EC 501: Problem Set 3, Solutions

1. (a) Comparing situations 1 and 2, we can see I is the same at \$90, but p_2 has fallen from \$1 to \$$\frac{1}{2}$. So

$$1 \to 2: \quad price\ effect\ of\ p_2 \downarrow from\ \$1\ to\ \$\frac{1}{2}.$$

We see that X_2 rises from 40 to 84 so

$$\Delta X_2 = 44.$$

If we compare situations 1 and 3, we see p_2 has fallen from \$1 to \$$\frac{1}{2}$, but U has remained constant. So

$$1 \to 3: \quad substitution\ effect\ of\ p_2 \downarrow from\ \$1\ to\ \$\frac{1}{2}.$$

We see that this change involves $\Delta X_2 = 70 - 40 = 30$, which is therefore the substitution effect. Since the total price effect is 44, the remainder of 14 units must be the income effect.

To find if X_2 is normal or inferior, compare situations 2 and 3. Prices are the same, but I is higher in situation 2. With higher I, we see that X_2 is also higher, so X_2 is normal.

(b) Filling in the blanks just needs application of the budget constraint; the missing numbers are:

$$row\ 1: 20,\ row\ 2: 36,\ and\ row\ 3: 41.$$

Now

$$\Delta X_2 = 36 - 20 = 16 \quad units,$$

of which

$$substitution\ effect = 34 - 20 = 14\ units$$

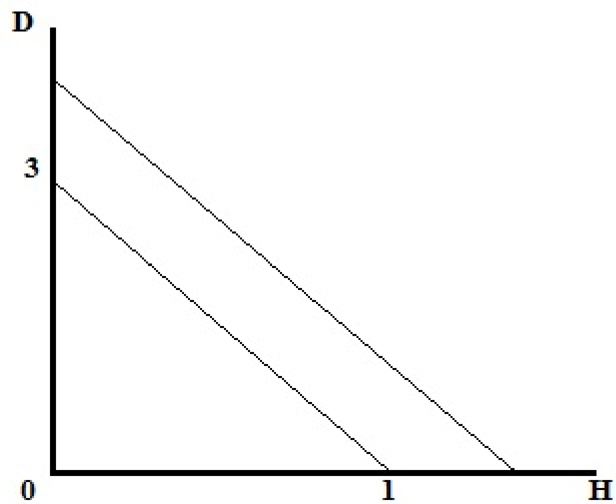
$$income\ effect = 16 - 14 = 2\ units.$$

And, since 36>34, X_2 is a normal good.

2. (a) Hamburgers and hot dogs are perfect substitutes with 3 hot dogs substituting for 1 hamburger. So the utility function is

$$U(H, D) = 3H + D.$$

(b)



The typical indifference curve is

$$3H + D = \bar{U} \quad or \quad D = \bar{U} - 3H.$$

So the slope of each indifference curve is -3. The budget constraint is

$$p_H H + p_D D = I$$

which can be rearranged as

$$D = \frac{I}{p_D} - \frac{p_H}{p_D} H,$$

so the slope is $-\frac{p_H}{p_D}$.

If the budget constraint is flatter than the indifference curve, Todd will buy only hamburgers; if it is steeper, he will buy only hot dogs. So the demand functions are

$$H = \begin{cases} \frac{I}{p_H} & if \quad \frac{p_H}{p_D} < 3 \\ 0 & if \quad \frac{p_H}{p_D} > 3 \end{cases}$$

and

$$D = \begin{cases} 0 & if \quad \frac{p_H}{p_D} < 3 \\ \frac{I}{p_D} & if \quad \frac{p_H}{p_D} > 3 \end{cases}$$

with an indeterminacy if $\frac{p_H}{p_D} = 3$.

