# EC 501: Mid-term Exam (Fall 2016), Solutions

1. (a) Since Olivia's utility function is of Cobb-Douglas form, with both coefficients equal to 1, we can write down her demand functions as

$$x = \frac{I}{2p_x} \quad and \quad y = \frac{I}{2p_y}.$$

(b) Substituting the data in the demand functions, we get

$$x_0 = 50 \quad and \quad y_0 = 10.$$

(c) After the price change, we find

$$x_1 = 12.5 \quad and \quad y_0 = 10.$$

To find the substitution effect, we need to find the income level needed, at the new prices, to obtain the original utility level. From the answer to (a), we can write down the indirect utility function:

$$V(I, p_x, p_y) = \left(\frac{I}{2p_x}\right)\left(\frac{I}{2p_y}\right) = \frac{I^2}{4p_xp_y}.$$

From the utility function and the answer to (b) we know that

$$U_0 = 50 \cdot 10 = 500.$$

So we need to find the income needed to achieve $U = 500$ when $p_x = 4$ and $p_y = 5$. That income level, say $I_2$ will satisfy the equation

$$I_2^2 = (500) \cdot (4 \cdot 4 \cdot 5) \qquad \rightarrow \qquad I_2 = 200.$$

The consumption pattern under the substitution effect will then be

$$x_2 = 25 \quad and \quad y_0 = 20.$$

The change in consumption of $x$ from $x_0 = 50$ to $x_1 = 12.5$ can then be divided into:

$$substitution\ effect\ = 25 - 50 = -25,$$

$$income\ effect\ = 12.5 - 25 = -12.5.$$

(d) The Compensating Variation in Income is the change in income needed *after* an economic change has taken place that would be just sufficient to restore the consumer to the original level of utility.

We have already calculated the income level needed after the change in price to restore Olivia to the original utility level. The CV is the change in income needed. So this is

$$CV = 200 - 100 = \$100.$$

2. (a) If Xandra could have bought year 2's bundle in year 1, she is better off in year 1. Let us first see what her income is in each time period:

$$I_1 = (2)(30) + (10)(4) = 100.$$

$$I_2 = (5)(10) + (10)(22) = 160.$$

Let's see whether she could have bought year 2's bundle in year 1:

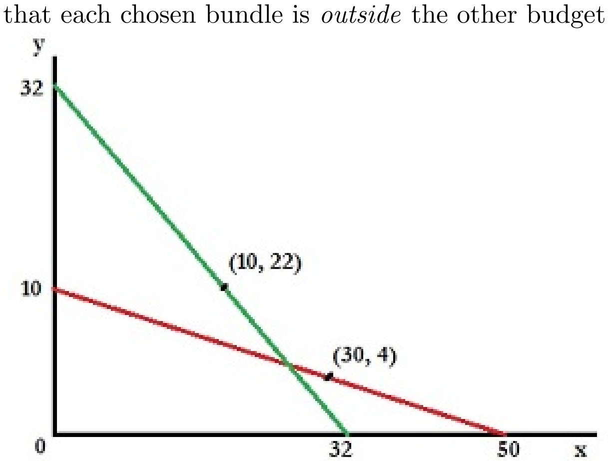
$$C(q_2, p_1) = (2)(10) + (5)(22) = 240.$$

Since this is greater than her income of 100, she could not have bought year 2's bundle in year 1.
Let's see if she could have bought year 1's bundle in year 2:

$$C(q_1, p_2) = (5)(30) + (5)(4) = 170.$$

Since this is greater than her income of 160, she could not have bought year1's bundle in year 2. Thus we do not have enough information to say in which year Xandra is better off.

The situation is illustrated in the Figure below. The key point is that each chosen bundle is *outside* the other budget constraint.



(b) The Laspeyres Price Index is

$$PI_L = \frac{C(q_1, p_2)}{C(q_1, p_1)} = \frac{170}{100} = 1.7 \quad or \quad 170.$$

The Paasche Price Index is

$$PI_P = \frac{C(q_2, p_2)}{C(q_2, p_1)} = \frac{160}{240} = 0.67 \quad or \quad 67.$$

