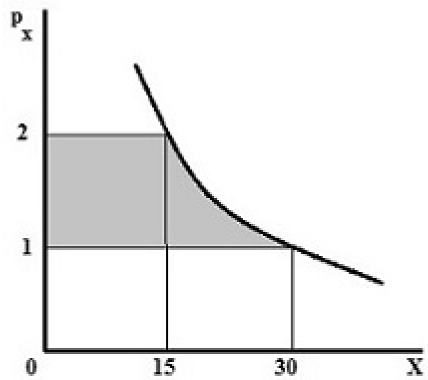
EC 501: Final Exam (Fall 2019), Solutions

1. (a) Since the utility function is Cobb-Douglas, we can write down the demand functions:

$$X = \frac{I}{3p_x} \qquad and \qquad Y = \frac{2I}{3p_y}.$$

Initially, $X_0 = 30$ and $Y_0 = 60$. After p_x rises, $X_1 = 15$ and $Y_1 = 60$.



The loss in consumer's surplus is the shaded area in the figure:

$$\Delta CS = \int_2^1 \frac{I}{3p_x} dp_x = \left[\frac{I}{3} ln p_x\right]_2^1 = -20.79.$$

If we had assumed the demand curve to be linear, we would have got

$$\Delta CS \approx -15 - \frac{1}{2}(15) = -22.5.$$

(b) Since only one price has changed, we can calculate CV as the area below the compensated demand curve between the price lines. To find the compensated demand curve, we need to minimize the expenditure needed to achieve the original utility level. That is, we need to satisfy the tangency condition and the utility function:

$$\frac{MU_x}{MU_y} = \frac{Y^2}{2XY} = \frac{p_x}{p_y} \qquad and \qquad U = XY^2.$$

This yields the compensated demand function for X:

$$X^h = \left(\frac{Up_y^2}{4p_x^2}\right)^{\frac{1}{3}}.$$

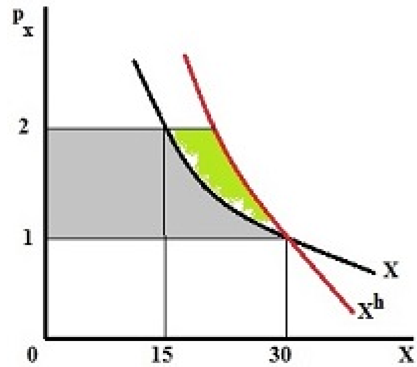
Since $U_0 = (30)(60)^2 = 108,000$ and $p_y = 1$, this reduces to

$$X^h = \left(\frac{27,000}{p_x^2}\right)^{\frac{1}{3}}.$$

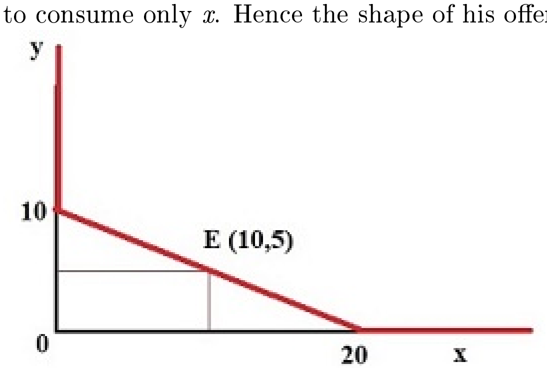
Then the change in welfare using the compensated variation measure (equal to –CV) can be calculated as

$$\Delta W_{CV} = \int_2^1 \frac{30}{p_x^{\frac{2}{3}}} dp_x = 30 \left[3 p_x^{\frac{1}{3}} \right]_2^1 = -23.39.$$

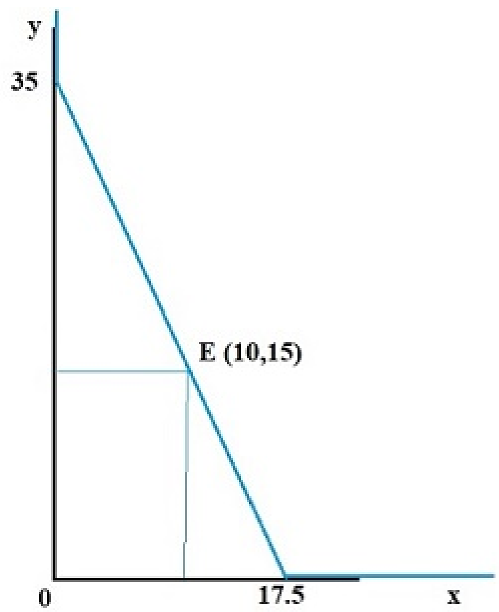
(c) The graph below shows the grey shaded area as the change in consumer's surplus. The compensating variation measure adds the green shaded area to the grey one.



