Leontief U F^n:

$$u(x_1, x_2) = \text{Min}\left[\frac{x_1}{\beta_1}, \frac{x_2}{\beta_2}\right]$$

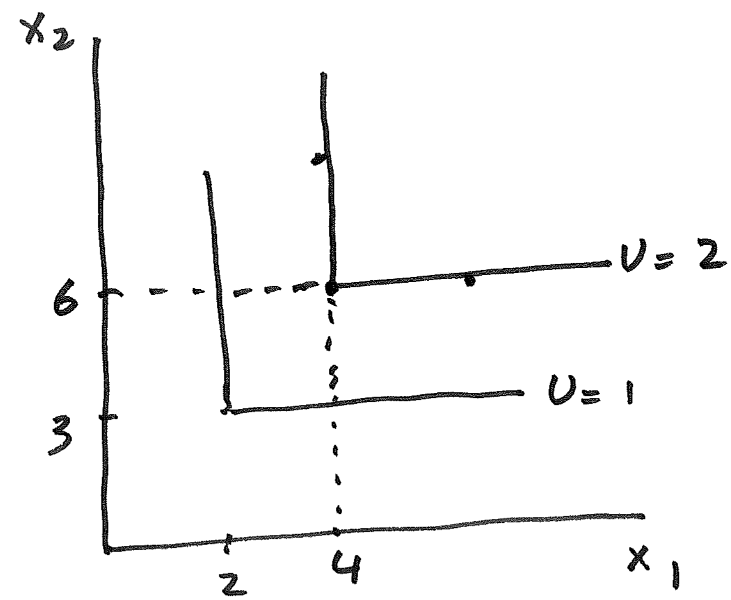
$$= \text{Min}\left[\frac{x_1}{2}, \frac{x_2}{3}\right]$$

