Uncertainty.

"Lotteries"

"Outcomes" $i = 1, \ldots, n$

Payoffs $\pi_1, \pi_2, \ldots, \pi_n$

Probabilities $p_1, p_2, \ldots, p_n$

- Objective
- Subjective

Expected Payoff $= \sum_{i=1}^{n} p_i \pi_i$

$EV(\text{lottery}) = -c + \sum_{i=1}^{n} p_i \pi_i$

"Fair" bet : $EV = 0$

St. Petersburg Paradox

1st :	prob	$1/2$	payoff	2
2nd :	"	$\frac{1}{2^2}$	"	2^2
3rd :	"	$\frac{1}{2^3}$	"	2^3

$$E(\pi) = \frac{1}{2} \cdot 2 + \frac{1}{2^2} \cdot 2^2 + \frac{1}{2^3} \cdot 2^3 + \ldots$$

$$\Rightarrow \infty$$

<u>Expected Utility Hypothesis</u>: People maximize their expected utility, not $E(\pi)$; further, $U(\cdot)$ is concave

<u>Why buy insurance?</u>

w_0 : 100

face possible loss of 64 with prob. $\frac{1}{10}$

w/ prob $\frac{9}{10}$, $w_1 = 100$; w/ prob $\frac{1}{10}$, $w_1 = 36$

$$E(w) = \bar{w} = \frac{9}{10} \cdot 100 + \frac{1}{10} \cdot 36 = 93.6$$

$$E(\text{loss}) = 6.4 = \underbrace{\frac{1}{10} \cdot 64}$$

$$U(w) = \sqrt{w}$$

$$EU_{\text{no insur}} = \frac{9}{10} \cdot \sqrt{100} + \frac{1}{10}\sqrt{36}$$

$$= \frac{9}{10} \cdot 10 + \frac{1}{10} \cdot 6 = 9.6$$

$$EU_{\text{w/ insur}} = \sqrt{w - m}$$

Max WTP: $\sqrt{100 - m} = 9.6$

$$100 - m = 92.16$$

$$m = 7.84$$

Cost of Risk : amount by which the Max WTP exceeds E(loss)

<u>Certainty Equivalent</u> (of a lottery) is the level of wealth that yields a utility equal to EU(lottery)

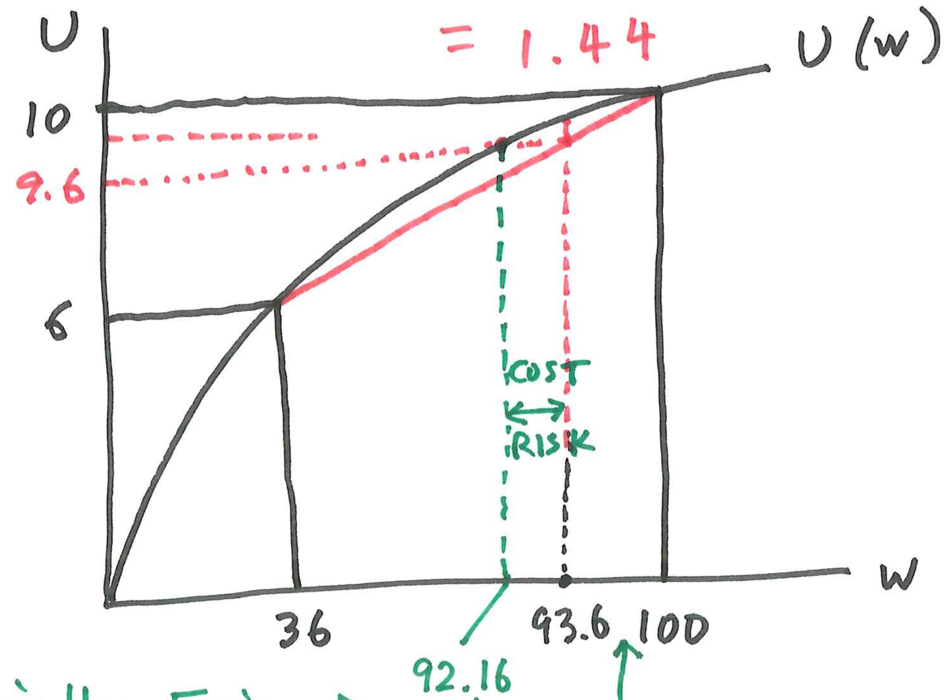
$$w_{CE} : \quad U(w_{CE}) = EU(\text{lottery})$$