(c) Plugging in the values into the demand functions, we see first that

$$\frac{p_H}{p_D} = \frac{5}{2} = 2.5 < 3.$$

Then

$$H = \frac{50}{5} = 10 \qquad and \qquad D = 0.$$

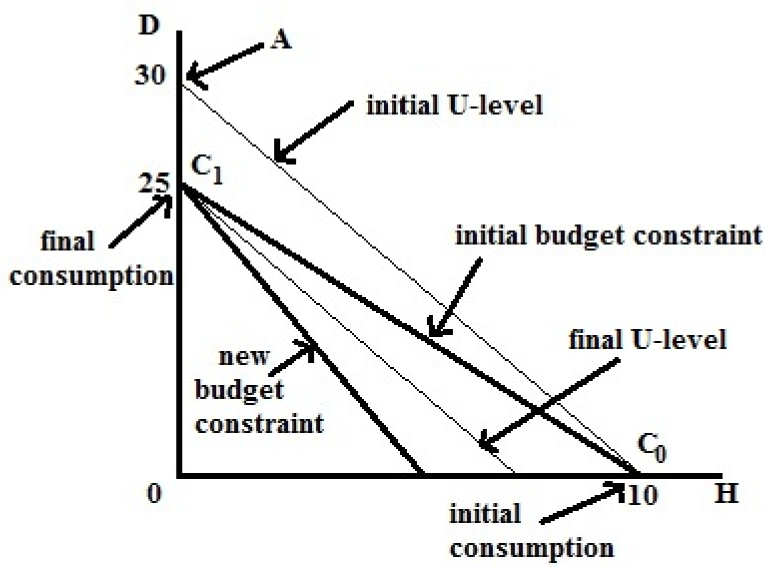
(d) If $p_H = 8$ and $p_D = 2$,

$$\frac{p_H}{p_D} = \frac{8}{2} = 4 > 3.$$

So now

$$H = 0 \qquad and \qquad D = \frac{50}{2} = 25.$$

To find the substitution effect, we need to find the cheapest way to achieve the initial U-level at the new prices. Clearly, this will be at point A in the figure, where $H = 0, D = 30$.



Thus, for H: price effect = -10 $(C_0 \to C_1)$,
of which substitution effect = -10 $(C_0 \to A)$
and income effect = 0 $(A \to C_1)$

and for D:	price effect = +25	($C_0 \to C_1$),
of which	substitution effect = +30	($C_0 \to A$)
and	income effect = -5	($A \to C_1$).

3. (a) This is a case of perfect complements, so

$$U(M, P) = min\left[M, \frac{P}{2}\right].$$

(b) At the optimum, we must have $M = \frac{P}{2}$ or $2M = P$. Now the budget constraint is

$$p_m M + p_p P = I.$$

Substituting the optimality condition, we get

$$p_m M + 2p_p M = I,$$

which simplifies to the demand function for movies:

$$M = \frac{I}{p_m + 2p_p}.$$

Then the demand function for popcorn is

$$P = \frac{2I}{p_m + 2p_p}.$$

(c)

$$M = \frac{60}{10+5} = 4 \qquad and \qquad P = \frac{2 \cdot 60}{10+5} = 8.$$

(d) Now

$$M = \frac{60}{10+10} = 3 \qquad and \qquad P = \frac{2 \cdot 60}{10+10} = 6.$$

Since there is no substitution effect in the case of perfect complements, all of the change in consumption:

$$\Delta M = -1 \qquad and \qquad \Delta P = -2$$

is due to the income effect.

(e) We know this is a case of perfect complements, with

$$U(M, P) = min\left[M, \frac{P}{2}\right].$$

At any optimum,

$$U = M = \frac{P}{2}. \tag{1}$$

Therefore we can write down Sarah's compensated demand function for popcorn as

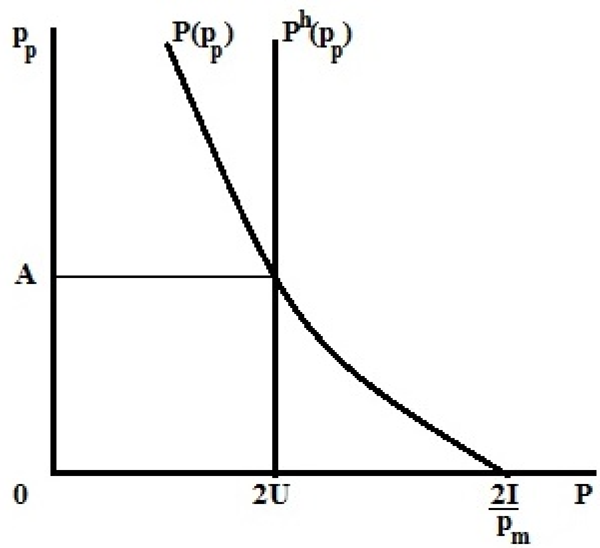
$$P^h = 2U.$$

We know that the ordinary demand function for popcorn is

$$P = \frac{2I}{p_m + 2p_p}.$$

Then the ordinary and compensated demand curves can be drawn as in the graph below. At A, the point at which the two curves intersect,

$$p_p = \frac{I}{2U} - \frac{p_m}{2}.$$



(f) To calculate CV and EV, we need to find the expenditure function. In turn, to find the expenditure function, we need the compendated demand functions. We have already found the compensated demand function for popcorn to be $P^h = 2U$. Applying the other part of equation (1), we can write down the compensated demand function for movies:

$$M^h = U.$$

Then the expenditure function, which simply substitutes the compensated demand functions in the expression for total expenditure $E = p_m M + p_p P$, is

