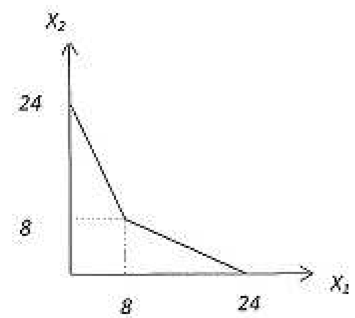
EC 501: Midterm Exam Solutions (Fall 2017)

1. (a) .



(b) The bundle (10,7) yields a utility of 24, so it is on the indifference curve drawn in part (a). Clearly, the bundle is to the right of the kink, so it lies on a linear segment of the indifference curve. The slope of this segment is $-\frac{1}{2}$, so the $MRS = \frac{1}{2}$.

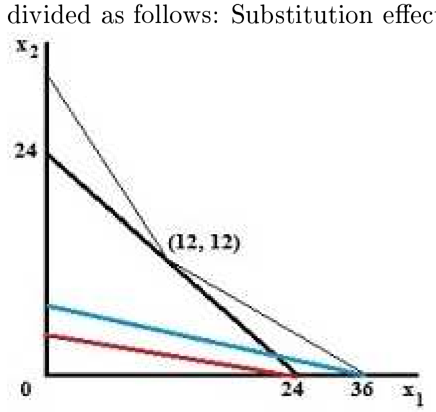
(c) The slope of the budget constraint is now -1, which lies in between the slopes of the two segments of the indifference curves. Therefore, the chosen bundle would be at the kink:

$$(x_1^*, x_2^*) = (12, 12).$$

(d) In this situation, the budget constraint is flatter than the indifference curve and so the chosen bundle would be at a corner:

$$(x_1^{**}, x_2^{**}) = (24, 0).$$

The new budget constraint is the red line in the diagram. The cheapest way to attain the original utility level at the new prices would be with the iso-expenditure line shown as the blue line; the chosen bundle would be (36,0). Therefore, the change in x_2 from 12 to 0 can be divided as follows: Substitution effect = -12, Income effect = 0.



2. (a) Mary's constraint is

$$H + L = 24,$$

where L is the number of hours she works. But her consumption is $c = wL$, which means $L = \frac{c}{w}$. Thus her constraint can be written as

$$H + \frac{1}{w}c = 24.$$

Mary's problem is to then maximize her utility subject to this constraint, which looks much like a budget constraint where her "income" is 24, $p_H = 1$ and $p_I = \frac{1}{w}$.

Since the utility function is Leontief, we know that Mary will always choose a point at the kink of one of her indifference curves. Therefore,

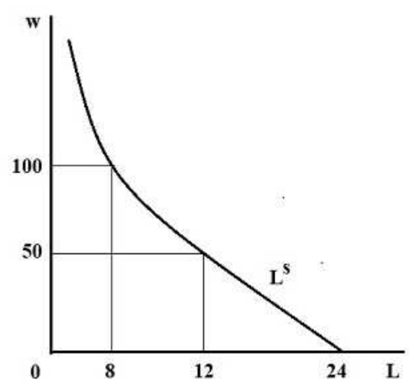
$$H = \frac{c}{50} \qquad or \qquad c = 50H.$$

Substituting this in her constraint, we find

$$H + \frac{50H}{w} = 24 \qquad or \qquad H = \frac{24w}{w+50}$$

Then her labor supply, which is (24-H), is

$$L^s(w) = \frac{1200}{w+50}.$$



(b) If $w = \$100, \quad L^s = 8\ hours.$

(c) If $w = \$50, \quad L^s = 12\ hours.$

(d) If Mary starts getting \$200 per day in addition to what she earns, her daily consumption will be

$$c = 200 + wL.$$

Then her constraint becomes

$$H + \frac{1}{w}c = 24 + \frac{200}{w}.$$

Solving in the same way as we did in part (a), we find the labor supply curve to be

$$L^s(w) = \frac{1000}{w + 50}.$$

Given this labor supply curve, we see that $w = \$50 \quad \rightarrow \quad L^s = 10\ hours.$

3. (a) The equation of a typical isoquant is

$$\frac{KL}{K+L} = q \qquad \rightarrow \qquad K = \frac{L\bar{q}}{L - \bar{q}}.$$

Note from this equation that, for K to be positive, we must have $L > \bar{q}$, which will be assumed in all that follows. To check the shape of this isoquant, we need to differentiate:

$$\frac{dK}{dL} = \frac{(L-\bar{q})\bar{q} - L\bar{q}}{(L-\bar{q})^2} = -\frac{\bar{q}^2}{(L-\bar{q})^2} < 0.$$

From this we see that the isoquant is downward-sloping. To check for its curvature, consider

$$\frac{d^2K}{dL^2} = \frac{2\bar{q}^2}{(L-\bar{q})^3} > 0, \quad since\ L > \bar{q}.$$

The positive second derivative means that the downward sloping isoquant is getting flatter; thus the isoquant has the usual shape.

