

EC 501: Problem Set 7
(Due in class on Tuesday, October 29)

1. In the economy of Ricardia, two consumer goods, X and Y, are produced from a single factor input, labor, according to the production functions:

$$Y = 3L_y \quad \text{and} \quad X = 3L_x$$

where L_y and L_x are the quantities of labor used in the production of Y and X respectively. The total amount of labor available is 66 units.

(a) Derive the equation for the economy's production possibility frontier. Confirm that the marginal rate of transformation is equal to MPL_y/MPL_x .

(b) All consumers in Ricardia have identical tastes which can be represented by a utility function

$$U(X, Y) = 5X^{2/3}Y^{1/3}$$

What are the Pareto optimal levels of output of X and Y and corresponding labor inputs L_x and L_y ?

2. Robinson Crusoe lives alone on a desert island. He works 9 hours per day gathering coconuts and hunting fish. His production functions for the two goods are:

$$C = 2 L_c \quad \text{and} \quad F = L_f^{1/2}$$

where C and F are the quantities of coconuts and fish produced (and consumed) per day and L_c and L_f are the number of hours Crusoe spends producing coconuts and fish respectively. Crusoe's utility function from consumption is

$$U(C, F) = CF.$$

- (a) Find Crusoe's production possibilities frontier. Find a formula for the marginal rate of transformation.
- (b) How many hours will Crusoe spend each day on each of the two production activities in order to maximize his utility? (*Allow fractional quantities of goods to be produced.*)
- (c) Show that, at the optimum you found in (b), the marginal rate of transformation is equal to the marginal rate of substitution. What price ratio would support this optimum as a competitive equilibrium?

... continued on next page

3. Robinson Crusoe can catch 1 fish per hour or can gather 10 coconuts in the same time. His utility function is

$$U = F^{1/2}C^{1/2}$$

where F and C represent the quantities of fish and coconuts he consumes each day.

(a) Assuming that Robinson has decided to work 8 hours per day, how many fish and coconuts will he produce and consume each day? What implicit price ratio would support this equilibrium?

(b) Suppose Robinson discovers that he can trade fish and coconuts freely on a neighboring island at the rate of 8 coconuts per fish. How many fish and coconuts will he produce and how many will he consume each day now?

(c) Suppose now that there is no neighboring island, but Robinson discovers Man Friday on his island. Friday can catch fish at the rate of 10 per day or can gather coconuts at the rate of 40 per day. His utility function is identical to Crusoe's. What is Friday's consumption pattern before he meets Crusoe?

(d) Now Friday and Crusoe are discussing the possibility of trading with one another. Is the price ratio of 8 coconuts per fish an equilibrium price ratio? Explain your answer fully.

(e) Can you find any other price ratio that would yield an equilibrium? If you can, find that price ratio (or ratios). If you cannot, explain why not.