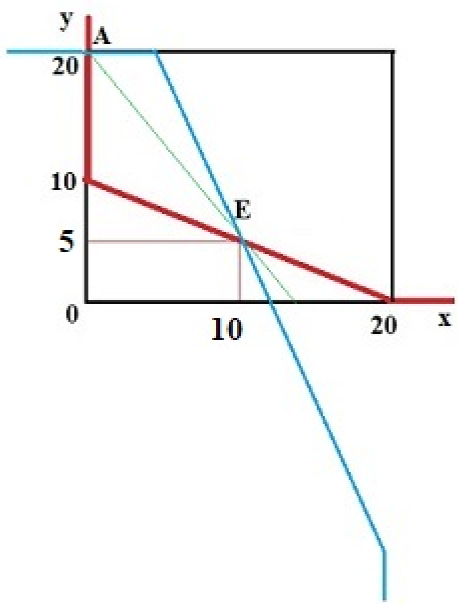
2. (a) The graph shows Adam's offer curve in red. E is his endowment (10,5). The diagonal line is his indifference curve through E; its slope is -0.5. For price ratios steeper than this, Adam will want to consume only y, while for price ratios flatter than this, he would want to consume only x. Hence the shape of his offer curve.



(b) The graph shows Becky's offer curve in blue. E is her endowment (10,15). The argument for its shape is the same as for Adam.



(c) The graph shows the Edgeworth box with the two offer curves drawn in.



We see that the offer curves intersect at A, which would be the Walrasian euilibrium. The price ratio necessary to attain that equilibrium is the thin green line; the slope is $\frac{p_x}{p_y} = \frac{3}{2}$.
Algebraically, we know that the equilibrium price ratio must lie between the slopes of Adam's and Backy's indifference curves. That

is,

$$\frac{1}{2} < \frac{p_x}{p_y} < 2.$$

For all such prices, Adam will want to consume only y and Becky will want to consume only x. Let us normalize the prices by setting $p_y = 1$. Then the supply of x will be 10 (Adam's entire endowment) and the demand for x will be $\frac{15}{p_x}$ (the amount of x Becky can buy after selling her entire endowment of y). At equilibrium, these must be equal. Therefore,

$$10 = \frac{15}{p_x} \qquad \to \qquad p_x = \frac{3}{2}.$$

At these prices, Adam will sell 10 units of x and buy 15 units of y. Becky's trade will be the mirror image of Adam's.

3. (a) The actuarially fair insurance premium would be equal to the expected loss:

$$\pi_a = (0.2)(64) = \$12.80.$$

(b) The maximum Bill would be willing to pay for insurance would be that premium which would reduce his exxpected utility to the level he has before buying any insurance. This would satisfy

$$\sqrt{100 - \pi_m} = (0.8)\sqrt{100} + (0.2)\sqrt{36} = 9.2$$

$$100 - \pi_m = 84.64 \qquad \to \qquad \pi_m = \$15.36.$$

(c) If Bill can buy a fractional policy, he would want to maximize his expected utility

$$E(U) = (0.8)\sqrt{100 - \pi_m x} + (0.2)\sqrt{36 - \pi_m x + 64x}$$

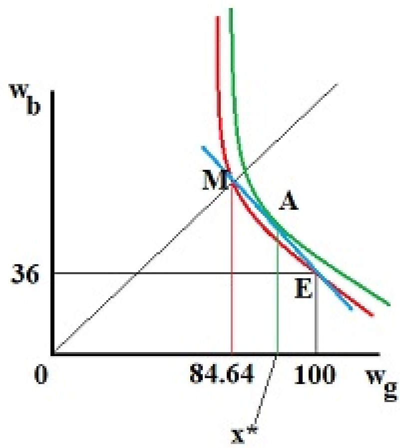
To maximize:

$$\frac{dE(U)}{dx} = \frac{(0.4)(-\pi_m)}{\sqrt{100 - \pi_m x}} + \frac{(0.1)(64 - \pi_m)}{\sqrt{36 - \pi_m x + 64x}} = 0.$$

This is the equation we need to solve to find the optimal value of x. Solving, we can find

$$x^* = 0.4578.$$

(d) The graph shows the solution graphically in the state-contingent utility space. E is the "endowment" showing the wealth levels in the good and bad states. M represents the point achieved if Bill pays his maximum willingness to pay for full insurance ... his utility level there is the same as at E. The blue line ME represents the insurance policy in which the premium is the maximum willingness to pay. Bill can determine the optimal fractional policy by finding an indifference curve tangent to the line ME. The tangency is at A, and $x^* = 100 - \pi_m x$.



