σ in C-D prod. fn.

$$q = AK^{\alpha} L^{\beta}$$

$$MP_K = A \cdot \alpha K^{\alpha - 1} L^{\beta}$$

$$MP_L = AK^{\alpha} \cdot \beta L^{\beta - 1}$$

$$MRTS = \frac{MP_L}{MP_K} = \frac{AK^{\alpha}\beta L^{\beta-1}}{A\alpha K^{\alpha-1}L^{\beta}} = \frac{\beta K}{\alpha L}$$

$$\frac{K}{L} = \frac{\alpha}{\beta}(MRTS)^{1}$$

$$\sigma = \frac{\partial(K/L)}{\partial MRTS} \cdot \frac{MRTS}{(K/L)}$$

$$= \frac{\alpha}{\beta} \cdot \frac{MRTS}{\frac{\alpha}{\beta} \cdot MRTS} = 1$$

Returns to Scale

$\cancel{q_0 = \lambda A K_0^{\alpha}}$ $\quad q_0 = F(K_0, L_0)$

$$q_1 = F(mK_0, mL_0)$$

$q_1 = m q_0$: constant returns to scale

$q_1 > m q_0$: increasing " " "

$q_1 < m q_0$: decreasing " " "

$$q_0 = A K_0^{\alpha} L_0^{\beta}$$

$$q_1 = A (m K_0)^{\alpha} (m L_0)^{\beta}$$

$$= A (m^{\alpha+\beta}) K_0^{\alpha} L_0^{\beta}$$

$$= m^{\alpha+\beta} q_0$$

Constant returns to scale if $\alpha+\beta = 1$

Increasing " " " " $\alpha+\beta > 1$

Decreasing " " " " $\alpha+\beta < 1$

<u>Cost Function</u> shows the <u>minimum</u> cost of producing any q, given input prices.

$$C = C(q, w, r)$$

corresponds to exp. fn.

