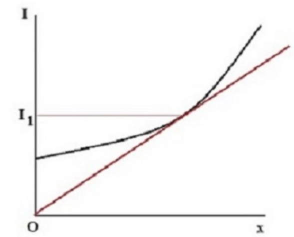
EC 501: Mid-term Exam (Fall 2019), Solutions

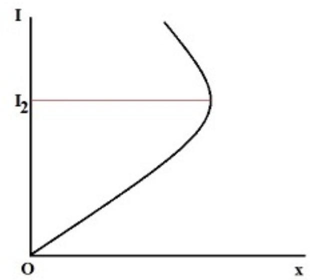
1. (a) An Engel curve is a curve showing the relationship between the quantity demanded and the consumer's budget.

   (b)

   The graph shows the desired Engel curve. At low levels of income, the ratio of I/x is falling, which means the budget share of x is rising, a property of luxuries. For income levels above I_1, the ratio of I/x is rising, which means the budget share of x is falling, a property of necessities.

   (c)

   The graph shows the desired Engel curve. At low levels of income, the ratio of I/x is rising, which means the budget share of x is falling, a property of necessities. For income levels above I_2, x is falling as I rises, a property of inferior goods.

2. (a) We know that Lily's utility function is linear; the indifference curves are downward-sloping straight lines with slope $-\frac{2}{3}$. Then her demand for x (whether compensated or ordinary) is zero for siuations where $p_x > \frac{2}{3}$, since $p_y = 1$, and she would consume only x if $p_x < \frac{2}{3}$. For $p_x = \frac{2}{3}$, her demand is indeterminate; I will set it to zero (arbitrarily).

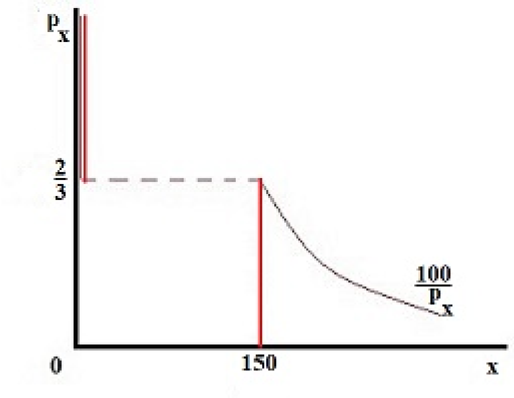
   Then Lily's demand curve for $I = 100$ is

$$x = \begin{cases} 0 & if \quad p_x \geq \frac{2}{3} \\ \frac{100}{p_x} & if \quad p_x \geq \frac{2}{3} \end{cases}$$

Since Lily's utility function is $U(x,y) = 2x+3y$, Lily can attain U=*300* by consuming either y=*100* or x=*150*. Therefore, her compensated demand curve for U=*300* is

$$x^h = \begin{cases} 0 & if \quad p_x \geq \frac{2}{3} \\ 150 & if \quad p_x \geq \frac{2}{3} \end{cases}$$

(b) The graph shows the two demand curves ... the black one is the ordinary demand curve and the red one is the compensated demand curve. Note that, at $p_x = \frac{2}{3}$, $\frac{100}{p_x} = 150$.



3. (a) To examine for returns to scale, let

$$q_0 = \frac{K_0 L_0}{K_0 + L_0}.$$

Now multiply each of the inputs by a factor λ:

$$q_1 = \frac{\lambda K_0 \cdot \lambda L_0}{\lambda K_0 + \lambda L_0} = \lambda q_0.$$

Therefore the production function exhibits constant returns to scale.

