EC 501: Problem Set 7, Solutions

1. (a) We can find the ppf by solving the problem:

$$Maximize \quad Y = 3L_y$$
$$subject\ to \quad X = 3L_x$$
$$and \quad L_x + L_y = 66.$$

In this case, because there is only one factor of production, the constraints combined become the ppf:

$$L_x + L_y = 66 \qquad \rightarrow \qquad \frac{X}{3} + \frac{Y}{3} = 66 \qquad \rightarrow \qquad X + Y = 198$$

which can be written

$$Y = 198 - X.$$

This is the ppf.

The marginal rate of transformation (MRT) is the absolute value of the slope of the ppf. Here, MRT=1. To confirm that this is equal to the ratio of marginal products, note that

$$MPL_y = 3 \qquad and \qquad MPL_x = 3, \qquad so\ \frac{MPL_y}{MPL_x} = 1.$$

(b) Since all consumers are identical, we can treat the economy as having a single consumer. Efficiency then requires the choice of a point on the ppf where the "social indifference curve" is tangent to the ppf, that is, where $MRS = MRT$. Now

$$MRS = \frac{MU_x}{MU_y} = \frac{5\left(\frac{2}{3}\right)X^{-\frac{1}{3}}Y^{\frac{1}{3}}}{5\left(\frac{1}{3}\right)X^{\frac{2}{3}}Y^{-\frac{2}{3}}} = \frac{2Y}{X}.$$

Setting this equal to MRT gives us

$$\frac{2Y}{X} = 1 \qquad \rightarrow \qquad X = 2Y$$

and, substituting this in the ppf gives us

$$3Y = 198 \qquad \rightarrow \qquad Y = 66.$$

This is the level of production of Y. From the ppf, we can get the production level for X:

$$X = 132.$$

Using the production functions, we can then find the levels of labor input:

$$L_x = \frac{132}{3} = 44 \qquad and \qquad L_y = \frac{66}{3} = 22.$$

2. (a) As in the previous question, we can get the ppf directly from the constraints since there is only one factor of production. We know

$$L_c + L_f = 9.$$

Using the production functions, this means

$$\frac{C}{2} + F^2 = 9 \qquad \rightarrow \qquad C = 18 - 2F^2.$$

This is the ppf.

The MRT is the absolute value of the slope of the ppf. Since

$$\frac{dC}{dF} = -4F \qquad therefore \qquad MRT = 4F.$$

(b) Crusoe wants to allocate his time in order to maximize his utility. That is, he wants to

$$Maximize \quad U = CF$$
$$subject\ to \quad C = 18 - 2F^2.$$

The Lagrangian for this problem is

$$\mathcal{L} = CF + \lambda \left[C - 18 + 2F^2\right].$$

Differentiating with respect to C, F:

$$\frac{\partial \mathcal{L}}{\partial C} = F + \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial F} = C + \lambda \cdot 4F = 0.$$

Combining these two equations gives us

$$\frac{F}{C} = \frac{1}{4F} \qquad \rightarrow \qquad 4F^2 = C.$$

Substituting for F^2 in the constraint then gives us

$$C = 18 - \frac{C}{2} \qquad \rightarrow \qquad C = 12,$$

which is the optimal level of production of coconuts. Substituting this back in the ppf gives us the optimal production level for fish:

$$2F^2 = 6 \qquad \rightarrow \qquad F = \sqrt{3}.$$

From the production functions, we can find the levels of labor input necessary to produce these output levels:

$$L_c = \frac{12}{2} = 6\, hours \qquad and \qquad L_f = \left(\sqrt{3}\right)^2 = 3\, hours.$$

(c) From (a) we know that

$$MRT = 4F = 4\sqrt{3}.$$

Now

$$MRS = \frac{MU_f}{MU_c} = \frac{C}{F} = \frac{12}{\sqrt{3}} = 4\sqrt{3}.$$

Thus *MRT*=*MRS*. The price ratio to support this as a competitive equilibrium would equal the *MRS*. Thus the price ratio we need is

$$\frac{p_f}{p_c} = 4\sqrt{3}.$$

3. (a) To find Crusoe's ppf, let's first write down his production functions:

$$F = L_f \qquad and \qquad C = 10L_c.$$

Since

$$L_f + L_c = 8,$$

we have

$$F + \frac{C}{10} = 8.$$

This is the ppf. Note that it looks much like a budget constraint, where Crusoe's "income" is 8 and $p_f = 1, p_c = \frac{1}{10}$. We can therefore treat his problem as one of a consumer maximizing his utility subject to a budget constraint. Since his utility function is Cobb-Douglas, we can write down his "demand functions":

