EC 501: Mid-term Exam (Fall 2015), Solutions

1. (a) The Lagrangian for this problem is

$$\mathcal{L} = x + ln\ y + \lambda \left[I - p_x x - p_y y \right].$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x} = 1 - \lambda p_x = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{y} - \lambda p_y = 0 \tag{2}$$

Dividing (1) by (2) and simplifying, we get

$$y = \frac{p_x}{p_y} \tag{3}$$

This is the demand function for y. Substituting this in the budget constraint gives us

$$p_x x + p_x = I,$$

which simplifies to

$$x = \frac{I - p_x}{p_x}.$$

This is the demand function for x.

(b) Substituting the given data into the demand functions gives us the quantities consumed:

$$x = \frac{100 - 20}{20} = 4 \qquad and \qquad y = \frac{20}{5} = 4.$$

(c) To find the compensated demand function for x, we need to minimize the expenditure needed to attain any given level of utility. The Lagrangian for this problem is

$$\mathcal{L} = p_x x + p_y y + \lambda \left[u - x - ln\ y \right].$$

Differentiating with respect to x and y yields the same tangency condition (3) as before. Substituting this in the utility function yields

$$u = x + ln\ \frac{p_x}{p_y}$$

and so the compensated demand function for x is

$$x = u - ln\ p_x + ln\ p_y.$$

(d) The elasticities form of the Slutsky equation is

$$\epsilon_x = \epsilon_x^h - \theta_x \eta_x.$$

We know the elasticity of demand is given by

$$\epsilon_x = \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{x}.$$

From the demand function for x:

$$\frac{\partial x}{\partial p_x} = \frac{p_x(-1) - (I - p_x) \cdot 1}{p_x^2} = \frac{-I}{p_x^2}.$$

Then

$$\epsilon_x = \frac{-I}{p_x^2} \cdot \frac{p_x}{x} = \frac{-I}{p_x^2} \cdot \frac{p_x}{(I - p_x)} \cdot p_x = -\frac{I}{I - p_x}.$$

Also from the demand curve, we can find

$$\frac{\partial x}{\partial I} = \frac{1}{p_x}$$

and so

$$\eta_x = \frac{1}{p_x} \cdot \frac{I}{(I - p_x)} \cdot p_x = \frac{I}{I - p_x}.$$

Also, rearranging the demand function, we can write

$$\theta_x = \frac{I - p_x}{I}.$$

Finally, from the compensated demand function, we can write

$$\frac{\partial x^h}{\partial p_x} = -\frac{1}{p_x}$$

and so

$$\epsilon_x^h = -\frac{1}{p_x} \cdot \frac{p_x}{x^h}.$$

Since at the original equilibrium $x = x^h$, this can be written as

$$\epsilon_x^h = -\frac{1}{p_x} \cdot \frac{p_x}{(I - p_x)} \cdot p_x = -\frac{p_x}{I - p_x}.$$

The right-hand side of the Slutsky equation then is

$$\epsilon_x^h - \theta_x \eta_x = -\frac{p_x}{I - p_x} - \frac{I - p_x}{I} \cdot \frac{I}{I - p_x} = -\frac{I}{I - p_x} = \epsilon_x,$$

thereby verifying the Slutsky equation.

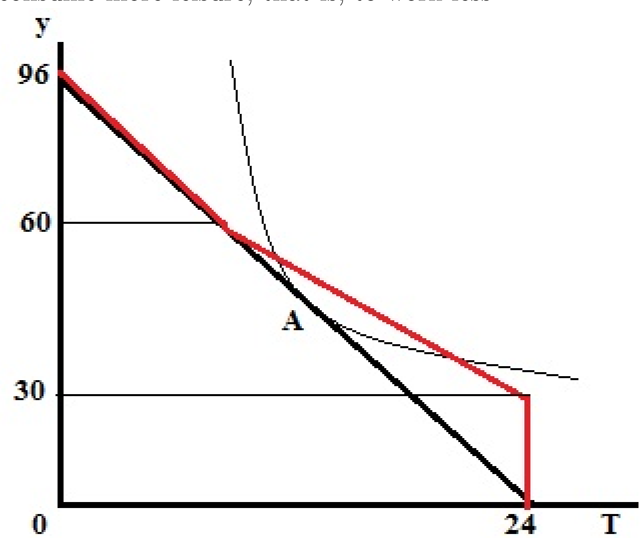
2. (a) Bill's problem is to

$$Maximize \quad U = yT$$
$$subject\ to \quad \frac{y}{4} + T = 24.$$

Since this utility function is Cobb-Douglas, we know he will "spend" half his "income" (of 24 hours) each on y and T. Therefore

$$T = 12, y = 48.$$

(b) Bill faced the solid black line originally as his constraint and chose point A. Under the welfare system, he faces the red line as his constraint. In his choice area, therefore, his constraint is flatter. On grounds of the substitution effect, he will choose a point on the original indifference curve somewhere to the right of A, and then, on grounds of income effect, he will choose even further to the right. Therefore, both substitution effect and income effect cause him to consume more leisure, that is, to work less



(c) Under the welfare system, Bill's constraint is

$$\frac{y-30}{2} + T = 24$$
$$i.e., \quad \frac{y}{2} + T = 39.$$

So now he will choose

$$T = \frac{39}{2} = 19.5.$$

He will therefore work 4.5 *hours*.
He will earn $4.5 * 4 = \$18$; and will receive welfare payments of $30 - 9 = \$21$. Therefore his income is $18 + 21 = \$39$.

