Niels Bohr's Generalization of Classical Mechanics

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Abstract

We clarify Bohr's interpretation of quantum mechanics by demonstrating the central role played by his thesis that quantum theory is a rational generalization of classical mechanics. This thesis is essential for an adequate understanding of his insistence on the indispensability of classical concepts, his account of how the quantum formalism gets its meaning, and his belief that hidden variable interpretations are impossible.

KEY WORDS: Bohr, Bohm, Quantum Mechanics, Classical Mechanics, Copenhagen Interpretation, Intertheoretic Relations

1 INTRODUCTION

In his 1994 article "A Bohmian Response to Bohr's Complementarity," Jim Cushing emphasizes the importance of evaluating Bohr's complementarity against the background of other interpretations of quantum theory, in particular, Bohm's interpretation. In his comparison, Cushing highlights two virtues of Bohm's the-

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ory: first, minimizing the radicalness of the departure from classical physics and, second, offering an intelligible interpretation of the quantum formalism. In this paper we argue that Bohr not only would have embraced these two virtues, but he would have claimed them for his own interpretation.

One of the least-discussed and least-understood parts of Bohr's interpretation is his thesis that quantum theory is a rational generalization of classical mechanics. In his own words, "quantum mechanics . . . may be regarded in every respect as a generalization of the classical physical theories" (Ref. [9], p. 4). We argue that an understanding of this thesis is essential for an adequate account of Bohr's philosophy, and we show how this thesis is closely intertwined with his better known views on complementarity, the correspondence principle, and the indispensability of classical concepts. Bohr's view that quantum theory is a generalization of classical theory shapes his understanding of the central interpretive problem facing quantum theory. This problem, which he refers to as the "measurement problem," is that of giving meaning to the quantum formalism by securing the validity of classical laws in measurement.

We conclude by showing how Bohm's theory can provide a useful lens through which to examine Bohr's interpretation. There are a number of surprising points of agreement between these two thinkers that are often overlooked in the literature. We argue that their fundamental disagreement concerns how concepts are to be connected to the quantum formalism—especially when measuring properties such as momentum. It is ultimately Bohr's commitment to the rational generalization thesis that leads him to reject the possibility of hidden variable theories such as Bohm's.

2 A RATIONAL GENERALIZATION OF CLASSICAL MECHANICS

The centrality of the rational generalization thesis to Bohr's philosophy is evidenced by the fact that it is a point that he makes repeatedly in his writings throughout his career. Perhaps surprisingly, this thesis appears in the context of both the old quantum theory and the new (post-1925) quantum theory. One of the earliest references to the rational generalization thesis is in Bohr's report to the third Solvay Congress in 1921:

> It may be useful first to examine the general features of the theory more closely and especially to elucidate, on the one hand, the radical departure of the quantum theory from our ordinary ideas of mechanics and electrodynamics as well as, on the other hand, the formal analogy with these ideas. ... [T]he analogy is of such a type that in a certain respect we are entitled in the quantum theory to see an attempt of a natural generalisation of the classical theory of electromagnetism. (Ref. [4], p. 366)

While Bohr did not view the old quantum theory as being yet a full rational generalization of classical mechanics, he did believe that it was making progress towards this aim.

Although Bohr sees Planck's discovery of the quantum of action as leading to the need for a fundamental revision in physics, in many ways he is more of a continuity theorist than a revolutionary. He is a continuity theorist in the sense that he tries to maintain and emphasize those features of the predecessor theory that are preserved in the transition to the successor theory. It is precisely his fundamental belief in this continuity that he is trying to call attention to by describing quantum theory as a rational generalization of classical mechanics. Bohr sees the old quantum theory and the new quantum theory as part of one continuous development. He emphasizes this point in 1929 when he writes, "We are dealing here with an unbroken development ... which, beginning with the fundamental works of Planck on black body radiation, has reached a temporary climax, in recent years, in the formulation of a symbolic quantum mechanics" (Ref. [8], p. 92). Bohr sees a continuity not only between the old and new quantum theories, but also between classical mechanics and these quantum theories. The point of the rational generalization thesis is to explain what the nature and extent of this continuity is.

In calling quantum theory a generalization of classical mechanics, Bohr is emphasizing that there is a sense in which classical mechanics has not been replaced, but rather survives the quantum revolution in a new form. He explains, "The problem with which physicists were confronted was therefore to develop a rational generalization of classical physics, which would permit the harmonious incorporation of the quantum of action" (Ref. [15], p. 309). In searching for a way to generalize classical mechanics, Bohr made central use of his correspondence principle. One understanding of the correspondence principle that appears repeatedly in his writings is the following: "The correspondence principle expresses the tendency to utilise in the systematic development of the quantum theory every feature of the classical theories in a rational transcription appropriate to the fundamental contrast between the postulates and the classical theories" (Ref. [6], p. 849). Note that Bohr's aim here is one of reconciliation—of bringing the classical and quantum theories together into a rational and consistent whole. We see Bohr reiterating this view of the correspondence principle in 1939 and connecting it more explicitly with his quest for a rational generalization of classical mechanics: "In the search for the formulation of such a generalization, our only guide has been the so-called correspondence argument, which gives expression for the exigency of upholding the use of classical concepts to the largest extent compatible with the quantum postulates" (Ref. [12], p. 13). On this understanding of the correspondence principle, Bohr is not simply saying that the quantum theory should "go over" to the classical theory in the appropriate limit. Rather, he is maintaining that quantum mechanics should be a theory that departs as little as possible from classical mechanics. Bohr's emphasis on continuity was not just a heuristic for theory construction, but was also an essential part of what he took to be the proper understanding and interpretation of quantum theory.

It is important to emphasize that Bohr takes this generalization of classical mechanics to be *rational*. His insistence on the rationality of this enterprise is somewhat surprising, since most physicists and historians have viewed Bohr's blending of classical and quantum ideas as being—at best—"clever bricolage."[1] In his writings, however, one can see that the issue of consistency was never far from Bohr's mind. He understands a rational quantum mechanics to be one that maximally incorporates classical concepts, suitably reinterpreted, in a consistent manner.