$$u(4,6) = \text{Min}\left[\frac{4}{2}, \frac{6}{3}\right] = 2$$

$$u(6,6) = \text{Min}\left[\frac{6}{2}, \frac{6}{3}\right] = 2$$

$$u(4,8) = \text{Min}\left[\frac{4}{2}, \frac{8}{3}\right] = 2$$

perfect complements



Quasi-Linear U F^n

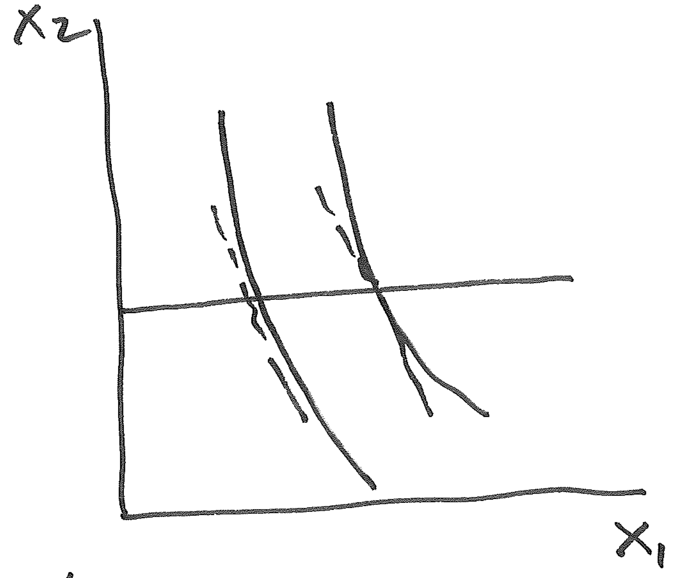
$$u(x_1, x_2) = B x_1 + f(x_2)$$

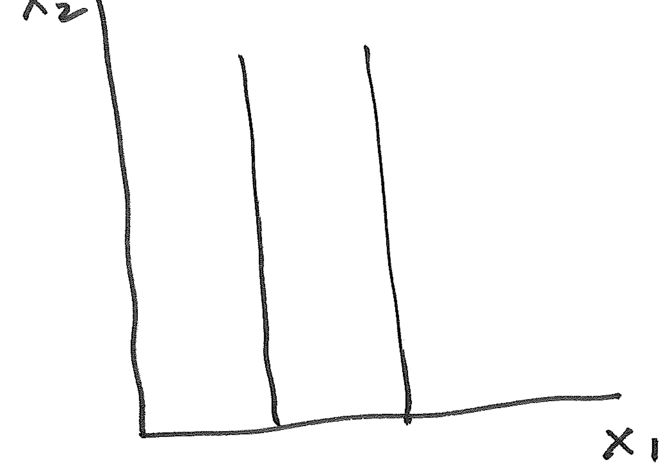
$$u_1 = B$$

$$u_2 = f'(x_2)$$

slope of IC =

$$-\frac{U_1}{U_2} = \frac{B}{F'(x_2)}$$





Hicks's Theory of Consumer Behavior

Max $U(x_1, x_2)$

subject to Budget : $P_1 x_1 + P_2 x_2 = I$

$$\mathcal{L} = U(x_1, x_2) - \lambda\left[P_1 x_1 + P_2 x_2 - I\right]$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = u_1 - \lambda P_1 = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = U_2 - \lambda P_2 = 0$$

Tangency:

$$\frac{u_1}{U_2} = \frac{P_1}{P_2} \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = P_1 x_1 + P_2 x_2 - I = 0 \quad (2)$$

$$\left\{ \begin{array}{l} x_1^* = x_1^*(P_1, P_2, I) \\ x_2^* = x_2^*(P_1, P_2, I) \end{array} \right.$$

Marshallian or Ordinary Demand Functions

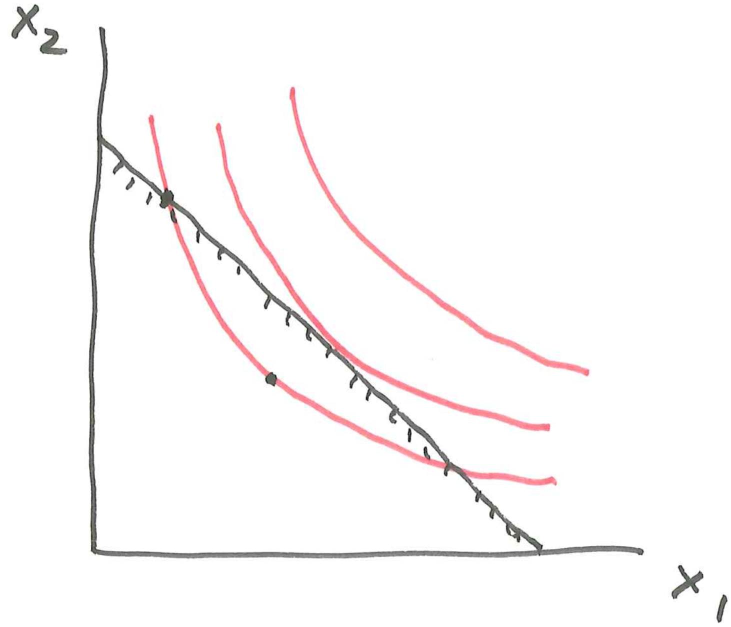
If we fix P_2, I:

$$x_1^* = x_1^*(P_1, \overline{P_2}, \overline{I})$$

Demand Curve

$$U\left(x_1^*(P_1, P_2, I),\ x_2^*(P_1, P_2, I)\right) \equiv V(P_1, P_2, I)$$

indirect utility function



Cobb-Douglas :

$$U(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2}$$

$$\frac{U_1}{U_2} = \frac{\alpha_1 x_1^{\alpha_1 - 1} x_2^{\alpha_2}}{x_1^{\alpha_1} \cdot \alpha_2 x_2^{\alpha_2 - 1}} = \frac{\alpha_1 x_2}{\alpha_2 x_1}$$

