$$\frac{P_1}{x_1} \cdot \frac{\partial x_1}{\partial P_1} = \frac{\partial x_1^h}{\partial P_1} \cdot \frac{P_1}{x_1^h} - \frac{P_1 x_1^h}{I} \frac{\partial x_1}{\partial I} \frac{I}{x_1}$$ SLUTSKY EQUATION

$$\underbrace{\varepsilon_1} = \varepsilon_1^h - \underbrace{\theta_1}\underbrace{\eta_1}$$ – in ELASTICITIES

Cobb-Douglas

$$U = x_1^2 x_2^3$$

$$\text{Min} \quad E = P_1 x_1 + P_2 x_2$$

$$\text{s.t.} \quad U = x_1^2 x_2^3$$

$$\frac{P_1}{P_2} = \frac{U_1}{U_2} = \frac{2x_2}{3x_1} \Longrightarrow x_2 = \frac{3}{2}\frac{P_1 x_1}{P_2}$$

$$U = x_1^2 \left(\frac{3}{2}\frac{P_1}{P_2} \cdot x_1\right)^3$$

$$= \left(\frac{3}{2}\right)^3 \left(\frac{P_1}{P_2}\right)^3 x_1^5$$

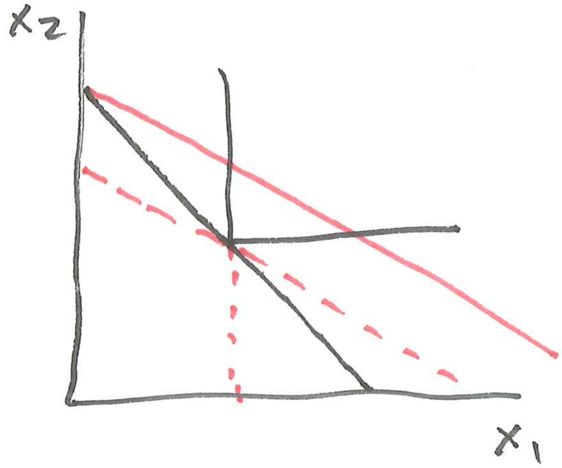
$$x_1^h = \left(\frac{2}{3}\right)^{\frac{3}{5}} \left(\frac{P_2}{P_1}\right)^{\frac{3}{5}} U^{1/5}$$

$$= \left(\frac{2}{3}\right)^{3/5} P_1^{-3/5} P_2^{3/5} U^{1/5}$$

$$x_2^h = \left(\frac{3}{2}\right)^{2/5} P_1^{2/5} P_2^{-2/5} U^{1/5}$$

Leontief :

$$U = \min\left[\frac{x_1}{\beta_1}, \frac{x_2}{\beta_2}\right]$$



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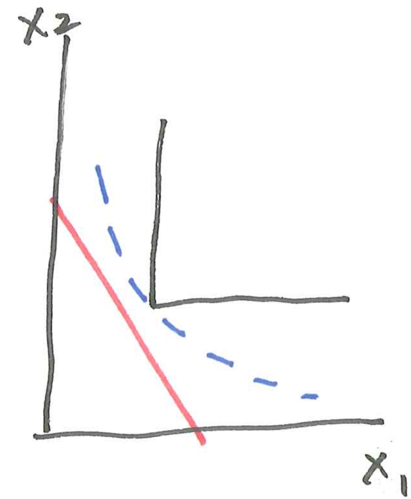
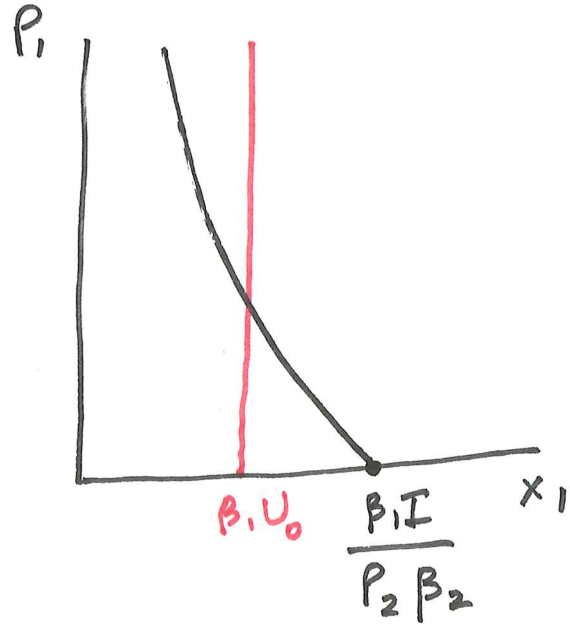