The key to a consistent and harmonious incorporation of the quantum postulate into classical mechanics lies in determining the proper scope and applicability of classical concepts. One of the central lessons of quantum theory, for Bohr, is that not all classical concepts can be simultaneously applied to a given experimental situation. The answer to where and when certain classical concepts can be applied is to be found in his viewpoint of complementarity. He explains, "[T]he indivisibility of the quantum of action . . . forces us to adopt a new mode of description designated as *complementary* in the sense that any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena" (Ref. [9], p. 10). While classically one can simultaneously apply all relevant classical concepts to a given physical system, quantum mechanically one can apply only half of the relevant classical con-

[1] Darrigol[23] uses this phrase to describe the physics community's perception of Bohr's old quantum theory.

cepts; or, more precisely, one can simultaneously apply the concepts associated with complementary observables only up to the degree of accuracy permitted by the uncertainty principle. Which concepts apply is determined by the concrete experimental situation in which the physical system is being investigated.

In some ways, Bohr's 'generalization' of classical mechanics to the quantum context might better be described as a *restriction* of classical mechanics. The restriction in question, however, is not one of the domain of applicability of classical mechanics to the domain of large quantum numbers. Bohr is not simply referring to the uncontroversial point that classical mechanics, while no longer universal, nonetheless continues to provide an empirically adequate description of large-scale phenomena. Rather, Bohr's generalization thesis can be understood as a restriction in the following sense: "[I]t is the combination of features which are united in the classical mode of description but appear separated in the quantum theory that ultimately allows us to consider the latter as a natural generalization of the classical physical theories" (Ref. [9], p. 19). To put the point more bluntly, one might say that quantum mechanics just *is* a restriction of classical mechanics in accordance with the viewpoint of complementarity.

Nonetheless, Bohr takes quantum theory to be a generalization, not a restriction, of classical mechanics. He tries to explain the sense in which it is more general as follows: "In representing a generalization of classical mechanics suited to allow for the existence of the quantum of action, quantum mechanics offers a frame sufficiently wide to account for empirical regularities which cannot be comprised in the classical way of description"(Ref. [13], p. 316). In other words, quantum mechanics is a generalization in the sense that it is an extension of the classical theory that, in addition to the usual classical phenomena, allows for the incorporation of a fundamental unit of action, $\hbar$, and the new phenomena that Bohr sees this quantum of action bringing about.

Bohr's thesis that quantum mechanics is a rational generalization of classical mechanics is closely connected with his infamous doctrine of the indispensability of classical concepts. There is some controversy, however, concerning what exactly Bohr means by a "classical concept."[2] The interpretation that is endorsed here is that by "classical concepts" Bohr means simply the concepts of classical mechanics, such as 'position,' 'momentum,' 'force,' 'electric field value,' etc. Support for this interpretation can be found in quotations such as the following: "the unambiguous interpretation of any measurement must be essentially framed in terms of the classical physical theories, and we must say that in this sense the language of Newton and Maxwell will remain the language of physicists for all time" (Ref. [10], p. 692). This interpretation of classical concepts also coheres with Bohr's rational generalization thesis.

Bohr's claim that classical concepts must be used for an unambiguous communication of experimental results has been met with considerable incredulity and puzzlement—both from his contemporaries and from subsequent scholars. One prominent trend in Bohr scholarship has been to try to make sense of this requirement in terms of a Kantian or neo-Kantian framework (a sample of such approaches can be found, for example, in Ref. [25]). The approach adopted here, however, is to try to understand Bohr's doctrine of the indispensability of classical concepts in terms of his belief that quantum theory is a rational generalization of classical mechanics.[3]

A point that has been overlooked in discussions of the doctrine of the indispensability of classical concepts is that there are really two distinct, though

[2]Don Howard has offered what he calls a "reconstruction" of the notion of a classical concept that "seeks to be faithful to Bohr's words." On his view, by classical concepts "Bohr did not mean simply the application of classical physics—the physics of Newton, Maxwell"; he argues instead that by 'classical' Bohr means "a description in terms of what physicists call 'mixtures'" (Ref. [26], p. 203).

[3]This approach can be seen as part of a general trend in recent Bohr scholarship that attempts to understand Bohr's views in terms of his concrete efforts to develop and interpret a physical theory, rather than in terms of a general philosophical framework. Another example of this approach can be found in Tanona's[30] treatment of Bohr's correspondence principle, which we believe is largely consistent with the position we are developing in this paper.

intertwined, ways in which Bohr takes these concepts to be indispensable. The first, and most often commented upon way, is Bohr's claim that the measuring instruments and the results of experiments must be expressed in terms of classical concepts. For example, Bohr writes, "However far quantum effects transcend the scope of classical physical analysis, the account of the experimental arrangement and the record of the observations must always be expressed in common language supplemented with the terminology of classical physics" (Ref. [13], p. 313). There is, however, a second and more subtle sense in which Bohr takes classical concepts to be indispensable. He expresses this second sense most clearly in his 1929 introductory essay:

> According to the view of the author, it would be a misconception to believe that the difficulties of the atomic theory may be evaded by eventually replacing the concepts of classical physics by new conceptual forms. . . . No more is it likely that the fundamental concepts of the classical theories will ever become superfluous for the description of physical experience. The recognition of the indivisibility of the quantum of action, and the determination of its magnitude, not only depend on an analysis of measurements based on classical concepts, but it continues to be the application of these concepts alone that makes it possible to relate the symbolism of the quantum theory to the data of experience. (Ref. [9], p. 16)

In this quotation we see Bohr emphasizing that it is not only in the analysis of measurements that classical concepts are essential. These concepts are also essential for giving meaning to the abstract formalism of quantum theory; that is, they are necessary for connecting up this formalism with experience.[4]

A further clue to Bohr's view on the importance of classical mechanics for quantum theory is found in a little-discussed paper where he engages in a bit

[4] This point is further elaborated in the next section.

of counterfactual history. He asks us to consider a history of physics in which quantum mechanics had been discovered *before* classical mechanics:

