EC 501 **Final Exam** **Dec 18, 2016**

Answer all questions, showing all your work. Try to use diagrams wherever possible. Time allowed: 2 hours. Each question is worth 20 points. Good luck!

1. (a) Suppose a consumer consumes only two goods, x and y. If ϵ_x and ϵ_{xy} are the own- and cross-price elasticities of demand for x, and η_x is the income elasticity of demand for x, prove that

$$\epsilon_x + \epsilon_{xy} + \eta_x = 0.$$

(b) John consumes only two goods, x and y. His utility function is

$$u(x, y) = x + \ln y.$$

Find John's demand functions for x and y.

(c) Using the demand functions you found in (b), verify the equation in (a) for both x and y.

2. Consider an exchange economy, with two consumers, A and B, and two commodities, X and Y. Initially, consumer A has 1 unit of X and 2 units of Y, while consumer B has 2 units of X and 1 unit of Y. The preferences of consumers A and B are represented by the utility functions

$$U_A(X_A, Y_A) = X_A Y_A \quad \text{and} \quad U_B(X_B, Y_B) = \min[X_B, Y_B].$$

respectively.

(i) Construct the Edgeworth Box. Label the axes. Label the endowment point with an "E". Sketch indifference curves for consumer A and for consumer B. Label the set of Pareto-efficient allocations with a "PS".
(ii) Find the set of allocations which are both Pareto efficient and provide at least as much utility as the initial endowment. Identify both endpoints of this set.
(iii) Calculate the competitive (Walrasian) equilibrium price vector and the equilibrium allocation.

3. Olivia's utility function is

$$U(I) = \sqrt{I}$$

where I is her monthly income. She presently has a job that pays her $3,600 per month. She has received a job offer that would pay her a base salary of $1,600 per month, but this would go up to $10,000 per month if she meets certain sales goals. Olivia estimates that her chance of meeting those sales goals is 30%.

(a) What is the expected income from the new job?
(b) Would Olivia accept this new job?
(c) What is her cost of risk if she accepts her new job?

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4. There are two firms, indexed 1 and 2, that produce widgets. Their marginal costs of producing widgets are respectively

$$MC_1 = q_1 \quad and \quad MC_2 = \frac{1}{2}q_2,$$

where q_1 and q_2 are their output levels.

(a) Suppose both firms act as perfect competitors. Find the supply curve of widgets.
(b) If the demand curve for widgets is

$$Q^d = 600 - 3p,$$

find the equilibrium price of widgets. How many widgets will each firm produce and how much profit will they each make?
(c) Suppose the two firms agree to merge; they would now be two plants belonging to the same company. How many widgets would be produced at each plant, what would be the price of widgets and how much profit will the combined firm make?
(d) How much better off or worse off is society as a result of the merger?

5. Appel and Bamsung are the only firms that produce smart phones. Each firm can produce phones at a cost of $5 each. They compete by setting prices. The demand for Appel phones is

$$D_a = 100 - 5p_a + 5p_b,$$

where p_a, p_b are the prices of Appel and Bamsung phones respectively.Similarly, the demand for Bamsung phones is

$$D_b = 100 - 5p_b + 5p_a$$

(a) Assume Appel and Bamsung choose their prices simultaneously. Find the Nash equilibrium prices for the two phone brands. How many phones will each firm produce and how much profit will they each make?

(b) Suppose instead that the firms play a two-stage game in which Appel chooses its price first and then Bamsung chooses its price. What will now be the equilibrium prices of the two phones, how many phones will each firm sell, and how much profit will each firm make?

(c) Draw a single clearly labeled graph showing the two solutions for (a) and (b) on the same graph.