

EC 501: Problem Set 1
(Due in class on Tuesday, September 10)

1. (a) Find the relationship between the Centigrade (C) and Fahrenheit (F) temperature scales, given that (i) the relationship is linear, (ii) water freezes at $C = 0$ or $F = 32$, and (iii) water boils at $C = 100$ or $F = 212$.
(b) What temperature is represented by the same number in the F and C scales?
2. If $u(y)$ represents the utility as a function of an individual's income (y), then
$$R = -y u''(y) / u'(y)$$
represents the coefficient of relative risk aversion. Find the value of R if
$$u(y) = y^{1/2}.$$
3. Suppose y is a differentiable function of x that satisfies the equation
$$2x^2 + 6xy + y^2 = 18.$$
Find y' and y'' at the point $(x,y) = (1,2)$.
4. Consider the function:
$$f(x) = \ln(2 + e^{x-3}).$$
For what values of x is the sign of $f'(x)$ positive and for what values of x is $f'(x)$ negative?
5. Find the first order Taylor approximation around $x = 1$ for the function
$$f(x) = (x^p - x^q) / (x^p + x^q), \text{ where } p > q > 0.$$
6. The number $N(t)$ of people who have fallen sick t days after being exposed to a virus is given by
$$N(t) = 1000 / (1 + 999 e^{-0.39t}).$$
(a) How many have fallen sick after 10 days?
(b) How many days does it take for 500 people to fall sick?
7. Consider the function
$$Q = A [aK^{-v} + bL^{-v}]^{-1/v}$$
where A , a , b , and v are positive constants.
(a) Is this function homogeneous or non-homogeneous? If it is homogeneous, find the degree of homogeneity. If it is non-homogenous, prove that is true.
(b) Find an expression for Q/L in terms of K/L .
8. A firm produces and sells widgets under two brand names in the amounts x and y . Its profits are given by
$$\pi(x,y) = -0.1 x^2 - 0.2 xy - 0.2 y^2 + 47 x + 48 y - 600.$$
(a) Find the values of x and y that maximize π . Confirm that your solution is a maximum with the second derivative test.
(b) If the total production of widgets can be no higher than 200, what values of x and y will maximize π ? There is no need to check the second-order condition now.

...continued

9. A monopolist faces a demand curve

$$P(Q) = 10 - (Q/2)$$

and a total cost curve

$$C(Q) = Q^2 + 8Q + 5.$$

- (a) If the firm's profit is given by $\pi = P(Q) \cdot Q - C(Q)$, what value of Q should it choose to maximize profit?
- (b) Now suppose the government imposes a tax of t per unit of the monopolist's output. Write down the new cost curve and a new expression for the firm's profit.
- (c) Find the monopolist's profit maximizing output level as a function of t .
- (d) Now find the value of t that would maximize the government's tax revenue, $T = tQ$.

10. If a firm's marginal revenue is

$$MR = 25 - 2Q,$$

use integration to find its total revenue. Be as precise as you can.