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Impact of Licensing on Investment and Financing of Technology Development*

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Abstract

Technology innovations continue to be one of the greatest drivers of economic growth. Realizing the value of such innovations, however, requires substantial follow-on investments in development and commercialization. The value of these investments depends on not only the demand, but also actions by potential competitors, which are uncertain at the time of the investment. This uncertainty creates difficulties in obtaining outside financing. Creative use of licensing contracts can capture the value and alleviate the financing problem. We develop an investment-timing model that includes a variety of licensing possibilities, including fixed fees, royalty schedules, and two-part licenses consisting of an upfront payment and a capped royalty schedule. In our model, investment in development efforts gives a firm the ability to license its technology and dissuade other firms from developing alternative technologies. We show that a royalty schedule that depends on realized demand dominates a fixed licensing fee. When investment funds are constrained, a two-part license can serve as a source of financing. We also study the investment problem conditional on the licensing and financing decisions.

Keywords: Licensing, investment under uncertainty, innovations, financing, real options, game theory

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1 Introduction

Investing in technology R&D and reaping the benefits of these investments are prime concerns to firms in a wide range of industries. Firms have to invest in development efforts while the potential value of an innovation is still uncertain. On the one hand, a new product developed around the innovation may fail to generate sufficient demand; on the other hand, when demand is high, other firms may enter the market and provide similar products, significantly reducing the innovating firm's profits. The presence of uncertainty may also cause difficulties in obtaining financing for the development efforts. In this paper, we explore how licensing arrangements can be used to dissuade potential competitors and mitigate the financing problem.

The environment for technological innovations exhibits the following features. First, innovations require significant irreversible investments in development and commercialization efforts. Technological breakthroughs lead to innovations that may form a key component vital for a new product.[1] Substantial further investment is needed to develop it into a form that can be used in the final new product. The investment may be incurred in engineering, testing, capacity building, manufacturing, or legal efforts. Such investment is often specialized in the new product and results in "complementary assets" that are crucial to the success of the product (Teece 1986). In sum, the "development" aspect of "research and development" can be as commercially important as the "research" (Green and Scotchmer 1995).

Second, innovating firms often deploy cooperation strategies such as licensing in face of market uncertainty. By licensing to potential entrants, the firm with the innovation is able to discourage other development efforts and appropriate value via licensing revenues. Worldwide, revenues from patents have soared from $15 billion in 1990 to $100 billion in 2000. Examples and cases of licensing strategies are observed in a wide range of industries. Teece (1986) provides details of several licensing cases in industries such as petrochemical, manufacturing, computer, and electronics. Recent case studies in biotechnology (Lerner and Merges 1998), information technology (Cusumano and Selby 1995), and

[1] Examples of innovations might be new delivery mechanisms for future drugs; new light sources for a big screen DLP TV; new equation editors in word processing software; and new features for upcoming models of automobiles.

chemical (Davis and Harrison 2001) industries describe both successful and failed intellectual property licensing. These cases show that firms' investment and licensing strategies depend critically on expected market conditions.

Third, a variety of licensing arrangements is observed, and royalty schedules are most often used. Rostoker (1984) finds 39% of the licensing contracts use royalties alone, 13% fixed fee alone, and 46% down payment plus running royalty. Goldscheider (1995) notes royalties account for most of the licensing remunerations. For instance, competing chemical firms are known to write licenses with declining step royalty rates for component pieces of a manufacturing process (e.g., binding agents for paints).

Fourth, licensing can play a key role in the financing of R&D investments for start-up firms that are often financially constrained (Gompers and Lerner 1999, Hall 2002). They may enter into licensing contracts with established firms in exchange for an upfront payment, which serves as a means of financing. The established firms that provide such financing to startups are often industry incumbents, suppliers, and distributors (Teece 1986, Gans et al. 2002).

We now examine several case examples that exhibit these features. Consider Qualcomm's decision to commercialize their innovative CDMA technology. In order to have the best chance of success, in the late 1980s, Qualcomm decided to license its technology while also producing and marketing its own version of CDMA products (Mock 2005). Specifically, Qualcomm licensed its CDMA technology to more established equipment manufacturers, like Motorola. Qualcomm and Motorola could be viewed as strategic competitors, because when the CDMA networks were deployed, Qualcomm also made its own wireless devices that embedded its CDMA chipsets.[2] When entering into the licensing deal, Qualcomm was incurring large R&D expenditures but being a privately held firm financially constrained. Mock (2005) points out how "financing was the lifeblood" for Qualcomm in its early stage when the cash flows did not cover salaries, let alone capital investment needs. Steve Altman, Executive

[2] Motorola at that time was producing cellular equipments based on its own AMPS technology.

VP and President, Qualcomm Technology Licensing, reinforced this point: "*License fees early on were extremely important because we needed money to survive.*" (Mock 2005, p. 160)

As is standard industry practice, specific details about Qualcomm's licensing terms are veiled in secrecy.[3] We are, however, able to obtain somewhat more detailed data on licensing deals in the pharmaceutical industry. For instance, we observe royalty rates that decline with sales. In 1998, Eli Lilly entered into an agreement granting the global rights for the anti cancer drug Ontak™ to Ligand Pharmaceuticals, under which Ligand would pay royalties according to the following schedule: no royalties on sales up to $38 million, a royalty rate of 20% on sales between 38 and 50 million, 15% on sales between 50 and 72 million, and 10% on sales in excess of $72 million.[4] In other cases, when the licensor is financially constrained, we often observe upfront payments coupled with royalty schedules. In their 2004 deal, Bio Delivery Sciences International (BDSI) licensed a topical formulation of its lead product BioNasal™ for the treatment of chronic sinusitis, to Accentia Biopharmaceuticals while retaining the rights for the oral and intravenous formulations of the drug. This deal allowed both firms to market the product in the same geographic market, albeit in different formulations. The terms of the license called for an upfront payment of $2.5 million, and effective royalty rates of 6% and 7% of net sales. In another example, in 2002 Biovail Corp. agreed to license from DepoMed, Inc the rights to manufacture and market a once-daily metformin product that was undergoing Phase 3 clinical trials for type 2 diabetes. The agreement provided for a multi-tiered royalty rate on sales, with an option at Biovail's election, to reduce certain of the royalties for a one-time payment to DepoMed of $35 million. These examples show a rich variety of license arrangements that exhibit tiered structures where the royalty rate varies with sales. Furthermore, smaller firms are willing to reduce the rate at which they receive royalties in exchange for an upfront fixed payment that can be used as a financing vehicle.

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[3] "*[T]he legal team at Qualcomm has been adamant about the confidentiality of the company's license terms, especially in the case of royalty points and financial terms*." (Mock 2005, p. 180) In fact, apart from aggregate data, details of licensing arrangements are usually treated as trade secrets and rarely revealed publicly. One exception is the pharmaceutical industry where some details about the deal terms are available through proprietary data services. Another exception is intellectual property (IP) conflicts that reach litigation, where deal terms may be required to be made public.

[4] Source for this and subsequent examples: PharmaDeals, an industry database owned by PharmaVentrues.

Our model stylizes these features and helps shed new insights. Suppose multiple firms engage in technological research races that lead to new products or services. One firm with a discernible lead (e.g., through a technology breakthrough) gains an opportunity window when it can develop the technology and then license it to others. We call such a firm the "leading" firm. While the passage of time will resolve some or all of the market uncertainty, it may also close the opportunity window as other firms make their own breakthroughs.

The leading firm has several vital decisions. First, should it attempt to seize its advantage by committing the development investment immediately, or should it wait? Second, should it license the innovation to others? If so, what kind of licensing arrangement (royalty, fees, or a combination) should it offer and when (now or later)?

To address these questions, we include a variety of license structures and extend the classic investment-timing problem by allowing investment to affect the market structure. The leading firm has an investment opportunity window that expires when uncertainty regarding market demand is resolved. A second firm (the new entrant) will be able to invest at this time and develop a perfectly substitutable product. We consider the leading firm using a licensing arrangement to dissuade the new entrant's investment in development and commercialization and determine the resulting royalty schedule.

We also explore licensing choices before uncertainty is resolved and the possibility of using licensing as a financing vehicle. Finally, we discuss the leading firm's investment decision, taking into account the licensing decisions.

We obtain several interesting results. First, we find that a royalty schedule consistent with individual rationality conditions dominates a fixed fee structure. Second, contracts involving a fixed upfront fee in exchange for a "cap" on the royalty rate can be used as a financing vehicle when the leading firm faces financial constraints. Third, there is generally a unique threshold level of expected demand above which investment should be committed immediately. High uncertainty and financial constraints usually raise this threshold. [5]

We draw on and contribute to several strands of the literature. First, the investment-timing problem has been extensively studied in the real options literature which emphasizes the time resolution of uncertainty but assumes that market structure is unaffected by firms' investment decisions.[6] The firm with an investment opportunity weighs the present value of benefits against the investment expenditure plus the value of the waiting-to-invest option (McDonald and Siegel 1986). The incentive to delay investment (and to raise the investment threshold) increases with uncertainty.

Second, the strategic investment literature allows investments to have an explicit impact on the imperfectly competitive market structure but does not deal with the effects of uncertainty and time (Dixit 1980, Spence 1984). In this case, committing an investment can have the strategic effects of dissuading market entry. Several recent papers have incorporated such strategic effects in real options models (Kulatilaka and Perotti 1998, 2000, Grenadier 1996). Although a decision to postpone an investment allows the firm to avoid regret by staying out of unprofitable markets, early investment confers a cost or timing advantage that can dissuade others from entering the market. Consequently, the incentives to postpone described in real options models are attenuated (or reversed).

In our model, investment allows the leading firm to use technology licenses in a strategic way. This creates a much more nuanced investment-timing problem. In fact, we have to examine the form of the licensing agreement before we solve the investment problem.

The possibility of licensing relates to a third strand of the literature. Like Katz and Shapiro (1985), Kamien and Tauman (2002), and Wang (2002), we focus on licensing an innovation owned by an

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[5] Except when the investment cost is too high and investment should always be postponed for any level of expected demand.

[6] Investment-timing models assume perfect competition and exogenously evolving prices. See Dixit and Pindyck (1994) for a comprehensive review of this literature.

industry incumbent.[7] While the licensing game in our model follows the standard structure in this literature, there are also several important points of departure.[8] First, we model the interactions between licensing and investment: 1) the entrant (potential licensee) chooses between licensing and investing in its own technology; and 2) the leading firm makes its investment decision depending on anticipated licensing arrangements. Second, there is demand uncertainty in our model, which affects both the licensing and the investment-timing decisions.

We now turn to the issue of the choice between different forms of licensing. The patent licensing literature examines three licensing methods: auction, fixed-fee per license, and royalty per unit of output. The optimal licensing method, however, depends critically on whether the inventor is an outsider or an industry incumbent. When the patent holder is an incumbent in the industry, royalty licensing is preferred to the other two methods, because the inventor-producer is interested not only in the licensing revenue but also in its profits from production (Kamien and Tauman 2002, Wang 2002).

Our research shows that licensing can be a source of external finance. The finance literature provides various explanations for costly external finance, including costs of bankruptcy and financial distress, adverse selection and moral hazard due to information asymmetries, and managers' private benefits from limiting their dependence on external investors (see, for example, Froot et al 1993, and Hubbard 1998). Licensing as a vehicle of external finance, however, has not been discussed in the literature before, as far as we know.