$$\text{Min } C = wL + rK$$

$$\text{s.t. } q(K, L) = \bar{q}$$

$$\Rightarrow L^*(w, r, q), \; K^*(w, r, q)$$

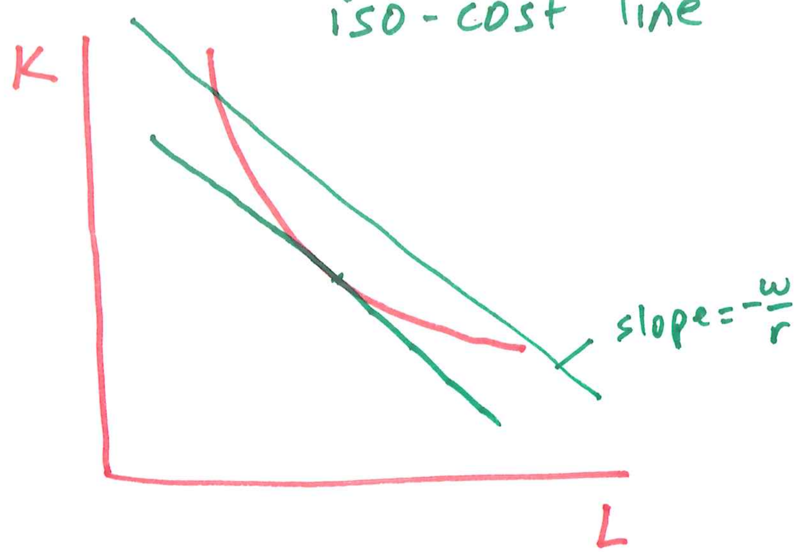
Conditional Input Demand Functions

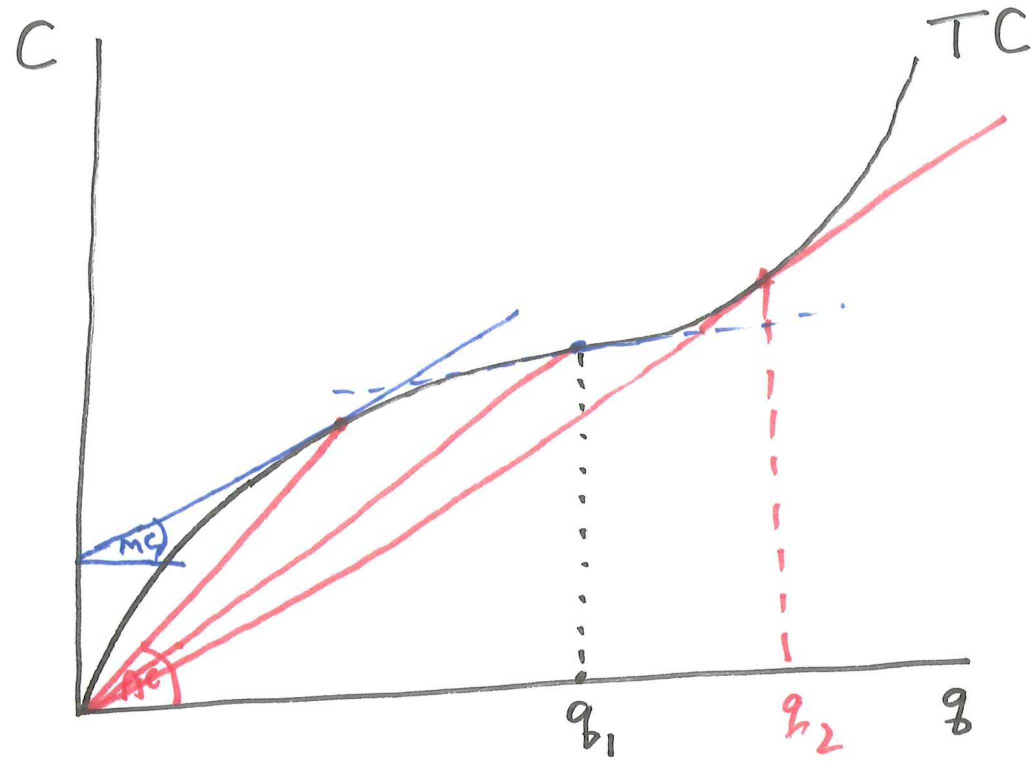
Long-Run (Total) Cost Function

$$C = w \cdot L^*(q, w, r) + r K^*(q, w, r)$$

$$\mathcal{L} = wL + rK + \lambda [\bar{q} - q(K, L)]$$

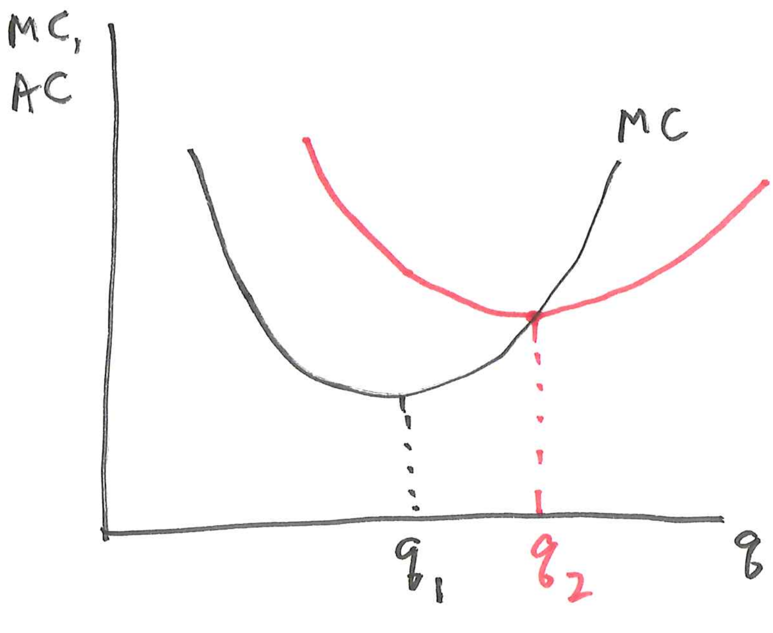
$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial L} &= w - \lambda q_L = 0 \\ \frac{\partial \mathcal{L}}{\partial K} &= r - \lambda q_K = 0 \end{aligned}\right\} \quad \frac{w}{r} = \frac{q_L}{q_K}$$