4. (a) Firm 1's profit:

$$\pi_1 = (50 - 5q_1 - 5q_2)(q_1) - 20 - 10q_1.$$

Differentiating with respect to q_1 and simplifying, we get firm 1's best response function:

$$q_1 = 4 - \frac{1}{2}q_2.$$

Similarly, we can find firm 2's best response function:

$$q_2 = \frac{38 - 5q_1}{10}.$$

Solving the two best response functions simultaneously gives us the Cournot Nash equilibrium:

$$q_1 = \frac{42}{15}, \qquad q_2 = \frac{36}{15}, \qquad Q = \frac{78}{15}, \qquad p = 24, \qquad \pi_1 = 19.2, \qquad \pi_2 = 18.8.$$

(b) To find the Stackelberg equilibrium, firm 1 will incorporate firm 2's best response function in its profit function:

$$\pi_1 = (50 - 5q_1 - \frac{38 - 5q_1}{2})(q_1) - 20 - 10q_1.$$

Differentiating and solving, we find $q_1 = \frac{21}{5}$. Then

$$q_2 = \frac{17}{10}, \qquad Q = \frac{59}{10}, \qquad p = \frac{41}{2}, \qquad \pi_1 = 24.1, \qquad \pi_2 = 4.45.$$

(c) If firm 1 could credibly commit to an output level before firm 2 enters, it might be able to pre-empt firm 2's entry. It would need to set q_1

such that firm 2's profits are driven to zero. Thus it must satisfy the following equation:

$$\pi_2 = \left(\frac{38-5q_1}{10}\right)\left(50-5q_1-\frac{38-5q_1}{2}\right)-10-12\left(\frac{38-5q_1}{10}\right)=0.$$

Solving, we get

$$q_1 = 4.77.$$

Since this would pre-empt entry, $q_2 = 0$ and so $p = 26.14$ and $\pi_1 = 57.023$.

Since this is higher than the profit in (b), firm 1 would indeed pre-empt entry. Firm 2's output and profit will be zero.

5. (a) The supply curve is the aggregation of the individual firm MC curves; in other words, its the private marginal cost curve (PMC). Now the supply curve can be written as

$$P = 5 + \frac{1}{2000}Q_s \qquad so \qquad PMC = 5 + \frac{1}{2000}Q_s.$$

The marginal external cost (MEC) of blodget production is \$6 per unit (=2 units of gunk produced x \$3 damage per unit of gunk). So the social marginal cost (SMC) is \$6 more than the PMC:

$$SMC = 11 + \frac{1}{2000}Q_s.$$

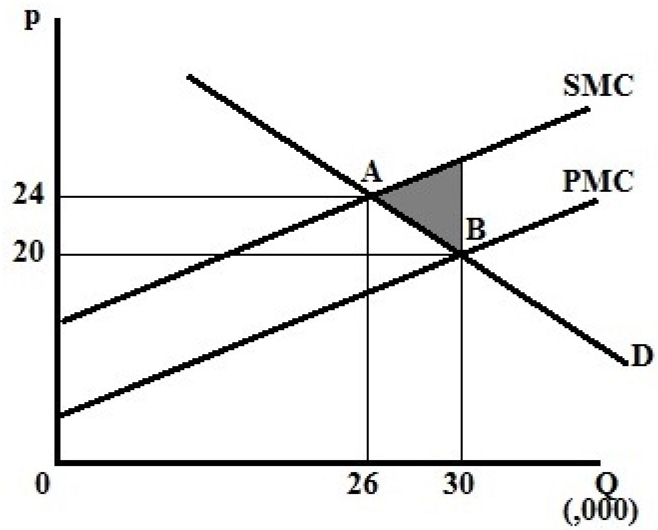
At the efficient solution, $p = SMC$, so

$$Q_s = 2000P - 22,000.$$

Equate this to Q_d to get the equilibrium:

$$2,000P-22,000 = 50,000-1,000P \qquad \rightarrow \qquad P = \$24 \;\; and\, so\, Q = 26,000.$$

This is the welfare-maximizing solution. The graph illustrates the situation, with point A representing the efficient solution.



(b) The competitive equilibrium would occur where demand and supply intersect:

$$2,000P - 10,000 = 50,000 - 1,000P \qquad \rightarrow \qquad P = \$20 \ \ and\, so\, Q = 30,000.$$

This equilibrium is point B in the graph. The deadweight loss in this equilibrium is the shaded area in the graph, since there is excessive production and this represents the amount by which social marginal cost exceeds the social marginal benefit over these excessive units of output. We can calculate this area (remember the vertical difference between PMC and SMC is \$6):

$$DWL = \frac{1}{2} \cdot 6 \cdot 4000 = \$12,000.$$

(c) The Pigouvian tax necessary to force the efficient solution is the MEC at the optimum, which is \$6.

(d) The marginal benefit of any gunk abatement is \$3 per unit, which is the damage that is prevented. Now the MC of abatement is

$$\frac{dA}{da} = \frac{a}{10,000},$$

so the optimal level of abatement (where MC=MB) will be where

$$\frac{a}{10,000} = 3 \qquad \rightarrow \qquad a = 30,000.$$

The optimal level of blodget production is still 26,000, since the MEC has remained at \$6.

The gain to society from this solution compared to the solution with the Pigouvian tax is the reduced external cost from the 30,000 units of gunk abated minus the cost of abatement:

$$\triangle W = 3 * 30,000 - \frac{(30,000)^2}{20,000} = \$45,000.$$