$$E(p, U) = (p_m + 2p_p)\, U.$$

Now we know that, with the given data at the intial prices, $M_0 = 4, P_0 = 8$ and therefore $U_0 = 4$. When p_p goes up to \$5, we had calculated in part (d) of question 6 on Problem Set 1 that $M_1 = 3, P_1 = 6$ and therefore $U_1 = 3$. Now

$$CV = E(p_1, U_0) - E(p_1, U_1)$$

and therefore we can calculate for this price change

$$CV = (10 + 2 \cdot 5) \cdot 4 - 60 = 20.$$

Similarly, we know that

$$EV = E(p_0, U_1) - E(p_0, U_0)$$

and therefore we can calculate for this price change

$$EV = \left(10 + 2 \cdot \frac{5}{2}\right) \cdot 3 - 60 = -15.$$

4. (a) The goods are perfect substitutes. A simple utility function that would describe his preferences is

$$U(x, y) = 0.75x + 2y$$

since from each x John gets 0.75 liter and from each y he gets 2 liters of water.

(b) We know that in this case John will buy only one commodity, depending upon which is cheaper per unit of water or utility. If

$$\frac{p_x}{p_y} < \frac{0.75}{2} = \frac{3}{8}, \quad he\,will\,buy\,only\,x,\,and\,if$$

$$\frac{p_x}{p_y} > \frac{3}{8}, \quad he\,will\,buy\,only\,y.$$

If $\frac{p_x}{p_y} = \frac{3}{8}$, he could buy any combination of x and y along the budget constraint. So John's demand function for y may be written as

$$y = \begin{cases} \frac{I}{p_y} & if \quad \frac{p_x}{p_y} > \frac{3}{8} \\ 0 & if \quad \frac{p_x}{p_y} < \frac{3}{8} \end{cases}$$

with an indeterminacy if $\frac{p_x}{p_y} = \frac{3}{8}$.

(c) To find the compensated demand function, note that the utility function is

$$U(x, y) = 0.75x + 2y$$

and we have already established John's consumption behavior depending upon the prices of the goods. Thus we can write down John's compensated demand function as

$$y^h = \begin{cases} \frac{U}{2} & if \quad \frac{p_x}{p_y} > \frac{3}{8} \\ 0 & if \quad \frac{p_x}{p_y} < \frac{3}{8} \end{cases}$$

(d) With the data given,

$$\frac{p_x}{p_y} = \frac{1}{2} > \frac{3}{8},$$

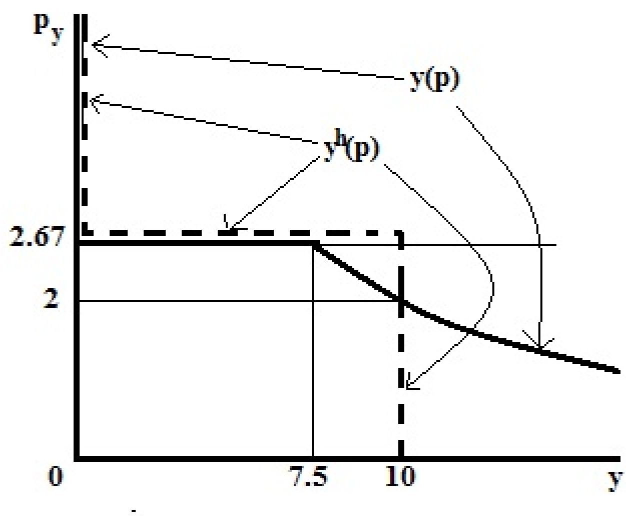
so John will consume only y. His consumption pattern will be $x_0 = 0, y_0 = 10$ and so $U_0 = 20$. Therefore his demand curve is

$$y = \begin{cases} \frac{20}{p_y} & if \quad p_y < \frac{8}{3} = 2.67 \\ 0 & if \quad p_y > \frac{8}{3} \end{cases}$$

and the compensated demand curve is

$$y^h = \begin{cases} 10 & if \quad p_y < \frac{8}{3} = 2.67 \\ 0 & if \quad p_y > \frac{8}{3} \end{cases}$$

The graph showing his ordinary and compensated demand curves is then shown below:



5. (a) To find the compensated demand functions, we need to solve the problem:

$$Minimize \qquad E = p_a A + p_b B$$

$$\textit{subject to} \qquad A^{\frac{1}{2}}B^{\frac{1}{2}} = U.$$

The Lagrangian for this problem is

$$\mathcal{L} = p_a A + p_b B + \lambda \left[U - A^{\frac{1}{2}}B^{\frac{1}{2}} \right].$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial A} = p_a - \lambda \cdot \frac{1}{2} A^{-\frac{1}{2}} B^{\frac{1}{2}} = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial B} = p_b - \lambda \cdot \frac{1}{2} A^{\frac{1}{2}} B^{-\frac{1}{2}} = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = U - A^{\frac{1}{2}}B^{\frac{1}{2}} = 0. \tag{4}$$

