c)}{\partial q} = 5p - 3p - 2(1-p) = 4p - 2.$$

   Then

$$q^* = \begin{cases} 0 & if \quad p < \frac{1}{2} \\ 1 & if \quad p > \frac{1}{2} \end{cases}$$

   and q^* can take any value in the interval [0,1] if $p = \frac{1}{2}$.
   The best response functions are shown in the graph below.



Thus there is a Nash equilibrium in mixed strategies at the point where the best response functions intersect, with the row player playing (T, B) with probabilities $\left(\frac{1}{2}, \frac{1}{2}\right)$ and the column player playing (L, R) with probabilities $\left(\frac{3}{4}, \frac{1}{4}\right)$.

(c) The normal form of the sequential game is as follows:

			Column	Player	
		LL	LR	RL	RR
Row	T	10,5	10,5	5,3	5,3
Player	B	5,0	20,2	5,0	20,2

(d) There are three Nash equilibria in pure strategies for this game: (T, LL), (B, LR) and (B, RR):
(T, LL) because 10>5 and 5>3
(B, LR) because 20>10 and 2>0
(B, RR) because 20>5 and 2>0.

Of these, (B, LR) is the subgame perfect equilibrium, because L is the best response to T (5>3) and R is the best response to B (2>0). (T, LL) involves the non-credible threat of L in response to B, while (B, RR) involves the non-credible threat of R in response to T; thus these two strategy pairs are not subgame perfect equilibria.

2. (i) Acme will set MR=MC. Now we know MC=5. To find MR, write the demand curve as:

$$p = 15 - \frac{1}{300}Q.$$

Then total revenue is

$$R(Q) = 15Q - \frac{1}{300}Q^2$$

so MR is:

$$MR = 15 - \frac{1}{150}Q.$$

Then profits are maximized where MR=MC, that is, where

$$15 - \frac{1}{150}Q = 5 \qquad \rightarrow \qquad Q = 1500.$$

To find the price, we use the demand curve:

$$p = 15 - \frac{1}{300}(1500) \qquad \rightarrow \qquad p = 10.$$

Then, since the variable cost of each unit of production is 5, annual profits will be:

$$\pi = (10 - 5) * 1500 - 3000 = 4500.$$

(ii) If the market becomes a Cournot duopoly, equilibrium will occur at the intersection of the two reaction functions. To find Acme's reaction function, we need to solve its profit-maximization problem. We index Acme by a and Better by b. Acme's profits are:

$$\pi_a = \left\{15 - \frac{1}{300}\left(Q_a + Q_b\right)\right\} Q_a - 5Q_a - 3000$$

Profit is maximized when

$$\frac{\partial \pi_a}{\partial Q_a} = 15 - \frac{1}{300}Q_b - \frac{1}{150}Q_a - 5 = 0 \qquad \rightarrow \qquad Q_a = 1500 - \frac{1}{2}Q_b.$$

This is Acme's best response function. By a similar process, we can find Better's best response function to be

$$Q_b = 1500 - \frac{1}{2}Q_a.$$

Solving the two reaction functions together, we find

$$Q_a = Q_b = 1000.$$

To find the price, we use the demand curve:

$$p = 15 - \frac{1}{300}(2000) \qquad \rightarrow \qquad p = \frac{25}{3}.$$

Profits per firm will be:

$$\pi_a = \pi_b = (\frac{25}{3} - 5) * 1000 - 3000 = \frac{1000}{3}.$$

Since profits are positive, Better would find it profitable to enter.