$$\frac{x_1^h}{\beta_1} = U$$

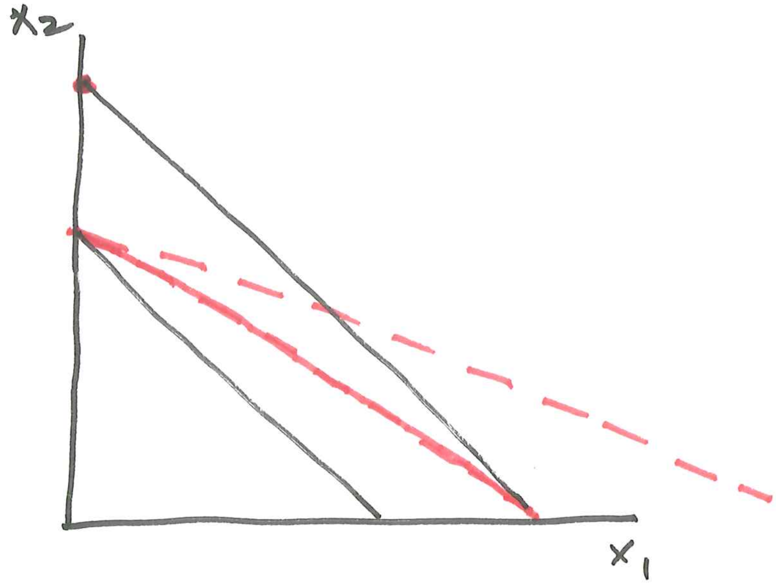
$$x_1^h = \beta_1 U_0 \quad ; \quad x_2^h = \beta_2 U_0$$

$$x_1^* = \frac{\beta_1 I}{P_1 \beta_1 + P_2 \beta_2}$$



Linear

$$U = \gamma_1 x_1 + \gamma_2 x_2$$

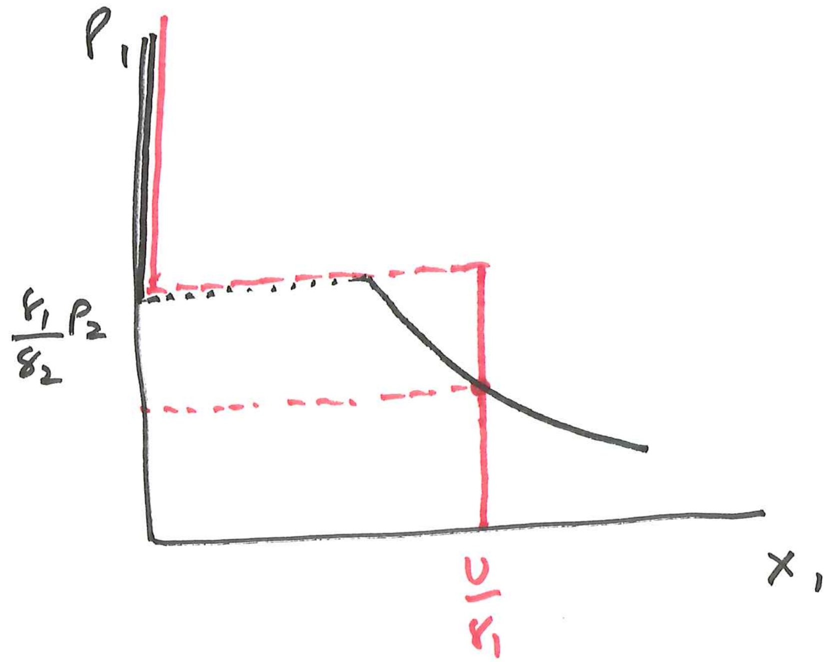


$$x_1^* = \begin{cases} \frac{I}{P_1} & \text{if } \frac{P_1}{P_2} < \frac{\gamma_1}{\gamma_2} \\ 0 & \text{if } \frac{P_1}{P_2} > \frac{\gamma_1}{\gamma_2} \end{cases}$$

$$x_1^h = \begin{cases} \frac{U}{\gamma_1} & \text{if } \frac{P_1}{P_2} < \frac{\gamma_1}{\gamma_2} \\ 0 & \text{if } \frac{P_1}{P_2} > \frac{\gamma_1}{\gamma_2} \end{cases}$$