2 Model

Two competing firms, M and N, are engaged in research and development efforts for a new product. Suppose at t=0, M's research makes a breakthrough, leading to an innovation. N continues its

[7] In related work, Kamien and Tauman (1986) and Katz and Shapiro (1986) examine the licensing decisions faced by an outsider inventor who is unable to observe the licensee's output level. In such cases, a fixed fee dominates a royalty rate.

[8] Kamien (1992) surveys the licensing literature, and recent research follows the same basic structure. A related literature on second-sourcing literature recognizes that a monopolist with a new product may license its technology to a competitor as a commitment mechanism to persuade consumers when it incurs setup costs (Farrell and Gallini 1988) or there are complementary goods (Kende 1998). Shepard (1987) sees licensing as an instrument for committing to product quality, thereby increasing demand. D'Aspremont and Jacquemin (1988) study innovations in a duopoly setting where the two firms cooperate in R&D but compete in product market. We consider non-cooperative R&D that result in new technologies.

research, which yields a comparable outcome at some future time, t=1. The time between t=0 and t=1 forms an opportunity window for M. As we focus on the strategic effects of investment timing, we assume zero interest rates and risk-neutrality (no systematic risk).[9] We assume the technical hurdles have been cleared with the innovation, so it is reasonable to assume that the development investment will have a successful result.

At t=0, M must decide whether to commit an irreversible investment, *I*, in order to develop the innovation. Such technical innovations occur often. The firm's challenge is to decide to which ones to commit development efforts while there remains substantial uncertainty about the demand for the new product. In our stylization, at t=1, all uncertainty regarding the demand for the new product is resolved when N follows with innovation, and M's opportunity window closes.[10]

The market for the new product opens only at t=1 with an inverse demand function $P = \theta - Q$, where Q is the total quantity of the product. The demand parameter θ is a random variable that can be interpreted as the maximum potential demand, which is uncertain at t=0 but is fully revealed at t=1. At t=0, M and N know the cumulative probability distribution $F(\theta)$ of $\theta \in [0,\infty)$. We use θ_0 to denote the expected value of θ at time 0, i.e., $\theta_0 \equiv E_0(\theta) = \int_0^\infty \theta dF(\theta)$.

Suppose M decides not to commit the investment *I* at t=0. Then, when N's research yields an innovation at t=1, both N and M have the ability to develop their own technologies at an investment cost *J*. We allow a general specification where $I \neq J$. This leaves the two firms in the identical position in the market. We assume they cannot coordinate and they Cournot-compete by producing perfectly substitutable products. As all uncertainty is resolved at this point, these investments, made under certainty, will be made only if the realized demand is high enough for both to be profitable.

Now suppose M decides to commit the investment at t=0. Once it has developed the technology, M can offer to license it and dissuade N from developing its technology at time 1. M's licensing decision

[9] Note that the time points t=0 and t=1 are defined by M's and N's innovations. Also note that M and N may achieve innovation either through their own research efforts or acquisition of research outcomes.
[10] In fact, all we need is for some amount of uncertainty to be resolved between t=0 and t=1.

may take place at different times; it can make licensing arrangements with N as early as the time of investment (t=0), or by the time N has also made the breakthrough (t=1), or at any time in between. The license may also take different forms: a fixed fee, a royalty rate or schedule, or a combination of fee and royalty. Table 1 summarizes the basic setup of the model.

In the rest of the paper, we first study the case where the licensing arrangement is made at t=1 and explore different forms of licensing agreement. We then examine M's investment decision, in expectation of the licensing outcome at t=1. We also investigate whether it is possible for M and N to agree on licensing arrangements at an earlier time, and if possible, the impact of such agreements on investment.

Table 1: Model Setup

	t=0	t=1
Research Outcome	M achieves an innovation	N achieves an innovation
Demand (θ)	Uncertain*	Known with certainty
Investment opportunity	M can invest I to develop its innovation	M and N can each invest J to develop their respective innovations
Production	Impossible	M and N choose production levels. (We assume zero production costs.)
Licensing	If M invests at t=0, it can enter into a licensing arrangement with N at t=0, t=1, or any time in between.	

* M and N know the distribution function of θ, $F(\theta)$.

Note that N's achieving or acquiring an innovation at t=1 is independent of M's investment and licensing decisions. In other words, we consider an environment with ongoing research programs, where research facilities have already been built, equipments installed, and researchers retained. As a result, M's investment and licensing will not prevent substitute innovations.[11] In our model, licensing is a mechanism to dissuade investment in development rather than in research. In fact, licensing deters development activities by encouraging the use of a developed innovation.

[11] Gallini (1984) views overlapping research programs as potentially inefficient. In our model, however, the market opens only at time 1, so keeping parallel research programs introduces an option value that offsets the cost of overlap.

3 *Ex post* Licensing

Suppose M has invested at t=0. Can licensing arrangements between M and N occur at t=1, and if so, what kind? We refer to licensing at t=1 as *ex post* licensing because the demand θ is fully realized, and there is no uncertainty.

At time 1, N is able to develop its own innovation by investing the amount of J. M has three choices: 1) Not to license its innovation; 2) License the innovation to N for a fixed fee; 3) License the innovation to N for a royalty. We now determine the payoffs under these choices and solve for the optimal choice.

3.1 Do Not License

If M does not license its innovation, N can either enter the market by investing J, or stay out of the market. If N invests, M and N engage in Cournot competition. Using subscripts 1 and 2 to denote firms M and N, their profits are given by $\pi_1 = q_1\left(\theta - q_1 - q_2\right)$ and $\pi_2 = q_2\left(\theta - q_1 - q_2\right)$.[12] The equilibrium quantities and profits are: $q_1^* = q_2^* = \frac{1}{3}\theta$ and $\pi_1^* = \pi_2^* = \frac{1}{9}\theta^2$.

For N, the net payoff from entry is: $\Pi_2 = \pi_2^* - J = \frac{1}{9}\theta^2 - J$. Since staying out of the market yields zero payoff, N will enter only when $\theta > 3\sqrt{J}$. This means that the realized market demand must exceed a threshold level for entry to be profitable. We call $\hat{\theta} \equiv 3\sqrt{J}$ the entry threshold. The resulting payoff function for N is:

$$\Pi_2^{NL} = \begin{cases} 0 & \text{when } \theta \le \hat{\theta} \\ \frac{1}{9}\theta^2 - J & \text{when } \theta > \hat{\theta} \end{cases}$$

We denote the no-licensing strategy by NL. Since N stays out of the market when the demand is below the entry threshold, M becomes the monopolist. Maximizing $\pi_1 = q_1\left(\theta - q_1\right)$ yields the monopoly quantity $q_1^* = \frac{1}{2}\theta$. Thus, M's payoff as a function of realized demand is given by:

[12] We assume zero production costs, which could describe industries such as software or music. Including production costs will not change the gist of our argument.

$$\Pi_1^{NL} = \begin{cases} \frac{1}{4}\theta^2 & \text{when } \theta \le \hat{\theta} \\ \frac{1}{9}\theta^2 & \text{when } \theta > \hat{\theta} \end{cases}$$

3.2 License for a Fixed Fee

If N does not invest to develop its own innovation, but instead licenses M's technology by paying M a fixed fee, φ, the firms engage in Cournot competition that yields the same quantities and profits as in the no-licensing case. With licensing, however, M and N use the same technology, while without licensing they produce perfectly substitutable products using different technologies. Their payoffs with a fixed fee license are given by:

$$\Pi_1^{\varphi} = \tfrac{1}{9}\theta^2 + \varphi;$$

$$\Pi_2^{\varphi} = \tfrac{1}{9}\theta^2 - \varphi.$$

Note that different levels of φ represent different fixed-fee licensing strategies.

3.3 License for a Royalty

The license may take the form of a royalty, where N pays M at a rate of l for each unit it produces. M and N thus engage in an asymmetric Cournot competition. Their profits are now given by: $\pi_1 = q_1\left(\theta - q_1 - q_2\right)$ and $\pi_2 = q_2\left(\theta - q_1 - q_2 - l\right)$. The equilibrium quantities are: $q_1^* = \frac{1}{3}\left(\theta + l\right)$, and $q_2^* = \frac{1}{3}\left(\theta - 2l\right)$. Note these quantities apply only when $l \le \frac{1}{2}\theta$. When $l > \frac{1}{2}\theta$, q_2^* remains zero, i.e., N will not use M's technology. Therefore, royalty licensing is in effect only when $l \le \frac{1}{2}\theta$. We confine our attention to this case.[13]

The firms' payoffs under royalty licensing, when M earns royalties in addition to profits, are:

$$\Pi_1^l = \pi_1^* + q_2^* l = \tfrac{1}{9}\left(\theta^2 + 5\theta l - 5l^2\right)$$

$$\Pi_2^l = \tfrac{1}{9}\left(\theta - 2l\right)^2.$$

[13] When $l = \frac{1}{2}\theta$, N does not produce and M is actually a monopolist; when l<0, M is in fact subsidizing N at the rate of -l for each unit N produces.

Obviously, $\frac{\partial \Pi_1^l}{\partial l} > 0$, and $\frac{\partial \Pi_2^l}{\partial l} < 0$. That is, M's payoff increases while N's payoff declines with the royalty rate.

3.4 Firms' Ex Post Licensing Choice

The firms may choose a strategy from any of the three categories: no licensing, fee licensing, and royalty licensing. Firms' payoffs under the no-licensing strategy serve as their reservation payoffs. We assume a firm may agree to a licensing agreement if and only if its payoff under this strategy is no less than that under the no-licensing strategy. An agreement can be reached if and only if both firms agree upon the same licensing strategy. If no agreement can be reached, the firms simply choose the no-licensing strategy. Formally, we denote a licensing strategy by *s*, which can be any $\varphi \in R$ or any $l \le \frac{1}{2}\theta$. M may agree to *s* if and only if $\Pi_1^s \ge \Pi_1^{NL}$. Similarly, N agrees to *s* if and only if $\Pi_2^s \ge \Pi_2^{NL}$. An agreement is defined as a licensing strategy such that both inequalities are satisfied.

First, we examine the licensing outcome when the realized demand does not exceed N's entry threshold. M earns monopolistic profits in this range without licensing, thus it will agree only to licenses that yield an equal or higher payoff. The licenses $\varphi = \frac{5}{36}\theta^2$ and $l = \frac{1}{2}\theta$ yield the same payoffs as monopolistic profits, while higher fees or royalty rates lead to even higher payoffs. Higher royalty rates are infeasible. Therefore, M only accepts $l = \frac{1}{2}\theta$ or $\varphi \ge \frac{5}{36}\theta^2$. As in this range of demand, without licensing N stays out of the market and earns a zero payoff, licenses that result in a zero payoff to N, namely, $\varphi = \frac{1}{9}\theta^2$ and $l = \frac{1}{2}\theta$, make N indifferent as to entering the market. For lower fees and lower rates, N can enter the market and retain some profits, realizing a positive payoff. Thus, N can accept $\varphi \le \frac{1}{9}\theta^2$ or $l \le \frac{1}{2}\theta$. Clearly, in this case, the only feasible agreement is $l = \frac{1}{2}\theta$. Note that when $l = \frac{1}{2}\theta$, N chooses an output level of zero, and M is in fact a monopolist, which is the same case as no licensing. Therefore, the no-licensing strategy and a royalty schedule of $l = \frac{1}{2}\theta$ are equivalent. It is reasonable to assume that the outcome is no licensing.

