Utility Functions:

$U(x_1, x_2, \ldots x_n) = U_1$

1. Complete
2. Transitivity

   $A \succ B$ & $B \succ C$ then $A \succ C$

3. Continuity

   $A \succ B$ then $A \succ B'$

4. Non-satiety

$$\frac{\partial U}{\partial x_1} > 0$$

$$\frac{\partial U}{\partial x_1} \geq 0 \longrightarrow \text{free disposal.}$$

5. Diminishing Marginal Rate of Substitution

$$u_2^2 u_{11} - 2u_1 u_2 u_{12} + u_1^2 u_{22} < 0$$

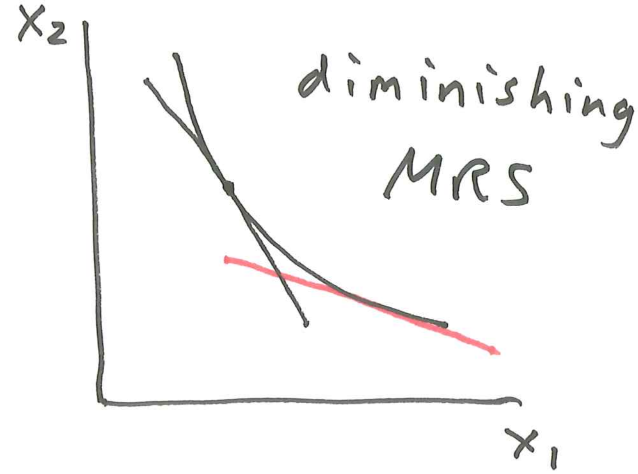
Notation: $u_1 \equiv \frac{\partial U}{\partial x_1}$, $u_2 \equiv \frac{\partial U}{\partial x_2}$

$u_{11} \equiv \frac{\partial^2 U}{\partial x_1^2}$, $u_{22} \equiv \frac{\partial^2 U}{\partial x_2^2}$

Young's Theorem:

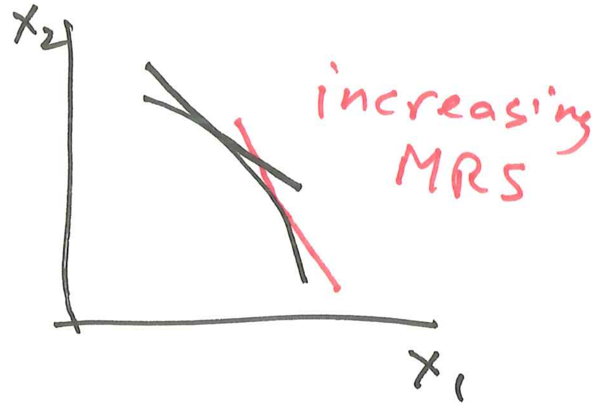
$u_{12} = u_{21} \longleftarrow u_{12} = \frac{\partial^2 U}{\partial x_1 \partial x_2}$ $\quad u_{21} = \frac{\partial^2 U}{\partial x_2 \partial x_1}$

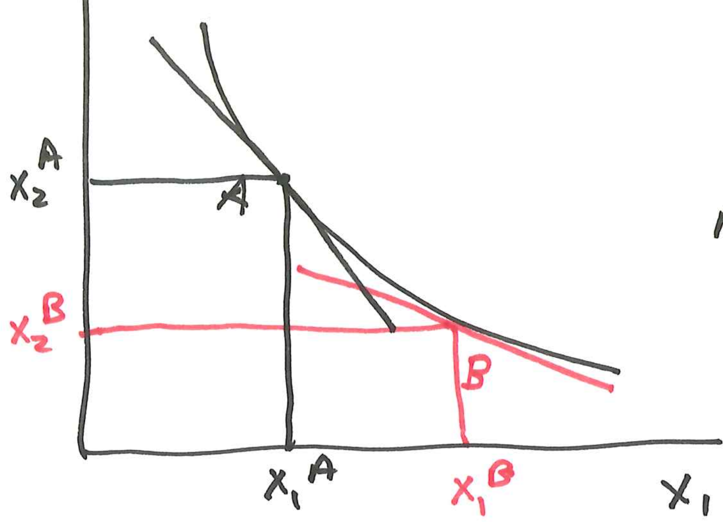
3. Convex to the origin



MRS ≡ absolute value of slope of I.C.

$$MRS \equiv \left| \left. \frac{dx_2}{dx_1} \right|_{\bar{u}} \right|$$





$A \rightarrow B$

$u_1 \downarrow$

$u_2 \uparrow$

$$u(x_1, x_2) = \bar{u}$$

$$d\bar{u} = 0 = u_1 dx_1 + u_2 dx_2$$

$$\frac{dx_2}{dx_1} = -\frac{u_1}{u_2}$$

$$MRS = \frac{u_1}{u_2}$$

$$MRS = \frac{U_1(x_1, x_2)}{U_2(x_1, x_2)}$$

$$\left.\frac{dMRS}{dx_1}\right|_{\bar{U}} = \frac{U_2\left[\frac{dU_1}{dx_1}\right] - U_1\left[\frac{dU_2}{dx_1}\right]}{U_2^2}$$

$$= \frac{U_2\left[U_{11} + U_{12}\frac{dx_2}{dx_1}\right] - U_1\left[U_{21} + U_{22}\frac{dx_2}{dx_1}\right]}{U_2^2}$$

$$= \frac{U_2\left[U_{11} + U_{12}\left(-\frac{U_1}{U_2}\right)\right] - U_1\left[U_{21} + U_{22}\left(-\frac{U_1}{U_2}\right)\right]}{U_2^2}$$

$$= \frac{U_2 U_{11} - U_1 U_{12} - U_1 U_{21} + \frac{U_1^2}{U_2} U_{22}}{U_2^2}$$