Average Cost: $AC = \frac{C}{q}$

Marginal Cost: $MC = \frac{dC}{dq}$



Cobb-Douglas :

$$q = AK^{\alpha}L^{\beta}$$

$$\frac{w}{r} = \frac{\beta K}{\alpha L} \implies K = \frac{\alpha}{\beta} \cdot \frac{w}{r} \cdot L$$

$$q = A\left[\frac{\alpha}{\beta} \cdot \frac{w}{r} \cdot L\right]^{\alpha} L^{\beta}$$

$$= A\left(\frac{\alpha}{\beta}\right)^{\alpha}\left(\frac{w}{r}\right)^{\alpha} L^{\alpha+\beta}$$

$$L^{\alpha+\beta} = \frac{q}{A}\left(\frac{\beta}{\alpha}\right)^{\alpha}\left(\frac{r}{w}\right)^{\alpha}$$

$$L^{*} = \left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\left(\frac{r}{w}\right)^{\frac{\alpha}{\alpha+\beta}}$$

$$K^{*} = \left(\frac{\alpha}{\beta}\right)\left(\frac{w}{r}\right)\left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\left(\frac{r}{w}\right)^{\frac{\alpha}{\alpha+\beta}}$$

$$= \left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}\left(\frac{w}{r}\right)^{\frac{\beta}{\alpha+\beta}}$$

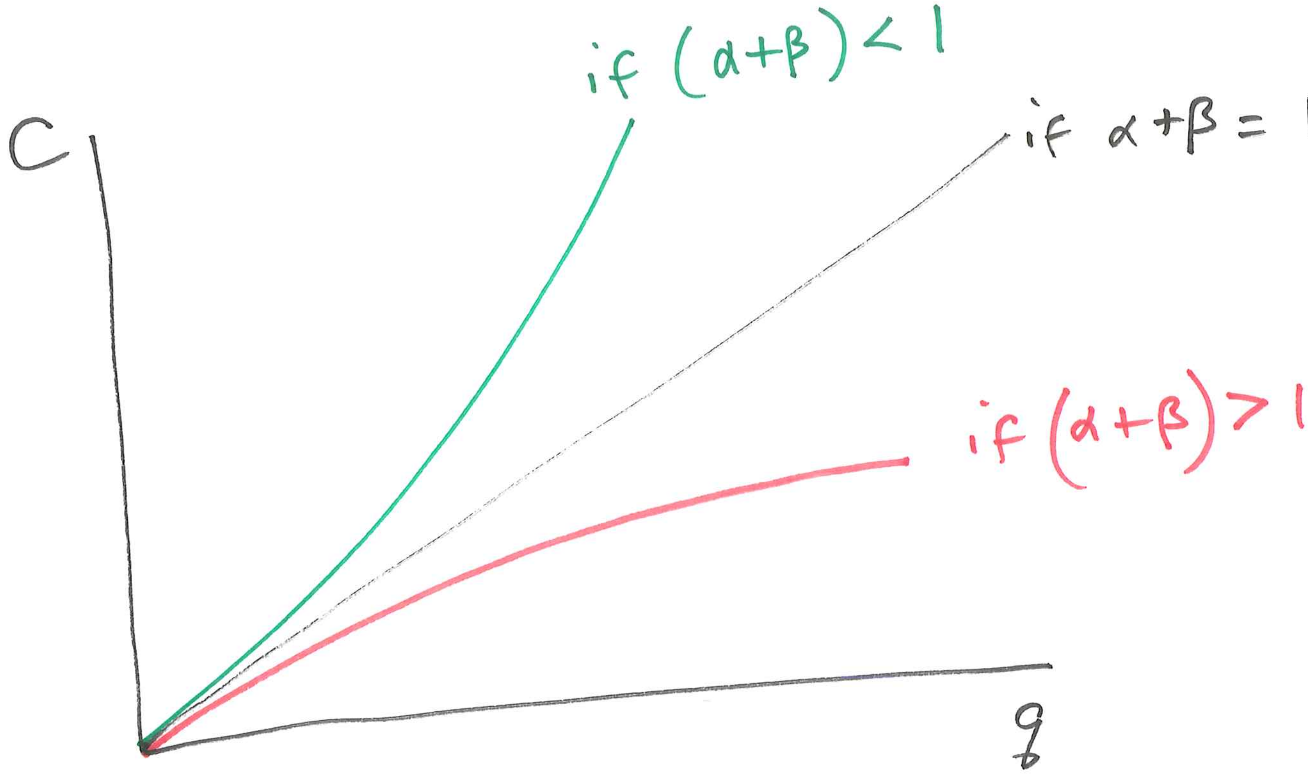
$$C(q,w,r) = wL^{*} + rK^{*}$$

$$=$$

$$C = \underbrace{\left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}} \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \underbrace{r^{\frac{\alpha}{\alpha+\beta}} \; w^{\frac{\beta}{\alpha+\beta}}}$$

$$+ \underbrace{\left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}} \left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \underbrace{w^{\frac{\beta}{\alpha+\beta}} \; r^{\frac{\alpha}{\alpha+\beta}}}$$