Next, we turn to the licensing agreements when the realized demand is above N's entry threshold. In this range of demand, M may agree to any non-negative fee and any royalty rate that $0 \le l \le \frac{1}{2}\theta$, and the higher the fee or the rate, the higher its payoff. M may agree to even a zero fee or royalty rate because it is profitable for N to enter the market by investment. Therefore, even if M does not license, N will still enter the market, and thus M can earn only a Cournot profit, which is the same as offering its technology to N for free. Any positive fee or rate makes M better off than not licensing, i.e. M will agree to any $0 \le l \le \frac{1}{2}\theta$ or $\varphi \ge 0$. In this range of demand, in the absence of licensing, it is optimal for N to enter the market by investment, earning a payoff of $\frac{1}{9}\theta^2 - J$. The licenses $\varphi = J$ and $l = \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)$ yield the same payoff and make N indifferent between investment and licensing. Fees or royalty rates lower than these licenses will effectively dissuade N from investment and induce it to license. Therefore, N will agree to a fee such that $\varphi \le J$ or a rate such that $l \le \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)$. What both parties can agree on, therefore, is any fee in the range of $0 \le \varphi \le J$ and any royalty rate in the range $0 \le l \le \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)$. Note that the two strategies $\varphi = 0$ and l=0 are equivalent; each strategy involves M allowing N to use its technology for free. For convenience, we use l=0 to represent this strategy. Thus, we rewrite the feasible agreements as $0 < \varphi \le J$ and $0 \le l \le \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)$. Although every strategy acceptable to both firms makes firms at least as well off as in the no licensing case, some strategies may lead to higher payoffs to firms than others. We now determine which agreements will emerge.

Definition: An agreement *s* is dominated if there exists another agreement s', such that $\Pi_1^s \le \Pi_1^{s'}$, $\Pi_2^s \le \Pi_2^{s'}$ and "<" holds for at least one of the two inequalities.

Assumption: A dominated agreement *s* will not emerge in equilibrium.

The Assumption simply suggests that as it tries to reach an agreement, if a firm can improve its payoff without hurting the other party, it will.[14]

Lemma 1: When $\theta > \hat{\theta} \equiv 3\sqrt{J}$, any fixed-fee licensing agreement is dominated. (See Appendix 1 for proof.)

Lemma 1 implies that the sum of M's and N's payoffs is higher under royalty licensing than under fee licensing. Under fee licensing, the firms' total payoff is $\frac{2}{9}\theta^2$, while under royalty licensing, $\frac{2}{9}\theta^2 + \frac{1}{9}l(\theta - l)$. M and N can both be better off by splitting the extra payoff in royalty licensing. There are higher payoffs to firms under royalty licensing because a fee does not affect the firms' profit-maximizing quantities. A royalty rate, however, which represents a marginal cost for N, reduces N's output level, and even though M increases its quantity in response, overall less is produced and the price is higher.[15] In other words, royalty licensing increases firms' payoffs at the cost of consumers.[16] The result in Lemma 1 is consistent with findings in the literature: royalty licensing is superior to fee licensing for firms.[17]

Proposition 1: (See Appendix 1 for Proof.)

The *ex post* equilibrium outcomes are:

(i) When $\theta \le \hat{\theta}$ where $\hat{\theta} \equiv 3\sqrt{J}$, no licensing between M and N occurs. M is the monopolist with a payoff of $\Pi_1 = \frac{1}{4}\theta^2$, and N stays out of the market with a zero payoff, $\Pi_2 = 0$;

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[14] Considering only the two firms, a dominated agreement simply means that it is not Pareto optimal, and Assumption 1 means that only Pareto optimal agreements will occur in equilibrium. This Pareto optimality interpretation, however, does not go through if consumers are also included.

[15] The total quantity and price under fee licensing are: $Q^{\varphi} = \frac{2}{3}\theta$ and $P^{\varphi} = \frac{1}{3}\theta$. Under royalty licensing, $Q^l = \frac{2}{3}\theta - \frac{1}{3}l$, and $P^l = \frac{1}{3}(\theta + l)$.

[16] For society, including both firms and consumers, royalty licensing causes a deadweight loss. The total consumer surplus is $\frac{1}{9}(2\theta - l)^2$ under royalty licensing, and $\frac{2}{9}\theta^2$ under fee licensing. Therefore, the total surplus is $\frac{4}{9}\theta^2 - \frac{1}{18}l(2\theta + l)$ under royalty licensing, which is lower than the total surplus under fee licensing, $\frac{4}{9}\theta^2$.

[17] See Kamien and Tauman (2002) and Wang (2002). In the literature, firms are competing in an established market, and the innovation reduces production cost. In our model, M's innovation opens a new market that N must enter by either licensing or investing.

(ii) When $\theta > \hat{\theta}$, M and N may agree to any royalty licensing rate *l* such that

$l \in \left\{ l : 0 \le l \le \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right) \right\}$ and their payoffs are given by $\Pi_1 = \frac{1}{9}\left(\theta^2 + 5\theta l - 5l^2\right)$

and $\Pi_2 = \frac{1}{9}\left(\theta - 2l\right)^2$.

Proposition 1 describes the conditions under which *ex post* licensing may occur and, when it occurs, what agreements may emerge. Figure 1 illustrates the *ex post* licensing outcomes. Figure 1a depicts the royalty rate *l* in the space of *θ*. Recall that when $\theta \le \hat{\theta}$, $l = \frac{1}{2}\theta$ is the only feasible licensing agreement, which is equivalent to the no licensing outcome whenM is the monopolist.

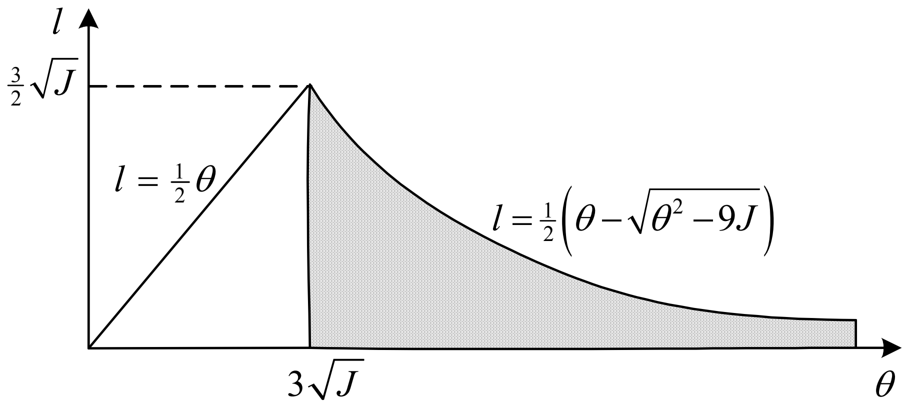
When $\theta > \hat{\theta}$, it is possible for M and N to agree upon any royalty rate such that $0 \le l \le \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)$ (represented by the shaded area). Recall that $l = \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)$ is the upper limit of royalty rates acceptable to N, which makes N indifferent between investment and licensing. Any royalty rate that lies below this upper limit makes N better off and M is worse off. The actual rate that emerges may depend on M's and N's relative bargaining power. Without other strategic considerations, however, the only credible threat from N is that it invests in its own innovation when the demand exceeds its entry threshold. Therefore, to dissuade N from investing, M can offer a take-it-or-leave-it (TIOLI) royalty rate that is infinitesimally lower than $l = \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)$, and N will accept it. For simplicity, we assume N will accept a royalty rate at that level.[18] Therefore, we can define the *ex post* licensing outcome as a royalty schedule that depends on the realized demand:

$$l(\theta) = \begin{cases} \frac{1}{2}\theta & \text{when } \theta \le \hat{\theta} \\ \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right) & \text{when } \theta > \hat{\theta} \end{cases}$$

[18] N may have other strategic considerations that make investment more attractive than licensing, driving down the royalty rate that makes it indifferent. Such strategic considerations do not affect our analysis on *ex post* licensing. The result that royalties dominate fixed fees holds for any bargaining power of M and N. Our discussion from this point on, however, excludes such strategic considerations and focuses on the expected payoffs from profits and licensing revenues/expenses under the assumption that M makes a TIOLI offer.

Figure 1: *Ex post* Licensing Outcome

(a) Royalty Rate Schedule



(b) M's and N's Payoffs

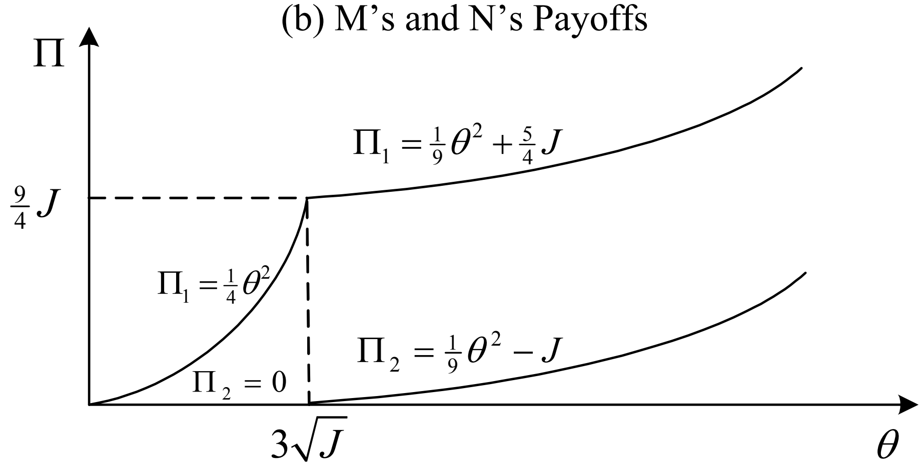


Table 2: *Ex post* Licensing Outcomes

Demand	$\theta \le \hat{\theta} \equiv 3\sqrt{J}$	$\theta > \hat{\theta} \equiv 3\sqrt{J}$
Royalty Schedule $l(\theta)$	$\frac{1}{2}\theta$	$\frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)$
M's payoff Π_1	$\frac{1}{4}\theta^2$	$\frac{1}{9}\theta^2 + \frac{5}{4}J$
N's payoff Π_2	0	$\frac{1}{9}\theta^2 - J$

This *ex post* royalty schedule and firms' payoffs are summarized in Table 2 and plotted in Figure 1. This royalty schedule is monotonically decreasing in θ. In practice, royalty schedules that decrease with quantities are widely observed. Rather than negotiate a single royalty rate, firms often agree upon royalty schedules in advance, and the shape of such schedules is consistent with our results. It is important to note that any agreement between parties reached at any time prior to t=1, which approximates this *ex post* royalty schedule, will be equivalent to licensing at t=1.