> Imagine for a moment that the recent experimental discoveries of electron diffraction and photonic effects, which fall in so well with the quantum mechanical symbolism, were made before the work of Faraday and Maxwell. Of course, such a situation is unthinkable, since the interpretation of the experiments in question is essentially based on the concepts created by this work. But let us, nevertheless, take such a fanciful view and ask ourselves what the state of science would then be. I think it is not too much to say that we should be farther away from a consistent view of the properties of matter and light than Newton and Huygens were. (Ref. [10], p. 692)

In considering whether quantum mechanics could have been discovered first, Bohr immediately runs into the objection that this would be impossible, since the interpretation of the experiments that led to the discovery of quantum theory requires the use of classical concepts. This is the first sense of Bohr's doctrine of the indispensability of classical concepts discussed above. Bohr, however, sets this objection aside and pursues the thought experiment further to draw attention to the second sense in which classical concepts are indispensable. His conclusion is the following: quantum mechanics *by itself* provides a less adequate account of light and matter than does classical mechanics. This is a surprising conclusion to draw, especially given our current understanding of quantum mechanics as the more adequate theory that replaced classical mechanics. Bohr's point seems to be that quantum mechanics—without classical mechanics—is an inadequate theory. He is, of course, not saying that quantum theory is incomplete in the sense of the EPR debate, that is, that there is some element of reality that it leaves out of its description. Rather, it is incomplete in the sense that quantum mechanics depends on classical mechanics for its meaning—

for connecting up its formalism with experience. Only by having classical and quantum mechanics together do we have an adequate physical theory.

When it comes to answering the question of what, according to Bohr, is the relation between classical and quantum mechanics, the usual options of reductionism (quantum mechanics reduces to classical mechanics in the appropriate limit) and theoretical pluralism (each theory has its own proper domain of application) are inadequate. Explicating Bohr's view of the relationship between these two theories is complicated by the fact that there are elements of his view that can be identified with both the reductionist and the pluralist.

On the one hand, Bohr's view is reductionistic in the sense that he takes quantum mechanics to be a universal mechanical theory. This is made particularly clear in his debates with Erwin Schrödinger over the reality of stationary states. For example, he writes,

> [I]n the limit of large quantum numbers where the relative difference between adjacent stationary states vanishes asymptotically, [classical] mechanical pictures of electron motion may be rationally utilised. It must be emphasized, however, that this connexion cannot be regarded as a gradual transition towards the classical theory in the sense that the quantum postulate would lose its significance for high quantum numbers. (Ref. [7], p. 589)

This quotation also brings out another sense in which Bohr is often seen as a reductionist. Frequently in the context of his discussions of the correspondence principle, Bohr notes that the laws of the classical theory are suitable for the description of phenomena in a limited region (e.g., Ref. [5], p. 22). This sounds a lot like reductionism in the sense that Thomas Nickles[28] has labeled "reduction$_2$." In the above quotation, however, Bohr makes it clear that it is not the case that classical mechanics is recovered in any robust sense in this limit; rather the classical algorithm simply provides an adequate approximation in this regime. In 1948 Bohr again emphasizes quantum theory's universal character:

> The construction and the functioning of all apparatus like diaphragms and shutters ... will depend on properties of materials which are themselves essentially determined by the quantum of action. Still ... we may to a very high degree of approximation disregard the molecular constitution of the measuring instruments. (Ref [13], p. 315)

In this sense Bohr—unlike Heisenberg—is not a theoretical pluralist; there is no regime for which classical mechanics is, strictly speaking, perfectly accurate or true.[5]

On the other hand, there is an aspect of Bohr's view of the relationship between classical and quantum mechanics that is more like theoretical pluralism than reductionism. Despite his assertion that quantum mechanics is a universal theory, Bohr is not an eliminativist—he does not think that classical mechanics, even in principle, can be eliminated. As we have seen in some detail, classical mechanics continues to play a very important role in physics, according to Bohr, and it is not just for "engineering purposes."[6] On his view, quantum theory, without classical mechanics, is an inadequate—perhaps even meaningless—theory.

Through his rational generalization thesis, Bohr is offering us a new way of viewing the relationship between classical and quantum mechanics. Quantum mechanics is not a rival to classical mechanics, but rather a modification of it—a modification that depends on the applicability and consistency of the classical theory. As we shall see in the next section, Bohr's view of quantum mechanics as a generalization of classical mechanics shapes his understanding of the central

[5] For a defense of this interpretation of Heisenberg see A. Bokulich Ref. [18].

[6] There is no one in the reductionism-pluralism debate who would deny the continued *practical* utility of classical mechanics. This is not, however, the sense in which Bohr or the theoretical pluralists take classical mechanics to be indispensable.

interpretive problems facing quantum theory.

3 BOHR'S MEASUREMENT PROBLEM

Although one can find references to the "measurement problem" in Bohr's writing, it is important to recognize that his understanding of this problem is quite different from our own. One of the clearest statements of Bohr's measurement problem is found in his 1933 paper on the measurability of quantum fields, coauthored by Léon Rosenfeld. In this paper he writes, "[Characteristic of the] quantum-mechanical measurement problem ... is the possibility of attributing to each individual measurement result a well-defined meaning in the sense of classical mechanics" (Ref. [17], p. 359). There are two points here that deserve emphasis: First, Bohr's primary worry is that of providing *meaning*; that is, he is concerned with providing an *interpretation* of the results of measurements. Second, this meaning comes from *classical* mechanics, and not from quantum theory, or some operational procedure. This measurement problem is crucial to Bohr's understanding of quantum theory, but it is almost completely unrecognized by commentators on his philosophy.

