An Approach to multi objective Facility Layout Planning

N. Sahoo, T. Shekhar, S. Sahu

[In this paper we discuss the facility layout problem. In this work we propose a system in which the analyst can specify the relationships between the facilities exactly as he perceives them to be. This can be seen as departure from the conventional way of specifying the relationship by numeric ratings. We also use a tabu search algorithm to iteratively solve the facility layout problem.]

1. Introduction

The facility layout problem deals with finding the most effective physical arrangement of facilities, personnel, and any resources required to facilitate the production of goods or services. Conventionally there are two basic criteria taken into consideration, while solving a facility layout problem - qualitative and quantitative. Both the qualitative and the quantitative methods have their own limitations.

In the qualitative approach, the interdepartmental closeness desirability are expressed in terms of closeness ratings. These ratings are decided by the designer after taking into consideration one or more of the qualitative factors, e.g. noise, heat, dust, flow of material etc. In solving the problem the overall subjective closeness rating is maximized. These subjective closeness ratings could be: A, E, I, O, U, X. They indicate the respective degrees of necessity that the two departments be located close to one another. Usually the layout designer assigns different numerical values to the ratings so that A>E>I>O>U>X. Seehof and Evans (1967), Lee and Moore (1967), Muther and McPherson (1970) and Muther (1973) have developed algorithms based on qualitative criteria to obtain final layouts. These different qualitative approaches are distinguished by the scoring methods used for the closeness ratings e.g. numerical values used by Sule (1994) and Harmonosky and Tothero (1992) for these ratings are A = 4, E = 3, I = 2, O = 1, U = 0 and X = -1. The ALDEP procedure presented by Seehof and Evans (1967) used the numerical values A = 64, E = 16, I = 4, 0 = 1, U = O and X = -1024.

In quantitative approach, usually the objective is to minimize the material handling cost. This problem is best represented by a Quadratic Assignment Problem (QAP). QAP is a problem of assigning n departments to n locations.

Both of the above methods have their own limitations. The ratings in the qualitative methods are dependent upon the analyst. Moreover the ratings are ordinal in nature, but the comparison of the alternatives are done on a metric scale. Also the subjective relationships are specified as numerals only which may not describe the dependency between the departments appropriately. It makes the solution inaccurate. Quantitative methods have, however, always been criticized for not taking into consideration anything that can not be quantified. So it might lead to practically inefficient solutions.

The modern approach is to take both the quantitative and qualitative factors into consideration. This falls under the category of Multi Objective Facility Layout (MOFL) problems, C. W. Chen and D. Y. Sha (1999). But it is only a combination of two old concepts. This approach retains most of the shortcomings of the old approaches, especially that of the qualitative factor.

In this paper we have tried to develop an alternative approach to specify the subjective rating. We have developed a language for specifying the subjective ratings as functions of various parameters.

Section 2 describes various developments in formulating mathematical models of facility layout problems by combining both the subjective and the objective ratings. It also throws light on various algorithms applied to solve the formulated problems. Section 3 describes the new system of specifying ratings. Section 4 describes in detail the Tabu search algorithm which has been used as our problem solving algorithm. Section 5 illustrates the new system with an example. Section 6 presents the summary and the conclusions.

2. Overview of earlier works

The QAP formulation of facility layout problem is shown in equation (1) to (4) below.

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} A_{ijkl} X_{ij} X_{kl} \quad (1)$$

Subject to,

$$\sum_{i=1}^{n} X_{ij} = 1 \quad (2)$$

$$\sum_{j=1}^{n} X_{ij} = 1 \quad (3)$$

$$X_{ij} \in \{0, 1\} \quad (4)$$

Where x_{ij} = 1, if facility i is located at location j
= 0, otherwise

A_{ijkl} = the cost of locating facility i at location j and facility k at location l.

Equation (2) ensures that only one facility is assigned to each location. Equation (3) ensures that one facility is assigned to one and only one location. The term A_{ijkl} in Equation (1) represents a combination of qualitative and quantitative measures in the facility layout problems. The formulation of this term has changed over time although the structure of the problem has remained the same. The various formulations of A_{ijkl} can be divided into four categories. They are presented in a chronological order below.

1. Rosenblatt (1979) and Dutta and Sahu (1982) defined A_{ijkl} as
   $A_{ijkl} = W_c C_{ijkl} - W_r R_{ijkl}$;
   Where C_{ijkl} = total material handling cost,
   R_{ijkl} = total closeness rating score
   W_c = weight assigned to the material handling cost
   W_r = weight assigned to total rating score
2. Fortenberry and Cox (1985) defined A_{ijkl} as
   $A_{ijkl} = f_{ik} d_{jl} r_{ik}$;
   Where f_{ik} = the work flow between two facilities
   d_{ji} = the distance between two locations and
   r_{ik} = closeness rating desirability of the two facilities
3. Urban (1987,1989) defined A_{ijkl} as
   $A_{ijkl} = d_{jl} (f_{ik} + C r_{ik})$;
   Where C is a constant weight that determines the importance of closeness rating before the workflow.
4. Khare et al. (1988) defined A_{ijkl} as
   $A_{ijkl} = W_1 r_{ik} d_{jl} + W_2 f_{ik} d_{jl}$
   W_1 and W_2 are the weights assigned to the closeness rating and workflow respectively.

