EC 501 **Final Exam** **Dec 15, 2019**

Answer all questions, showing all your work. Try to use diagrams wherever possible. Time allowed: 2 hours. Good luck! (Numbers in parentheses refer to the number of points attached to each question and suggested time allocation.)

1. *(20 points, 25 minutes)* Mary consumes only two goods, X and Y. Her utility function is

$$U(X, Y) = XY^2$$

Initially, her income is \$90 and the prices of both goods are \$1. The price p_x then rises to \$2.

(a) Using consumer's surplus as your welfare measure, find how much better off or worse off is Mary.
(b) Using the compensating variation as your welfare measure, find how much better off or worse off is Mary.
(c) Illustrate your answers to (a) and (b) on one clearly labeled diagram.

2. *(20 points, 20 minutes)* Adam consumes only two goods, X and Y, and his utility function is

$$U(x_A, y_A) = x_A + 2y_A$$

where x_A, y_A are the quantities of the two goods he consumes. Adam is endowed with 10 units of X and 5 units of Y.

Becky also consumes only X and Y, but her utility function is

$$U(x_B, y_B) = 2x_B + y_B$$

where x_B, y_B are the quantities of the two goods she consumes. Becky is endowed with 10 units of X and 15 units of Y.

(a) Draw Adam's offer curve in a clearly labeled diagram, explaining how you found it. Your graph should make it clear what Adam's offers would be at all possible relative price ratios.
(b) Draw Becky's offer curve in a clearly labeled diagram, explaining how you found it.
(c) Suppose Adam and Becky are in a pure exchange economy. Draw the Edgeworth Box for this problem and draw the two offer curves in it. What would be the equilibrium price ratio if Adam and Becky engaged in a Walrasian tâtonnement process (i.e., the perfectly equilibrium price ratio if they engaged in trade)? What would be the quantities of X and Y traded and by whom?

3. *(20 points, 25 minutes)* Bill has wealth of \$100 but faces the possibility that, with probability 0.2, he could lose \$64 of that. His utility function is

$$U(w) = \sqrt{w},$$

where *w* is his wealth, and he maximizes expected utility.

(a) What is the actuarially fair insurance premium for Bill?
(b) What is the maximum amount Bill would be willing to pay for full insurance? Explain fully.
(c) Let π_m represent the maximum willingness to pay that you found in (b). Suppose the insurance company offers Bill the option to buy fractional insurance, in which, for a premium of $x\pi_m$, he gets paid $\$64x$ if he suffers the loss. Write down the equation you would need to solve to find the value of x that Bill would choose.
(d) Draw a single diagram in state-contingent utility space to illustrate your answers to (b) and (c).

Continued on back …

4. *(20 points, 25 minutes)* There are 2 firms in the widget industry. Their cost functions are

$$C_1(q_1) = 20 + 10q_1 \quad and \quad C_2(q_2) = 10 + 12q_2$$

The inverse market demand curve for widgets is

$$p = 50 - 5Q,$$

where Q is the total number of widgets produced.

(a) Suppose the firms choose their quantities simultaneously. Find the Cournot-Nash equilibrium. How much profit does each firm make?
(b) Suppose instead that firm 1 can choose its quantity first, and firm 2 follows. How much will each firm produce now? How much profit will each firm make?
(c) Finally, suppose firm 1 enters the market first and can credibly commit to a production level before firm 2 has actually entered the market. What will now be the production levels and profit levels of both firms? (Note: If firm 2 enters, it must spend the fixed cost of 10, but it does not need to spend this if it does not enter.)

5. *(20 points, 25 minutes)* The demand and supply for blodgets are given by

$$Q_d = 50{,}000 - 1{,}000p \quad and \quad Q_s = 2{,}000p - 10{,}000.$$

The production of blodgets involves the simultaneous production of gunk, a toxic by-product. Two units of gunk are produced for each blodget produced. When discharged into the environment, each unit of gunk causes $3 worth of environmental damage.

(a) How many blodgets should be produced, and what price should be charged for them, in order to maximize welfare?
(b) What will be the equilibrium number of blodgets produced, and what will be the price, if the blodget market is perfectly competitive and is unregulated? What is the deadweight loss to society of this equilibrium, compared to the efficient one of part (a)?
(c) What would be the Pigouvian tax that would force the competitive market to the efficient solution?
(d) Suppose that it is possible for blodget producers to clean up the gunk before it is discharged into the environment at a cost of

$$A = \frac{a^2}{20{,}000},$$

where *a* is the amount of gunk abated. What is now the efficient level of production of blodgets, and how much gunk should be abated? What is the gain in welfare, if any, in this solution as compared to the solution in part (c)?