$\therefore$ Tangency : $\dfrac{p_1}{p_2} = \dfrac{\alpha_1 x_2}{\alpha_2 x_1}$

$$\underbrace{\frac{\alpha_2}{\alpha_1} p_1 x_1} = p_2 x_2$$

$$p_1 x_1 + \underbrace{p_2 x_2} = I$$

$$p_1 x_1 \left(1 + \frac{\alpha_2}{\alpha_1}\right) = I$$

$$x_1^* = \frac{\alpha_1}{\alpha_1 + \alpha_2} \cdot \frac{I}{p_1}$$

$$x_2^* = \frac{\alpha_2}{\alpha_1+\alpha_2} \cdot \frac{I}{P_2}$$

$$\theta_2 = \frac{P_2 x_2^*}{I} = \frac{\alpha_2}{\alpha_1+\alpha_2}$$

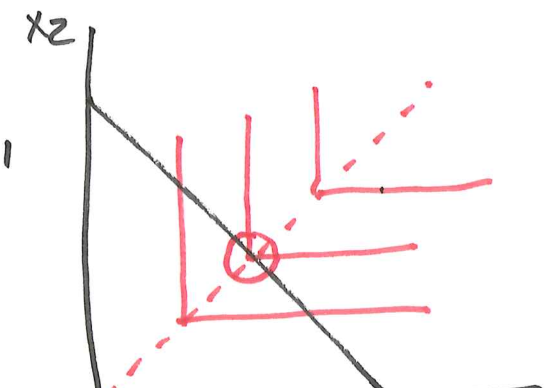
Budget share of good 2

<u>Engel's Law</u> : Budget share of food $\downarrow$ as $I \uparrow$

<u>Leontief</u>

$$\text{Max } U = \text{Min}\left[\frac{x_1}{\beta_1}, \frac{x_2}{\beta_2}\right]$$

$$\text{s.t. } P_1 x_1 + P_2 x_2 = I$$



$$\frac{x_1}{\beta_1} = \frac{x_2}{\beta_2} \rightarrow x_2 = \frac{\beta_2}{\beta_1} x_1$$

$$P_1 x_1 + P_2\left(\frac{\beta_2}{\beta_1} x_1\right) = I$$

$$\left(\frac{P_1\beta_1 + P_2\beta_2}{\beta_1}\right) x_1 = I$$

$$x_1^* = \frac{\beta_1 I}{p_1\beta_1 + p_2\beta_2}$$

$$x_2^* = \frac{\beta_2 I}{p_1\beta_1 + p_2\beta_2}$$

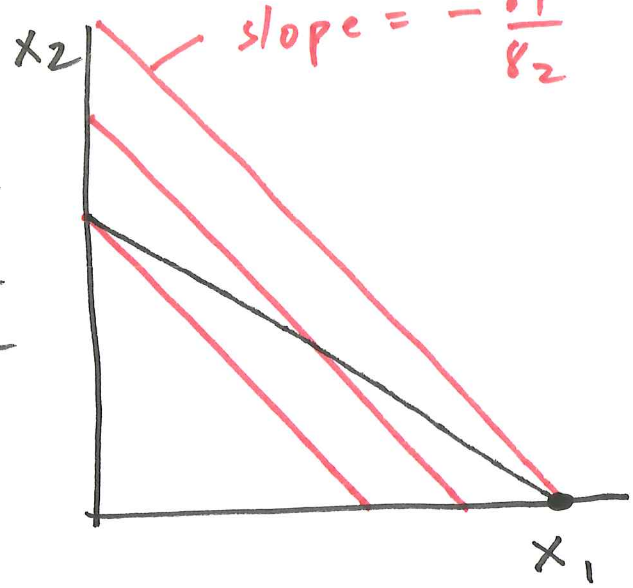
Linear U F^n.

$$\text{Max } U(x_1, x_2) = \gamma_1 x_1 + \gamma_2 x_2$$

$$\text{s.t. } p_1 x_1 + p_2 x_2 = I$$

$$x_1^* = \begin{cases} \frac{I}{p_1} & \text{if } \frac{p_1}{p_2} < \frac{\gamma_1}{\gamma_2} \\ 0 & \text{if } \frac{p_1}{p_2} > \frac{\gamma_1}{\gamma_2} \end{cases}$$

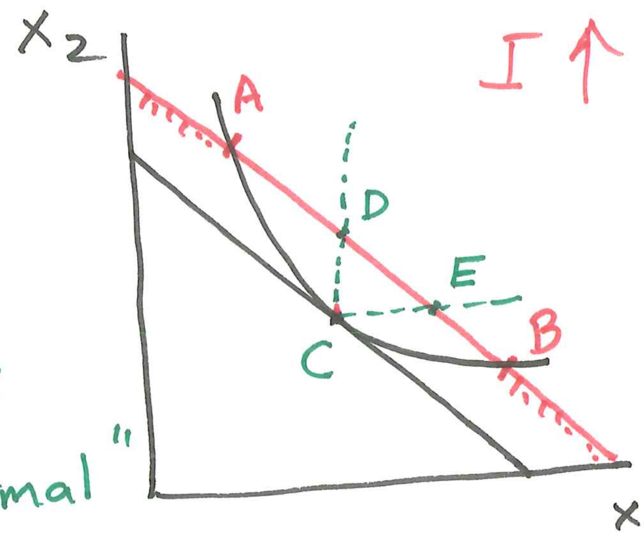
$$x_2^* = \begin{cases} 0 & \text{if } \frac{p_1}{p_2} < \frac{\gamma_1}{\gamma_2} \\ \frac{I}{p_2} & \text{if } \frac{p_1}{p_2} > \frac{\gamma_1}{\gamma_2} \end{cases}$$