By way of clarification, let us contrast Bohr's measurement problem with the standard measurement problem—a problem that is typically considered to be the central interpretive issue facing quantum theory. Suppose we have a device that measures some property, such as whether a particle passes through the upper or lower slit in a diaphragm. By definition, this means that if the particle is localized in the upper slit of the diaphragm, then the interaction of the particle with the apparatus will leave the apparatus in a state indicating this; we describe this situation by saying that the pointer needle of the apparatus goes from a "ready" state to an "up" state. If we assume that the state of the particle is left unchanged, we can represent this evolution as

$$|\uparrow\rangle|\text{“ready”}\rangle \Longrightarrow |\uparrow\rangle|\text{“up”}\rangle. \quad (1)$$

Likewise, if the particle passes through the lower slit, the pointer needle will evolve into a "down" state:

$$|\downarrow\rangle|\text{"ready"}\rangle \Longrightarrow |\downarrow\rangle|\text{"down"}\rangle. \qquad (2)$$

The difficulty arises from the fact that quantum evolution is linear, which implies that if $|\uparrow\rangle$ and $|\downarrow\rangle$ are both solutions to the equations of motion, then a linear superposition of these states such as $|\uparrow\rangle + |\downarrow\rangle$ (ignoring normalization factors) will also be a solution. However, if the evolution represented by "$\Longrightarrow$" is the unitary evolution of quantum theory, then such a state will evolve as

$$\Big(|\uparrow\rangle + |\downarrow\rangle\Big)|\text{"ready"}\rangle \Longrightarrow |\uparrow\rangle|\text{"up"}\rangle + |\downarrow\rangle|\text{"down"}\rangle. \qquad (3)$$

The state we are left with seems to describe a superposition of "up" and "down" states for the *macroscopic* pointer needle; however, we never observe such superposed macroscopic states. The standard measurement problem, then, is to reconcile the lack of macroscopic superpositions in our world with a quantum theory that seems to demand such superpositions.

By contrast, Bohr's measurement problem concerns "the possibility of attributing to each individual measurement result a well-defined meaning in the sense of classical mechanics." He would claim that what we have called the standard measurement problem is ill formed from the very start. According to Bohr, the glib identification of state vectors such as $|\uparrow\rangle$ with physical properties such as having a location within the upper slit of a diaphragm stands in need of *justification*. Mathematical objects in the quantum formalism, such as a wave function or a vector in a Hilbert space, must be interpreted with great care.

Throughout his career, Bohr emphasizes that the quantum formalism is a "purely symbolic scheme," an abstract symbolism that requires careful interpretation. He gives the following account of how the formalism is given meaning:

> [T]he appropriate physical interpretation of the symbolic quantum-mechanical formalism amounts only to predictions, of determinate or

statistical character, pertaining to individual phenomena appearing under conditions *defined* by classical physical concepts" (Ref. [14], p. 238, emphasis added).

The central interpretive move here is not that of limiting the quantum theory to a merely instrumental role, but rather of connecting up the quantum mechanical formalism with phenomena through the use of classical concepts. He believes that it is only through the application of classical concepts that the formalism can be given an unambiguous meaning. It is our ability to secure the legitimacy of the classical laws in certain contexts—and especially in the context of measurement—that allows us to apply classical concepts.

Bohr's measurement problem is a special case of what he more frequently refers to as the "observation problem." This is the general problem of how to connect up a physical formalism with empirical data. Bohr notes that the observation problem is not specific to quantum theory and often draws an analogy to the observation problem in relativity. The generality of this problem leads one to question whether there are further complications that arise in the particular case of quantum theory. Bohr's answer to this is yes—and no. On the one hand, Bohr writes, "[T]he observation problem in quantum theory involves, however, certain novel epistemological aspects as regards the analysis and synthesis of physical experience" (Ref. [12], pp. 18-19). He is here referring to the revolutionary lesson of complementarity that we can no longer synthesize the observations gained from different experimental contexts into a single coherent picture. On the other hand, Bohr also tells us, "As all measurements . . . concern bodies sufficiently heavy to permit the quantum to be neglected in their description, there is, strictly speaking, no new observational problem in atomic physics" (Ref. [16], p. 170). It is precisely by solving his measurement problem—i.e., by being able to interpret measurement results classically—that Bohr is able to reduce the quantum observation problem to the classical one.

Note that the solution to the measurement problem requires producing a measurement arrangement that allows one to neglect any quantum uncertainties in the analysis of its functioning. That is, we need to guarantee that we are safely in the classical limit. As we shall see in the following section, this problem can sometimes pose substantial challenges.

4 THE APPLICATION OF CLASSICAL CONCEPTS IN MEASUREMENT

Much of the confusion over Bohr's use of classical concepts grows out of his attempts to avoid technical details and use simple examples that will be accessible to non-physicists. Thus his discussions of complementarity are nearly always restricted to cases such as the two-slit experiment, in which the classical concepts are the familiar ones of position and momentum, and the incompatibility between the contexts in which these concepts can be applied rests on the obvious fact that a diaphragm cannot simultaneously be rigidly fixed and also free to move in response its interaction with the particle passing through it. The simplicity of Bohr's preferred example, however, presents a challenge for those who are looking for a rigorous exposition of his position.

A much more helpful example of Bohr's application of classical concepts in quantum measurements is his treatment of the measurability of quantum field values, which he published together with Rosenfeld in 1933. This paper has received very little attention from historians and philosophers of physics, and when it is discussed, it is often misinterpreted.[7] The value of Bohr and Rosenfeld's treatment lies in the fact that the application of classical concepts in this case is highly non-trivial: they are forced to argue carefully for the applicability of the concepts in question, which provides us with a much more rigorous account

[7] An account of the (mis)understanding of Bohr and Rosenfeld's argument can be found in a forthcoming work by one of us (P.B.).

of how these concepts are characterized and applied. We shall see below that the lessons extracted from the case of measuring fields can be straightforwardly extended to clarify Bohr's comments about the two-slit experiment.

Bohr and Rosenfeld begin by explaining that the measurement of a component of a quantum electromagnetic field must, in principle, follow the same procedure that would be used to measure the components of a classical electromagnetic field. Specifically, we must place a charged test body in the region of interest and measure the amount of momentum transferred to this body. This, according to Bohr and Rosenfeld, is required by the very *definition* of the electromagnetic field: "The measurement of electromagnetic field quantities rests by definition on the transfer of momentum to suitable electric or magnetic test bodies" (Ref. [17], p. 368).

