EC 501: Problem Set 9, Solutions

1. (a) Under perfect competition in the long run, each firm will produce at the minimum point of the AC curve. Now

$$AC = \frac{200}{q} + 0.5q$$

and so AC is at a minimum where

$$\frac{dAC}{dq} = -\frac{200}{q^2} + 0.5 = 0 \qquad \rightarrow \qquad q = 20.$$

When $q = 20$, $AC = 20$, so each firm will produce 20 units at an average cost of 20.

Then the industry supply curve will be perfectly elastic at a price of 20. At $P = 20$, the quantity demanded will be $Q = 200$. Since each firm produces 20 units of output, there will be 10 bakeries.

(b) Since the monopoly can have any number of bakeries, its MC and AC curves are essentially flat at a cost of 20, since it would operate each bakery at the minimum point of its AC curve. Now the demand curve can be written as

$$P = 60 - \frac{1}{5}Q$$

and so the monopoly's profits are

$$\pi = 60Q - \frac{1}{5}Q^2 - 20Q$$

and profits are maximized where

$$\frac{d\pi}{dQ} = 60 - \frac{2}{5}Q - 20 = 0 \qquad \rightarrow \qquad Q = 100.$$

Then $P = 40$ and, since each bakery produces 20 units, there will be 5 bakeries. The firm's profit will be

$$\pi = 40 \cdot 100 - 20 \cdot 100 = 2000.$$

(c) Social welfare would be maximized with the competitive solution, where $Q = 200, P = 20$.

(d) If the monopoly can separate the markets and charge different prices in them, its profit will be

$$\pi = \frac{100}{3}Q_1 - \frac{1}{3}Q_1^2 + 100Q_2 - \frac{1}{2}Q_2^2 - 20\left(Q_1 + Q_2\right).$$

The first-order conditions for a profit maximum yield

$$\frac{\partial \pi}{\partial Q_1} = \frac{100}{3} - \frac{2}{3}Q_1 - 20 = 0 \qquad \rightarrow \qquad Q_1 = 20$$

$$\frac{\partial \pi}{\partial Q_2} = 100 - Q_2 - 20 = 0 \qquad \rightarrow \qquad Q_2 = 80.$$

Thus the monopolist's total output hasn't changed. To find its profit now, first find the prices from the respective demand curves:

$$P_1 = \frac{80}{3} \qquad and \qquad P_2 = 60.$$

Then the firm's profit is

$$\pi = \frac{80}{3} \cdot 20 + 60 \cdot 80 - 20 \cdot 100 = 3333.33,$$

which is considerably higher than before, showing the advantage of price discrimination.

2. (a) Since MNE has two plants, first we need to determine its joint MC curve. Note that

$$MC_h = 10 \qquad and \qquad MC_f = 5 + S_f.$$

So the most efficient thing for MNE to do is to produce the first 5 units of output in the foreign plant, and, for any additional production, to produce in the home plant at a cost of $MC = 10$.

Let's find MR in each of the markets and then we can set MR=10 in each market. In the home market:

$$P_h = 30 - Q_h \qquad \rightarrow \qquad MR_h = 30 - 2Q_h$$

so profit would be maximized where

$$30 - 2Q_h = 10 \qquad \rightarrow \qquad Q_h = 10 \quad and\, so\, P_h = 20.$$

In the foreign market:

$$P_f = 20 - \frac{1}{2}Q_f \qquad \rightarrow \qquad MR_f = 20 - Q_f$$

so profit would be maximized where

$$20 - Q_f = 10 \qquad \rightarrow \qquad Q_f = 10 \quad and\ so\ P_f = 15.$$

Total sales are $Q_h + Q_f = 20$, so production would have to be $S_f = 5, S_h = 15$. Thus 5 units of output will be produced at home and sold abroad; there will be trade.

Profit is

$$\pi = 200 + 150 - 150 - 37.5 = 162.5.$$

(b) If trade is banned, MNE will set $Q_h = S_h$ and $Q_f = S_f$ and will maximize profit in each market separately by setting MR=MC in each market.