(iii) The figure shows the key points of the two solutions. Welfare is taken as the sum of consumers' surplus and firm profits. When the market is a monopoly:

$$Consumers'\,surplus = area\,CKE \quad and \quad Profit = area\,KELN - 3000(fixed\,cost).$$

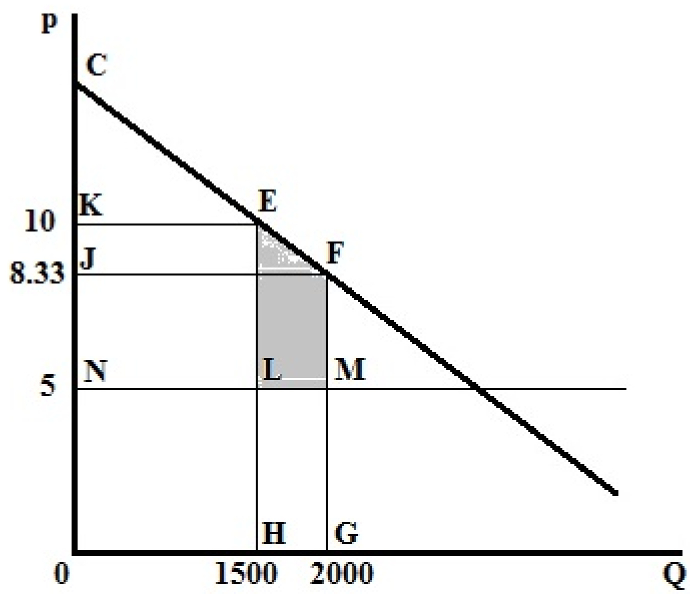
When the market is a duopoly:

$$Consumers'\,surplus = area\,CJF \quad and \quad Profit = area\,JFMN - 6000(fixed\,cost).$$

So the net gain from entry is area EFML - 3000. Now

$$area\,EFML = \frac{1}{2} \cdot \frac{5}{3} \cdot 500 + \frac{10}{3} \cdot 500 = 2083.33.$$

So the net welfare effect is a loss of 3000-2083.33=916.67.



Of this, consumers gain:

$$area\,KEFJ = \frac{5}{3} \cdot 1500 + \frac{1}{2} \cdot \frac{5}{3} \cdot 500 = 2916.67.$$

Better Drug gains 333.33 (its profit, calculated in part ii), while Acme loses 4500 - 333.33 = 4166.67 (its profit in part i minus its profit in part ii). The net then is

$$\Delta W = 2916.67 + 333.33 - 4166.67 = 916.67.$$

So this adds up to the net welfare effect we calculated earlier.

(iv) If Acme played as a Stackelberg leader, it would take Better's reaction function and optimize over that. The expression for π_a we used at the start of part (ii) will then be modified by substituting Better's reaction function for Q_b. Thus Acme's profit can be written as:

$$\pi_a = \left\{15 - \frac{1}{300}\left(Q_a + \left\{1500 - \frac{1}{2}Q_a\right\}\right)\right\} Q_a - 5Q_a - 3000,$$

which can be simplified to

$$\pi_a = 5Q_a - \frac{1}{600}Q_a^2 - 3000.$$

Acme will then maximize profits where

$$\frac{d\pi_a}{dQ_a} = 5 - \frac{1}{300}Q_a = 0 \qquad or \qquad Q_a = 1500,$$

which is actually the same output level it had chosen in the monopoly solution.

Using Better's reaction function, we can see how much it will produce:

$$Q_b = 1500 - \frac{1}{2}Q_a = 750.$$

Total output is therefore 2250 and so

$$p = 15 - \frac{1}{300}(2250) \qquad \rightarrow \qquad p = \frac{15}{2}.$$

In that case, Better's profits are

$$\pi_b = (\frac{15}{2} - 5) * 750 - 3000 = -1175.$$

Thus Better would be making losses and so would not want to enter. The solution would therefore revert to the one we found in part (i).

3. (i) The ith firm's profit is

$$\pi_i = 38q_i - q_i^2 - 0.2\tilde{q}_i q_i - 72 + 10q_i - q_i^2.$$

Profit is maximized where:

$$\frac{\partial \pi_i}{\partial q_i} = 38 - 2q_i - 0.2\tilde{q}_i + 10 - 2q_i = 0 \qquad \rightarrow \qquad q_i = 12 - 0.05\tilde{q}_i.$$

This is firm i's reaction function.

