Bertrand Competition

$$\pi = p(q) \cdot q - c(q)$$

$$\frac{d\pi}{dq} = \underbrace{p + q\frac{dp}{dq}} - c'(q) = 0$$

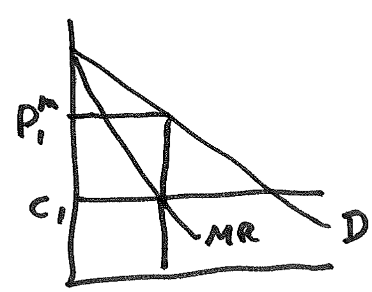
$$\pi = p \cdot q(p) - c[q(p)]$$

$$\frac{d\pi}{dp} = q + p\frac{dq}{dp} - c' \cdot \frac{dq}{dp} = 0$$

$$\frac{dq}{dp}\left[\underbrace{q \cdot \frac{dp}{dq} + p - c'}\right] = 0$$

Homogeneous goods model.

- constant AC + MC
- if $P_1 < P_2$ then $q_2 = 0$
- if $P_1 = P_2$ then $q_1 = q_2$



If $c_1 < c_2$ then

(1) if $c_2 > P_1^m$ then $P_1 = P_1^m$

(2) If $c_2 < P_1^m$ then $P_1 = c_2 - \epsilon$

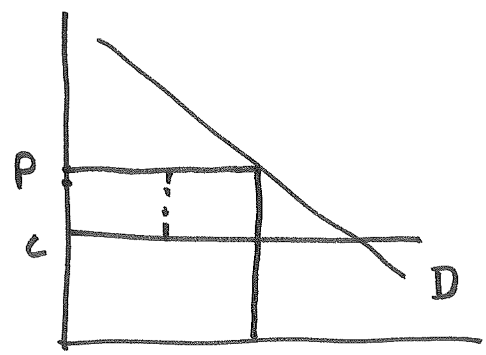
If $c_1 = c_2 = c$, then $P_1 = P_2 = c$

(1) $P_1 = P_2$. Suppose $P_1 > P_2 \rightarrow$ contradiction

(2) $P_1 = P_2 = c$. Suppose $P_1 = P_2 < c \rightarrow$ losses - no eqm.

Suppose $P_1 = P_2 > c$

Eqm: $P_1 = P_2 = c$



Capacity Constraints

Stage 1: Build capacity

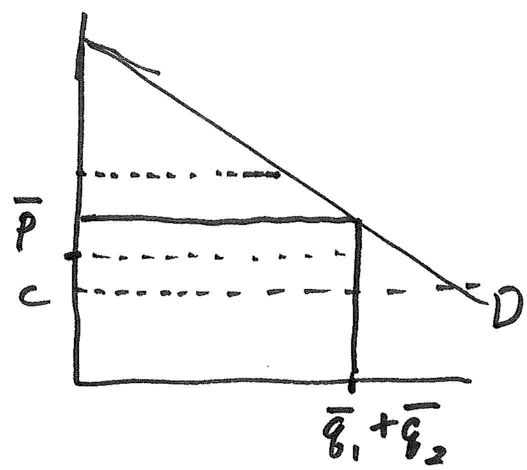
Stage 2: Bertrand competition

$\bar{q}_1, \bar{q}_2$ capacity has been built.

what are P_1, P_2?

Suppose $P_1 > P_2$

$q_1 \quad q_2$



$$\max_{\bar{q}_1} \pi = \bar{q}_1 \cdot P(\bar{q}_1 + \bar{q}_2) - c\bar{q}_1$$

$\rightarrow$ Cournot solution

Differentiated Goods

$$q_1 = q_1(P_1, P_2) \quad , \quad \frac{\partial q_1}{\partial P_1} < 0 \quad , \quad \frac{\partial q_1}{\partial P_2} > 0$$

$$\Pi_1 = P_1 \cdot q_1(P_1, P_2) - c_1 q_1(P_1, P_2)$$

$$\frac{\partial \Pi_1}{\partial P_1} = q_1(\cdot) + P_1 \frac{\partial q_1}{\partial P_1}(\cdot) - c_1 \frac{\partial q_1}{\partial P_1}(\cdot) = 0$$