Bohr and Rosenfeld's treatment deals with a number of difficulties and subtleties that we need not go into here; fortunately, the essential lessons can be extracted from a fairly straightforward example: the measurement of a single component of the electric field. To avoid the infinities introduced by the treatment of point charges, they consider an extended test body of volume V and uniform charge density ρ. The electric field value averaged over volume V and time period T will then be given by the amount of momentum transferred to the test body during this period, as specified by the following equation:

$$p''_x - p'_x = \rho \bar{\mathfrak{E}}_x V T. \tag{4}$$

According to Bohr and Rosenfeld, this equation of classical electrodynamics is our *definition* of the average electric field value $\bar{\mathfrak{E}}_x$. Therefore, this must serve as the foundation for an account of a *quantum* field component as well. If we claim to have measured a value of the x-component of the quantum electric field, the very meaning of this statement implies that we have measured the momentum of the test body at two times, and have reliably been able to invoke Equation (4), or some equivalent classical law, to infer the value of $\bar{\mathfrak{E}}_x$.

As Bohr and Rosenfeld emphasize, however, the applicability of this equation faces a number of limitations. To begin with, it requires that the acceleration of the test body be negligible during time T. This can be assured by using a sufficiently heavy test body. As Bohr often points out, using objects with very large mass often secures for us the validity of classical descriptions because we then face only negligible limitations imposed by the Heisenberg uncertainty principle,

$$\Delta x \Delta p_x \geq \hbar/2. \tag{5}$$

However, the precision of a measurement of the value of $\bar{\mathfrak{E}}_x$ depends both on the precision of the initial and final momentum measurements, p'_x and p''_x, and on the precision of the location of our test body, which is supposed to be measuring the average field in region V. Thus merely increasing the mass of the test body does not secure the precision we need. Fortunately, however, we also have the charge density of the test body at our disposal, and by increasing its value we can decrease the uncertainty in the inferred value of $\bar{\mathfrak{E}}_x$ to any degree desired, despite the fundamental limitations imposed by (5).

There is, however, a further difficulty that arises from the fact that our charged test body will itself alter the field we are trying to measure. In classical electrodynamics, we can consider the limit in which the charge, mass, and volume of a test body all go to zero, but as we have just seen, this is not possible in the quantum case. When our strongly charged test body is fixed in region V, we can compensate for its effect on the field by adding another body with an equal and opposite charge density, $-\rho$, to the region. However, at the beginning of time period T, the test body will have to be freed and subjected to a momentum measurement. In response to this momentum measurement, Bohr and Rosenfeld tell us, the test body "experiences a simple non-uniform translation in the x-direction" (Ref. [17], p. 380), the order of magnitude of which will be specified by the uncertainty relation (5).[8]

[8] This way of describing the situation gives the appearance that the test body has a definite

Because the test body is no longer aligned with the neutralizing body, the field will now be altered by the new charge distribution, and this additional field strength will then have an effect on the amount of momentum that is transferred to our test body. Bohr and Rosenfeld calculate the magnitude of this back-reaction of the field on the test body and are able to determine that its strength will be proportional to the "unpredictable displacement" of the test body. Thus, even though we are unable to measure the displacement of the test body (on pain of rendering useless our initial momentum measurement), we can install a carefully calibrated spring that will precisely *compensate* for the change in the momentum that is due to the unwanted field effects.

With this careful arrangement of neutralizing charge distributions and springs in place, we can now legitimately apply Equation (4) to infer the average field strength we are interested in, and Bohr and Rosenfeld declare success:

> Without further correction, the measurement results obtained by means of the experimental arrangement described thus appear as the desired field averages for testing the theoretical statements. Such a view of the measuring results . . . is also suggested by the fact that all measurements of physical quantities, by definition, must be a matter of the application of classical concepts; and that, therefore, in field measurements any consideration of limitations on the strict applicability of classical electrodynamics would be in contradiction with the measurement concept itself. (Ref. [17], p. 387)

The measurement problem for this simple case is therefore solved:[9] we have, in Bohr and Rosenfeld's words, attributed to the "individual measurement result

position at all times, a position that is unknown only because it is *disturbed* in the momentum measurement. P. Bokulich[19] argues that although this is a natural reading, it is mistaken precisely because it fails to recognize that Bohr's arguments, here and elsewhere, are fundamentally about the limitations of our *concepts*, and only secondarily about the limitations of our ability to discover values in measurements.

[9]Measurements of more than one field component, perhaps over different regions and at

a well-defined meaning in the sense of classical mechanics" (Ref. [17], p. 359). The abstract quantum formalism of (source-free) quantum electrodynamics has been connected with the possibilities of observation by formulating a context in which the appropriate laws of classical physics apply.

One way that Bohr formulates his insistence on classically describable measuring arrangements is in terms of the requirement that our use of classical concepts be "unambiguous," and this lack of ambiguity is essentially tied to our ability to ignore the existence of Planck's constant and remain in the classical domain:

> [T]he possibility of an *unambiguous* use of these fundamental [classical] concepts solely depends upon the self-consistency of the classical theories from which they are derived and . . . therefore, the limits imposed upon the application of these concepts are naturally determined by the extent to which we may, in our account of the phenomena, disregard the element which is foreign to classical theories and symbolized by the quantum of action. (Ref. [9], p. 16)

The above example of a field measurement provides us with an explicit example of the efforts that we may have to go through to exorcize $\hbar$ from our description of the measuring arrangement and secure the use of a classical concept such as that of an average component of the electric field in some spatiotemporal region.

With these lessons in mind we can now gain a deeper understanding of Bohr's preferred example, the two-slit experiment. A position measurement involves

differing times, require a more complicated treatment, making use of more auxiliary bodies, springs, levers, and light signals—the fundamental lessons regarding the applicability of classical concepts, however, remains unchanged. A further worry that should be mentioned here is that in the 1950s, Corinaldesi—a student of Rosenfeld's—discovered an error in the 1933 calculations that seems to imply that the measurement results described here will in fact be masked by quantum fluctuations. This is discussed by Darrigol[22], and more extensively in a forthcoming work by one of us (P.B.).

