EC 501: Final Exam (Fall 2016), Solutions

1. (a) We know that the demand function is homogeneous of degree zero. Using a property of homogenous function we can take the demand function

$$x(p_x, p_y, I)$$

and write

$$p_x \frac{\partial x}{\partial p_x} + p_y \frac{\partial x}{\partial p_y} + I \frac{\partial x}{\partial I} = 0.$$

Dividing through by x gives us

$$\frac{p_x}{x} \cdot \frac{\partial x_x}{\partial p_x} + \frac{p_y}{x} \cdot \frac{\partial x}{\partial p_y} + \frac{I}{x} \cdot \frac{\partial x}{\partial I} = 0,$$

which is

$$\epsilon_x + \epsilon_{xy} + \eta_x = 0.$$

(b) Tangency condition is:

$$\frac{u_x}{u_y} = \frac{p_x}{p_y} \qquad \rightarrow \qquad y = \frac{p_x}{p_y} \qquad \rightarrow \qquad p_y y = p_x.$$

Substituting in the budget constraint:

$$p_x x + p_x = I \qquad \rightarrow \qquad x = \frac{I - p_x}{p_x}.$$

This is the demand function for x. The second equation above is the demand function for y:

$$y = \frac{p_x}{p_y}.$$

(c) From the demand function for x,

$$\frac{\partial x}{\partial p_x} = \frac{-I}{p_x^2} \qquad \rightarrow \qquad \epsilon_x = \frac{-I}{I - p_x}.$$

$$\frac{\partial x}{\partial p_y} = 0 \qquad \rightarrow \qquad \epsilon_{xy} = 0.$$

$$\frac{\partial x}{\partial I} = \frac{1}{p_x} \qquad \rightarrow \qquad \eta_x = \frac{I}{I - p_x}.$$

Then

$$\epsilon_x + \epsilon_{xy} + \eta_x = 0.$$

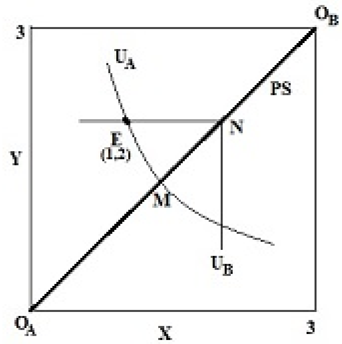
From the demand function for y, since it is of Cobb-Douglas form, we can read off the elasticities directly:

$$\epsilon_y = -1, \quad \epsilon_{yx} = 1, \quad \eta_y = 0,$$

and so

$$\epsilon_y + \epsilon_{yx} + \eta_y = 0.$$

2. (a) A's indifference curves have the usual convex shape, but B's indifference curves are L-shaped. The contract curve PS is the diagonal of the box.



(b) Pareto improvements over the endowment E that are also efficient are on the line segment MN where A's allocations are, at M: $(\sqrt{2}, \sqrt{2})$, and at N: $(2, 2)$.

(c) If the Walrasian auctioneer announces prices p_x and p_y, the income levels for A and B will be respectively

$$I_A = p_x + 2p_y \qquad and \qquad I_B = 2p_x + p_y.$$

Now, given his Cobb-Douglas utility function, A will want to spend half his income on x. His demand function for x is therefore

$$x_A = \frac{p_x + 2p_y}{2p_x}.$$

B has a Leontief utility function and, in equilibrium, would want to set $x_B = y_B$. Therefore, his demand function for x is

$$x_B = \frac{2p_x + p_y}{p_x + p_y}.$$

In Walrasian equilibrium, we must have

$$x_A + x_B = 3.$$

Substituting the demand functions in this equation and rearranging gives us the equilibrium price ratio:

$$\left(\frac{p_x}{p_y}\right)^* = 1.$$

Setting $p_x = p_y = 1$, we can use the demand functions to solve for the chosen consumption bundles:

$$x_A^* = 1.5, y_A^* = 1.5; \quad x_B^* = 1.5, y_B^* = 1.5.$$

3. (a) $E(I_1) = (0.7)(1600) + (0.3)(10000) = \$4,120.$
   (b) With her existing job, her utility is:

$$U(I_0) = \left(\sqrt{3600}\right) = 60.$$

   With the new job, her expected utility would be:

$$EU(I_1) = (0.7)\left(\sqrt{1600}\right) + (0.3)\left(\sqrt{10000}\right) = 28 + 30 = 58.$$