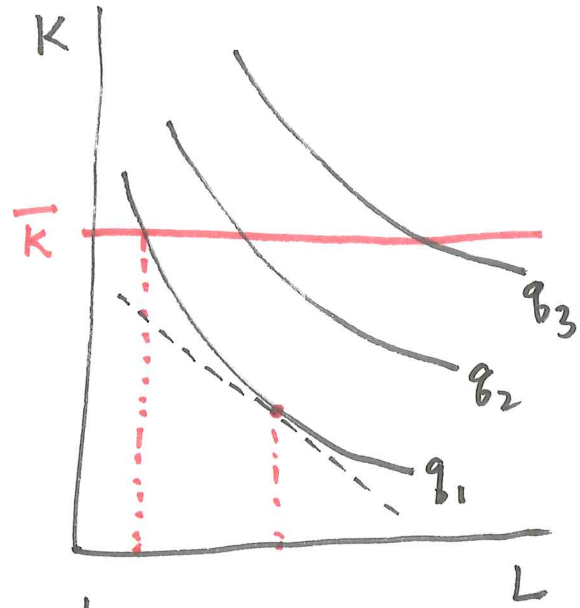
$$= \underbrace{\left[\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} + \left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}\right] \cdot \frac{1}{A^{\frac{1}{\alpha+\beta}}}}_{\text{constant}} \cdot q^{\frac{1}{\alpha+\beta}} \, w^{\frac{\beta}{\alpha+\beta}} \, r^{\frac{\alpha}{\alpha+\beta}}$$



