EC 501: Final Exam (Fall 2015), Solutions

1. (a) Since the utility function is Leontief, we can write down the demand functions as

$$x = \frac{2I}{2p_x + 3p_y} \qquad and \qquad y = \frac{3I}{2p_x + 3p_y}.$$

   (b) Substituting the given data into the demand functions we get:

$$x_0 = 14 \qquad and \qquad y_0 = 21.$$

   (c) Once again, substituting the revised data into the demand functions we get:

$$x_1 = 10 \qquad and \qquad y_1 = 15.$$

   Since there is no substitution possible with a Leontief utility function, the entire $\triangle x = -4$ is due to the income effect.

   (d) The CV is the change in income needed after the price change that would restore John to the original utility level. The cheapest way to achieve u_0 would be to buy the original bundle again. So

$$CV = C(q_0, p_1) - 175 = 140 + 105 - 175 = +\$70.$$

   The EV is the change in income needed before the price change that would force John to the final utility level. The cheapest way to achieve u_1 would be to buy the final bundle. So

$$EV = C(q_1, p_0) - 175 = 50 + 75 - 175 = -\$50.$$

2. (a) $E(\pi_A) = (0.2)(225) = \$45$ and $E(\pi_B) = (0.1)(500) = \$50$.

   (b) We need to find Alex's expected utility in the two scenarios:

$$E(U_A) = (0.2)\left(\sqrt{625}\right) + (0.8)\left(\sqrt{400}\right) = 5 + 16 = 21.$$

   and

$$E(U_B) = (0.1)\left(\sqrt{900}\right) + (0.9)\left(\sqrt{400}\right) = 3 + 18 = 21.$$

   Therefore Alex is indifferent between the two options.

   (c) The minimum price t at which Alex would be willing to sell the ticket would have to satisfy

$$\sqrt{(400 + t)} = 21 \qquad or \qquad 400 + t = 441 \qquad so \qquad t = \$41.$$

3. (a) Since there are 100 boys and 100 girls, the total market demand for rides is:

$$Q = 100q_b + 100q_g = 2000 - 100p + 1000 - 100p = 3000 - 200p.$$

(b) The firm's profits are

$$\pi = Q \cdot \left(15 - \frac{Q}{200}\right),$$

so profits are maximized where

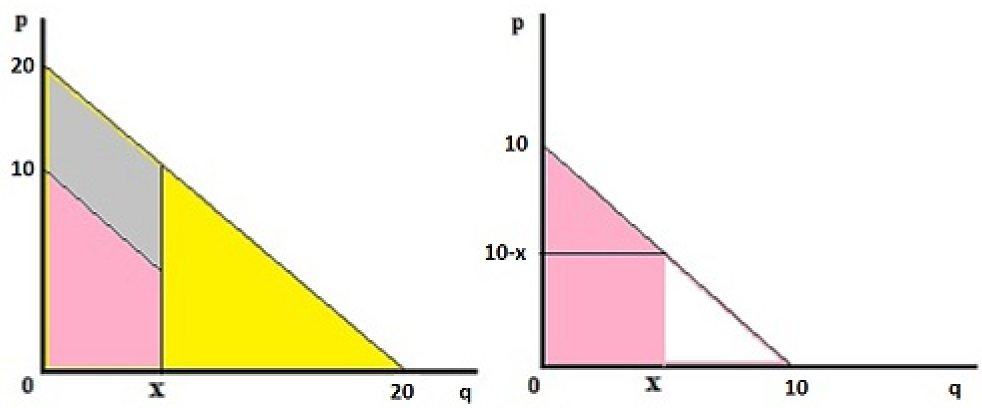
$$\frac{d\pi}{dQ} = 15 - \frac{Q}{100} = 0 \qquad \rightarrow \qquad Q_1 = 1500.$$

Then

$$p_1 = 7.5, \pi_1 = 11,250.$$

(c) GRCo could try to implement second-degree price discrimination, offering bundles of rides in two sizes: small bundles to try to entice the girls and then bundles of 20 rides to sell to the boys. Suppose the small bundles consist of x rides. Then the price that can be charged for this bundle would be the pink area, which is

$$F_1 = x(10 - x) + \frac{1}{2}x^2 = 10x - \frac{1}{2}x^2.$$



The boys would get a surplus of the grey area if they bought the small bundle. Therefore, the maximum they can be charged for the large bundles of 20 rides is the sum of the pink and yellow areas. This is

$$F_2 = \frac{1}{2}(20)(20) - 10x = 200 - 10x.$$

GRCo's profits then are

$$\pi = 100\left(10x - \frac{1}{2}x^2\right) + 100(200 - 10x) = 20,000 - 50x^2.$$

This is maximized when

$$\frac{d\pi}{dx} = -100x = 0 \qquad \rightarrow \qquad x = 0.$$

Thus Grandpa will sell just one bundle of rides: 20 rides at a price of \$200. Only boys will buy these bundles. Then $\pi^* = \$20,000$.