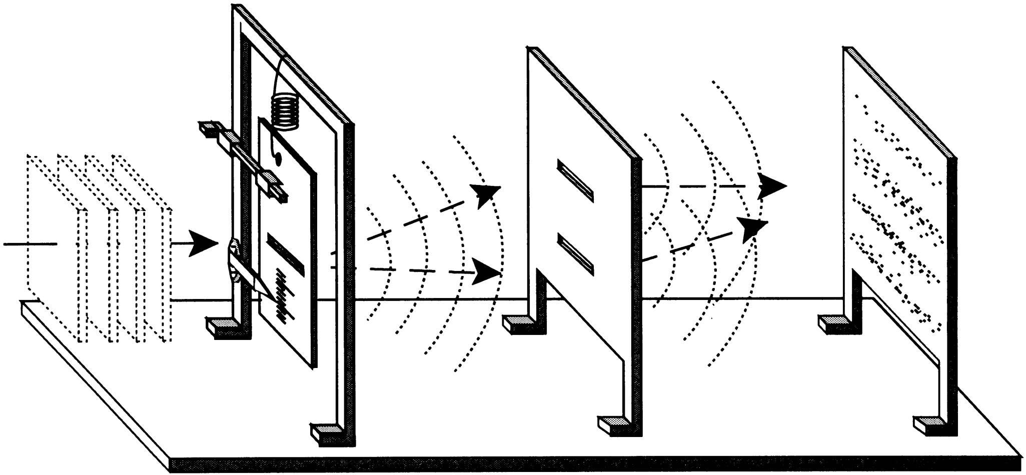

Figure 1: Bohr's two-slit thought experiment.

correlating the position of the particle with some piece of lab apparatus, such as the slit of a diaphragm—rigidly bolted to a lab bench—through which the particle passes (see Figure 1, which is based on the diagrams in Bohr Ref. [14]). From the fact that the particle has passed through the diaphragm, we conclude that it had a position within the slit, and the position of the slit relative to the lab bench can easily be determined using a ruler, for example. This is a clear example of what Bohr means when he tells us that the property of the measuring apparatus that is to be correlated with the property we are interested in measuring must be "directly determinable according to its definition in everyday language or in the terminology of classical physics" (Ref. [12], p. 19). Here we are concerned with the position of the electron, and we succeed in measuring it once we correlate it with the slit of the fixed diaphragm, for the position of this slit can be straightforwardly (and unambiguously) specified by a phrase such as 'ten centimeters above the lab bench.'[10]

Now let us consider the case in which we wish to establish the momentum

[10]For an analysis of Bohr's reliance on reference frames in his treatment of position and momentum measurements, see Dickson Ref. [24].

of the electron after it passes through the slit. This requires us to allow the diaphragm to move freely in response to its interaction with the electron. We can then measure the momentum of the diaphragm before and after the electron passes through the slit, and thereby predict the outcome of any future momentum measurement using conservation of momentum. (In the two-slit experiment this would allow us to determine which slit the electron subsequently passed through.) Measuring the momentum of the diaphragm is straightforward. In Bohr's words,

> such measurements of momentum require only an unambiguous application of the classical law of conservation of momentum, applied for instance to a collision process between the diaphragm and some test body, the momentum of which is suitably controlled before and after the collision. It is true that such control will essentially depend on an examination of the space-time course of some process to which the ideas of classical mechanics can be applied; if, however, all spatial dimensions and time intervals are taken sufficiently large, this involves clearly no limitation as regards the accurate control of the momentum of the test bodies, but only a renunciation as regards the accuracy of the control of their space-time coordination. (Ref. [11], 698)

Notice that we correlate the momentum of the diaphragm with that of the test body via the application of the *classical* conservation law. The momentum of this test body is then measured (or "controlled") once we can apply the "ideas of classical mechanics" to the "space-time course of some process."

To see what Bohr means by this, consider how we would go about measuring the momentum of our test body. A typical procedure would be to use a charged test particle and measure its deflection in a magnetic field of known strength B.

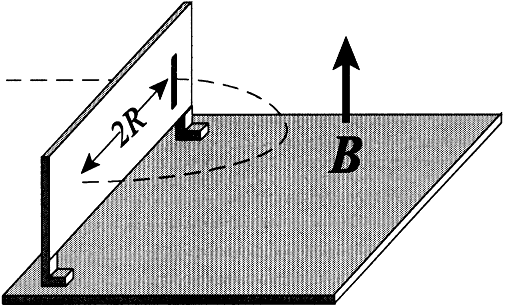
2R
B

Figure 2: A momentum measurement.

The momentum of the particle is then given by the relation:

$$p = eB/cR, \tag{6}$$

where R is the radius of the circle along which the particle travels, and e is the charge of the particle. A simple arrangement for such a measurement is pictured in Figure 2. The particle is allowed to enter the detector through a slit, and it registers on a screen after its direction is reversed by the magnetic field. The radius R is then half the distance from the slit to the registration point on the screen. The precision of this measurement will be limited by the width of the entrance slit and the precision of our final position measurement.

Notice that this measurement involves, in Bohr's words, "an examination of the space-time course of some process to which the ideas of classical mechanics can be applied." The magnetic field here must be described by classical electromagnetism with an accuracy sufficient for the precision desired in our experiment. Then, given the classically described positions of the entrance slit and final registration spot, we can infer the momentum of the electron. This is possible because our classical electrodynamical calculation is valid in this arrangement—again, to the degree of accuracy desired for our measurement.

This would not be the case if, for example, R were the same order of magnitude as the width of our slit. Our actual measurement involves taking a ruler and measuring the distance from entrance slit to the registration spot, but we can use the "ideas" of classical mechanics legitimately to assign the particle a value of momentum—where now this momentum can be thought of as an actual, well-defined quantity, in the same sense that it is well defined in classical mechanics.

