EC 501: Problem Set 8, Solutions

1. Without insurance, Liz's expected utility is

$$(EU)_0 = \frac{1}{100}U(0) + \frac{99}{100}U(100,000).$$

With insurance at a premium of π her expected utility is

$$(EU)_1 = U(100,000 - \pi).$$

The largest premium she would be willing to pay would be that value of π where

$$(EU)_0 = (EU)_1.$$

(i) If $U = Y^{\frac{1}{4}}$, this condition becomes

$$(100,000 - \pi_1)^{\frac{1}{4}} = \frac{1}{100} \cdot 0 + \frac{99}{100} \cdot (100,000)^{\frac{1}{4}} = 17.6.$$

Solving for the premium, we get

$$\pi_1 = 3,940.4.$$

(ii) If $U = Y^4$, this condition becomes

$$(100,000 - \pi_2)^4 = \frac{1}{100} \cdot 0 + \frac{99}{100} \cdot (100,000)^4.$$

Solving for the premium, we get

$$\pi_2 = 250.94.$$

(iii) If $U = Y$, this condition becomes

$$(100,000 - \pi_3) = \frac{1}{100} \cdot 0 + \frac{99}{100} \cdot (100,000).$$

Solving for the premium, we get

$$\pi_3 = 1000.$$

2. (a) There are two options open to Wildcat: drill or no drill. Wildcat will choose the option with the higher expected utility. Under the no drill option:

$$U_{no\,drill} = y = 100.$$

Under the drill option:

$$E(U)_{drill} = 0.6 \cdot 200 + 0.4 \cdot 0 = 120.$$

Since 120>100, Wildcat would drill.

If the probability of success were p, Wildcat's expected utility would be

$$E(U)_{drill} = p \cdot 200 + (1-p) \cdot 0 = 200p$$

and Wildcat would be indifferent between drilling and not if this expected utility were equal to its expected utility from the no drill option, that is, when

$$200p = 100 \qquad or \qquad p = 0.5.$$

(b) If $U = 2y^{\frac{1}{2}}$,

$$\frac{dU}{dy} = y^{-\frac{1}{2}} \qquad and \qquad \frac{d^2U}{dy^2} = -\frac{1}{2}y^{-\frac{3}{2}} < 0.$$

Thus the utility function is concave and reflects risk-aversion. With this utility function,

$$U_{no\,drill} = 2(100)^{\frac{1}{2}} = 20,$$

$$E(U)_{drill} = 0.6 \cdot 2(200)^{\frac{1}{2}} + 0.4 \cdot 0 = 12\sqrt{2} = 16.97 < 20.$$

Now Wildcat will not drill.

Wildcat will be indifferent between drilling and not drilling if

$$p \cdot 2(200)^{\frac{1}{2}} + (1-p) \cdot 0 = 20 \qquad or \qquad p = 0.71.$$

(c) If Wildcat does not run the test, we know from (a) that its optimal action is to drill and thereby achieve a utility level of 120. If it runs the test and the field is dry, it will not drill, while, if the field is wet, it will drill. If c is the cost of the test, its income in the two scenarios is:

$$y_{dry} = 100 - c$$

$$y_{wet} = 100 - c - 100 + 200 = 200 - c.$$

Then

$$E(U)_{test} = 0.6(200-c) + 0.4(100-c) = 160 - c.$$

When c=*20*, $E(U)_{test} = 140$. This is higher than its expected utility under the no test (and therefore drill) option, which was 120; therefore Wildcat will run the test.

The maximum Wildcat would be willing to pay for the test would be the value of c when its expected utility from the test is 120, that is, where

$$160 - c = 120 \qquad or \qquad c = 40.$$

(d) When $U = 2y^{\frac{1}{2}}$, we know that Wildcat's optimal action (in the absence of a test) is to not drill and thereby achieve a utility level of 20. Now the EU under the test is

$$E(U)_{test} = 0.6 \cdot 2(200 - c)^{\frac{1}{2}} + 0.4 \cdot 2(100 - c)^{\frac{1}{2}}.$$

When *c=20*, $E(U)_{test} = 23.26 > 20$; therefore, Wildcat would run the test.

The maximum amount Wildcat would pay for the test would be the value of c where

$$0.6 \cdot 2(200 - c)^{\frac{1}{2}} + 0.4 \cdot 2(100 - c)^{\frac{1}{2}} = 20.$$

This equation is hard to solve analytically, but it can be solved by trial and error (say by using a spreadsheet) to find $c \approx 53$.

The maximum amount Wildcat is willing to pay for the test is greater when it is risk-averse than when it is risk-neutral (53>40). When it is risk-neutral, Wildcat is willing to pay only the expected gain from the test. But, when it is risk-averse, Wildcat is willing to pay that plus an extra amount equal to the cost of risk associated with the uncertainty it faces.

3. (i) The cost of drilling is 10 (millions will be suppressed), while the potential gain (with probability 0.2) is 100. Therefore

$$E(net\ gain) = -10 + 0.2(100) = \$10\, million.$$

(ii) If JR doesn't drill,

$$W = 20 \qquad and \qquad U(W) = ln20 = 2.9957.$$

If JR does drill,

$$W = \begin{cases} 10\ with\ probability\ 0.8\ (no\ oil) \\ 110\ with\ probability\ 0.2\ (oil\ found) \end{cases}$$

and so

$$EU(W) = 0.8ln10 + 0.2ln110 = 2.7821.$$

Thus JR is better off not drilling, since his EU is higher.

(iii) If JR doesn't let Bobby in the deal, he will not drill and his utility level will be 2.9957.

If JR does let Bobby in the deal, his cost of drilling will be only \$5 million and the possible payoff will be \$50 million. Then

$$W = \begin{cases} 15\ with\ probability\ 0.8\ (no\ oil) \\ 65\ with\ probability\ 0.2\ (oil\ found) \end{cases}$$

and so

$$EU(W) = 0.8ln15 + 0.2ln65 = 3.0013.$$

Thus JR now finds it advantageous to drill.

This is an example of risk-spreading. By dividing the large risky project into two parts, the cost of risk is reduced to the point where JR finds it advantageous to adopt the risky project.