EC 501: Problem Set 2, Solutions

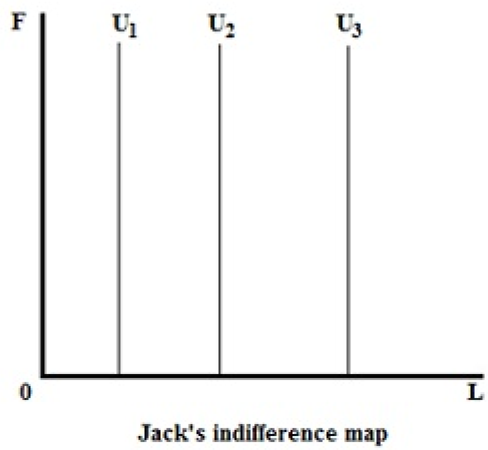
1. Suppose Jack's utility function is $U_J(L_J, F_J)$, where L_J is the amount of "lean" and F_J is the amount of "fat" he consumes. Since he never actually eats any fat, his utility function must be

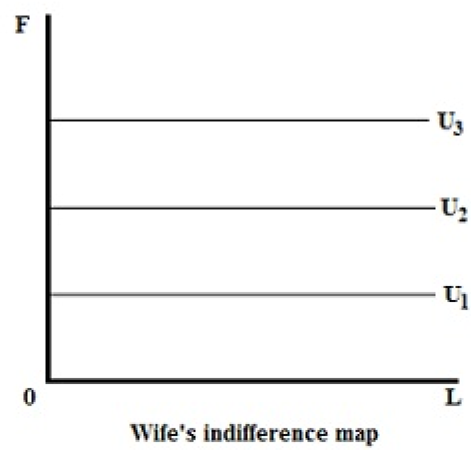
$$U_J(L_J, F_J) = L_J,$$

or any positive monotonic transformation of that. Similarly, his wife's utility function must be

$$U_W(L_W, F_W) = F_W.$$

The indifference maps are illustrated below.





2. (i) The Lagrangian for this problem is

$$\mathcal{L} = X_1^a X_2^b X_3^c + \lambda \left[I - p_1 X_1 - p_2 X_2 - p_3 X_3 \right].$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial X_1} = a X_1^{a-1} X_2^b X_3^c - \lambda p_1 = 0 \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial X_2} = b X_1^a X_2^{b-1} X_3^c - \lambda p_2 = 0 \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial X_3} = c X_1^a X_2^b X_3^{c-1} - \lambda p_3 = 0 \qquad (3)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_1 X_1 - p_2 X_2 - p_3 X_3 = 0. \qquad (4)$$

Dividing (1) by (2) and simplifying, we get

$$p_2X_2 = \frac{b}{a} \cdot p_1X_1 \tag{5}$$

Similarly, combining (1) and (3) gives us

$$p_3X_3 = \frac{c}{a} \cdot p_1X_1. \tag{6}$$

Substituting (5) and (6) in (4), we get

$$p_1X_1 + \frac{b}{a} \cdot p_1X_1 + \frac{c}{a} \cdot p_1X_1 = I,$$

which may be simplified to

$$X_1 = \frac{a}{a+b+c} \cdot \frac{I}{p_1}. \tag{7}$$

(7) is the demand function for X_1. By substituting (7) in (5) and (6) respectively, we get the demand functions for X_2 and X_3:

$$X_2 = \frac{b}{a+b+c} \cdot \frac{I}{p_2}$$

$$X_3 = \frac{c}{a+b+c} \cdot \frac{I}{p_3}.$$

(ii) To find the quantities consumed, we enter the values of a, b, c, p_1, p_2, p_3 and I in the demand functions to find

$$X_1 = 5, \qquad X_2 = 15, \qquad X_3 = 5.$$

To check our answer, look at the cost of this bundle:

$$(10 * 5) + (2 * 15) + (4 * 5) = 100,$$

which is the value of I.

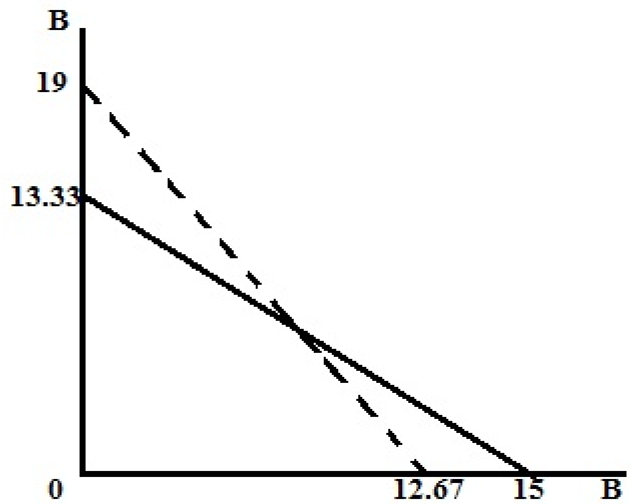
3. (a) Using our answer to the previous question, we can write down the demand functions:

$$A = \frac{1}{2} \cdot \frac{I}{p_A} \qquad and \qquad B = \frac{1}{2} \cdot \frac{I}{p_B}.$$

Substituting the data, we get

$$A = 7.5 \qquad and \qquad B = 6.67.$$

(b) We now have a second constraint, the dashed line in the figure and so the constraint is now the kinked line consisting of the inner parts of the two constraints.



We can see if Adam's choice in part (a) can still be achieved under the coupon constraint. The bundle $(A = 7.5, B = 6.67)$ would require

$$(7.5 * 3) + (6.67 * 2) = 35.84 \quad coupons.$$

This is less than the endowment of 38 coupons, so he will continue to buy the bundle $(A = 7.5, B = 6.67)$.

(c) For the original bundle, Adam now needs

$$(7.5 * 6) + (6.67 * 3) = 65 \quad coupons.$$

This is clearly not attainable.

So let us ignore the budget constraint and see what Adam's choice would be if he faced the coupon constraint alone. That is like facing a "budget" constraint with $"I" = 48, "p_A" = 6, \ and \ "p_B" = 3$. Using the demand functions from part (a), we see that Adam's chosen bundle would be

$$A = 4 \qquad and \qquad B = 8.$$

The money cost of this bundle is

$$(8 * 4) + (9 * 8) = \$104.$$

This is less than his available budget of \$120, so this bundle is attainable and Adam would therefore consume the bundle $(A = 4, B = 8)$ in this case.

(d) The simplest way to solve this problem is to consolidate the money and coupons, since they are interchangeable. It is as if Adam has an "income" of

$$I = 120 + 48 = 168$$

and the prices are

$$p_A = 8 + 6 = 14 \qquad and \qquad p_B = 9 + 3 = 12.$$

Using this data in the original demand functions gives

$$A = \frac{1}{2} \cdot \frac{168}{14} = 6 \qquad and \qquad B = \frac{1}{2} \cdot \frac{168}{12} = 7.$$

The money cost of this bundle (6, 7) is

$$(8 * 6) + (9 * 7) = \$111$$

and the coupon cost is

$$(6 * 6) + (3 * 7) = 57 \quad coupons.$$

With an actual income of \$120, Adam will buy this bundle and have \$9 left over, and he can use this to buy 9 coupons, giving him 48+9 = 57 coupons, precisely what he needs.