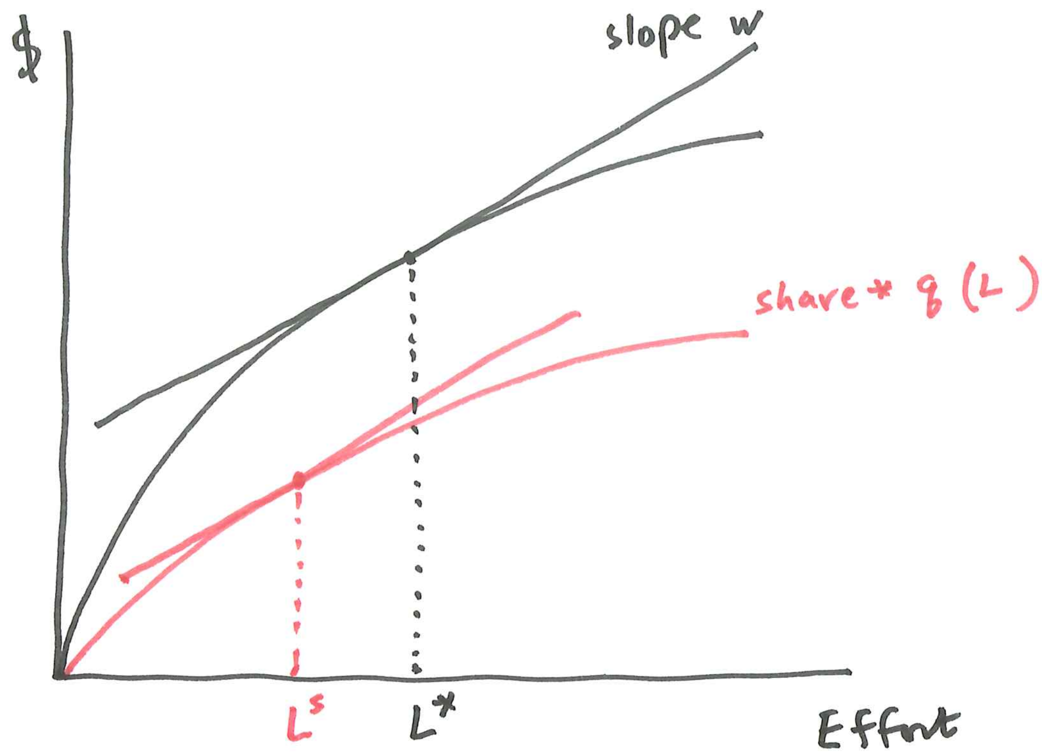
Principal-Agent Problems

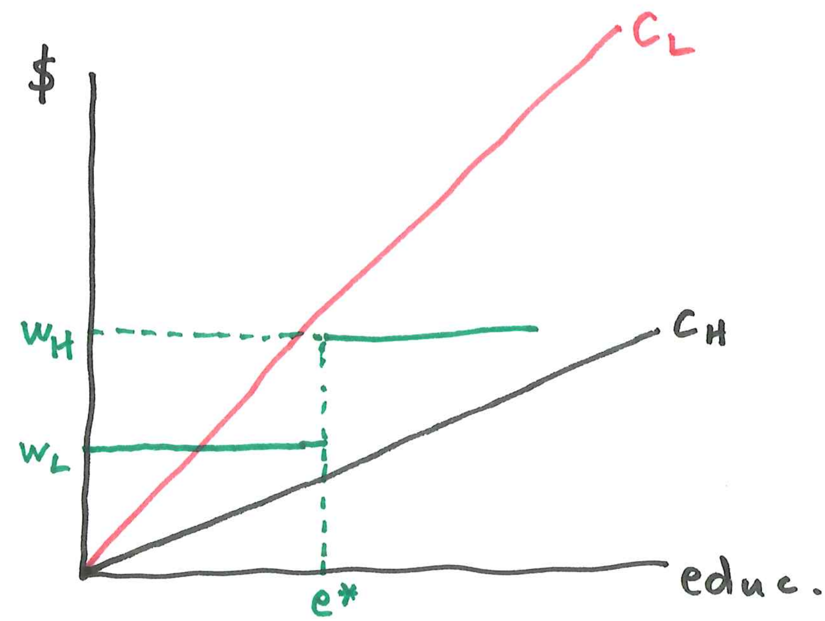
sharecroppers.