4. (a) Twin's profit is

$$\pi_T = q_1 \cdot \left(100 - \frac{q_1}{10}\right) - 10q_1.$$

This will be maximized where

$$\frac{\partial \pi_T}{\partial q_1} = 100 - \frac{q_1}{5} - 10 = 0 \qquad \rightarrow \qquad q_1 = 450.$$

Then

$$\pi_T = (450)(55) - (10)(450) = 450 \cdot 45 = \$20,250.$$

(b) Twin's profit is now

$$\pi_T = q_T \cdot \left(100 - \frac{q_T + q_P}{10}\right) - 10q_T.$$

This will be maximized where

$$\frac{\partial \pi_T}{\partial q_T} = 100 - \frac{q_T + q_P}{10} - \frac{q_T}{10} - 10 = 0.$$

But, effectively, Twin's and Plain Bridge's costs are the same, \$10 per unit. The Cournot equilibrium will therefore be a symmetric equilibrium and $q_T = q_P$. Imposing this on Twin's first order condition, we find

$$q_T = q_P = 300.$$

Since Taxachusetts is getting \$4 per unit of Plain Bridge's output, the estimated tax revenue per day would be \$1200.

(c) If Twin plays Stackelberg to Plain Bridge's Cournot, it will incorporate Plain Bridge's best-response function for q_P in its objective function. Now Plain Bridge's best response function will be:

$$100 - \frac{q_T + q_P}{10} - \frac{q_P}{10} - 10 = 0 \qquad or \qquad q_P = 450 - \frac{q_T}{2}.$$

Substituting this into Twin's profit, we get

$$\pi_T = 45q_T - \frac{q_T^2}{20}.$$

This will be maximized where

$$\frac{d\pi_T}{dq_T} = 45 - \frac{q_T}{10} = 0 \qquad \rightarrow \qquad q_T = 450.$$

Then Plain Bridge's best response will be

$$q_P = 450 - 225 = 225.$$

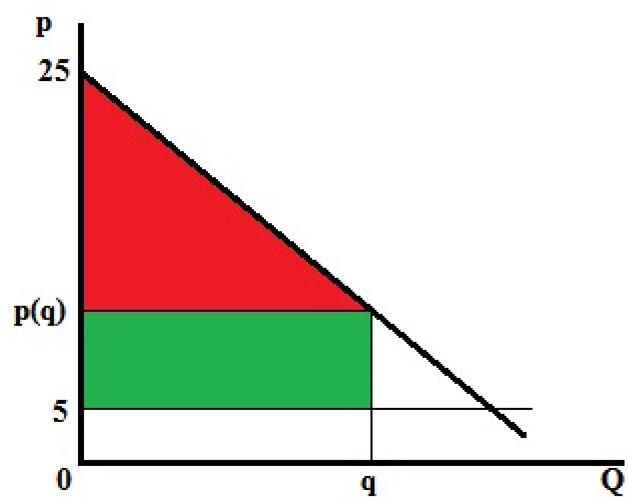
Since Taxachusetts is getting \$4 per unit of Plain Bridge's output, the tax revenue per day would be \$900.

5. (a) Since the MC of producing lead is constant at 5, social welfare would be maximized if the price were set equal to MC and the level of production chosen accordingly. The socially optimal equilibrium is therefore

$$Q^* = 20,000, p^* = 5.$$

(b) We know that the inverse demand curve is

$$p(Q) = 25 - \frac{Q}{1,000}.$$



For any value of Q, say q, the graph shows the Consumer Surplus (CS) as the red area and the firm profits (π) as the green area. If the environmental damage caused by the toxic waste is D, we can write social welfare as

$$W(Q) = CS + \pi - D = \frac{Q^2}{2,000} + \left(20Q - \frac{Q^2}{1,000}\right) - \frac{Q^2}{8,000} = 20Q - \frac{5Q^2}{8,000}.$$

This is maximized where

$$\frac{dW}{dQ} = 20 - \frac{Q}{800} = 0 \qquad \rightarrow \qquad Q^e = 16,000.$$

Then $p^e = 9$.

(c) Social welfare can now be written as

$$W = \frac{Q^2}{2,000} + \left(20Q - \frac{Q^2}{1,000}\right) - \frac{(Q-A)^2}{8,000} - \frac{A^2}{2,000}.$$

This is maximized where

$$\frac{\partial W}{\partial Q} = 20 - \frac{Q}{1,000} - \frac{(Q-A)}{4,000} = 0,$$

$$and \qquad \frac{\partial W}{\partial A} = -\frac{(Q-A)}{4,000}(-1) - \frac{A}{1,000} = 0.$$

Solving these two first-order conditions simultaneously, we get the optimal solution:

$$Q^a = \frac{100,000}{6}, p^a = \frac{50}{6}.$$