5 WHAT BOHM'S THEORY CAN TEACH US ABOUT BOHR

Bohr's views on the application of classical concepts can be brought into sharper focus by contrasting them with Bohm's hidden variable interpretation.[2] Standard characterizations of the conflict between these two thinkers typically overlook the important similarities between their views and mistakenly identify the issue of determinism vs. indeterminism as their fundamental point of disagreement. One key question here is why Bohr thought that hidden variable theories such as Bohm's were impossible. This question becomes even more pressing when one realizes that Bohr took a dismissive attitude towards von Neumann's "proof" that any hidden variable theory would be inconsistent with the predictions of quantum mechanics.[11] Some have approached this question by focusing on contingent historical facts, such as which interpretation arrived first on the scene, or the political dynamics between the physicists involved (see Cushing Ref. [20] and Beller Ref. [1]). While these "external" considerations certainly played a role, we argue that a complete explanation must also take into consideration points internal to Bohr's philosophy. In particular, we argue that the primary motivation for Bohr's rejection of hidden variable theories is to be

[11] See Kalkar Ref. [27] pp. 262-263. For a discussion of von Neumann's theorem and the attitude of the physics community toward it, see Cushing Ref. [20], pp. 131-134.

found in his view that quantum theory is a rational generalization of classical mechanics.

It is standard to view Bohr as an advocate of indeterminism; indeed, this is often viewed as the crux of his debate with Einstein. However, this view of Bohr's position is far too simplistic, and Bohr himself repeatedly rejects this characterization of the debate: "The view-point of complementarity allows us indeed to avoid any futile discussion about an ultimate determinism or indeterminism of physical events, by offering a straightforward generalization of the very ideal of causality" (Ref. [12], p. 25). Bohr's view seems to be that the choice between determinism and indeterminism is a false dichotomy. The notion of causality, like classical mechanics itself, is not to be *abandoned*, but rather is to be *generalized* in such a way as to allow for the existence of the quantum of action. According to Bohr, the appropriate generalization of causality is complementarity. In his "Discussion with Einstein ... " Bohr again emphasizes this point: "[T]he viewpoint of complementarity may be regarded as a rational generalization of the very ideal of causality" (Ref. [14], p. 211). Given Bohr's understanding of this issue, it would be a mistake to view his rejection of hidden variables as stemming from some sort of opposition to determinism.

The fundamental difference between Bohr and Bohm can perhaps best be brought out by asking the following simple question: *Do momentum measurements measure momentum*?[12] Bohr's answer to this is an unequivocal yes. As we have seen, he claims that in ascribing a momentum to a quantum particle we are, *by definition*, saying that we have been able to secure the validity of a classical law that allows us to infer the momentum of that particle. The validity of the classical law in this context rests on our being able to neglect quantum uncertainties in our analysis of the measuring procedure, i.e, on our being in

[12]By momentum measurement we mean a process by which the observable associated with the operator $\hat{p} = -i\hbar\partial/\partial x$ is measured. An example of such a process was given at the end of Section 4.

the classical limit. Thus, the value of momentum revealed in this procedure is the actual momentum of the particle. This is because momentum is a classical concept and we have secured the necessary classical context for its application. Without this connection to classical mechanics we would not even know what we *mean* by the term 'momentum.'

Bohm's answer to this question, however, is *no*: so-called momentum measurements generally do *not* reveal the actual (pre-measurement) value of the particle's momentum. Bohm's interpretation of the quantum formalism stipulates that the *actual* momentum of a quantum particle is given by the gradient of the phase of the wave function. That is, if we decompose the particle's wave function ψ into two real functions R and S (such that $\psi = Re^{iS/\hbar}$), then we define the particle's momentum as

$$p = \nabla S. \tag{7}$$

While the *meaning* of momentum in this case is exactly the classical meaning (the velocity of the particle multiplied by its mass) this value of momentum will not generally be revealed by a procedure in which we use the laws of classical physics, while securely in the classical limit, to infer a momentum value.

This can be readily seen from the nature of the guidance condition (7). Neither the phase of a wave function, nor the gradient of that phase, will in general be constant over space; instead, Equation (7) will generate a vector field that assigns a momentum to each point of space. Thus, a typical ensemble of particles with identical wave functions will consist of particles with different momenta—where each momentum is determined by the wave function and the precise position of the particle. The exception is when the wave function of our particle(s) is a "momentum" eigenstate $\psi = e^{ipx/\hbar}$. In this case it is obvious that the gradient of the phase is simply the value p, so all particles associated with this wave function, regardless of their position, will have the momentum p. Momentum eigenstates are also the only wave functions that will remain unchanged by "momentum measurements." In general, such procedures will drive some arbitrary wave function into separated packets that individually will approximate momentum eigenstates—but *which* of these packets the particle actually ends up in will sensitively depend on the original position of the particle. Thus "momentum measurements," according to Bohm's interpretation, actually "reveal" only a combination of the gradient of the phase of the wave function and the original position of the particle.

It is worth noting here that Bohm's theory relies on the claim that fundamentally all measurements are *position* measurements. This allows him to privilege the positions of particles as their primary, real intrinsic properties (and, incidently, the positions are actually revealed by "standard" position measurements). In this claim that all measurements ultimately boil down to position (and time) measurements, Bohm and Bohr—somewhat surprisingly—agree. Bohr writes,

> Strictly speaking, every reference to dynamical concepts implies a classical mechanical analysis of physical evidence which ultimately rests on the recording of space-time coincidences. Thus, also in the description of atomic phenomena, use of momentum and energy variables for the specification of initial conditions and final observations refers implicitly to such analysis and therefore demands that the experimental arrangements used for the purpose have spatial dimensions and operate with time intervals sufficiently large to permit the neglect of the reciprocal indeterminacy expressed by [$\Delta q \cdot \Delta p = h/4\pi$]. (Ref. [13], p. 315)

Bohr here is telling us that even when we describe the state of a system in terms of momentum, this ultimately amounts to taking position (and time) measurements and applying the laws of classical physics to attribute momentum values to the system. The measurement of the displacement of a diaphragm hanging from a spring with a known (classically described) spring constant, and the measurement of the position of a charged particle after it has been deflected in a magnetic field are both examples of such an analysis of "space-time coincidences."

In the above quotation, however, we can also see the principle divide between Bohr and Bohm. For while the Bohmian route from the measured positions (or "recording of space-time coincidences") to the actual momenta of the particles will require an essentially *quantum mechanical analysis*, Bohr insists that the route from this evidence to the "momentum and energy variables" must be by way of a "*classical mechanical analysis*."