   This is lower than her utility in the existing job and therefore Olivia would not accept the new job.
   (c) The cost of risk is the difference between the expected income and the certainty equivalent income. The cetainty equivalent of her new job would be that I_2 that resulted in:

$$EU(I_2) = \sqrt{I_2} = 58 \qquad \rightarrow \qquad I_2 = 3364$$

   Therefore, Olivia's cost of risk in the new job would be

$$C = 4120 - 3364 = \$756.$$

4. (a) For both firms, the supply curve is just their MC curve. Therefore their supply curves are

$$q_1 = p \qquad and \qquad q_2 = 2p.$$

   Then the market supply curve is the sum of these:

$$Q^s = 3p.$$

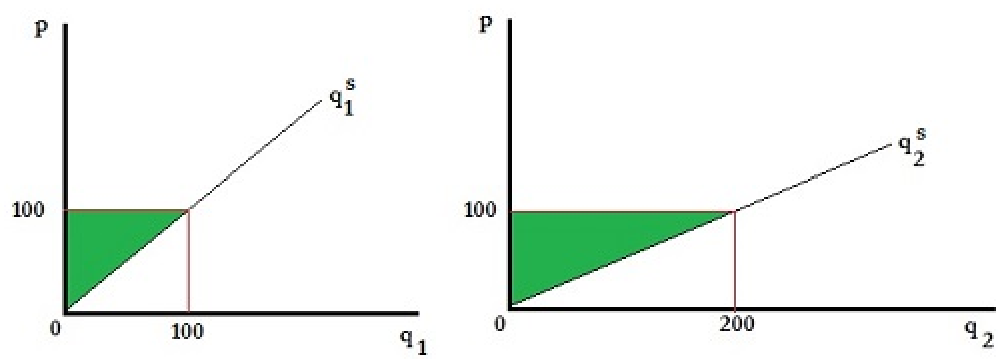
   (b) Equilibrium will be where supply equals demand:

$$3p = 600 - 3p \qquad \rightarrow \qquad p = 100 \qquad and \qquad Q = 300.$$

   From the individual supply curves, given that $p = 100$, we find

$$q_1 = 100 \qquad and \qquad q_2 = 200.$$

   The green areas in the diagram indicate the profits of the two firms.



Then profits are

$$\pi_1 = \frac{1}{2}(100)(100) = \$5000 \qquad \textit{and} \qquad \pi_2 = \frac{1}{2}(100)(200) = \$10,000.$$

(c) If the two firms merged, they would operate as a multi-plant monopoly. The marginal cost curve would be the old supply curve we found in part (a):

$$MC = \frac{1}{3}Q.$$

The demand curve can be written as

$$p = 200 - \frac{1}{3}Q^d,$$

so the marginal revenue is then

$$MR = 200 - \frac{2}{3}Q.$$

Profit is maximized when MC=MR:

$$200 - \frac{2}{3}Q = \frac{1}{3}Q \qquad \rightarrow \qquad Q = 200.$$

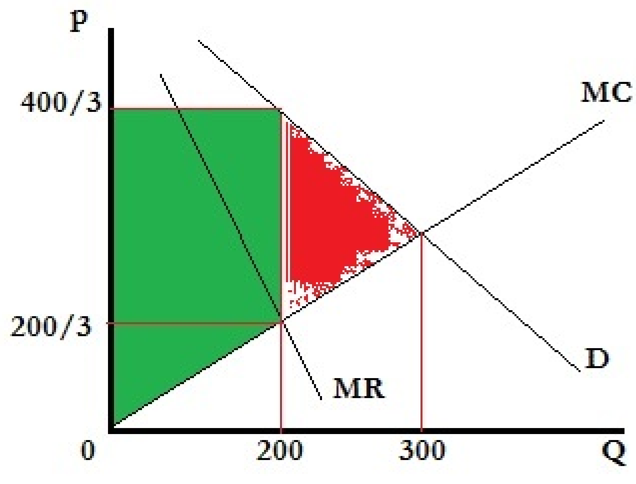
Then

$$MC = \frac{200}{3} \qquad \textit{and so} \qquad q_1 = \frac{200}{3} \qquad \textit{and} \qquad q_2 = \frac{400}{3}.$$

From the demand curve,

$$p = 200 - \frac{200}{3} = \frac{400}{3}.$$

The graph illustrates the situation.



The combined firm's profit (the green area) is

$$\pi = (200)\left(\frac{400}{3}\right) - \frac{1}{2}(200)\left(\frac{200}{3}\right) = \$20,000.$$

(d) Society is worse off after the merger by the red area in the graph above. So

$$\Delta W = -\frac{1}{2}(100)\left(\frac{200}{3}\right) = -\frac{10000}{3}.$$

5. (a) Appel's profit is

$$\pi_a = (p_a - 5) \cdot (100 - 5p_a + 5p_b).$$

This will be maximized where

$$\frac{\partial \pi_a}{\partial p_a} = 100 - 5p_a + 5p_b + (p_a - 5)(-5) = 0 \qquad \rightarrow \qquad p_a = 12.5 + \frac{1}{2}p_b.$$

This is Appel's best-response function. Because the problem is symmetrical, we can write down Bamsung's best-response function:

$$p_b = 12.5 + \frac{1}{2}p_a.$$

The equilibrium is where the two best-response functions intersect, which yields

$$p_a = p_b = 25.$$

Then

$$q_a = q_b = 100 \qquad \textit{and} \qquad \pi_a = \pi_b = 2000.$$

(b) In the two-stage game, Appel will choose its price while taking into account Bamsung's best-response function. Thus

$$\pi_a = (p_a - 5) \cdot \left(100 - 5p_a + 5\left\{12.5 + \frac{1}{2}p_a\right\}\right).$$

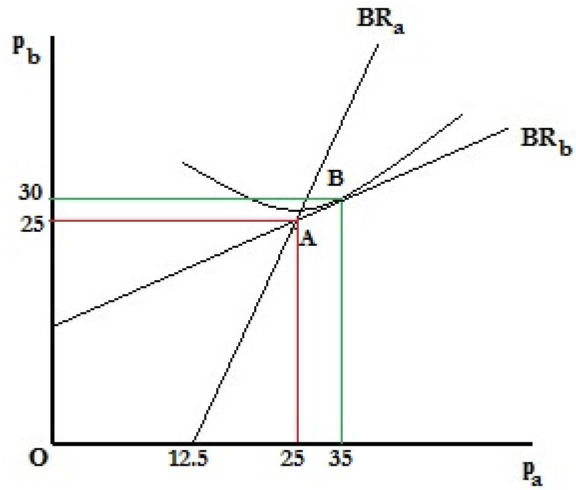
Differentiating this with respect to p_a, setting equal to zero and solving, yields

$$p_a = 35 \qquad \textit{and so} \qquad p_b = 30.$$

Then

$$q_a = 75, q_b = 125 \qquad \textit{and} \qquad \pi_a = 2250, \pi_b = 3125.$$

(c) The graph follows; A is the initial Nash equilibrium, while B is the equilibrium in part (b):


BR_a
p_b
BR_b
B
30
25
A
O
12.5
25
35
p_a