Properties of Demand Functions.

$$x_1 = x_1(P_1, P_2, I)$$

Income changes



sign $\frac{\partial x_1}{\partial I}$?

If choice is on DE :

both goods are "normal"

Normal good : $\frac{\partial x}{\partial I} > 0$

Inferior good : $\frac{\partial x}{\partial I} < 0$

If choice is on AD :

x_1 is inferior

x_2 is normal

If choice is on EB :

x_1 is normal

x_2 is inferior

Income elasticity of demand:

$$\eta = \frac{\partial x}{\partial I} \cdot \frac{I}{x}$$

Normal good : $\eta > 0$

Inferior " : $\eta < 0$

Necessity : $1 > \eta > 0$

Luxury : $\eta > 1$

$$\theta = \frac{p X(I)}{I}$$

$$\frac{\partial \theta}{\partial I} = \frac{I \cdot p \cdot \frac{\partial X}{\partial I} - pX \cdot \frac{\partial I}{\partial I}}{I^2}$$

$$= \frac{p x \cdot \frac{\partial x}{\partial I} \cdot \frac{I}{x} - pX}{I^2}$$

$$= \frac{px(\eta - 1)}{I^2}$$

Necessity : $\frac{\partial \theta}{\partial I} < 0$, Luxury: $\frac{\partial \theta}{\partial I} > 0$

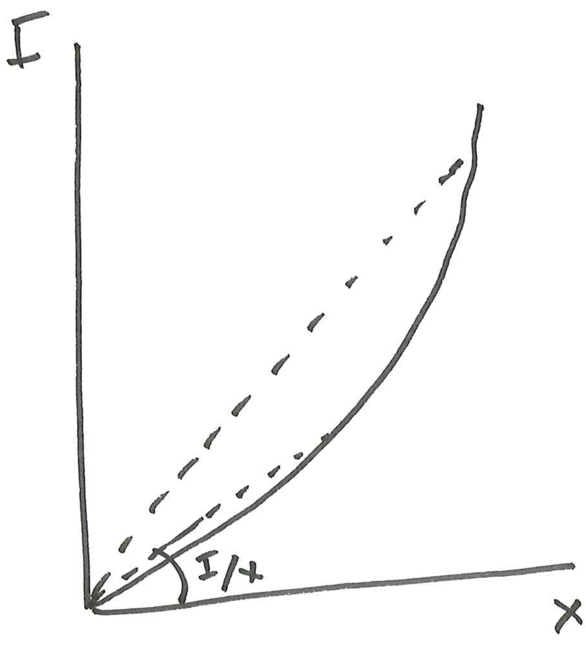
$$\sum_{i=1}^{n} p_i x_i = I$$

$$\sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial I} = 1$$

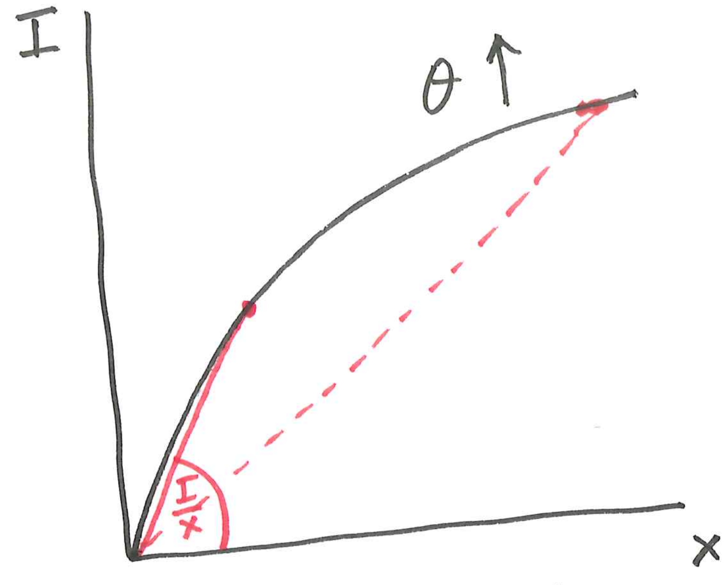
$$\sum_{i=1}^{n} \frac{p_i x_i}{I} \cdot \frac{\partial x_i}{\partial I} \cdot \frac{I}{x_i} = 1$$