$$\rightarrow BR_1$$

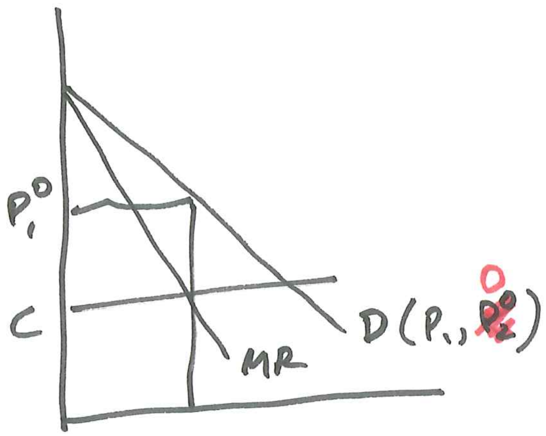
$$\Pi_1'(P_1, P_2) = 0$$

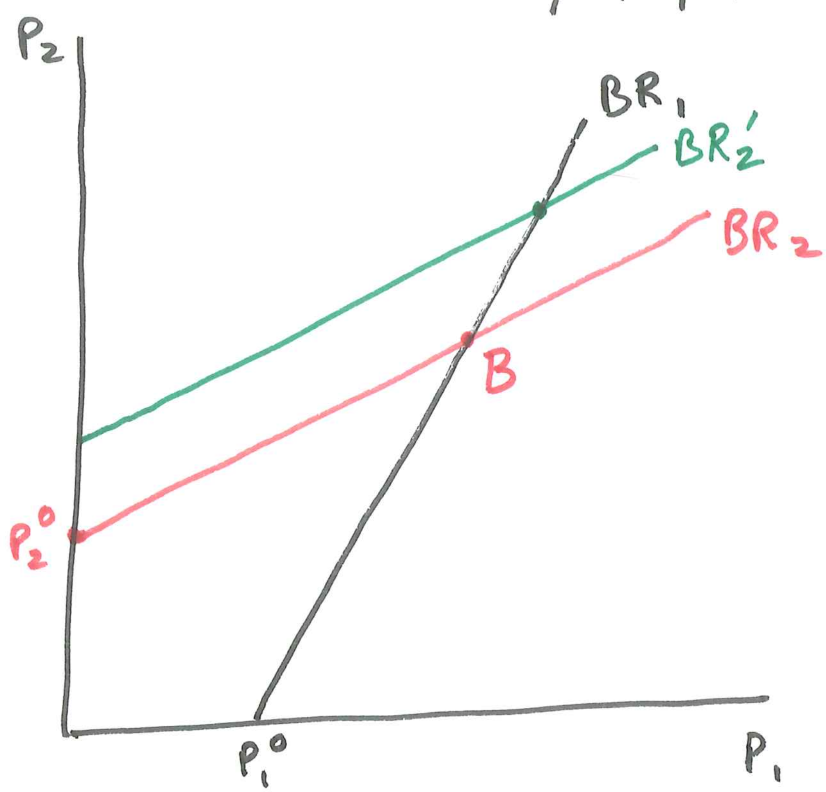
$$\frac{\partial \Pi_1'}{\partial P_1} dP_1 + \frac{\partial \Pi_1'}{\partial P_2} dP_2 = 0$$

$$\frac{dP_2}{dP_1} = - \frac{\partial \Pi_1' / \partial P_1}{\partial \Pi_1' / \partial P_2}$$

