EC 501 **Final Exam** **Dec 17, 2015**

Answer all questions, showing all your work. Try to use diagrams wherever possible. Time allowed: 2 hours. Each question is worth 20 points. Good luck!

1. John consumes only two goods, x and y, and his utility function is

$$u(x, y) = min\left[\frac{x}{2}, \frac{y}{3}\right].$$

Let p_x and p_y represent the prices of *x* and *y* respectively. *I* is John's income.

(a) Find John's demand functions for *x* and *y*.
(b) Suppose John's income is \$175, $p_x = \$5$ and $p_y = \$5$. How much *x* and *y* will John buy?
(c) Now suppose the price of *x* goes up to \$10. How much *x* and *y* will John buy now? Of the change in demand for *x*, how much is due to the substitution effect and how much is due to the income effect?
(d) Find the compensating variation and equivalent variation in income of the price change in *x*. Pay attention to the signs!

2. Alex has a wealth of \$400. Her utility function is

$$U(w) = \sqrt{w}$$

where *w* is her wealth. Alex's parents want to give her a gift of a lottery ticket and offer her a choice between two tickets. Ticket A gives her a 20% chance of winning \$225, while ticket B gives her a 10% chance of winning \$500.

(a) What are the expected payoffs from each of the lottery tickets?
(b) Which ticket will Alex choose? Explain fully.
(c) If Alex's parents gave her ticket A, what is the minimum price at which she would be willing to sell the ticket?

3. Grandpa's Ride Company (GRCo) has a Ferris wheel on which it sells rides. Every day, on average, 100 boys and 100 girls come to take rides. Each boy's demand curve is

$$q_b = 20 - p$$

where p is the price of one ride. The demand curve of each girl is

$$q_g = 10 - p.$$

The marginal cost of producing rides is zero.

(a) Find the market demand curve for rides.
(b) What price for rides should GRCo set if it wishes to maximize its profits? How much profit will it make per day?
(c) Suppose now that GRCo has hired you as a consultant to see if they could increase their profits by using a more sophisticated pricing strategy. It would of course be illegal to charge boys and girls different prices. What pricing strategy would you recommend and how much profit would Grandpa's make now per day?

… continued over

4. Twin Casino operates a casino in the state of Road Island. The average revenue it can earn (think demand curve) per day as a function of the number of slot machines it operates is given by

$$p = 100 - \frac{Q}{10},$$

where *p* is the average revenue (think price) and *Q* is the number of slot machines it operates. It costs Twin $10 per day to operate a slot machine.

   (a) How many slot machines will Twin operate and what will be its daily profit?
   (b) Now suppose Plain Bridge Casino builds a casino just across the state line in the state of Taxachusetts. Plain Bridge will be serving the same market as Twin, since they are very close to each other, even though they are in different states. Plain Bridge's cost of operating a slot machine is $6 per day. The government of Taxachusetts, in granting Plain Bridge a license to build the casino, imposed a tax of $4 per day per slot machine operated. The government assumes that Twin and Plain Bridge will compete according to the Cournot model. How much tax revenue per day will the government of Taxachusetts estimate it will earn?
   (c) Suppose that, once Plain Bridge enters the market, Twin actually plays as a Stackelberg leader by pre-committing to the number of slot machines it operates. How much tax revenue will the government of Taxachusetts actually collect per day?

5. National Lead Co. (NLC), a monopoly public enterprise, produces lead at a cost of $5 per unit. Demand for lead is given by

$$Q = 25{,}000 - 1{,}000\ P.$$

   (a) How much lead should NLC produce, and what price should it charge, to maximize welfare?

   (b) Suppose environmentalists discover that that for each unit of lead it produces, NLC also produces one unit of a toxic waste (W) that causes a total damage given by

$$D = W^2/8000$$

   where W is the amount of waste discharged. What would be your answer to part (a) given this additional information?

   (c) Suppose NLC now discovers that it could eliminate waste at a cost of

$$C = A^2/2000$$

   where A is the amount of waste abated. Answer again the questions in part (a).