EC 501: Problem Set 6, Solutions

1. Rearrange the demand curves as follows:

$$p = 50 - 2q_a \quad \rightarrow \quad 2q_a = 50 - p \quad \rightarrow \quad q_a = 25 - \frac{1}{2}p$$

$$p = 50 - q_b \quad \rightarrow \quad q_b = 50 - p.$$

Then the market demand curve is the sum of the two:

$$Q_{market} = q_a + q_b = 75 - \frac{3}{2}p$$

which can also be written as

$$p = 50 - \frac{2}{3}Q_m.$$

2. A perfectly inelastic demand curve is a vertical line at the relevant quantity level. Since there are 20 people with an inelastic demand for 50 units each (for a total of 20*50=1000 units) and 30 people with an inelastic demand for 40 units each (for a total of 40*30=1200 units), there is a total inelastic demand for 2200 units.

A perfectly elastic demand curve is a horizontal line at the relevant price level. For example, the 40 people with a perfectly elastic demand at price \$2 have a demand for zero at any price above \$2 and an infinite demand at any price at or below \$2. The 10 people with a perfectly elastic demand at price \$3 have a demand for zero at any price above \$3 and an infinite demand at any price at or below \$3. Collectively, these 50 people's demands aggregate to a perfectly elastic demand at price \$3.

Putting these groups together yields the demand curve in the graph.

3. (a) To solve this problem, we use a discrete version of the formula

$$\frac{dp_d}{dT} = \frac{\epsilon_s}{\epsilon_s - \epsilon_d}.$$

We use

$$\triangle p_d = \frac{\epsilon_s}{\epsilon_s - \epsilon_d} \cdot \triangle T.$$

Substituting the values in this case, we get

$$\triangle p_d = \frac{0.8}{0.8 + 1.7} \cdot 1.80 = 0.576.$$

Therefore the new demand price is

$$p_d^{'} = \$30.58$$

and the supply price is

$$p_s^{'} = \$28.78.$$

To find the new quantity, we could use a discrete version of the elasticity of demand:

$$\epsilon_d = \frac{\triangle Q_d}{\triangle p_d} \cdot \frac{p_d}{Q_d}.$$

Using the initial values of p_d, Q_d we get

$$\triangle Q_d = \frac{(-1.7)(0.576)(2,000,000)}{30} = -65,280$$

and therefore the new quantity traded is

$$Q^{'} = 1,934,720.$$

(b) The tax revenue collected is

$$Tax\ R = (1.80)Q^{'} = \$3,482,496.$$

Of this, buyers pay a fraction of $\frac{0.576}{1.80}$, or 32%, which amounts to

$$Buyers'\ burden = \left(\frac{0.576}{1.8}\right)(3,482,496) = \$1,114,399.$$

4. A simple first-order approximation for this problem would calculate the effect of a 75 cent tax (=15% of \$5), using the method used in the previous problem. Thus

$$\triangle p_d = \frac{2}{2 + 0.7} \cdot 0.75 = 0.556$$

and therefore the new demand price is

$$p_d^{'} = \$5.56,$$

the supply price is

$$p_s^{'} = \$4.81,$$

and the new quantity is

$$Q^{'} = 1000 + \frac{(-0.7)(0.556)(1000)}{5} = 922.22.$$

Alternatively, we could assume the demand and supply curves are of constant elasticity form:

$$Q_d = ap^{-0.7} \qquad and \qquad Q_s = cp^2,$$

where a and c are constants. In this case, we know that

$$p_d^{'} = p_0 \left(1 + t\right)^{\frac{\epsilon_s}{\epsilon_s - \epsilon_d}}.$$