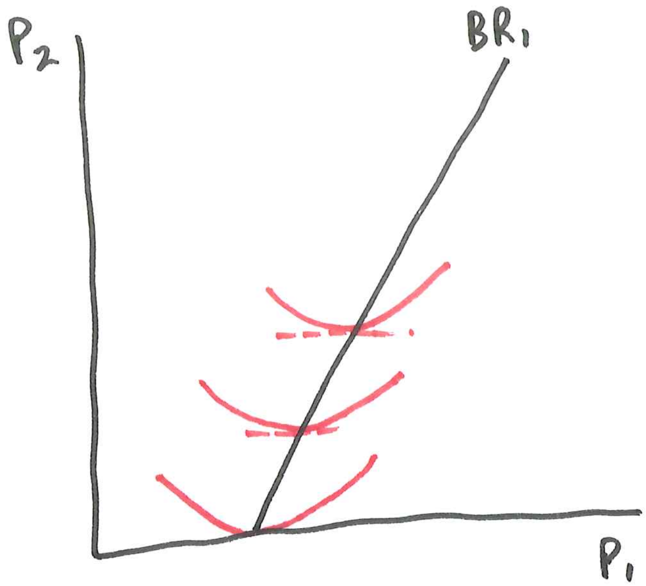
$$\frac{dP_2}{dP_1} = - \frac{\frac{\partial q_1}{\partial P_1} + \frac{\partial q_1}{\partial P_1} + P_1 \frac{\partial^2 q_1}{\partial P_1^2} - c_1 \frac{\partial^2 q_1}{\partial P_1^2}}{\frac{\partial q_1}{\partial P_2} + P_1 \frac{\partial^2 q_1}{\partial P_1 \partial P_2} - c_1 \frac{\partial^2 q_1}{\partial P_1 \partial P_2}}$$

$$\simeq - \frac{2 \frac{\partial q_1}{\partial P_1}^{\ominus}}{\frac{\partial q_1}{\partial P_2}^{\oplus}} > 0$$

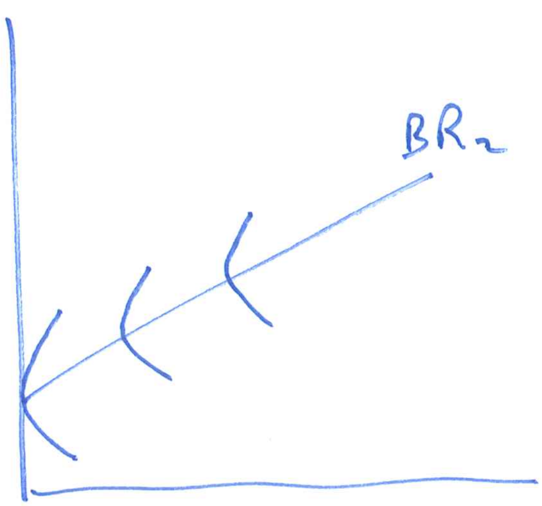


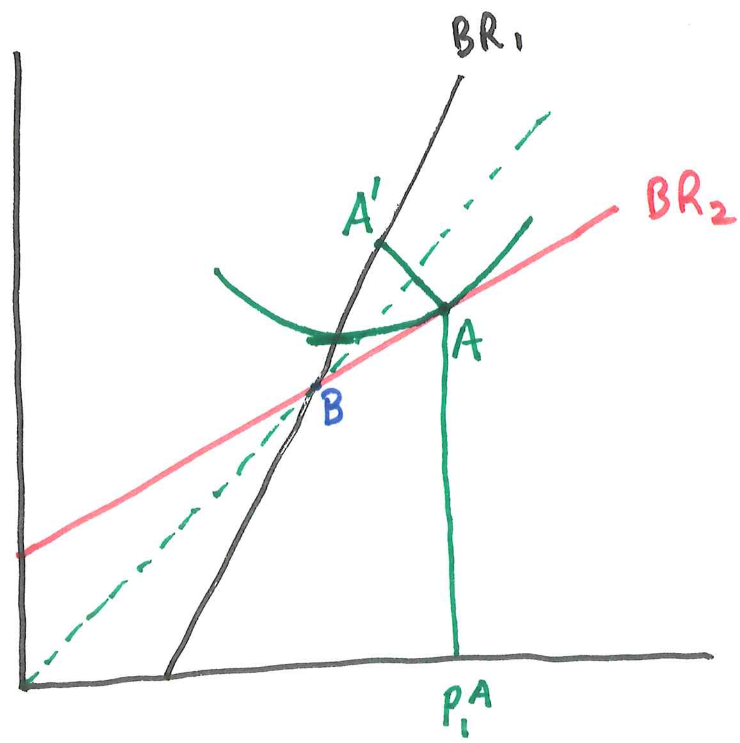


"strategic complements"



Firm 1 moves first





$\Pi_2^{A'} = \Pi_1^{A}$

Externalities

Pigouvian tax