Cost of Risk : $\bar{w} - w_{CE}$

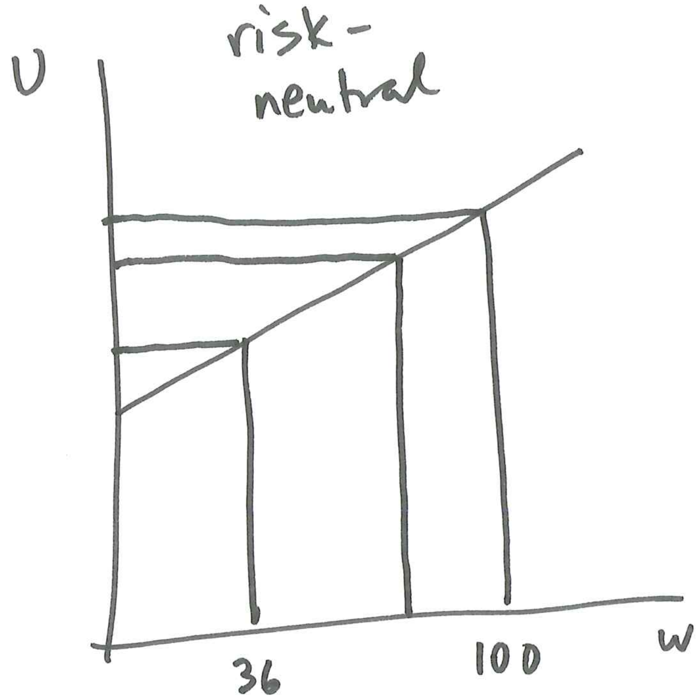
$= 93.6 - 92.16$

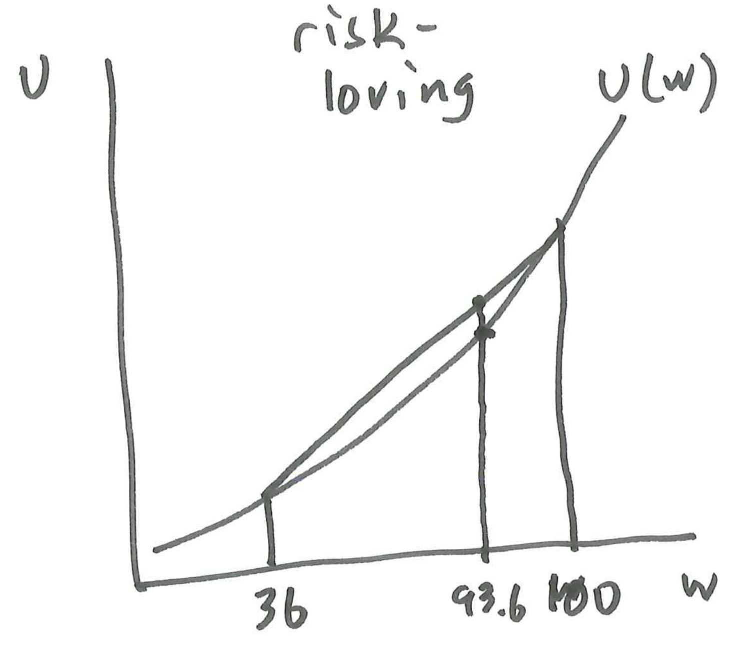
$= 1.44$

"Risk-averse"



Actuarially Fair Premium





$U(w) = a + bw$

State-contingent approach to Uncertainty

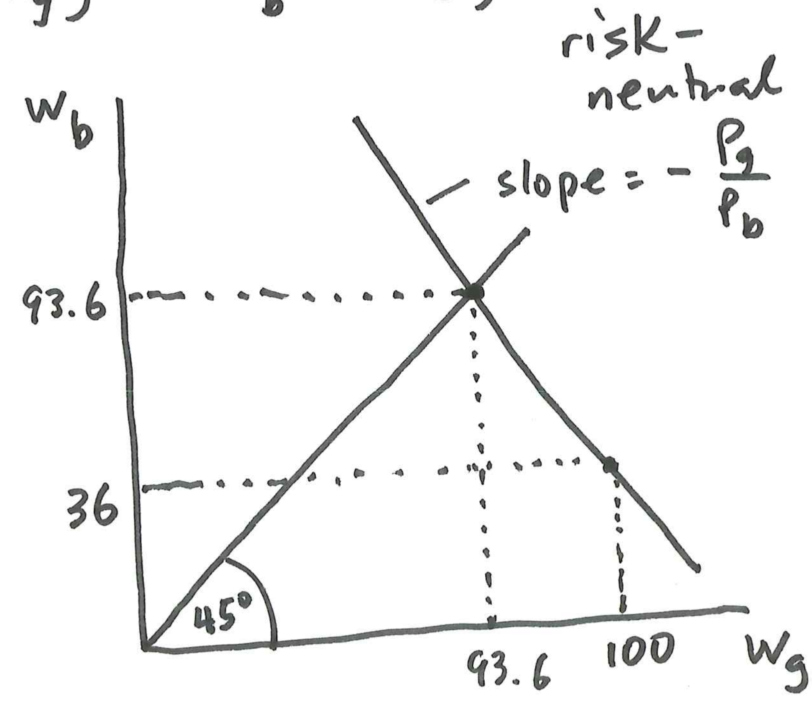
$$U = U(\underset{100}{w_g}, \underset{36}{w_b})$$

$$= P_g U(w_g) + P_b U(w_b)$$

$$P_g w_g + P_b w_b = \bar{U}$$

$$P_b w_b = \bar{U} - P_g w_g$$

$$w_b = \frac{\bar{U}}{P_b} - \frac{P_g}{P_b} \cdot w_g$$



$$\bar{U} = P_g U(w_g) + P_b U(w_b)$$

$$0 = P_g U'(w_g)\,dw_g + P_b U'(w_b)\,dw_b$$

$$\frac{dw_b}{dw_g} = -\frac{P_g U'(w_g)}{P_b U'(w_b)}$$

on 45^0 line, $w_g = w_b$

$$\frac{dw_b}{dw_g} = -\frac{P_g}{P_b}$$