4 *Ex ante* Licensing and Its Impact on Financing

Our discussion of firms' licensing decisions at t=1 (*ex post* licensing) shows that a royalty license is better than a fixed fee for the firm. We now consider whether M can pre-commit to a specific royalty rate through the following contract. Before time 1, N pays M a lump sum C, and M pre-commits to a royalty rate L at which it will later license its technology to N. At t=1, N can license M's technology at the stipulated royalty rate, L, but N is not obliged to license, and may invest to develop its own technology instead. We call C the down payment, L the royalty cap, and the contract (C, L) a royalty cap contract. For N, accepting the contract is similar to buying a call option on royalty rate. We investigate the possibility of firms agreeing upon such a contract under two timing scenarios: 1) between t=0 and t=1; and 2) at t=0.

At t=1, after the demand uncertainty is resolved external financing would generally be available. In contrast, at t=0 the presence of uncertainty may cause financial constraints. In such cases, a licensing arrangement with a down payment could be used as a form of external financing.

4.1 Licensing between t=0 and t=1

To determine whether a royalty cap contract can be agreed upon between t=0 and t=1, we need to derive firms' expected payoffs both with and without the contract. Prior to t=1, the market demand θ is still uncertain and only the cumulative distribution function of θ, $F(\theta)$, is known. Without any pre-agreement, at any time between t=0 and t=1, M and N take expectations of their time-1 payoffs. Based on previous results (see Table 2), we have:

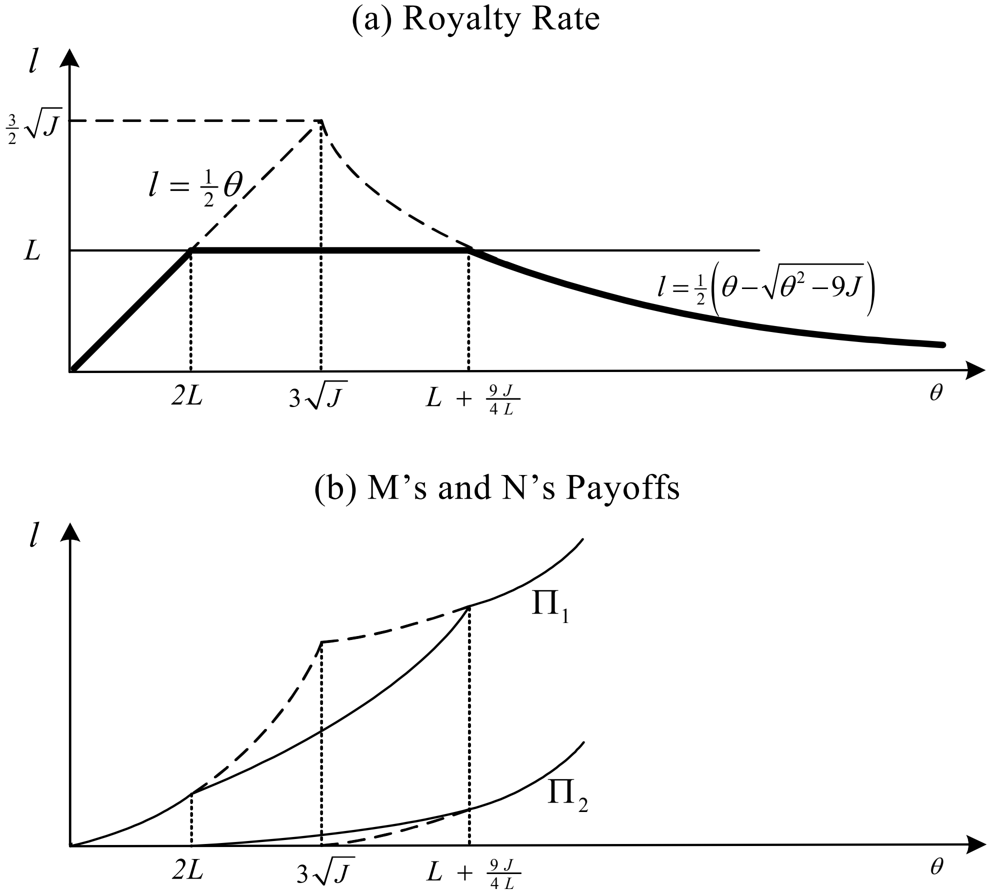
$$E\Pi_1 = \int_0^{3\sqrt{J}} \tfrac{1}{4}\theta^2 dF + \int_{3\sqrt{J}}^{\infty} \left(\tfrac{1}{9}\theta^2 + \tfrac{5}{4}J\right) dF$$

$$E\Pi_2 = \int_{3\sqrt{J}}^{\infty} \left(\tfrac{1}{9}\theta^2 - J\right) dF .$$

We now consider firms' expected payoffs under the royalty cap contract. Note that the *ex post* royalty schedule has a maximum, $\max\, l(\theta) = \frac{3}{2}\sqrt{J}$. Considering only non-negative royalty caps *L*, we treat three cases of *L* (Case 2 is illustrated in Figure 2):

1. $L = 0$, so $L < l(\theta)$ for all positive θ.
2. $0 < L < \frac{3}{2}\sqrt{J}$, so $L < l(\theta)$ for $\theta \in \left(2L, L + \frac{9J}{4L}\right)$ and $l(\theta) \le L$ elsewhere.
3. $L \ge \frac{3}{2}\sqrt{J}$, so $L \ge l(\theta)$ everywhere, and equality holds only at $\theta = 3\sqrt{J}$.

Figure 2: Impact of Royalty Cap†



†In both panels, solid lines represent the case under contract (*C*, *L*), while dotted lines represent the case of *ex post* licensing.

Recall that N would accept a royalty schedule only at or below $l(\theta)$, and lower rates yield higher payoffs. In Case 1, since the cap *L* lies below the entire *ex post* royalty schedule, N will require M to honor the contract, and M's technology will be licensed at the stipulated royalty rate *L*, – in this case zero. The same is true in the range of $\theta \in \left(2L, L+\frac{9J}{4L}\right)$ in Case 2, where *L* is binding in this range of realized demand. M and N will engage in Cournot competition, where the quantities and payoffs are given by:

$$q_1^* = \tfrac{1}{3}(\theta + L),\; q_2^* = \tfrac{1}{3}(\theta - 2L)$$

$$\Pi_1 = \tfrac{1}{9}\left(\theta^2 + 5\theta L - 5L^2\right),\; \Pi_2 = \tfrac{1}{9}(\theta - 2L)^2$$

In Case 2, $L > l(\theta)$ for the range of $\theta \in (0, 2L)$, so it is not profitable for N to license M's technology at the rate *L*. Therefore, no licensing occurs. Since the demand is below the entry threshold $\hat{\theta} \equiv 3\sqrt{J}$, N stays out of the market, and M remains the monopolist. For the range of $\theta \in \left(L+\frac{9J}{4L}, \infty\right)$, since $L > l(\theta)$ and the demand exceeds the entry threshold, N would rather invest in its own innovation than license at the rate of *L*. N's investment will cause M to earn Cournot profits. Therefore, M has an incentive to reduce the rate to $l(\theta)$ in order to dissuade N from investment (in other words, induce N to license). Thus $l(\theta)$ will be the effective royalty schedule, and the payoffs will be the same as in *ex post* licensing, $\Pi_1 = \frac{1}{9}\theta^2 + \frac{5}{4}J$ and $\Pi_2 = \frac{1}{9}\theta^2 - J$. Figure 2 illustrates the effective *ex post* royalty schedule (a) and the firms' payoffs (b) under a royalty cap contract.

For any royalty cap $L > l(\theta)$, the contract (*C*, *L*) has no effect, and the outcome remains the same as in *ex post* licensing. Therefore, any $L \geq \frac{3}{2}\sqrt{J}$ (Case 3) has no effect on the outcome. We can then concentrate on contracts where $L < \frac{3}{2}\sqrt{J}$. In this case, the firms' expected payoffs with a contract (*C*, *L*) are:

$$E\Pi_1^{CL} = C + \int_0^{2L} \tfrac{1}{4}\theta^2 dF + \int_{2L}^{L+\frac{9J}{4L}} \tfrac{1}{9}\left[\theta^2 + 5\theta L - 5L^2\right] dF + \int_{L+\frac{9J}{4L}}^{\infty} \left(\tfrac{1}{9}\theta^2 + \tfrac{5}{4}J\right) dF$$

$$E\Pi_2^{CL} = -C + \int_{2L}^{L+\frac{9J}{4L}} \tfrac{1}{9}(\theta - 2L)^2\, dF + \int_{L+\frac{9J}{4L}}^{\infty} \left(\tfrac{1}{9}\theta^2 - J\right) dF$$

Will M and N agree to any royalty cap contract at any point between time 0 and time 1? With their expected payoffs under *ex post* licensing as their reservation payoffs, M will accept a contract (C, L) if and only if its expected payoff under the contract is no less than its payoff from *ex post* licensing, i.e., $E\Pi_1^{CL} \geq E\Pi_1$. Similarly, N will accept (C, L) if and only if $E\Pi_2^{CL} \geq E\Pi_2$. A contract can be agreed upon if the terms are acceptable to both M and N. In other words, M and N may sign a royalty cap contract (C, L) prior to time 1 if and only if (C, L) satisfies both $E\Pi_1^{CL} \geq E\Pi_1$ and $E\Pi_2^{CL} \geq E\Pi_2$.

Proposition 2: For any distribution of θ with a strict positive support in the range $[0, +\infty)$, a royalty cap contract cannot be agreed upon between t=0 and t=1. (See Appendix 2 for proof.)

Proposition 2 suggests that if M has invested at time 0, it is never optimal for it to license its technology by pre-committing to a royalty cap in exchange for a lump sum payment, because the most N is willing to pay is always lower than what would make M indifferent with regard to licensing. As we have shown, a royalty license results in lower total quantities and a higher price, and thus higher payoffs for firms, than a fee license. Furthermore, the higher the royalty rate, the stronger these effects (see Footnote 16). In other words, a higher royalty rate increases producers' surplus while lowering consumer surplus. When M pre-commits to a royalty rate that may be lower than the *ex post* rate it can charge, M and N end up producing more in total, resulting in a higher consumer surplus. Therefore, N cannot expect to fully capture the benefits of a royalty cap, making M's expected loss always greater than N's expected gain. As a result, after Firm M has invested at time 0, both firms will simply wait until time 1, when N is able to develop an alternative technology.

4.2 Licensing and Financing at the Time of Investment (t=0)

Proposition 2 shows that a royalty cap contract cannot be reached between t=0 and t=1. Is it possible for the firms to agree upon such a contract at t=0? From the proof of Proposition 2, we know that the minimum down payment M asks is its expected loss by pre-committing to a royalty cap, *L*. Denote this amount by C^M. For each *L*:

$$C^M(L) \equiv \int_{2L}^{3\sqrt{J}} \tfrac{5}{36}(\theta - 2L)^2\, dF + \int_{3\sqrt{J}}^{L+\frac{9J}{4L}} \left[\tfrac{5}{4}J - \tfrac{5}{9}\theta L + \tfrac{5}{9}L^2 \right] dF$$

Similarly, the amount N is willing to pay M to remain indifferent between *ex ante* and *ex post* licensing is denoted by C^N, and

$$C^N(L) \equiv \int_{2L}^{3\sqrt{J}} \tfrac{1}{9}(\theta - 2L)^2\, dF + \int_{3\sqrt{J}}^{L+\frac{9J}{4L}} \left[J - \tfrac{4}{9}\theta L + \tfrac{4}{9}L^2 \right] dF$$

We see that C^M is $\frac{5}{4}$ times C^N for any L, resulting in a bid-ask spread between M and N. Obviously, if M is risk-neutral and has unlimited financial resources, a royalty cap contract cannot be reached at t=0. If M has financial constraints at t=0, and seeks external finance for the investment I, such a contract may become a possible source of costly external finance.[19] In other words, M can pre-commit to a royalty cap L and receive funds, C, from N. The amount M receives is then determined by N's individual rationality constraint, i.e., C equals $C^N(L)$. The cost of such external financing is therefore $C^M(L)/C^N(L)$, which equals $\frac{5}{4}$ for any (C, L).[20] Thus, we can state:

Proposition 3: A royalty rate cap contract can be used as a source of external finance. Under the current setup of linear demand and Cournot competition, for every \$1 of capital provided by the licensee, the licensor gives up \$1.25 of time-1 licensing revenues. This holds for any distribution of θ with a strict positive support in the range $[0, +\infty)$.