$$F = \frac{I}{2p_f} \qquad and \qquad C = \frac{I}{2p_c}.$$

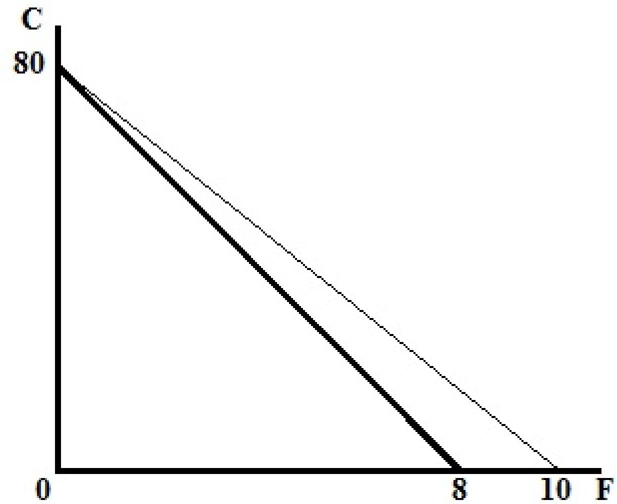
Substituting the values of "income" and "prices" we get his "demand" for each good, which will be the amount he will choose to produce and consume:

$$F = \frac{8}{2(1)} = 4 \qquad and \qquad C = \frac{8}{2\left(\frac{1}{10}\right)} = 40.$$

The implicit price ratio that would support this equilibrium would be

$$\frac{p_f}{p_c} = \frac{1}{\left(\frac{1}{10}\right)} = 10.$$

(b) If Crusoe can trade with a neighboring island at a price ratio $\frac{p_f}{p_c} = 8$, which is a lower relative price of fish, he would specialize in coconut production and then trade for fish. In the graph, the heavy line represents his ppf and the thin line represents his trading possibilities. Clearly, he expands his choice set by specializing in coconut production and then trading.



If we let the price of fish be the numeraire, the prices Crusoe faces are $p_f = 1, p_c = \frac{1}{8}$ and his income is now $I_1 = \frac{1}{8} \cdot 80 = 10$. With this "income" and "prices" he will produce $C = 80, F = 0$ and consume

$$F = \frac{10}{2(1)} = 5 \qquad and \qquad C = \frac{10}{2\left(\frac{1}{8}\right)} = 40.$$

(c) Since Friday's utility function is the same as Crusoe's, his demand functions will also be the same. We are given his productivity "per day." To make the calculations simple, let's suppose he works 10 hours per day. Then, implicitly, his "income" is 10 and his "prices" are $p_f = 1, p_c = \frac{1}{4}$. Substituting in the demand functions, we find the levels of fish and coconuts that he would produce and consume:

$$F = \frac{10}{2(1)} = 5 \qquad and \qquad C = \frac{10}{2\left(\frac{1}{4}\right)} = 20.$$

(d) The price ratio would be an equilibrium price ratio if the excess demands for each good are zero, that is, if the trade offers of Crusoe and Friday matched. We have already seen in part (b) that, at these prices, Crusoe would want to sell 40 coconuts and buy 5 fish. Let's see what Friday would like to do.

First, note that Friday would want to specialize in fish production. AT these prices, the value of his daily fish output (10 fish) would be 10 (since $p_f = 1$) while the value of his daily coconut output (40 coconuts) would be 5 (since $p_c = \frac{1}{8}$). Thus he would produce only fish and his "income" (i.e., the value of his production) would be 10. With income of 10 and

$p_f = 1, p_c = \frac{1}{8}$, his consumption would be

$$F = \frac{10}{2(1)} = 5 \qquad and \qquad C = \frac{10}{2\left(\frac{1}{8}\right)} = 40.$$

Thus Friday would want to sell 5 fish and buy 40 coconuts, which exactly mirrors what Crusoe wants to trade. Therefore $\frac{p_f}{p_c} = 8$ is indeed an equilibrium price ratio.

(e) Any equilibrium price must result in desired trades matching (or excess demands being zero). Suppose $p_f = 1, p_c = \bar{p}$ is an equilibrium set of prices. Since Crusoe has the comparative advantage in the production of coconuts and Friday has the comparative advantage in the production of fish, it must be the case that $\bar{p}$ is such that Crusoe specializes in coconut production and Friday in Fish production.

Then Crusoe would produce 80 coconuts and his income would be

$$I^C = 80\bar{p}$$

Therefore his desired consumption pattern would be

$$F^C = \frac{80\bar{p}}{2(1)} = 40\bar{p} \qquad and \qquad C^C = \frac{80\bar{p}}{2\left(\bar{p}\right)} = 40$$

and his excess demands would be

$$XD_f^C = 40\bar{p} \qquad and \qquad XD_c^C = 40 - 80 = -40.$$

Friday would produce 10 fish and his income would be

$$I^F = 10$$

Therefore his desired consumption pattern would be

$$F^F = \frac{10}{2(1)} = 5 \qquad and \qquad C^F = \frac{10}{2\left(\bar{p}\right)} = \frac{5}{\bar{p}}$$

and his excess demands would be

$$XD_f^F = 5 - 10 = -5 \qquad and \qquad XD_c^F = \frac{5}{\bar{p}}.$$

Combined excess demands are therefore

$$XD_f = 40\bar{p} - 5 \qquad and \qquad XD_c = \frac{5}{\bar{p}} - 40.$$

For $\bar{p}$ to be an equilibrium, these excess demands must be zero. Solving, we see that the only value for which this is true is $\bar{p} = \frac{1}{8}$. Thus the price ratio $\frac{p_f}{p_c} = 8$ is the only equilibrium price ratio.