Screening

Signalling

$w_L < w_H$



A $\begin{cases} \$2500 & w/\ p = .33 \\ \$2400 & w/\ p = .66 \\ \$0 & w/\ p = .01 \end{cases}$ / B \$2400

/ C $\begin{cases} \$2500 & w/\ p = .33 \\ 0 & w/\ p = .67 \end{cases}$ D $\begin{cases} 2400 & w/\ p = .34 \\ 0 & w/\ p = .66 \end{cases}$

B over A: $u(2400) > .33\,u(2500) + .66\,u(2400) + .01\,u(0)$

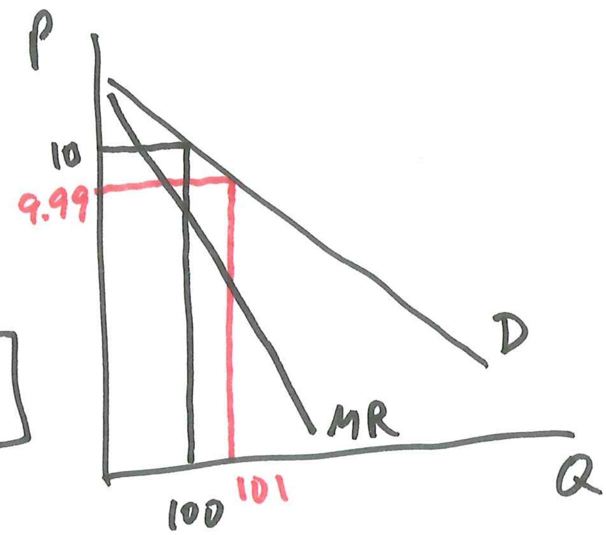
C over D: $.33\,u(2500) + .67\,u(0) > .34\,u(2400) + .66\,u(0)$

$$.33\,u(2500) > .34\,u(2400) - .01\,u(0)$$

$$.33\,u(2500) < .34\,u(2400) - .01\,u(0)$$

Monopoly.

$$R(Q) = P(Q) \cdot Q$$



$$MR = \frac{dR}{dQ} = P + Q\frac{dP}{dQ}$$

$$= P\left[1 + \frac{dP}{dQ} \cdot \frac{Q}{P}\right]$$

$$= P\left[1 + \frac{1}{\varepsilon_d}\right]$$

$MR_{101} = 9.99 - 1$

$= 8.99$

$R_{100} = 1{,}000$

$R_{101} = 1{,}008.99$

$$\pi = R(Q) - C(Q)$$

$$\frac{d\pi}{dQ} = MR - MC = 0$$

$$MR = MC$$

$$P\left(1 + \frac{1}{\varepsilon_d}\right) = c$$

$$P + \frac{P}{\varepsilon_d} = c$$

Mark-up $\left\{ \frac{P - c}{P} = -\frac{1}{\varepsilon_d} = \frac{1}{|\varepsilon_d|} \right.$

Degree of monopoly, Lerner monopoly index

$$\varepsilon_d = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

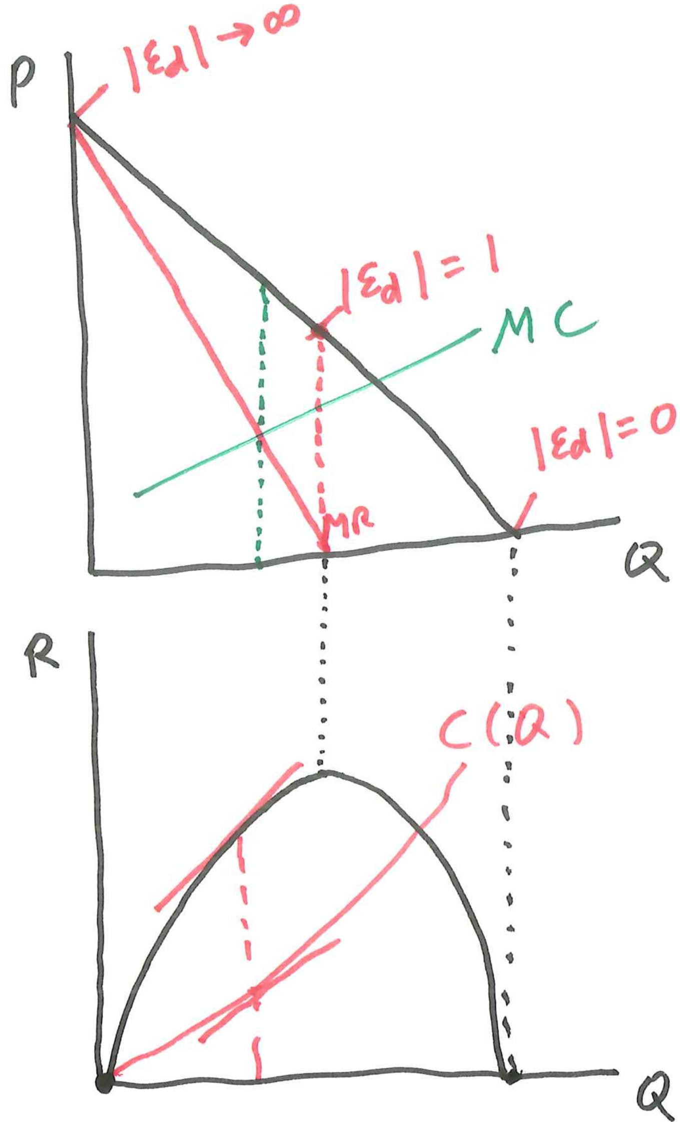
$$P\left(1 + \frac{1}{\varepsilon_d}\right) = MR$$