Interestingly, the financing cost of such contracts is independent of the terms of the contract and the distribution of demand. M's commitment to a royalty cap L changes the market structure for the demand range $\theta \in \left(2L, L + \frac{9J}{4L}\right)$. For each θ in this range, the cap leads to a loss for M and a gain for N (see Figure 2b). The ratio between M's loss and N's gain is *the same* for *any* θ, because it is intrinsic to

[19] In our model, M will not agree to a royalty cap contract unless it has financial constraints. This is also true for a variation of our model. Suppose N has uncertainty in its research efforts and may not make a breakthrough at t=1. Now N has further incentive to license M's technology to gain access to the market. Since M now has a greater probability to be the monopolist, it demands an even higher upfront payment, while the maximum amount N is willing to pay remains the Cournot profit. There remains a "bid-ask" spread, and M still will not agree to a royalty cap contract unless it is financially constrained.

[20] Note that the financing cost associated with a royalty cap contract is *not* a deadweight loss to society; consumers gain from such a royalty rate cap.

the market structure both with and without the cap. Therefore, no matter how the cap extends or constrains the range of demand being affected, and no matter how the probability mass of demand shifts, the ratio between the expected loss and gain, which is the financing cost, remains unchanged.

The royalty cap *L*, however, does affect the down payment *C*, i.e. the amount M can finance. It can be proved that *C* declines with *L*.[21] Any $L \geq \frac{3}{2}\sqrt{J}$ results in zero finance. By lowering *L*, M can increase the amount of down payment it receives upfront. In particular, M receives the maximum amount by committing to *L*=0, i.e. allowing N to use its technology for free at t=1. This maximum amount is given by $\overline{C} \equiv C^N\left(L=0\right) = \int_0^{3\sqrt{J}} \frac{1}{9}\theta^2 dF + \int_{3\sqrt{J}}^{\infty} J dF$.

Obviously, the terms of a royalty cap contract depend on the distribution of demand. We illustrate the behavior of (C, L) by simulations under a set of lognormal distributions. A lognormal distribution of demand θ can be characterized by the expected demand θ_0 and the shape parameter σ, which represents uncertainty in our simulations.[22]

Figure 3 plots the simulated down payment *C* on the vertical axis for a family of royalty caps *L*, with the expected demand θ_0 on the horizontal axis, for a fixed level of uncertainty ($\sigma = 0.5$).[23] Note that higher expected demand does not always lead to higher down payment! Firm N is willing to pay M a down payment at t=0 because it expects to gain from a lower royalty rate at t=1. Figure 2b shows that N gains only when the realized demand θ falls in the range of $\left(2L, L+\frac{9J}{4L}\right)$. Therefore, for any positive *L*, if N expects θ to be too low (below $2L$) or too high (above $L+\frac{9J}{4L}$), it is not willing to pay a high amount,

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[21] We know $C = \int_{2L}^{3\sqrt{J}} \frac{1}{9}\left(\theta - 2L\right)^2 dF + \int_{3\sqrt{J}}^{L+\frac{9J}{4L}} \left[J - \frac{4}{9}\theta L + \frac{4}{9}L^2\right] dF$. In the first term, when *L* increases, both the integration interval and the value of the integrand (positive) drop, which means that the value of this term declines. In the second term, the upper bound of the integration interval decreases with *L* for $0 \leq L < \frac{3}{2}\sqrt{J}$; the value of the integrand also decreases with *L* because $\theta \geq 3\sqrt{J} > 2L$ in this integration interval. Therefore the second term also decreases with *L*. Overall, *C* declines with *L*.

[22] The specification we use ensures that a change in σ is a mean-preserving spread of the distribution.

[23] The parameter σ should be interpreted as total volatility over the period M has a lead. This period may range from months to several years.

so *C* starts to decrease for very high θ_0. The only exception is when $L = 0$: now the range of demand where N can gain becomes $(0, +\infty)$, therefore higher expected demand always results in a higher down payment. In other words, $\overline{C}$ is monotonically increasing in θ_0.

Figure 3: Simulation of Royalty Cap Contract (*J*=1, σ=0.5)

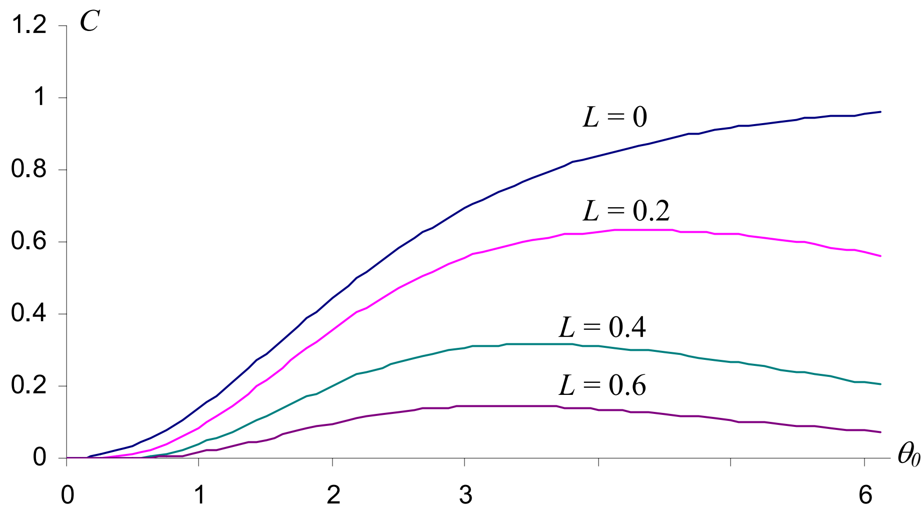

Figure 4: Simulation of Royalty Cap Contract (*J*=1, θ_0 =1)

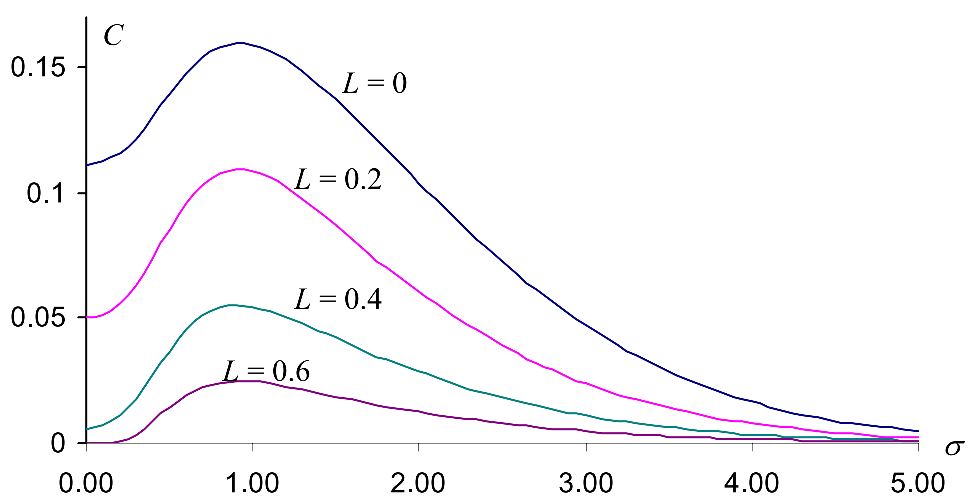

Figure 4 illustrates how the terms of a royalty cap contract change with the level of uncertainty σ under a fixed expected demand $\theta_0 = 1$ (Figures A1 and A2 in Appendix 2 show the results for $\theta_0 = 10$

and $\theta_0 = 3$). Clearly, the effect a reduction in royalty cap has on the amount of down payment varies with the level of uncertainty. In particular, the figure indicates a royalty cap contract tends to raise more money for σ between 0.5 and 1.5 than lower and higher levels of σ.[24]

When uncertainty is extremely high, a royalty cap contract can hardly raise any money for the financially constrained firm, no matter how low the cap is. When the prospects of a new market are highly uncertain, it often means other sources of financing are unavailable and the firm is more likely to resort to a licensing contract. In such a situation, however, a royalty cap contract also has little value to the potential licensee, and therefore cannot raise much funds either.

More interestingly, by committing to the same royalty cap, the licensing firm may even receive more money under some uncertainty than under almost certainty! Figure 4 shows that given $\theta_0 = 1$ and $J = 1$, the same royalty cap induces the highest down payment when $\sigma \approx 0.9$ rather than when $\sigma \to 0$. The reason again is that Firm N gains from this contract only when θ is expected to be in the range of $\left(2L, L + \frac{9J}{4L}\right)$. A low uncertainty means that the probability mass of θ is concentrated at the expected value, and if the expected value is out of the range of $\left(2L, L + \frac{9J}{4L}\right)$, C tends to be very low (Figures 4 and A1 in Appendix 2 provide examples of θ concentrated at low and high levels, respectively). In fact, a level of uncertainty that puts the most probability mass of θ in the range of $\left(2L, L + \frac{9J}{4L}\right)$ induces the highest down payment from N. In the case of $\theta_0 = 1$ and $J = 1$, this proper level of uncertainty is approximately $\sigma \approx 0.9$ for all levels of royalty cap (see Figure 4).

This result has implications for firms seeking funding to develop their innovations. While financial resources tend to diminish with the level of uncertainty, financing via a royalty cap contract does not always follow this rule. Some level of uncertainty may in fact help make more resources available to the firm with the innovation. The uncertainty cannot be too high, though. It is also important to note that the per-dollar cost of this financing vehicle remains high and constant.

[24] These values are constant with implied volatility of early-stage technology firms.

5 Investment Decision

We have shown that a firm can capture the value of an innovation by *ex post* licensing. To develop the innovation, however, an investment *I* must be made at t=0, when there is still uncertainty around the future demand. At t=0, even if the firm has unlimited financial resources, it still has to answer this question: is the expected value from production and *ex post* licensing worth the upfront investment? If the firm is financially constrained and has to use *ex ante* licensing to finance the investment, the question is: given that the firm has to give up some future licensing revenues in return for a down payment, is the expected payoff still worth the investment and financing costs? This section addresses these questions and examines factors that influence the investment decision.