(ii) If there are 11 identical firms, each with the same reaction function, the equilibrium will be symmetric. Thus, in the reaction function, $\tilde{q}_i = 10q_i$. Substituting this in the reaction function, we get

$$q_i = 12 - 0.05(10q_i) \qquad \rightarrow \qquad q_i = 8.$$

Thus each firm will produce 8 units of output, total output will be 88 units and the price will be

$$p = 38 - 8 - 0.2(80) = 14.$$

(iii) Suppose there are (n+1) firms in the final equilibrium. Using the ith firm's reaction function, we can write

$$q_i = 12 - 0.05(nq_i) \qquad or \qquad nq_i = \frac{12 - q_i}{0.05}.$$

Then the price will be

$$p = 38 - q_i - 0.2\left(\frac{12 - q_i}{0.05}\right) = 3q_i - 10.$$

In this case, the ith firm's profits will be

$$\pi_i = 3q_i^2 - 10q_i - 72 + 10q_i - q_i^2 = 2q_i^2 - 72.$$

In free entry equilibrium, each firm's profits will be zero (or just above zero). Therefore, in equilibrium,

$$2q_i^2 = 72 \qquad \rightarrow \qquad q_i = 6.$$

Then the price will be

$$p = 3q_i - 10 \qquad \rightarrow \qquad p = 8$$

and the number of firms can be found by noting that

$$n = \frac{12 - q_i}{0.05q_i} = 20.$$

Since we had assumed the number of firms to be (n+1), that figure is 21 firms.

4. (a) Able's profit, as a function of its level of production, is

$$\pi = (14 - W) \cdot W - 2W.$$

Differentiating with respect to W and setting equal to zero yields the profit-maximizing output level: W=6. Then p=8 and π=36.

(b) Let's assume Baker finds it profitable to enter and see what the Cournot equilibrium would be. Let W_a and W_b represent the output levels of Able and Baker respectively. Baker's profit would be

$$\pi_b = (14 - W_a - W_b) \cdot W_b - 2W_b.$$

Differentiating with respect to W_b, setting equal to zero, and simplifying, yields Baker's best-response function:

$$W_b^* = \frac{12 - W_a}{2}.$$

Able's best-response function will be symmetrical to Baker's. Solving the two functions simultaneously yields the Cournot equilibrium:

$$W_a = W_b = 4.$$

Then p=6 and profits are $\pi_a = 16, \pi_b = 12$, (remember that Baker has the fixed entry cost of 4).

(c) If Able can pre-commit to an output level, it needs to decide whether to accommodate or deter Baker's entry. If it accommodates entry, it will play as a Stackelberg leader, taking Baker's best-response function into account when maximizing its profit:

$$\pi_a = \left\{14 - W_a - \frac{12 - W_a}{2}\right\} \cdot W_a - 2W_a.$$

Optimizing over W_a and then completing all the calculations, we get:

$$W_a = 6, W_b = 3, p = 5, \pi_a = 18, \pi_b = 5.$$

Since $\pi_b > 0$, Baker would find it profitable to enter and this equilibrium would emerge.

In order to deter entry, Able would have to pre-commit to an output level high enough so as to reduce Baker's operating profit to 4, thereby making its entry unprofitable. Suppose this output level is W_a^*. Then, using Baker's best-response function, we can infer that Baker would produce

$$W_b = \frac{12 - W_a^*}{2}.$$

Then Baker's profit will be

$$\pi_b = \left\{14 - W_a^* - \frac{12 - W_a^*}{2}\right\} \cdot \left\{\frac{12 - W_a^*}{2}\right\} - 2\left\{\frac{12 - W_a^*}{2}\right\}.$$

Setting this equal to 4 and solving, we find

$$W_a^* = 8.$$

If Able pre-committed to this output level, Baker would not enter and therefore

$$p = 6 \quad and \quad \pi_a = 32.$$

Able is therefore better off deterring entry and this will be the equilibrium.