(b) To find the cost function, we must solve the firm's cost minimization problem:

$$Minimize \qquad C = wL + rK$$
$$subject\ to \qquad \frac{KL}{K+L} = q.$$

The Lagrangian for the problem is

$$\mathcal{L} = wL + rK + \lambda\left[q - \frac{KL}{K+L}\right].$$

The first-order conditions are:

$$\frac{\partial\mathcal{L}}{\partial L} = w + \lambda\frac{(K+L)K - KL}{(K+L)^2} = 0 \qquad \rightarrow \qquad w = -\lambda\frac{K^2}{(K+L)^2}$$

$$\frac{\partial\mathcal{L}}{\partial K} = r + \lambda\frac{(K+L)L - KL}{(K+L)^2} = 0 \qquad \rightarrow \qquad r = -\lambda\frac{L^2}{(K+L)^2}$$

Dividing one equation by the other, we get

$$\frac{w}{r} = \frac{K^2}{L^2} \qquad \to \qquad K = \sqrt{\frac{w}{r}} \cdot L.$$

Substituting this in the production function, we find

$$q = \frac{\sqrt{\frac{w}{r}} \cdot L \cdot L}{\sqrt{\frac{w}{r}} \cdot L + L} = \frac{\sqrt{\frac{w}{r}} \cdot L}{\sqrt{\frac{w}{r}} + 1}.$$

Rearranging,

$$L = \left(1 + \sqrt{\frac{r}{w}}\right) q.$$

This is the conditional demand function for L. Substituting in the expression we found earlier for K and simplifying, we get the conditional demand function for K:

$$K = \left(1 + \sqrt{\frac{w}{r}}\right) q.$$

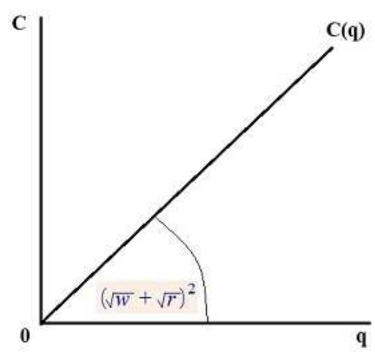
Substituting the conditional input demand functions back into the expression for total cost gives us the cost function:

$$C(q, w, r) = w\left(1 + \sqrt{\frac{r}{w}}\right) q + r\left(1 + \sqrt{\frac{w}{r}}\right) q,$$

which can be simplified to

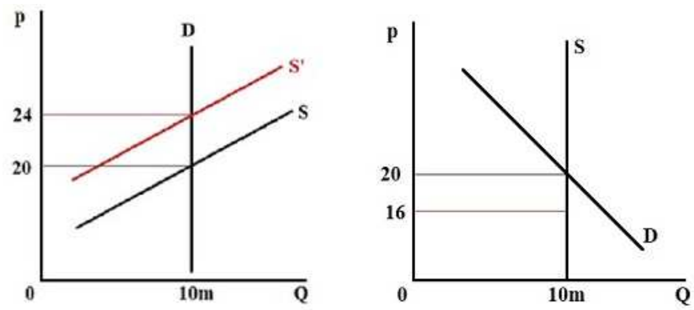
$$C(q, w, r) = \left(\sqrt{w} + \sqrt{r}\right)^2 q.$$

(c) .



4. (a) The conclusion that the government will collect $40 million revenue from this tax implies that the quantity transacted in the market remains at 10 million units after the imposition of the tax. This could happen if either the demand or the supply is perfectly inelastic. If

the demand is perfectly inelastic, the entire \$4 tax is passed on to consumers and they do not change their demand at all. If the supply is perfectly inelastic, the sellers will absorb the entire \$4 tax. The graphs below show the two situations that could prevail.



Demand inelastic   Supply inelastic

(b) Using the formulae for the effects of taxes in competitive markets, we find

$$dp_d = \frac{\varepsilon_s}{\varepsilon_s - \varepsilon_d} dT = \frac{3}{4} \cdot 4 = 3.$$

$$dp_s = \frac{\varepsilon_d}{\varepsilon_s - \varepsilon_d} dT = \frac{-1}{4} \cdot 4 = -1.$$

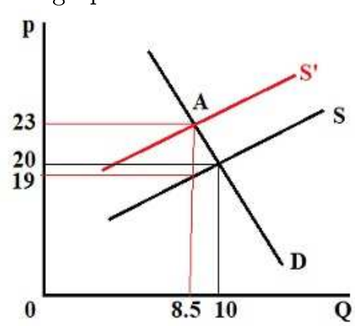
$$dQ_s = \varepsilon_s \cdot \frac{Q_0}{p_0} \cdot dp_s = 3 \cdot \frac{10}{20} \cdot (-1) = -1.5.$$

Therefore, the new equilibrium is: $Q^* = 8.5\ million, \quad P_d = \$23, \quad P_s =$ \$19.

The Tax Revenue collected is

$$TR = 4(8.5) = \$34\ million.$$

(c) The graph shows the effects of the tax.



Since sales taxes and excise taxes have the sane effect, let us think of this problem in terms of a percentage excise tax. For a percentage tax to have the exact same effect as the \$4 per-unit tax, it must be the case that the new supply curve must pass through the point A in the graph. In other words, if t represents the percentage tax rate, it must be the case that

$$19(1+t) = 23 \qquad \textit{or} \qquad t = \frac{23}{19} - 1 = 0.2105 = 21.05\%.$$