Substituting the values in this example, we get

$$p_d^{'} = (5)\left(1.15\right)^{\frac{2}{2+0.7}} = \$5.55$$

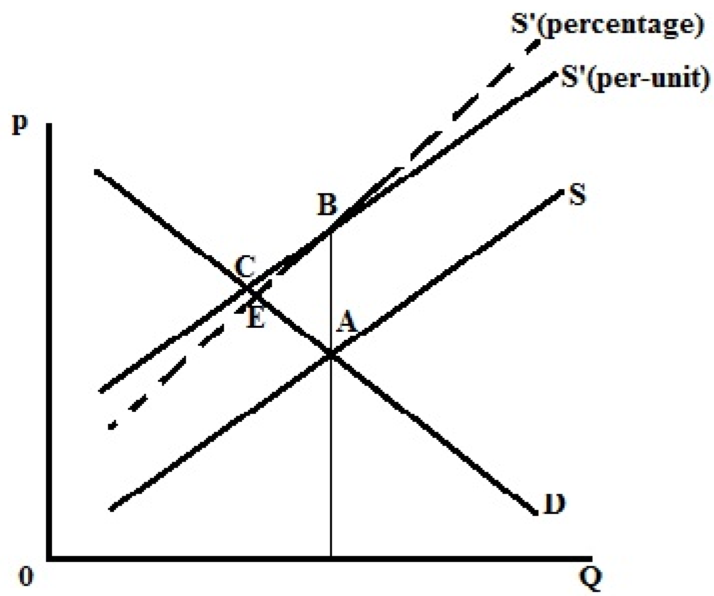
$$p_s^{'} = \frac{p_d^{'}}{1.15} = \$4.82$$

$$Q^{'} = \frac{1000}{(5)^{-0.7}} \cdot (p_d^{'})^{-0.7} = 929.5.$$

So these numbers are slightly different from those found with the first-order approximation.

5. The answer to the previous question suggests that the percentage tax will lead to a smaller distortion, and we can confirm that conclusion by drawing a graph of the situation. In the Figure, D and S represent the pre-tax demand and supply curves respectively, with the initial equilibrium being at A. We know that a per-unit excise tax will cause the S curve to move in a parallel way, as shown by S'(per-unit). A percentage tax will cause the supply curve to shift in a proportional way, as shown by S'(percentage). The key point is that the two S' curves will cross at point B, directly above point A, since it is only when p=$10 (as at A) that the vertical shift of S'(percentage) will equal exactly $1. Thus we see that the final equilibrium under the per-unit tax will be at C, while under the percentage tax it will be at E ... showing the bigger distortion is caused

by the per-unit tax.



6. (a) If MC falls by \$4 per unit, the supply curve will shift downwards by \$4 everywhere. The original supply curve was

$$p = \frac{1}{50}Q_s + 12,$$

so the new supply curve will be

$$p = \frac{1}{50}Q_s^{'} + 8$$

which can be written as

$$Q_s^{'} = 50p - 400.$$

Equilibrium is where $Q_s^{'} = Q_d$, that is where

$$50p - 400 = \frac{1000}{p}.$$

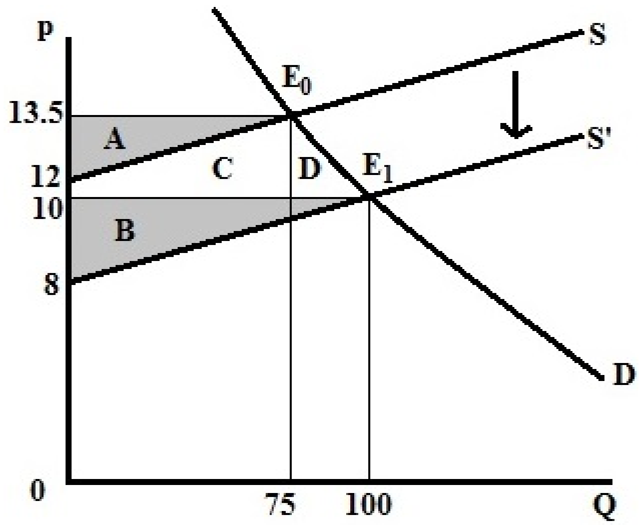
Solving for p:

$$50p^2 - 400p - 1000 = 0$$
$$p^2 - 8p - 20 = 0$$
$$(p + 2)(p - 10) = 0$$
$$p = 10$$

Then

$$Q = \frac{1000}{10} = 100.$$

(b) The situation is illustrated in the figure. The initial equilibrium is at E_0, and the final equilibrium is at E_1. Producers' surplus has gone up by area B minus area A, while consumers' surplus has gone up by area A+C+D.



To calculate these:

$$\triangle PS = \frac{1}{2}(100)(2) - \frac{1}{2}(75)(1.5) = 43.75.$$

$$\triangle CS = \int_{10}^{13.5} \frac{1000}{p} dp = 1000\, ln\, p \mid_{10}^{13.5} = 300.10.$$

Thus the total gain from this cost reduction is $43.75 + 300.10 = 343.85$, which is less than the cost of dissemination of 400. The government should not distribute the information.

If we treated the line segment E_0E_1 as linear, we could measure the change in consumers' surplus as follows:

$$\triangle CS = (75)(3.5) + \frac{1}{2}(25)(3.5) = 306.25.$$

This would not change our answer.