Then, in the home market:

$$30 - 2Q_h = 10 \qquad \rightarrow \qquad Q_h = 10 \quad and\, so\, P_h = 20$$

and, in the foreign market:

$$20 - Q_f = 5 + Q_f \qquad \rightarrow \qquad Q_f = 7.5 \quad and\, so\, P_f = 16.25.$$

Profit is now

$$\pi = 200 + 121.875 - 100 - 65.625 = 156.25.$$

(c) If MNE can't price discriminate, it effectively faces one market, so we need to combine the two demand curves:

$$Q_h = 30 - P_h$$

$$Q_f = 40 - 2P_f.$$

So, remembering that in this scenario $P_h = P_f$, the combined demand curve is

$$Q = 70 - 3P \qquad or \qquad P = \frac{70}{3} - \frac{1}{3}Q.$$

The MR curve then is

$$MR = \frac{70}{3} - \frac{2}{3}Q$$

and profit is maximized where MR=MC, that is, where

$$\frac{70}{3} - \frac{2}{3}Q = 10 \qquad \rightarrow \qquad Q = 20 \quad and\, so\, P = \frac{50}{3}.$$

Production is divided as before $S_f = 5, S_h = 15$.
From the demand curves we find that sales are $Q_f = \frac{20}{3}, Q_h = \frac{40}{3}$. Thus 1.67 units are traded (exported from home to the foreign country).

Profit is now

$$\pi = 20 \cdot \frac{50}{3} - 150 - 37.5 = 145.83,$$

so profit is lower than in (a) by 16.67.

3. (a) If Big Boy is a price leader, we must find the supply curve of the fringe firms, which is the sum of their MC curves. For each firm, differentiating the cost function, we get:

$$MC(q) = 2q + 5$$

so each firm's supply curve is

$$q = \frac{1}{2}p - \frac{5}{2}.$$

Then the combined supply curve of the fringe firms is 100 times this:

$$Q_c = 50p - 250.$$

Now the demand curve for numos is

$$Q_d = 1000 - 50p,$$

so the residual demand curve faced by Big Boy is

$$Q_b = Q_d - Q_c \quad \rightarrow \quad Q_b = 1250 - 100p \quad \rightarrow \quad p = 12.5 - \frac{1}{100}Q_b.$$

To maximize profit, Big Boy will set MR=MC:

$$12.5 - \frac{1}{50}Q_b = \frac{1}{50}Q_b \quad \rightarrow \quad Q_b = 312.5.$$

From the residual demand curve of Big Boy, we can find the price:

$$p = 12.5 - 3.125 = 9.375.$$

From the supply curve of the fringe firms, we can find the output level of the fringe:

$$Q_c = 50(9.375) - 250 = 218.75$$

and so the output of each fringe firm is $q = 2.1875$.

(b) In the long run, since there is free entry into the fringe, each fringe firm will produce at the minimum point of its AC curve and the fringe supply curve will be perfectly elastic at that minimum AC. Now the AC of each fringe firm is

$$AC(q) = q + 5 + \frac{10}{q}$$

and this will be minimized when

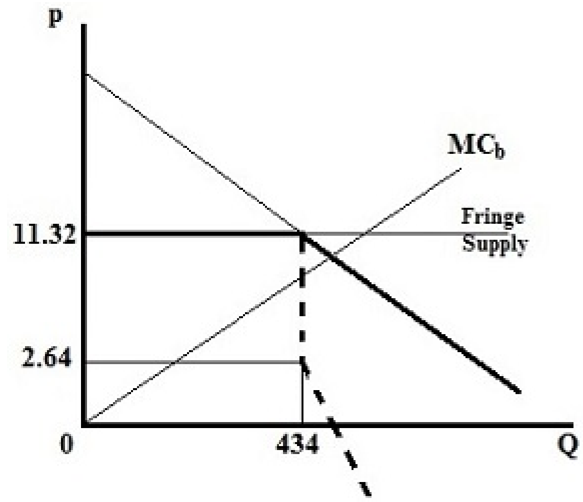
$$\frac{dAC}{dq} = 1 - \frac{10}{q^2} = 0 \quad \rightarrow \quad q = \sqrt{10} = 3.16.$$

At this output level, $AC(q) = 11.32$, and so the fringe supply will be perfectly elastic at a price of 11.32.