The investment decision in our model involves tradeoff between the strategic benefits and costs of investing early. The investment rule is that M should invest at t=0 if the expected value of immediate investment (V^I) exceeds the expected value of waiting (V^{NI}), i.e. $V^I > V^{NI}$, while M should wait if $V^I < V^{NI}$.[25] Investing immediately allows the firm with an innovation to license its technology and collect royalty revenues (and receive funds if financing is needed). In addition, the royalty rate serves as a marginal cost for the licensee, rendering the licensor a cost advantage.[26] Waiting until a later time, on the hand, resolves the uncertainty around market demand and allows the firm to invest a lower amount ($J < I$).

In this section, we first assume that M has no financial constraint at t=0, and examine how M's optimal investment decision is influenced by the level of requirement investment, the expected demand, and uncertainty. At the end, we relax the assumption and discuss the impact of financial constraint on the investment decision.

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[25] Because $V^{NI} \geq 0$, this investment rule ensures that only positive NPV investments are made. Such a rule also considers the value of waiting to invest, and a positive-NPV investment will be made only when it also exceeds the expected value of waiting. When $V^I = V^{NI}$, M is indifferent between investing immediately and waiting. For convenience, we assume that in this case M waits.

[26] In Kulatilaka and Perotti (1998), early investment confers a pure cost advantage onto the leading firm. In this paper, however, the strategic benefits of early investment stem from both the revenue side and the cost side, and more importantly, the optimal investment decision is conditional on the optimal licensing decision.

When M is not financially constrained, we find that when the required time-0 investment is too high compared to the time-1 amount, M should never consider investing immediately, *no matter how promising the market is*.

Lemma 2: If $I \geq \frac{9}{4}J$, M does not invest at t=0 for *any* distribution of demand. (See Appendix 3 for proof)

For lower time-0 investment levels ($I < \frac{9}{4}J$), M's optimal investment decision depends on the distribution of demand. We derive an investment rule in terms of the expected demand.

Proposition 4: Suppose the distribution of θ is such that higher expected value of θ, θ_0, implies strong first-order stochastic dominance, the density of distribution goes to infinity at $\theta = 0$ if $\theta_0 \to 0$, and the density goes to zero in the range $\left[0, 3\sqrt{J}\right]$ if $\theta_0 \to \infty$. When $I < \frac{9}{4}J$, there exists a unique threshold θ_0^* so that M invests at t=0 if $\theta_0 > \theta_0^*$, and waits until t=1 if $\theta_0 \leq \theta_0^*$. (See Appendix 3 for proof)

Proposition 4 implies that the strategic value of an innovation with a large potential market can be so high that it justifies an immediate investment more than twice as much as the investment required at a later time! This is because an unprecedented innovation, if developed immediately, allows the firm to either operate as a monopolist or earn licensing revenues from a competitor. If the expected market demand is high enough a.k.a. exceeds a threshold level, a significant upfront investment (up to 2.25 times of the investment required at t=1) can be justified. In Appendix 3, we demonstrate how to determine such a threshold of expected demand using simulations.

The level of uncertainty also impacts M's investment decision. Appendix 4 explains in detail how uncertainty shifts the threshold of expected demand for immediate investment. Table 3 presents the simulated threshold of demand θ_0^* for different levels of uncertainty and ratios of required investment (assuming M has no financial constraint). From Table 3, we see that uncertainty, represented by σ, has ambiguous impact on the investment threshold θ_0^*.

Table 3: Investment Threshold				
σ	Threshold of Expected Demand for Immediate Investment θ_0^* ($J=1$)			
	$I=1$	$I=1.25$	$I=1.5$	$I=2$
$\sigma \to 0$	2.00	2.24	2.45	2.83
0.1	1.99	2.23	2.44	2.89
0.2	1.97	2.21	2.46	3.13
0.3	1.95	2.24	2.54	3.48
0.4	1.96	2.30	2.68	3.96
0.5	2.00	2.40	2.87	4.57
0.6	2.07	2.54	3.12	5.35
0.7	2.16	2.72	3.44	6.36
0.8	2.29	2.95	3.83	7.65
0.9	2.45	3.24	4.32	9.30
1.0	2.64	3.59	4.93	11.5

The threshold of expected demand for immediate investment generally increases with uncertainty with a few exceptions. Insights from contingent claims valuation can shed some light on the implications of this result. In our setting, early commitment of an investment in development lets the firm capture the upside in the potential market via licensing an innovation. Therefore, the strategic value of the innovation can be interpreted as a growth option, whose value increases with uncertainty. However, the value of this growth option must be offset against the value of the waiting-to-invest option, which also increases with uncertainty. As the innovating firm's ability to collect royalties is limited by the competitor's *ex post* entry threat, the value of the growth option generally increases with uncertainty at a lower rate than the waiting-to-invest option. Hence, the investment threshold increases with uncertainty.[27]

[27] An exception occurs when σ is low and the expected demand is below the entry threshold. Then the investment threshold drops slightly with increasing uncertainty.

We now discuss the impact of financial constraints on the investment decision. Suppose M has access to only $w < I$ amount of capital at zero cost, and no other sources of financing except a royalty cap contract.[28] The investment decision is changed in two ways.

First, a financial constraint may render investment infeasible, because there is an upper limit to the down payment a royalty cap can induce and it may not be enough to fill the gap between available capital and the required investment. Therefore, under a financial constraint, Firm M starts its investment decision process with a feasibility analysis. The maximum amount of down payment $\overline{C}$ can be derived by setting the royalty cap at zero.[29] If $\overline{C} < I - w$, investment is infeasible and no investment can be made at t=0. In other words, a financial constraint imposes a condition that must be satisfied for immediate investment to be feasible: $\overline{C} \geq I - w$. It can be proved that the expected demand must exceed a certain threshold, which we denote by θ_0^{**}, to make immediate investment feasible.[30] This threshold, imposed by the financial constraint, can be called the feasibility threshold. Note that this is different from the threshold for immediate investment, θ_0^{*}. Only when investment is feasible does the firm continue with the investment decision process.

Second, when a royalty cap contract can overcome the financial constraint (i.e. $\overline{C} \geq I - w$), making investment *feasible*, the firm then decides whether it is *optimal* to invest immediately. While M's

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[28] Generally, the cost of financing is an increasing function of the amount of funds needed. Our assumption implies a financing cost curve that is 0 for financing needs in the range of $\left(0, w\right]$, 0.25 for $\left(w, w+\overline{C}\right]$, and infinity for $\left(w+\overline{C}, \infty\right)$.

[29] Recall that $\overline{C} = \int_0^{3\sqrt{J}} \frac{1}{9}\theta^2 dF + \int_{3\sqrt{J}}^{\infty} J dF$.

[30] The existence and uniqueness of θ_0^{**} can be proved under the same conditions for Proposition 4. The logic for this result is also similar to that in the proof of Proposition 4. Intuitively, because higher expected value of θ implies strong first-order stochastic dominance, the feasibility condition requires that the expected value of θ be high enough such that $\overline{C} \geq I - w$. For a non-constrained firm, $w \geq I$, thus the feasibility condition is satisfied for any distribution, even when $\overline{C} = 0$ (as a result of $\theta_0 = 0$). Therefore, the feasibility threshold θ_0^{**} for a firm without a financial constraint can be considered zero. For any firm with a financial constraint, however, θ_0^{**} is positive.

investment rule now is the same as that in absence of financial constraint, i.e. to invest if $V^I > V^{NI}$, the calculation of V^I is completely different. Since financing through a royalty cap is costly, M will finance no more than the difference between *I* and *w*. For a given $F(\theta)$, M chooses the optimal royalty cap L^* such that $C^N(L^*) = I - w$. M then derives the value of immediate investment V^I, where expected revenues from royalties and M's own production are now calculated based on L^*, rather than the *ex post* licensing schedule. The comparison between V^I thus derived and V^{NI} then yields a threshold of expected demand for immediate investment to be optimal. This threshold has the same meaning as the threshold for immediate investment introduced earlier in this section, and is therefore denoted by θ_0^*. Overall, a firm with a financial constraint invests at t=0 only when the expected demand exceeds both the feasibility threshold and the investment threshold, i.e. $\theta_0 > \max(\theta_0^*, \theta_0^{**})$.

How does the investment threshold for a financially constrained firm compare with that for a non-constrained firm? We know that financing via a royalty cap contract restricts the firm's ability to capture value through a license, eroding the value of a developed innovation. This translates into a higher cost of capital, which is similar to having an increased level of required investment *I*. Table 3 shows, not surprisingly, that a higher *I* leads to a higher threshold of expected demand for immediate investment. Therefore, the investment threshold is higher for a firm with a financial constraint (but investment is still feasible) than for one without.

In sum, the strategic value of developing and licensing an innovation may justify very high upfront investment when the market is promising. While this investment opportunity represents a growth option for the innovating firm, the entry threat by competitors limits the growth opportunity. Therefore, higher uncertainty makes the investment less attractive (relative to waiting). In addition, financial constraints also restrict the innovating firm's ability to capture value by licensing, requiring much higher expectations on market demand to make immediate investment optimal or just feasible.

6 Concluding Remarks

Our results have several broad implications for the development and management of innovations. A firm with an innovation can reap a higher value by offering to license the intellectual property to others than keeping it proprietary when a potential competitor is going to come up with a substitute innovation. The licensing arrangement that yields the highest reward and satisfies *ex post* rationality conditions is a royalty schedule that depends on the level of demand. Licensing practices in many industries feature royalties that decline with the level of demand, which is consistent with our theoretical result.

We also show that such a licensing schedule can be used to dissuade the competitor from investing to develop its technology. If, as in our model, the demand uncertainty is fully resolved by the time the competitor makes its breakthrough, a single royalty rate based on this realized demand can be determined.

A financially unconstrained firm's decision to invest in development depends on the licensing schedule. In some cases, when the level of investment is sufficiently high, no level of forecasted demand would justify immediate investment. Instead, the firm should wait for the resolution of some uncertainty even if this means the opportunity window will close. In other cases, we solve for the unique threshold level of expected demand above which investment should be committed immediately. This threshold level usually increases with uncertainty, suggesting that development efforts in highly uncertain markets must clear a higher hurdle. This result implies that in a highly uncertain situation, the option to wait becomes more valuable than the growth opportunity of developing and licensing an innovation. An interesting extension occurs when the innovation may lead to network standards. In this case, early investment creates a single standard as well as a larger market. The resulting network effects will be greater than the network effects in two smaller markets based on two separate standards. The growth option thus created may dominate the waiting-to-invest option that is forgone. Hence, the investment threshold may actually drop with increasing uncertainty if the innovation leads to a network standard.

If the leading firm faces financial constraints and requires costly external financing, we show that funds can be obtained through a licensing arrangement, where the licensee pays the leading firm a down payment and in return, the leading firm commits to a capped royalty schedule. This result provides a rationale for the observation that smaller firms often finance their technological development from users of their products. The down payment is, in effect, an option premium the licensee pays for imposing a cap on the licensing schedule. The leading firm's ability to finance via this type of licensing arrangement depends critically on the licensee's expectation of the potential market. A special feature of financing via licensing is that higher uncertainty sometimes allows the leading firm to raise more funds, making this financing vehicle particularly useful for some range of uncertainty. Our results thus provide guidelines for the use of license structures for financially constrained firms. We also find that financial constraints limit the innovating firm's ability to capture value from innovations, which may make investment infeasible or unprofitable.