$$y = \frac{I}{x}$$

$$xy = I$$



How to measure Changes in Welfare

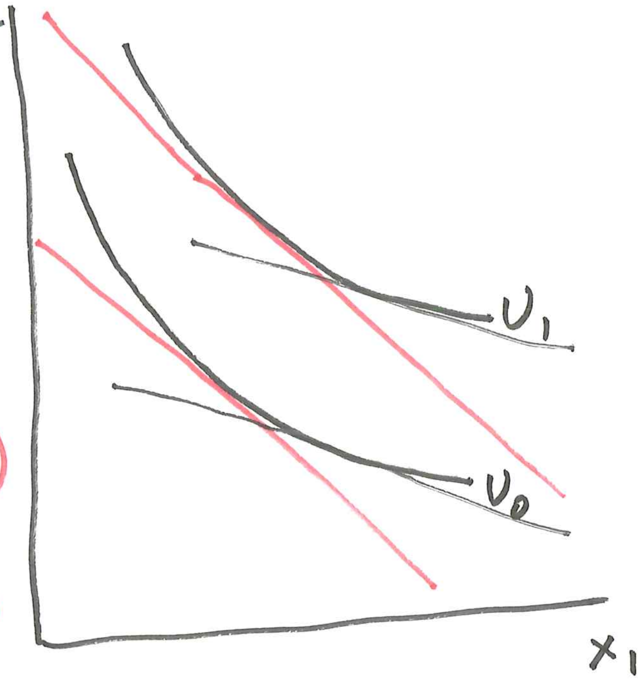
$\Delta W = E(P, U_1) - E(P, U_0)$

$\Delta W_{EV} = E(P_0, U_1) - E(P_0, U_0)$

EV: Equivalent Variation

$\Delta W_{CV} = E(P_1, U_1) - E(P_1, U_0)$

CV: Compensating Variation



Compensating Variation in Income is the change in I necessary after an economic change to restore the consumer to U_0

$$CV = E(P_1, U_0) - E(P_1, U_1)$$
$$= -\Delta W_{CV}$$

<u>Equivalent Variation in Income</u> is the change in I needed <u>before</u> an economic change that would push the consumer to U_1.

<u>Example</u> Cobb-Douglas: $U = x_1 x_2$

$$x_1^* = \frac{I}{2P_1} \qquad x_2^* = \frac{I}{2P_2}$$

$$V = \frac{I}{2P_1} \cdot \frac{I}{2P_2} = \frac{1}{4} \frac{I^2}{P_1 P_2}$$

$$x_1^h = \left(\frac{U P_2}{P_1}\right)^{1/2}, \quad x_2^h = \left(\frac{U P_1}{P_2}\right)^{1/2}$$

$$E = P_1 x_1^h + P_2 x_2^h$$

$$= 2U^{1/2} P_1^{1/2} P_2^{1/2}$$

$I_0 = \$100$, $P_1^0 = \$2$, $P_2^0 = \$1$

$$V_0 = \frac{1}{4} \frac{100^2}{2 \cdot 1} = 1250$$

$$V_1 = \frac{1}{4} \cdot \frac{100^2}{\ldots} = 625$$

Suppose $P_1 \uparrow$ to \$4

How much worse off?

$$\Delta W_{EV} = E(p^0, U_1) - \underset{\$100}{E(p^0, U_0)}$$

$$= 2(625)^{1/2}(2)^{1/2}(1)^{1/2} - 100$$

$$= 70.71 - 100$$

$$= -29.29$$

$$\Delta W_{CV} = E(p', U_1) - E(p', U_0)$$

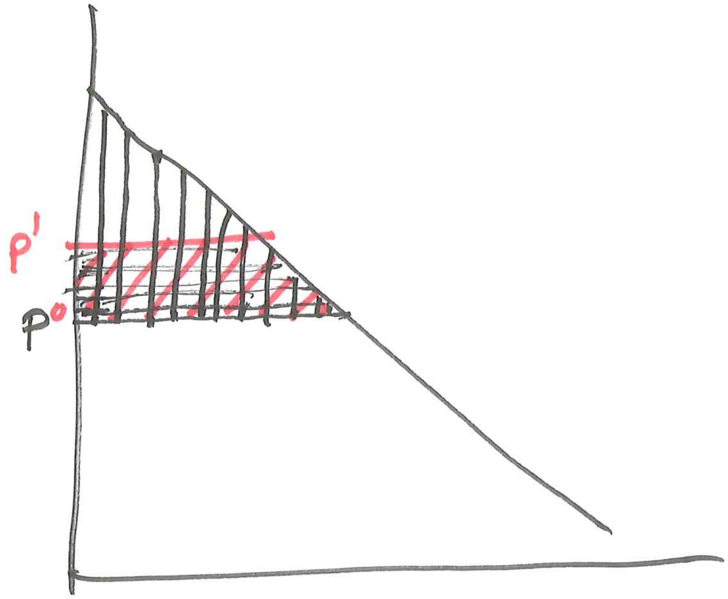
$$= 100 - 2(1250)^{1/2}(4)^{1/2}(1)^{1/2}$$

$$= 100 - 141.42$$

$$= -41.42$$

Marshall :

$$x_1^* = \frac{I}{2P_1}$$



$$\Delta W_{marshall} = \int_4^2 \frac{I}{2P_1} \, dP_1$$

$$= 50 \ln P_1 \Big|_4^2$$

$$= 50 (\ln 2 - \ln 4)$$

$$= 50 (0.693 - 1.386)$$

$$= -34.66$$