Big Boy's residual demand curve is therefore flat at a price of 11.32, and for prices lower than that, Big Boy will have the entire market to itself. From the demand curve, we can see that demand at a price of 11.32 is

$$Q_d = 1000 - 50(11.32) = 434.$$

The solid kinked line in the figure then represents Big Boy's residual demand curve. The MR curve coincides with the demand curve for $Q<434$ and is the dotted line after that.



At $Q=434$, the MR jumps down. To find the point where it jumps to, we need to find the MR at $Q=434$. At this point, Big Boy will have the whole market to itself, so its MR is simply the MR from the original demand curve. That demand curve can be written as

$$p = 20 - \frac{1}{50}Q,$$

so we know that

$$MR = 20 - \frac{1}{25}Q.$$

Substituting $Q=434$ in this expression gives us $MR=2.64$. Thus this is the point of the second kink in the MR curve.

The question is, where does Big Boy's MC curve cut its MR curve? To see this, we can calculate its MC at $Q_b = 434$. We see then that

$$MC_b = \frac{434}{50} = 8.68.$$

Thus the MC curve cuts MR in the vertical section of the MR curve. Big boy will therefore produce 434 units of output and set the limit price of 11.32. The fringe firms will be driven from the market.

4. (a) The monopolist would set MR=MC. Now total revenue is

$$R(Q) = p(Q) \cdot Q = \left(30 - \frac{Q}{2}\right) \cdot Q$$

so marginal revenue is

$$MR(Q) = 30 - Q.$$

Profit is maximized where MR=MC:

$$30 - Q = 20 \qquad \rightarrow \qquad Q = 10.$$

From the demand curve, we can find P=25, and so the firm's profits are

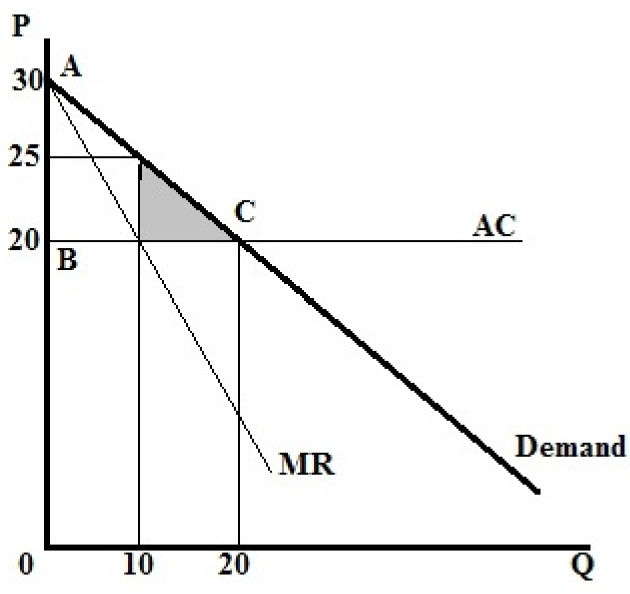
$$\pi = 250 - 200 = 50.$$

(b) Efficiency would require p=MC, so

$$p = 20, Q = 20.$$

(c) The situation is illustrated in the figure. The monopoly produces 10 units of output and charges a price of 25, while the efficient solution is to produce 20 and charge 20. The shaded area represents the deadweight loss due to monopoly. It is equal to

$$DWL = \frac{1}{2}(5)(10) = 25.$$



(d) From part (a), we know the monopoly's profits are 50 per year. If there are additional overhead costs of 75, the monopoly would shut down.

If the government took over the industry and set price and quantity to maximize welfare, it would generate consumer surplus equal to the area ABC in the figure. This is equal to

$$Surplus = \frac{1}{2}(10)(20) = 100.$$

Since this is greater than the overhead cost of 75, government should take over the industry and operate it in the social interest.