3. (a) To find the cost function, we must solve the firm's cost minimization problem:

$$\begin{aligned} &\textit{Minimize} \qquad C = wL + rK \\ &\textit{subject to} \qquad \frac{KL}{K+L} = q. \end{aligned}$$

The Lagrangian for the problem is

$$\mathcal{L} = wL + rK + \lambda \left[q - \frac{KL}{K+L} \right].$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial L} = w + \lambda \frac{(K+L)K - KL}{(K+L)^2} = 0 \qquad \rightarrow \qquad w = -\lambda \frac{K^2}{(K+L)^2}$$

$$\frac{\partial \mathcal{L}}{\partial K} = r + \lambda \frac{(K+L)L - KL}{(K+L)^2} = 0 \qquad \rightarrow \qquad r = -\lambda \frac{L^2}{(K+L)^2}$$

Dividing one equation by the other, we get

$$\frac{w}{r} = \frac{K^2}{L^2} \qquad \rightarrow \qquad K = \sqrt{\frac{w}{r}} \cdot L.$$

Substituting this in the production function, we find

$$q = \frac{\sqrt{\frac{w}{r}} \cdot L \cdot L}{\sqrt{\frac{w}{r}} \cdot L + L} = \frac{\sqrt{\frac{w}{r}} \cdot L}{\sqrt{\frac{w}{r}} + 1}.$$

Rearranging,

$$L = \left(1 + \sqrt{\frac{r}{w}}\right) q.$$

This is the conditional demand function for L. Substituting in the expression we found earlier for K and simplifying, we get the conditional demand function for K:

$$K = \left(1 + \sqrt{\frac{w}{r}}\right) q.$$

Substituting the conditional demand functions in the expression for total cost, we get the cost function:

$$C(q, w, r) = \left\{ w \left(1 + \sqrt{\frac{r}{w}}\right) + r \left(1 + \sqrt{\frac{w}{r}}\right) \right\} q = \left(\sqrt{w} + \sqrt{r}\right)^2 q.$$

(b) If w=4,r=9, the cost function becomes

$$C(q) = 25q.$$

This cost function shows an AC that is constant at 25. Thus the market supply curve will be perfectly elastic at $p^* = 25$.

(c) Since the supply curve is perfectly elastic, the equilibrium price will be $p = 25$. Substituting this in the demand curve yields the equilibrium quantity traded:

$$Q^* = 200 - 2(25) = 150.$$

4. (a) The price of imported widgets would be \$18. At this price, demand would be $Q_d = 4$ and domestic supply would be $Q_s = 12$. Thus this price cannot be sustained as there would be excess domestic supply; the tariff is prohibitive. The equilibrium will therefore be one where there are no imports and $Q_d = Q_s$:

$$40 - 2p^* = \frac{2}{3}p^* \qquad \rightarrow \qquad p^* = 15 \qquad \rightarrow \qquad Q^* = 10.$$

(b) If the tariff is eliminated, imports can take place at a price of \$9. This would be the equilibrium price. At this price, demand is

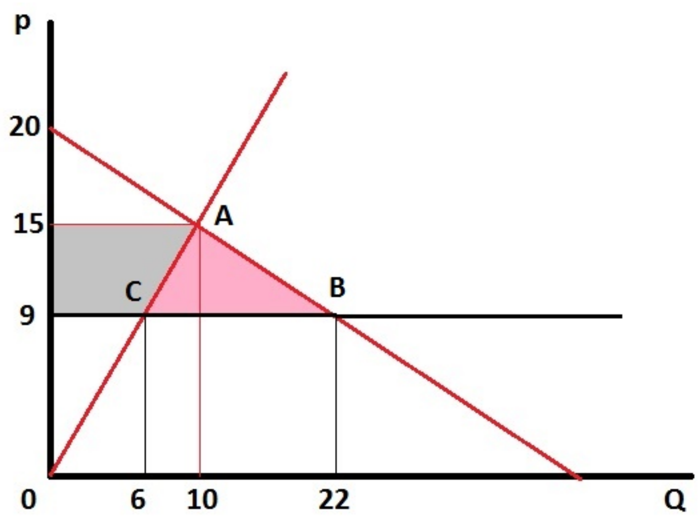
$$Q_d^{**} = 40 - 2(9) = 22,$$

and domestic supply is

$$Q_s^{**} = \frac{2}{3}(9) = 6.$$

Therefore the quantity of imports will be 16 (million units).

(c) The graph shows the two equilibria.



Domestic producers lose surplus equal to the grey area in the graph:

$$\triangle PS = -6\left(\frac{6+10}{2}\right) = -48.$$

Consumers gain the grey area plus the pink area:

$$\triangle CS = 6\left(\frac{10+22}{2}\right) = +96.$$

Therefore the net gain for society is

$$\triangle W = -48 + 96 = +48.$$