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Appendix 1: Ex Post Licensing

1. Set Notations

For clarity, we use set notations in the proofs. We denote the set of fixed fee licensing strategies by $\mathcal{F} = \{\varphi : \varphi \in R\}$ and the set of feasible royalty licensing strategies by $\mathcal{L} = \{l : l \le \frac{1}{2}\theta\}$. The no-licensing strategy belongs to a singleton set, $\mathcal{N} = \{NL\}$. The set of licensing strategies that M may agree to is defined as: $S_1 = \{s : s \in \mathcal{F} \cup \mathcal{L}, \text{ and } \Pi_1^s \ge \Pi_1^{NL}\}$. Similarly, the set for N is defined as: $S_2 = \{s : s \in \mathcal{F} \cup \mathcal{L}, \text{ and } \Pi_2^s \ge \Pi_2^{NL}\}$. The set of licensing strategies upon which both M and N may agree, called the "agreement set," is $S = S_1 \cap S_2$. When $S = \varnothing$, it is impossible for M and N to reach an agreement and the outcome is no licensing.

When $\theta \le \hat{\theta} \equiv 3\sqrt{J}$, $S_1 = \{\varphi : \varphi \ge \frac{5}{36}\theta^2\} \cup \{l : l = \frac{1}{2}\theta\}$ and $S_2 = \{\varphi : \varphi \le \frac{1}{9}\theta^2\} \cup \{l : l \le \frac{1}{2}\theta\}$. The agreement set is $S = S_1 \cap S_2 = \{l : l = \frac{1}{2}\theta\}$. Since the no-licensing strategy and a royalty schedule of $l = \frac{1}{2}\theta$ are equivalent, we assume that the outcome is no licensing.

When $\theta > \hat{\theta} \equiv 3\sqrt{J}$, $S_1 = \{\varphi : \varphi \ge 0\} \cup \{l : 0 \le l \le \frac{1}{2}\theta\}$ and

$S_2 = \{\varphi : \varphi \le J\} \cup \left\{l : l \le \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)\right\}$. Therefore,

$S = \{\varphi : 0 \le \varphi \le J\} \cup \left\{l : 0 \le l \le \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)\right\}$. Since strategies $\varphi = 0$ and l=0 are equivalent, the agreement set is rewritten as: $S = \{\varphi : 0 < \varphi \le J\} \cup \left\{l : 0 \le l \le \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)\right\}$.

2. Proof of Lemma 1

For any $\varphi \in \{\varphi : 0 < \varphi \le J\}$, we can find a royalty rate $l \in S$, such that $l = \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9\varphi}\right)$, which makes N indifferent between the fixed fee agreement of φ and the royalty agreement of l, i.e., $\Pi_2^{\varphi} = \Pi_2^{l} = \frac{1}{9}\theta^2 - \varphi$. Under this rate l, however, we can show that $\Pi_1^{\varphi} = \frac{1}{9}\theta^2 + \varphi < \Pi_1^{l} = \frac{1}{9}\theta^2 + \frac{5}{4}F$, i.e., M does strictly better than under the fixed fee agreement. Therefore, any $\varphi \in \{\varphi : 0 < \varphi \leq J\}$ is dominated. *Q.E.D.*

3. Proof of Proposition 1

We have shown that when $\theta \leq 3\sqrt{J}$, the outcome is no licensing. When $\theta > 3\sqrt{J}$, we know the agreement set is given by $S = \{\varphi : 0 < \varphi \leq J\} \cup \left\{l : 0 \leq l \leq \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)\right\}$. By Lemma 1 and the Assumption, no $\varphi \in \{\varphi : 0 < \varphi \leq J\}$ will emerge as an equilibrium outcome. Next, we need to show that no $l \in \left\{l : 0 \leq l \leq \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)\right\}$ is dominated. This is true because $\frac{\partial \Pi_1^l}{\partial l} > 0$ and $\frac{\partial \Pi_2^l}{\partial l} < 0$ for any *l* in the agreement set, therefore for a given *l*, any change in *l* will make one party worse off. Furthermore, when *l*=0, M is indifferent between licensing and not licensing while N is strictly better off licensing; when $l = \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)$, N is indifferent between licensing and not licensing (a.k.a. investing in its own technology) while licensing makes M strictly better; for all other royalty rates, both M and N are strictly better off by licensing than not licensing. *Q.E.D.*

Appendix 2: Ex Ante Licensing

1. Proof of Proposition 2:[31]

A contract that both M and N can accept must satisfy $E\Pi_1^{CL} \geq E\Pi_1$, i.e.

$$C \geq \int_{2L}^{3\sqrt{J}} \tfrac{5}{36}\left(\theta - 2L\right)^2 dF + \int_{3\sqrt{J}}^{L+\frac{9J}{4L}} \left[\tfrac{5}{4}J - \tfrac{5}{9}\theta L + \tfrac{5}{9}L^2\right] dF \qquad \text{(A1.)}$$

It must also satisfy $E\Pi_2^{CL} \geq E\Pi_2$, i.e.

$$C \leq \int_{2L}^{3\sqrt{J}} \tfrac{1}{9}\left(\theta - 2L\right)^2 dF + \int_{3\sqrt{J}}^{L+\frac{9J}{4L}} \left[J - \tfrac{4}{9}\theta L + \tfrac{4}{9}L^2\right] dF \qquad \text{(A2.)}$$

[31] The proof here assumes $l(\theta) = \frac{1}{2}\left(\theta - \sqrt{\theta^2 - 9J}\right)$ when $\theta > 3\sqrt{J}$. In fact, the result is true for any possible bargaining power between M and N (proof available on request).

Clearly, the right-hand-side of (A1) is $\frac{5}{4}$ times the right-hand-side (RHS) of (A2). If these are positive, then obviously the two inequalities cannot be satisfied simultaneously. It suffices to show that the RHS of (A2) is positive. We know that any effective L must satisfy $L < \frac{3}{2}\sqrt{J}$, so the first term on the RHS of (A2) is positive. For the second term on the RHS of (A2), we can prove that $J - \frac{4}{9}\theta L + \frac{4}{9}L^2 > 0$ for $\theta \in \left(2L,\, L + \frac{9J}{4L}\right)$, so this term is also positive. Therefore, (A1) and (A2) cannot be satisfied together. This result is true for any distribution of θ with a strict positive support in the range $[0, +\infty)$. *Q.E.D.*

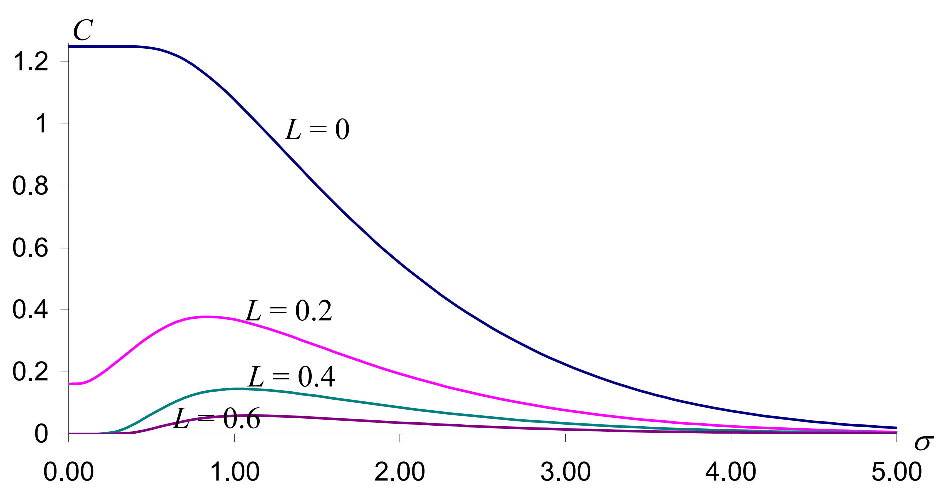
2. Simulation Results

For simulations, we use a set of lognormal distributions, where each distribution is $LN\left(\ln\theta_0 - \frac{1}{2}\sigma^2, \sigma^2\right)$.[32] Formally, the set is given by:

$$\Psi_{LN} = \left\{F(\theta) : F(\theta) = \Phi\left[\frac{1}{\sigma}\left(\ln\theta - \ln\theta_0\right) + \frac{\sigma}{2}\right], \theta_0 \in [0,\infty), \sigma = const.\right\}$$

where $\Phi(\cdot)$ is the c.d.f. of standard normal distribution.

Figure A1: Simulation of Royalty Cap Contract (J=1, θ_0=10)



[32] In our notation, $\theta \sim LN\left(\ln\theta_0 - \frac{1}{2}\sigma^2, \sigma^2\right)$ is equivalent to $\ln\frac{\theta}{\theta_0} \sim N\left(-\frac{1}{2}\sigma^2, \sigma^2\right)$, where $N(\cdot)$ represents standard normal distribution.

In Figure A1, for any positive *L*, *C* is low when $\sigma \to 0$, because the probability mass of θ is concentrated at a very high level out of the range of $\left(2L, L+\frac{9J}{4L}\right)$. For any positive *L*, the down payment *C* is maximized in the medium range of uncertainty.

Figure A2: Simulation of Royalty Cap Contract (J=1, θ_0=3)

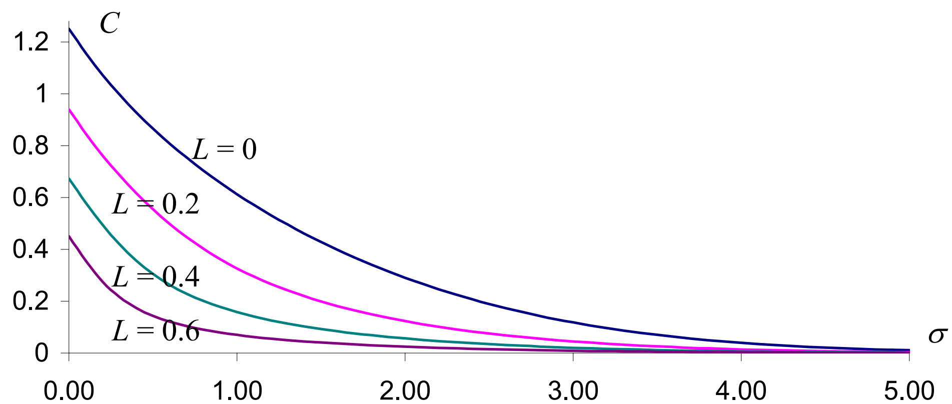


Figure A2, however, does show that the down payment *C* is maximized when $\sigma \to 0$. This is because here $\theta_0 = 3 = 3\sqrt{J}$. We know N gains from the contract when $\theta \in \left(2L, L+\frac{9J}{4L}\right)$, and within this range the gain is maximized at $\theta = 3\sqrt{J}$. Therefore, N expects to gain most if the probability mass of θ is concentrated at $3\sqrt{J}$, i.e. $\theta_0 = 3\sqrt{J}$ and $\sigma \to 0$.

Appendix 3: Proofs of Lemma 2 and Proposition 4.