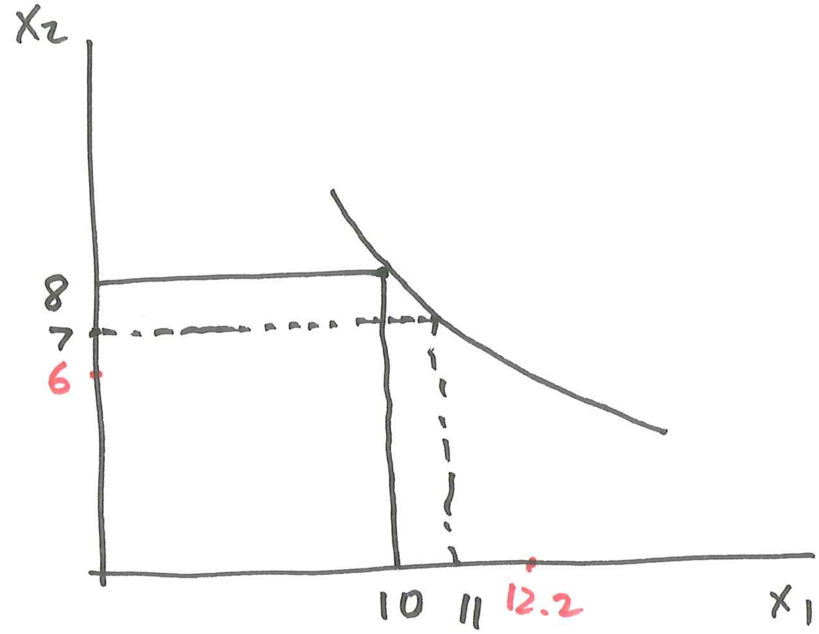
$$= \frac{U_2^2 U_{11} - U_1 U_2 U_{12} - U_1 U_2 U_{21} + U_1^2 U_{22}}{U_2^3}$$

$$= \frac{\overset{+}{U_2^2}\,\overset{-}{U_{11}} - 2\overset{+}{U_1}\overset{+}{U_2} U_{12} + \overset{+}{U_2^2}\,\overset{-}{U_{22}}}{\underset{+}{U_2^3}}$$

---

eg $U = x_1^2 x_2^3$

$U_1 = 2x_1 x_2^3$



Examples of Utility Functions

Cobb-Douglas

$$U(x_1, x_2) = A\,\underbrace{x_1^{\alpha_1} x_2^{\alpha_2}}\;\left[\, x_3^{\alpha_3}\; x_4^{\alpha_4} \dots \right.$$

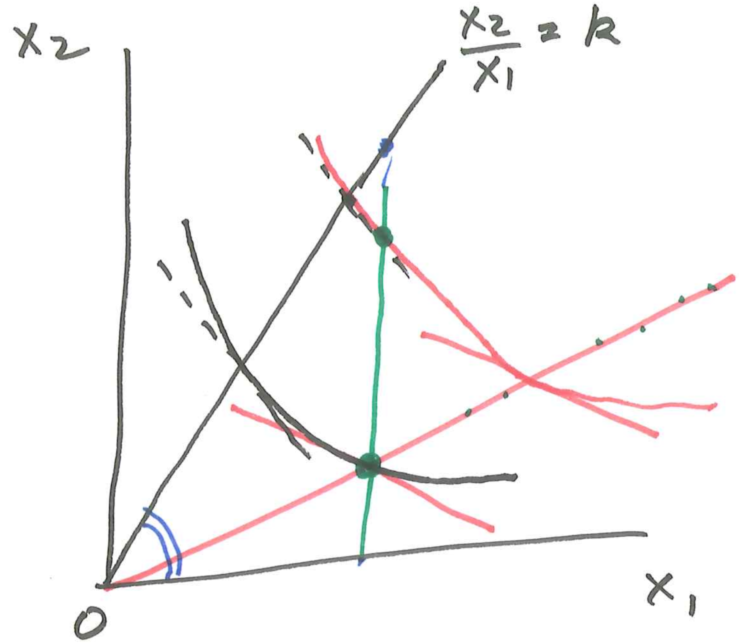
Homotheticity (Homothetic Function).

$$\left.\frac{dx_2}{dx_1}\right|_{\bar{U}} = -\frac{U_1}{U_2}$$

$$U_1 = \alpha_1\, x_1^{\alpha_1 - 1}\; x_2^{\alpha_2}$$

$$U_2 = \alpha_2\, x_1^{\alpha_1}\; x_2^{\alpha_2 - 1}$$

$$\text{slope of IC} = -\frac{U_1}{U_2} = -\frac{\alpha_1 x_1^{\alpha_1 - 1}\, x_2^{\alpha_2}}{\alpha_2\, x_1^{\alpha_1}\, x_2^{\alpha_2 - 1}} = -\frac{\alpha_1 x_2}{\alpha_2 x_1}$$



$$\text{slope} = -\frac{\alpha_1}{\alpha_2}\left(\frac{x_2}{x_1}\right)$$

Linear Utility Fn.

$$U(x_1, x_2) = \gamma_1 x_1 + \gamma_2 x_2$$

$$u_1 = \gamma_1, \quad u_2 = \gamma_2$$

Along IC:

$$\gamma_1 x_1 + \gamma_2 x_2 = \bar{U}$$

$$x_2 = \frac{\bar{U}}{\gamma_2} - \frac{\gamma_1}{\gamma_2} x_1$$