If our reading of Bohr's use of classical concepts is correct, then the primary motivation that he would have for rejecting a hidden variable theory such as Bohm's would be the manner in which such a theory goes about trying to supply the quantum formalism with meaning. Bohr's insistence on the necessity of utilizing classical theories to interpret the abstract quantum formalism leaves little room for maneuvers such as Bohm's that drive a wedge between our ascriptions of properties and our classical methods of measurement. Bohr would presumably see such maneuvers as cutting off the only branch we have to stand on.

Unfortunately, Bohr apparently never commented on Bohm's theory in print, so this characterization of Bohr's criticisms rests on some degree of speculation. However, our reading of Bohr, which focuses on the interpretative significance of the application of classical concepts, gains some support from the criticisms leveled against Bohm by Rosenfeld, one of Bohr's most devoted disciples. Of course, one needs to exercise caution in attributing a spokesperson's views to Bohr, but in this case Rosenfeld's complaints clearly flow from truly Bohrian concerns:

> Bohm's argument is very cleverly contrived. One would look in vain

> for any weakness in its formal construction. What a paradox! Here is a faithful translation of all the formulae of quantum mechanics into a language which to all appearances is that of classical mechanics. . . . Yet, all this seductive construction is just a sham. It is Bohm's pleasure to give his 'hidden parameters' such names as coordinate and momentum, but it is a far cry from the name to the thing. In order to be sure that such and such a parameter really represents the position of a particle it is necessary to examine its relation to the spatial system of reference of some observer, in other words to analyse the *measurement* of the position. But then, as one would expect and as Bohm conscientiously proves, one finds that the identification of the parameters with the corresponding physical concepts is only justified within the limits of the uncertainty relations. Thus, in the end, this subtle and laborious circuit leads us back again to complementarity. (Ref. [29], pp. 402-403)

If Rosenfeld were more familiar with the theory he might have realized that his example of a position measurement is inappropriate, for this is one case where Bohm's theory tells us that an ideal measurement actually does reveal the pre-measurement value of the particle's property. A position measurement actually measures position, even by Bohr's standards. If anything deserves the label "sham" it is the relationship between the particle *momentum* postulated by the theory, and the "momentum" revealed in ideal measurements—for here indeed Bohm's theory does not accord with Bohr's analysis of the proper attribution of classical concepts. Nonetheless, Rosenfeld's insistence that we must "analyse a *measurement*" (emphasis Rosenfeld's) before we can attribute a meaning to a parameter is a faithful reiteration of Bohr's dictum that "experimental conditions . . . constitute in fact the only basis for the definition of the concepts by which the phenomenon is described" (Bohr Ref. [12], p. 24); and, if our reading of Bohr is correct, it is precisely this principle that is violated by hidden variable theories such as Bohm's.

A Bohmian response to Bohr requires less speculation: in a centenary volume dedicated to Bohr, Bohm contributed a paper in which he outlines the similarities and differences between their views. Bohm begins by distinguishing Bohr's view from von Neumann's—two views that are to this day still often conflated under the rubric of the "Copenhagen interpretation."[13] He criticizes von Neumann for continuing to speak of a "quantum system" as being composed of parts that exist separately from each other and separately from the measuring instrument. He praises Bohr for avoiding this mistake and recognizing that "what is relevant instead is the *wholeness* of the form of the experimental conditions and the content of the experimental results" (Ref. [3], p. 154). It is on this issue of wholeness that Bohm identifies Bohr's views as being in closest agreement with his own. Bohm sees himself siding with Bohr in taking this wholeness to be the central lesson that emerged out of the EPR-Bohr debate.

Bohm furthermore seems to recognize that the fundamental disagreement between him and Bohr lies in the doctrine of the indispensability of classical concepts. He writes,

> Thus far, Bohr's views are in general harmony with those adopted in my discussion of 'hidden variables.' But now we come to an important difference between Bohr's views and my own. For Bohr went on to say that the terms of discussion of the experimental results were *necessarily* those of 'everyday language,' suitably 'refined' where necessary, so as to take the form of classical dynamics. (Ref. [3], p. 158).

[13]It is important to recognize that nowhere in Bohr's many writings does he ever refer to a collapse of the wave function, and this notion plays no role in his interpretation. Thus, it might be useful to refer to von Neumann's collapse view by some other label and reserve the term "Copenhagen interpretation" for Bohr's views.

Although Bohm is correct in identifying this as the crux of their disagreement, it is not clear that he really comprehends what leads Bohr to this view. Bohm, like numerous other readers of Bohr, finds this commitment to classical concepts incomprehensible. He also levels the common, but mistaken, criticism that "Bohr was led to the conclusion that the 'quantum' implies absolute contingency—that is, the necessity for 'complete randomness'" (Ref. [3], p. 159). Bohm concludes by painting his program as one that tries to free quantum physics from this "unreasonable" constraint. He writes, "What is called for, in my view, is therefore a movement in which physicists freely explore novel forms of language, which take into account Bohr's very significant insights but which do not remain fixed statically to Bohr's adherence to the need for classical language forms" (Ref. [3], p. 159). Bohm sees his own hidden variable interpretation as introducing these needed new language forms.

Bohm's theory teaches us that we need not apply classical concepts in the way that Bohr prescribes. In this sense, Bohr's insistence on the doctrine of the indispensability of classical concepts would indeed be "an (unwarranted) slide from consistency to necessity" (Cushing Ref. [21], p. 59). Most commentators, however, would question whether Bohr's doctrine is even consistent. We have here argued that the root of the confusion over this doctrine lies in the tendency to consider this aspect of Bohr's philosophy in isolation from his general position on the relationship between classical and quantum mechanics. Once we recognize that Bohr takes quantum theory to be a *generalization* of classical physics, his insistence on the use of classical concepts loses much of its mystery. While this certainly does not establish the necessity of Bohr's interpretation, it does bring us one step closer to recognizing its consistency.

Acknowledgments

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