Dividing (2) by (3) and simplifying, we get

$$B = \frac{p_a}{p_b} A \tag{5}$$

Substituting (5) in (4), we get

$$A^{\frac{1}{2}} \left(\frac{p_a}{p_b} A \right)^{\frac{1}{2}} = U,$$

which may be simplified to

$$A^h = U \left(\frac{p_b}{p_a} \right)^{\frac{1}{2}} \tag{6}$$

(6) is the compensated demand function for A. By substituting (6) in (5) and simplifying, we get the compensated demand function for B:

$$B^h = U \left(\frac{p_a}{p_b} \right)^{\frac{1}{2}}. \tag{7}$$

(b) Adam's ordinary demand functions are:

$$A = \frac{1}{2} \cdot \frac{I}{p_a} \qquad \textit{and} \qquad B = \frac{1}{2} \cdot \frac{I}{p_b}.$$

Substituting the data, we get

$$A = 25 \qquad \textit{and} \qquad B = 10.$$

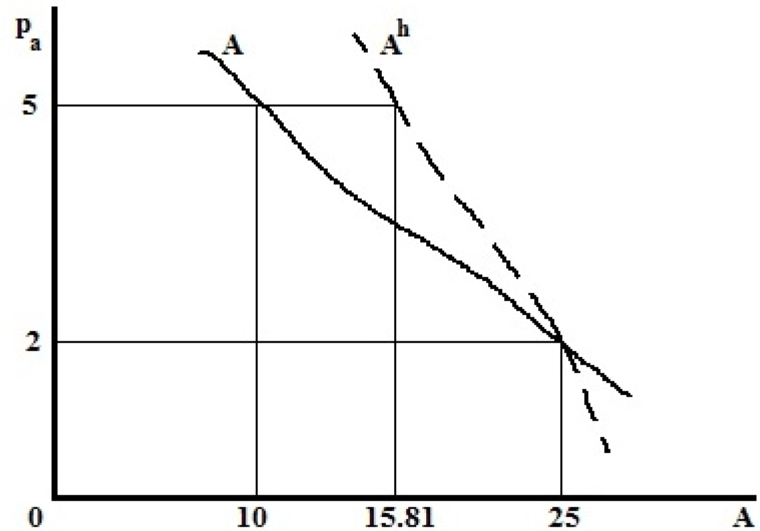
These are the amounts of the goods that Adam would buy. Note that, with these quantities, his utility level is $U_0 = \sqrt{250} = 15.81$.
(c) If p_a goes up to \$5, we can substitute the data in the ordinary demand functions to find

$$A = 10,$$

while B will remain at 10 as before. Now the final utility is
$U_1 = \sqrt{100} = 10$.
If Adam's income were compensated in order to keep his utility constant, we look at his compensated demand function to find how much he would buy. Entering the data in (6), we get

$$A^h = \sqrt{250} \cdot \left(\frac{5}{5}\right)^{\frac{1}{2}} = 15.81.$$

The graph of the ordinary and demand curves is shown below.



(d) To find the CV and EV, we need to find the expenditure function.
To do this, we use (6) and (7) to substitute for A and B in the definition of expenditure to get

$$E = p_a \cdot U \left(\frac{p_b}{p_a}\right)^{\frac{1}{2}} + p_b \cdot U \left(\frac{p_a}{p_b}\right)^{\frac{1}{2}}$$

which simplifies to

$$E = 2U p_a^{\frac{1}{2}} p_b^{\frac{1}{2}}. \tag{8}$$

This is the expenditure function. Now

$$CV = E(p_1, U_0) - E(p_1, U_1)$$

$$EV = E(p_0, U_1) - E(p_0, U_0).$$

We know

$$E(p_1, U_1) = E(p_0, U_0) = 100$$

in this example, since income has remained unchanged at 100. So, to find CV, we must calculate $E(p_1, U_0)$ and, to find EV, we must calculate $E(p_0, U_1)$. Now, using (8), we have

$$E(p_1, U_0) = 2 \cdot \sqrt{250} \cdot (5)^{\frac{1}{2}} \cdot (5)^{\frac{1}{2}} = 158.11$$

and therefore

$$CV = 158.11 - 100 = 58.11.$$

To find EV, note that

$$E(p_0, U_1) = 2 \cdot 10 \cdot (2)^{\frac{1}{2}} \cdot (5)^{\frac{1}{2}} = 63.25$$

and therefore

$$EV = 63.25 - 100 = -36.75.$$