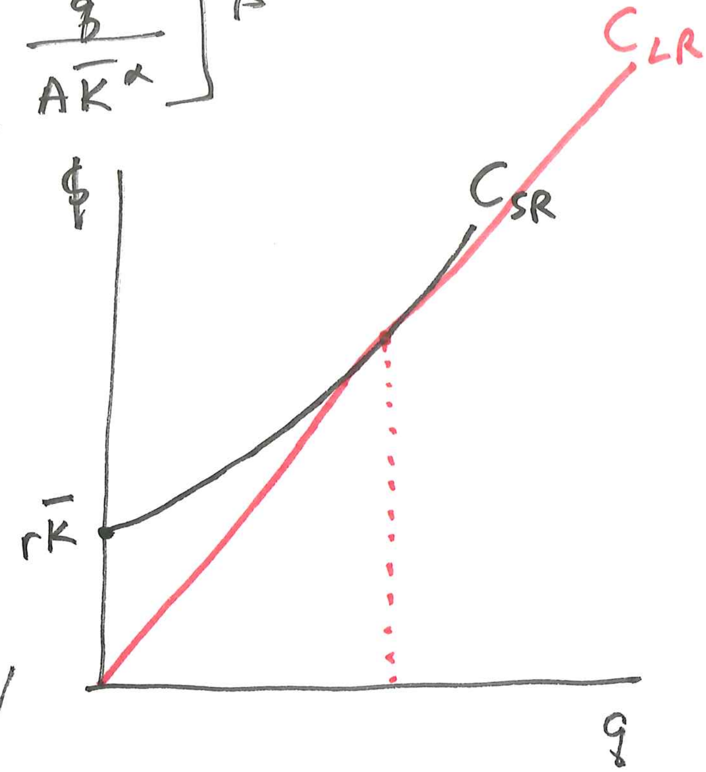
$$q = A\bar{K}^{\alpha}L^{\beta}$$

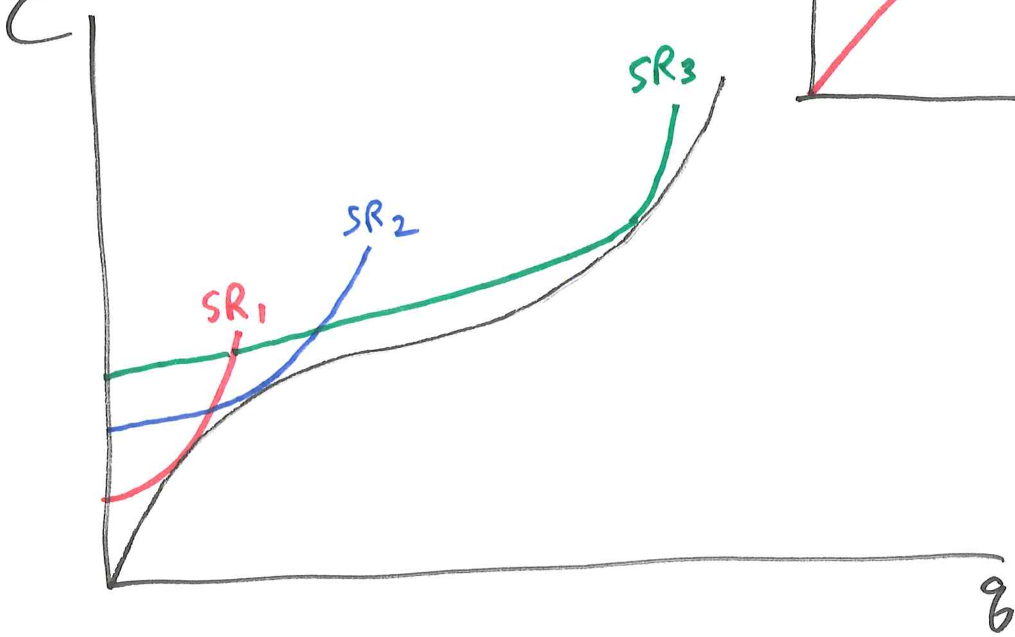
$$L^{\beta} = \frac{q}{A\bar{K}^{\alpha}}$$

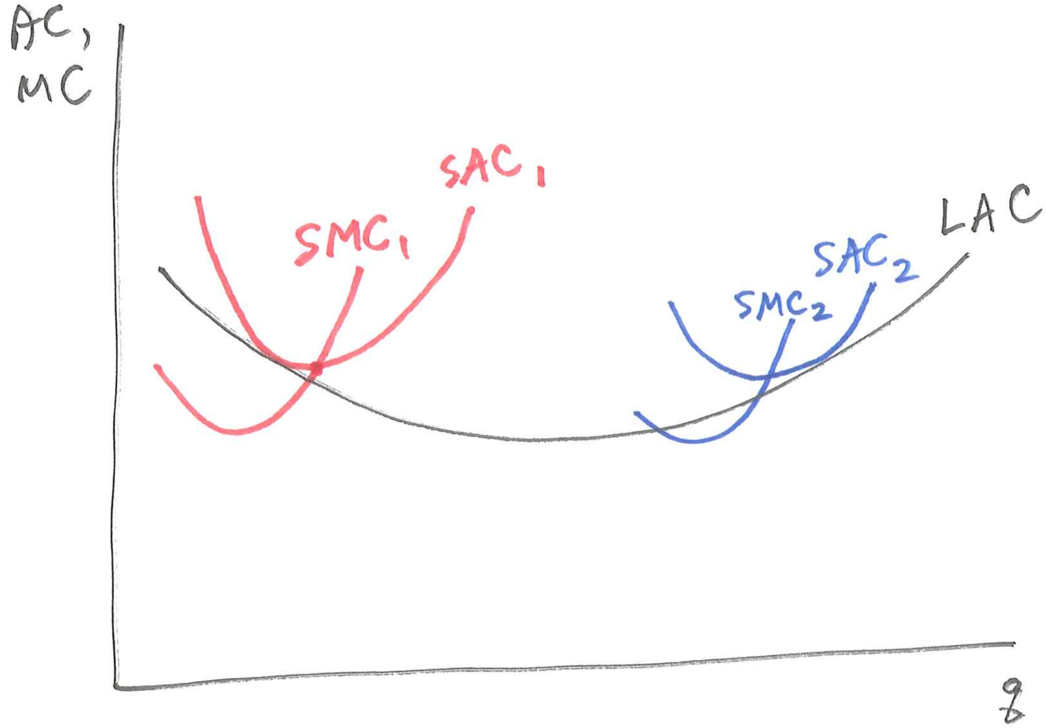
$$L^{*} = \left[\frac{q}{A\bar{K}^{\alpha}}\right]^{\frac{1}{\beta}}$$



$$C^{*}_{SR} = r\bar{K} + w\left[\frac{q}{A\bar{K}^{\alpha}}\right]^{\frac{1}{\beta}}$$






AC,
MC
SAC,
SMC,
LAC
SAC2
SMC2
q