(b) To find the long run cost function, we must solve the problem of minimizing cost subject to the production function as a constraint. The Lagrangian for the problem is

$$\mathcal{L} = wL + rK + \lambda \left[q - \frac{KL}{K+L} \right].$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial L} = w - \lambda \left[\frac{K^2}{(K+L)^2} \right] = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial K} = r - \lambda \left[\frac{L^2}{(K+L)^2} \right] = 0 \tag{2}$$

Dividing (1) by (2) and simplifying, we get

$$K = L\sqrt{\frac{w}{r}}. \tag{3}$$

Substituting (3) in the production function and simplifying gives us the conditional demand function for labor:

$$L = q\left[1 + \sqrt{\frac{r}{w}}\right].$$

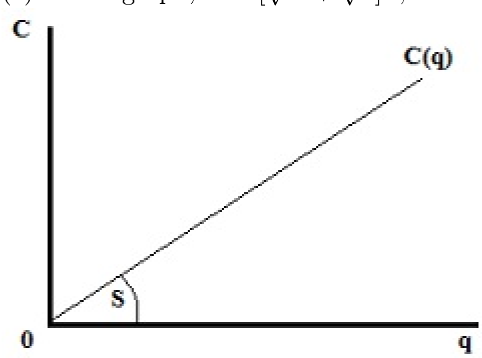
Substituting back in 3gives us the conditional demand function for capital:

$$K = q\left[\sqrt{\frac{w}{r}} + 1\right].$$

Substituting the conditional input demand functions into the expression for cost gives us the cost function:

$$C = wL + rK = q\left[\sqrt{w} + \sqrt{r}\right]^2.$$

(c) In the graph, $S = \left[\sqrt{w} + \sqrt{r}\right]^2$, the slope of the cost curve.



4. (a) Equilibrium will occur where $Q_d = Q_s$, that is, where

$$5000 - 100p = 150p \qquad \rightarrow \qquad p = 20.$$

Then $Q_d = Q_s = 3000$.

(b) If gadgets can be imported and sold at a price of \$10, the price would have to be \$10. At this price, the demand would be

$$Q_d = 5000 - 1000 = 4000,$$

and the domestic supply will be

$$Q_s = 150 \cdot 10 = 1500.$$

The remainder $4000 - 1500 = 2500$ would be imported.

(c) If the government imposes an import tax of \$5, the price of the imported gadgets will go up to \$15, and this would now be the market price. Now the demand would be

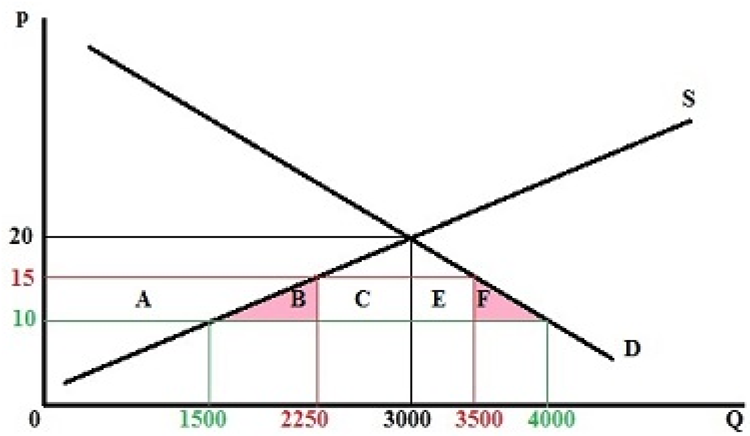
$$Q_d = 5000 - 1500 = 3500,$$

and the domestic supply will be

$$Q_s = 150 \cdot 15 = 2250.$$

The remainder $3500 - 2250 = 1250$ would be imported.

(d)



The diagram shows the various equilibria. With the tax, consumers lose, since the price they face rises from \$10 to \$15. The net welfare effect on consumers is the negative of the area A+B+C+E+F in the diagram:

$$\Delta CS = -[5 * 3500 + \frac{1}{2} * 5 * 500] = -18,750.$$

Producers are better off, since the price they receive rises from \$10 to \$15. The net welfare effect on domestic producers is the area A:

$$\Delta PS = 5 * 1500 + \frac{1}{2} * 5 * 750 = 9,375.$$

Government also benefits, since it collects tax revenue of the area C+E:

$$Tax = 5 * 1250 = 6,250.$$

The net welfare effect then is

$$\Delta W = -18,750 + 9,375 + 6,250 = -3,125.$$

This is equal to the sum of the two shaded areas in the diagram (B+F), which show the quantity distortions caused by the tax and the resulting deadweight losses associated with them.