3. (a) First find each firm's marginal cost:

$$MC = \frac{dC}{dq} = 2q.$$

Next, find the average variable cost. From the cost function, we know that

$$VC = q^2,$$

so

$$AVC = \frac{VC}{q} = \frac{q^2}{q} = q.$$

We see that $MC>AVC$ for all $q$ and so the entire MC curve is the firm's supply curve. Therefore, each firm's supply curve is

$$p = 2q \qquad or \qquad q = \frac{1}{2}p.$$

To find the short run price and quantity of widgets, we need to find the market supply curve and solve for the equilibrium. Now the market supply, $Q_s$, will simply be 12 times the individual firm supply. Therefore

$$Q_s = 12 \cdot \frac{1}{2}p = 6p.$$

At equilibrium, supply must equal demand, so

$$6p = 28 - p \qquad \rightarrow \qquad p = 4$$

is the equilibrium price and the quantity will be

$$Q = 6 * 4 = 24.$$

(b) In the long run, each firm would produce at the minimum point of the AC curve. Now,

$$AC = \frac{C(q)}{q} = \frac{q^2+1}{q} = q + \frac{1}{q}.$$

Then AC will be minimized where

$$\frac{dAC}{dq} = 1 - \frac{1}{q^2} = 0 \qquad \rightarrow \qquad q = 1.$$

To confirm this is a minimum, check

$$\frac{d^2AC}{dq^2} = \frac{2}{q^3} = 2 > 0,$$

so we do indeed have a minimum. Thus each firm will produce 1 unit of output. At *q=1*,

$$AC = q + \frac{1}{q} = 2$$

and so $p = 2$ in the long run equilibrium. At this price, demand will be $Q_d = 26$ and so 26 firms will operate in the long run.

4. (a) A subsidy is just a negative tax; we can therefore employ the usual formulae to calculate the effect of taxes. We know that

$$\triangle p_s \simeq \frac{\epsilon_d}{\epsilon_s - \epsilon_d} \cdot \triangle T.$$

Substituting the relevant values, we have

$$\triangle p_s \simeq \frac{-0.2}{1.8 + 0.2} \cdot (-5) = 0.5.$$

Thus the supply price after the subsidy will be $p_s = 10.50$.
The demand price will be \$5 lower: $p_d = 5.50$. We could have found this also as

$$\triangle p_d \simeq \frac{\epsilon_s}{\epsilon_s - \epsilon_d} \cdot \triangle T = \frac{1.8}{1.8 + 0.2} \cdot (-5) = -4.5.$$

We can use the formula for elasticity of demand to calculate the new quantity:

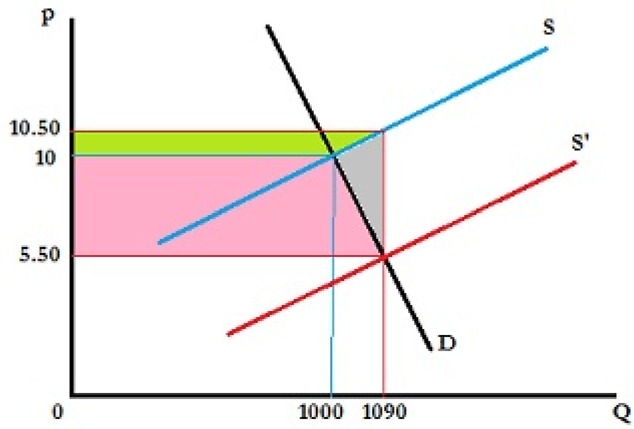
$$\triangle Q \simeq \frac{\epsilon_d \cdot \triangle p_d \cdot Q_0}{p_0} = \frac{(-0.2)(-4.5)(1000)}{10} = 90.$$

Therefore the new quantity is $Q_1 = 1090$.

(b) The cost of the subsidy program would be the per unit subsidy multiplied by the number of widgets produced:

$$C = (5)(1090) = \$5450.$$

(c) Consumers gain because prices fall and producers gain because their profits rise. Government loses because the subsidy program has a cost. The situation is illustrated in the Figure below.




The consumers gain the pink area and the producers gain the green area. The gain in consumers' surplus is approximately (assuming linearity of the curves):

$$\triangle CS = (4.50)(1000) + \frac{1}{2}(4.50)(90) = 4702.50.$$

The gain in producers' surplus is:

$$\triangle PS = (0.50)(1000) + \frac{1}{2}(0.50)(90) = 522.50.$$

The net change in welfare is the sum of the changed in CS and PS minus the cost of the subsidy program:

$$\triangle W = 4702.5 + 522.5 - 5450 = -225.$$

This is the net loss in welfare, equal in area to the grey-shaded triangle.