1. Deriving V^I and V^{NI}

Assume that M does not have financial constraints. The net expected value of investing at t=0 is given by:

$$V^I = -I + E_0\left[\Pi_1^I\right] = -I + \int_0^{3\sqrt{J}} \tfrac{1}{4}\theta^2 dF + \int_{3\sqrt{J}}^{\infty}\left(\tfrac{1}{9}\theta^2 + \tfrac{5}{4}J\right)dF$$

If it does not invest at t=0, M will be identical to N at t=1, when each firms has the opportunity to invest the amount *J*. The firms will make the same *ex post* investment decision, and engage in Cournot competition if both invest. There will be no licensing between the two firms. Therefore, M will stay out of the market ($\Pi_1^{NI} = 0$) when $\theta \leq 3\sqrt{J}$, and invest when $\theta > 3\sqrt{J}$, earning a payoff of $\Pi_1^{NI} = \frac{1}{9}\theta^2 - J$. By postponing the investment decision to time 1, M avoids investing under unfavorable market conditions. Note that M's *ex post* entry decision and payoffs are the same as N when M has invested at t=0. That is, M has the same entry threshold as N, $3\sqrt{J}$, and its *ex post* payoff Π_1^{NI} is the same as N's payoff Π_2 .

The expected value of not investing at t=0 (i.e., waiting until t=1) is given by:

$$V^{NI} = E\left(\Pi_1^{NI} - J\right)^+ = \int_{3\sqrt{J}}^{\infty}\left(\tfrac{1}{9}\theta^2 - J\right)dF$$

We define the *ex post* net gain of investment as: $\delta\left(\theta\right) \equiv -I + \Pi_1^I - \Pi_1^{NI}$.

The difference between the expected values of the two investment-timing choices is then the expectation of $\delta\left(\theta\right)$, i.e., $E_0\left[\delta\left(\theta\right)\right] = V^I - V^{NI}$. The investment rule can also be stated as: M should invest if $E_0\left[\delta\left(\theta\right)\right] > 0$, and wait if $E_0\left[\delta\left(\theta\right)\right] \leq 0$.

2. Proof of Lemma 2

We know that

$$\delta\left(\theta\right) \equiv -I + \Pi_1^I - \Pi_1^{NI} = \begin{cases} -I + \frac{1}{4}\theta^2 \text{ when } \theta \leq 3\sqrt{J} \\ -I + \frac{9}{4}J \text{ when } \theta > 3\sqrt{J} \end{cases}$$

Obviously, when $I > \frac{9}{4}J$, $\delta\left(\theta\right) < 0$ for any *θ*. Therefore, for any distribution of *θ*,

$E_0\left[\delta\left(\theta\right)\right] < 0$.

When $I = \frac{9}{4}J$, $\delta(\theta) \le 0$ and "=" holds for $\theta \ge 3\sqrt{J}$. Therefore, $E_0\left[\delta(\theta)\right] \le 0$ for any distribution of θ, and "=" only holds for distributions that have no mass in M's monopoly range, i.e., $\theta \in \left[0, 3\sqrt{J}\right]$.

Overall, for any distribution of θ, $E_0\left[\delta(\theta)\right] \le 0$ when $I \ge \frac{9}{4}J$. *Q.E.D.*

3. Proof of Proposition 4:

First, we define the conditions given in Proposition 4 more formally.

Since $\theta \in [0, \infty)$, a cumulative probability distribution (c.d.f.) of θ can be represented as $F(\theta): R_+ \to [0,1]$, with $F(0) = 0$ and $F(\theta \to \infty) = 1$. A set of distributions of demand, $\Psi = \{F(\theta)\}$ should satisfy the following regularity conditions:

(i) Higher expected value of θ implies strong first-order stochastic dominance. Formally, two distributions $F_A, F_B \in \Psi$, $F_A \ne F_B$, $\theta_0^A \equiv \int_0^\infty \theta dF_A$, $\theta_0^B \equiv \int_0^\infty \theta dF_B$, have the following property: $\theta_0^A > \theta_0^B$ implies $F_A(\theta) \le F_B(\theta)$ for any $\theta \in [0, \infty)$, and strict inequality holds for some interval of θ in the range $\left[0, 3\sqrt{J}\right]$;

(ii) If the expected value of θ goes to zero, the density goes to infinity at $\theta = 0$. Formally, if a distribution $F_A \in \Psi$ is such that $\theta_0^A \to 0$, then $F_A(\theta) = 1$ for $\theta > 0$;

(iii) If the expected value of θ goes to infinity, no θ is distributed in the range where M will be a monopolist. Formally, if a distribution $F_A \in \Psi$ is such that $\theta_0^A \to \infty$, then $F_A(\theta) \to 0$ for $\theta \in \left[0, 3\sqrt{J}\right]$.

We define $\Delta \equiv E_0\left[\delta(\theta)\right] = V^I - V^{NI}$. For a set of distributions $F(\theta)$ that satisfies the above regularity conditions, we have

$$\Delta = -I + \int_0^{3\sqrt{J}} \tfrac{1}{4}\theta^2 dF + \int_{3\sqrt{J}}^{\infty} \tfrac{9}{4} J dF$$

Integrating by parts, we get:

$\int_0^{3\sqrt{J}} \frac{1}{4}\theta^2 dF = \frac{1}{4}\theta^2 F(\theta)\Big|_0^{3\sqrt{J}} - \int_0^{3\sqrt{J}} F(\theta)\, d\left(\frac{1}{4}\theta^2\right)$, and $\int_{3\sqrt{J}}^{\infty} \frac{9}{4} J dF = \frac{9}{4} J \cdot F(\theta)\Big|_{3\sqrt{J}}^{\infty}$.

Therefore, $\Delta = -I + \frac{9}{4} J - \int_0^{3\sqrt{J}} \frac{1}{2}\theta \cdot F(\theta)\, d\theta$ (A3.)

Given the regularity conditions of $F(\theta)$, we have:

(a) $\Delta(\theta_0 \to 0) = -I + \frac{9}{4} J - \frac{1}{4}\theta^2\Big|_0^{3\sqrt{J}} = -I$;

(b) $\Delta(\theta_0 \to \infty) = -I + \frac{9}{4} J$;

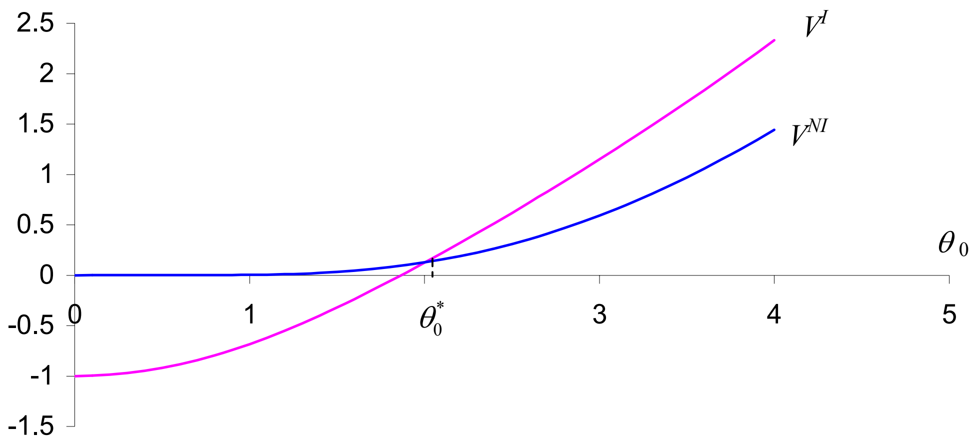
(c) Since $\theta_0^A > \theta_0^B$ implies $F_A(\theta) \le F_B(\theta)$ for any $\theta \in [0,\infty)$, then from (A3) we know that $\Delta(\theta_0^A) \ge \Delta(\theta_0^B)$, i.e., Δ is non-decreasing in θ_0. Furthermore, in $F_A(\theta) \le F_B(\theta)$, "<" holds for some interval of θ in the range $\theta \in \left[0, 3\sqrt{J}\right]$, thus, Δ is strictly increasing in θ_0.

Summarizing (a), (b), and (c), we have: $\Delta(\theta_0 \to 0) = -I < 0$, $\Delta(\theta_0 \to \infty) = -I + \frac{9}{4} J$, and Δ is strictly increasing in θ_0. Since $I < \frac{9}{4} J$, by (a), (b), and (c), there exists a unique θ_0 that makes $\Delta = 0$. Denote it by θ_0^*. Obviously, $\Delta > 0$ for $\theta_0 > \theta_0^*$, and $\Delta < 0$ for $\theta_0 < \theta_0^*$. *Q.E.D.*

4. Determining the Threshold of Expected Demand for Immediate Investment:

The threshold for immediate investment θ_0^* is the θ_0 value such that $V^I = V^{NI}$. To demonstrate how to determine such a threshold of expected demand, we simulate the behaviors of the value functions, again using the set of lognormal distributions Ψ_{LN}. Figure A3 illustrates the uniqueness of the investment threshold.

Figure A3: Investment threshold (I=J=1, σ =0.5)



Appendix 4: The Impact of Uncertainty on Threshold

We consider an increase in uncertainty as a mean-preserving spread of the demand, which shifts the probability mass to more extreme values without changing the mean.

The *ex post* payoff of immediate investment Π_1^I is a continuous and increasing function of θ with a kink at the entry threshold $\hat{\theta} \equiv 3\sqrt{J}$. Π_1^I is neither convex nor concave, so the impact of greater uncertainty on V^I is ambiguous. Specifically, when uncertainty increases, greater probability mass in the low range of demand reduces the expected payoff of early investment, while in the high range of demand the expected payoff rises. Because the payoff function is not convex – it increases at a higher rate on the left side of the kink than it does on the right side - the net effect of higher uncertainty is ambiguous.

The *ex post* payoff of not investing at t=0 (waiting until t=1), Π_1^{NI} , is a non-decreasing function of θ, also with a kink at the entry threshold. Because it is a convex function, by Jensen's inequality V^{NI} increases with uncertainty. Greater probability mass in the high range of demand increases the expected value while greater mass in the low range has no impact on the expected value.

The *ex post* net gain function, $\delta(\theta)$, with a kink at the entry threshold, equals the monopolist profits to the left of the kink and remains constant to the right. Clearly, it is neither convex nor concave, so the impact of greater uncertainty on the investment threshold is ambiguous. The net gain function is increasing on the left side of the entry threshold while it remains constant on the right, so as higher uncertainty shifts the probability mass to the extreme values, the expected net gain of early investment declines. Therefore, in most cases (exceptions discussed in the next paragraph), under greater uncertainty, the expected demand needs to be higher to justify early investment. Intuitively, when higher uncertainty makes N more likely to enter the market, the extra value captured through licensing usually cannot compensate for the down side of higher uncertainty.

An exception occurs when the probability mass of θ lies almost entirely to the left of $\hat{\theta}$, the kink in $\delta\left(\theta\right)$. When a small increase in uncertainty spreads the probability but still keeps most of the mass within this range, the investment threshold decreases. This is because the *ex post* net gain of immediate investment is convex in this range. Intuitively, when uncertainty is slightly higher but not too high to make N likely to enter and thus M is still likely to be a monopolist, M is more inclined to invest early.

Again, we use the set of lognormal distributions Ψ_{LN} to simulate the impact of uncertainty. Figure A4 shows how uncertainty shifts the expected values of different investment-timing decisions V^I and V^{NI}, and changes the investment threshold.

Figure A4: Effect of Uncertainty on Investment Threshold