5. (a) If Bigfoot wants to maximize profits, it will set the marginal expense (ME) from each type of labor equal to the marginal revenue product (MRP) of labor. Since men and women are equally productive, the MRP for both is the same, and is equal to

$$MRP = 2 * 20 = \$40\, per\ day.$$

To find the ME for men, rewrite the supply curve as

$$w_m = 10 + \frac{1}{25}L_m.$$

Then total expense on men is

$$E_m = 10L_m + \frac{1}{25}L_m^2$$

and therefore

$$ME_m = 10 + \frac{2}{25}L_m.$$

Setting $ME_m = MRP$, we get

$$10 + \frac{2}{25}L_m = 40 \quad \rightarrow \quad L_m = 375 \quad \rightarrow \quad w_m = 25.$$

For women:

$$w_w = 20 + \frac{1}{50}L_w$$

$$E_w = 20L_w + \frac{1}{50}L_w^2$$

$$ME_w = 20 + \frac{2}{50}L_w$$

$$20 + \frac{2}{50}L_w = 40 \quad \rightarrow \quad L_w = 500 \quad \rightarrow \quad w_w = 30.$$

Since each worker (whether man or woman) generates revenue of \$40 per day, we can calculate Bigfoot's profit as follows:

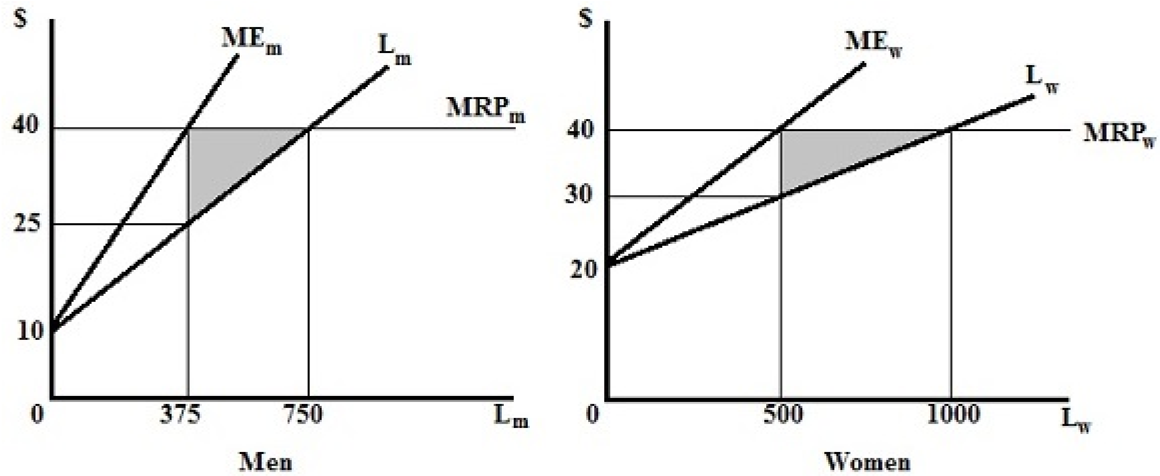
$$\pi = (40 - w_m) * L_m + (40 - w_w) * L_w = 15 * 375 + 10 * 500 = \$10,625\, per\ day.$$

(b) For social efficiency, labor should be hired to the point where $w = MRP$, so the wage should be \$40 for both men and women. Then

$$w_m = w_w = 40 \quad and \quad L_m = 750,\, L_w = 1000.$$

Bigfoot's profit is now 0, since it would be paying workers exactly what they produce.

However, workers are better off. The net gain to society (equal to the deadweight loss due to monopsony in the previous solution) is shown as the shaded areas in the two graphs below. In each case, it is the amount by which the MRP of labor exceeds the opportunity cost of the workers' time.



Using the data from the graph, we can see this welfare gain is

$$\Delta W = \frac{1}{2} \cdot 15 \cdot 375 + \frac{1}{2} \cdot 10 \cdot 500 = 5312.5$$