$$P\left(1 + \frac{1}{\varepsilon_d}\right) = 0$$

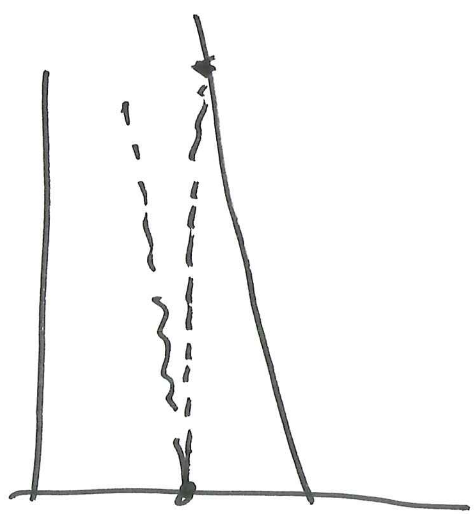
$$P + \frac{P}{\varepsilon_d} = 0$$

$$\frac{P}{\varepsilon_d} = -P$$

$$\varepsilon_d = -1$$



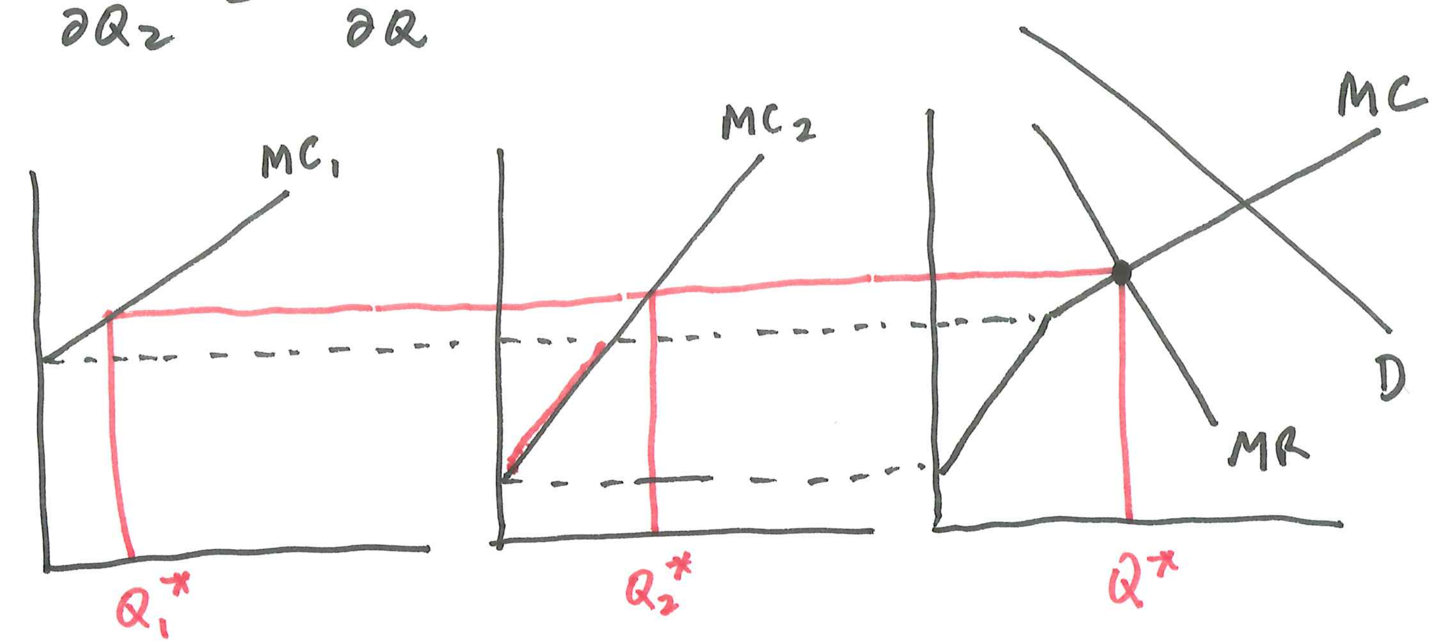
Compare Competition + Monopoly



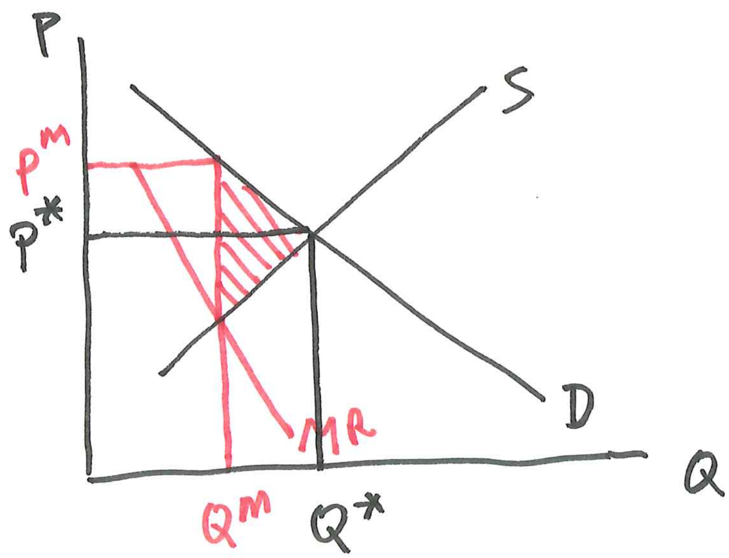
Multi-plant monopoly

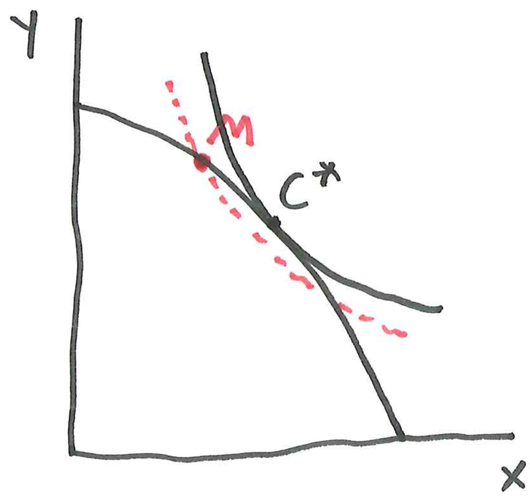
$$\pi = R(Q_1 + Q_2) - C_1(Q_1) - C_2(Q_2)$$

$$\left.\begin{aligned} \frac{\partial \pi}{\partial Q_1} &= \frac{\partial R}{\partial Q} \cdot - C_1' = 0 \\ \frac{\partial \pi}{\partial Q_2} &= \frac{\partial R}{\partial Q} \cdot - C_2' = 0 \end{aligned}\right\} C_1' = C_2' = MR$$



Competition vs Monopoly.





Cartels

$$\Pi = pQ - \sum_{i=1}^{n} C_i$$

$$= (q_i + \tilde{q}_i) \cdot p(q_i + \tilde{q}_i) - C_i(q_i) - \sum_{j \neq i} C_j(q_j)$$

$$\frac{\partial \Pi}{\partial q_i} = p + (q_i + \tilde{q}_i)\frac{dp}{dQ} - C_i' = 0$$

$$p + Q \cdot \frac{dp}{dQ} = C_i'$$

$$\underbrace{p + q_i \frac{dp}{dQ} - C_i'}_{} = -\tilde{q}_i \frac{dp}{dQ}$$

$$\Pi_i = q_i \cdot p(q_i + \tilde{q}_i) - C_i(q_i)$$

$$\frac{\partial \Pi_i}{\partial q_i} = p + q_i \frac{dp}{dQ} - C_i' > 0$$

$$= -\tilde{q}_i \frac{dp}{dQ} > 0$$