$$\sum_{i=1}^{n} \theta_i \underline{\underline{\eta_i}} = 1$$

Engel curve

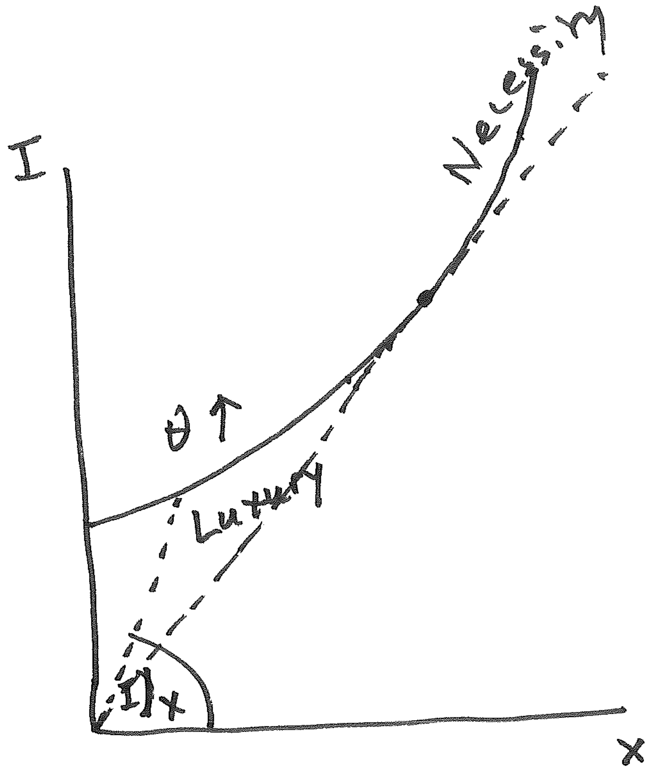


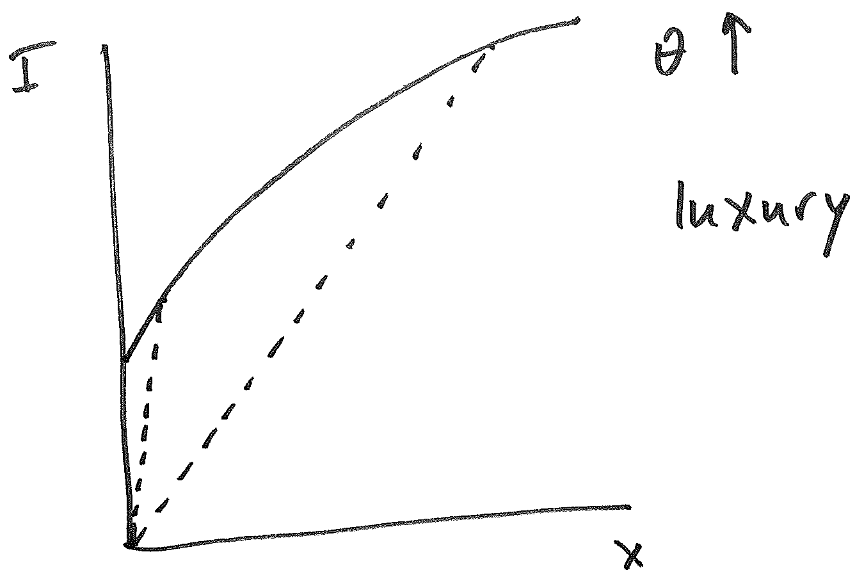
Necessity.



~~Necessity~~
Luxury

$$\theta = \frac{px}{I}$$


I
Necessity
θ ↑
Luxury
I/x
x


I
θ ↑
luxury
x