The above formulations of A_{ijkl} are similar in that, they all target to minimize the material handling cost and they also try to place together those departments which should be placed together considering non quantitative factors. But they all vary in the way A_{ijkl} is formulated out of qualitative and quantitative factors. These different combinations can be appropriate in different situations. However we feel there is a need of a system which allows the analyst to construct his own 'cost term' out of the qualitative and quantitative factors, in a way, he perceives to be appropriate in the given situation. We present an alternative system in section (4) using which the analyst can write a piece of code to construct the cost term A_{ijkl} from the qualitative and the quantitative factors.

3. Tabu Search

Once the problem has been formulated as QAP, various heuristics can be applied to get the optimal value. Among the recently developed intelligent search methods, Tabu Search algorithm is a metaheurstic method, which has proved to be very popular along with simulated annealing. Tabu search is basically characterized by the following points:

a) The neighboring solutions from a solution are generated from the current solution by exchanging the locations of two departments, or, "making a move".
b) The recent moves are stored in a tabu list and are normally not considered for the next move. It helps prevent the cycling behavior of the algorithm.
c) Among all the possible moves, the best improving move is checked for tabu status. If the move is in the tabu list and does not satisfy aspiration criteria, it is dropped from consideration and the second best move is considered.
d) Aspiration criteria are provided to allow the exceptionally good moves to be considered even if they belong to the tabu list. If the best improving move is tabu but satisfies aspiration criteria, it is eligible for the next move. These aspiration criteria may be the minimum reduction in cost or may be a combination of factors.
e) Diversification is needed to test the unexplored solution spaces, namely moves that have not been made so far. The "Diversification Strategy" penalizes the frequently occurring moves and makes the algorithm diversify.
f) Maximum portion of improvement is achieved during the initial few moves. At the later stages, even a comparatively small improvement may prove to be very significant. Intensification strategy makes sure that the moves resulting to such improvements have higher probabilities of being present in the final solution. It is done by keeping the good moves in the tabu list for a longer period.
g) The algorithm can be terminated by specifying the maximum number of moves since the beginning or maximum number of moves since the last improvement in the solution.

For a detailed discussion on Tabu-search the reader is referred to CHIANG, W. and CHIANG, C. (1998).

The proposed Algorithm consists of the following steps:

Step 1 (initialization)

1.1. get flow and distance matrices
1.2. read initial layout

1.3. read the set of specifications for the interdepartmental relationships.
1.4. compute the total cost (C(loc (σ)) of the layout (loc(σ)), set it as minimum cost. (*min_cost*)
1.5. initialize long-term memory
1.6. get *max_iter*

Step 2 (evaluate potential candidates)

2.1. calculate the cost

2.1.1. calculate material handling cost for every pair(i,j) of departments (m_{ij}). Calculate the total cost $M = \sum_i \sum_j m_{ij}$

2.1.2. for every subjective factor (k) execute the instruction for every pair (i,j) of departments to get subjective factor cost(s_{ijk}).
Calculate the total subjective factor cost $Sk = \sum_i \sum_j s_{ijk}$.

2.1.3. calculate total cost $C(loc(\sigma')) = \sum_k S_k + M$

2.2. if $C(loc(\sigma')) < min\_cost$ accept the move

2.3. if $C(loc(\sigma')) > min\_cost$ and the move is not in tabu list, compute
$\Delta = C(loc(\sigma')) - C(loc(\sigma)) + freq(loc(\sigma'), loc(\sigma))'$
and accept the move with minimum Δ.

Step 3 (update)

3.1. set $loc(\sigma) = loc(\sigma')$ and $C(loc(\sigma)) = C(loc(\sigma'))$
3.2. update the tabu list and long term memory
3.3. if $C(loc(\sigma)) < min\_cost$
min_cost = C(loc(σ)), *unchange* = 0
else *unchange* = *unchange* + 1
3.4. *cur_iter* = *cur_iter* + 1

Step 4

If *unchange* > *max_iter*
Stop, print the best solution, minimum cost and number of iterations.
else goto Step 2.

4. Proposed Subjective Rating Specification System

Keeping the requirements cited in section (2) in mind, we developed an alternative subjective rating specification system, which enables the analyst to specify his idea of the relationship between two departments through a piece of code. For this purpose we designed a language and developed an interpreter to interpret the same.

In this system the analyst enters the flow and distance matrix and the relationship between the departments. After the analyst has input the relationship between every pair of departments, the program parses the input and stores the instructions for all the pairs. This is carried out for every factor. The tabu search engine at the heart of the program, calls a routine to execute these stored procedures/instructions for every pair of department to evaluate the cost incurred due to the subjective factors. In addition to this it also takes the flow and the distance matrices and calculates the material handling cost. All these together constitute the total layout cost. The objective of the tabu search engine is to minimize this total cost.

A typical input file specifying the relationship between the departments will have the following general form.

```
NDEP - n
NLOC - n
NFAC - m

start { ..................................}
distances {
        read distancefile
}

flows {
        read flowfile
}

factor 1 {
        [a to b] [c to d]{
        ............................
        ............................
        return expr
}
.........................
.........................
}

factor 2 {
    .....................
    .....................
}

.....................
.....................

factor m {
    .....................
    .....................
}
```

Where,

NDEP = *n* specifies the number of departments.
NLOC = *n* specifies the number of locations.
NFAC = *m* specifies the number of subjective factors in consideration

start { .... }gives the starting solution
distances {
read *distancefile*
} instructs the program to read the distance matrix from the

distancefile .
flows {
    read flowfile
} instructs the program to read the flow matrix from the *flowfile*.

factor 1 { .... } specifies the effect of the factor 1 on every pair of departments. Similar is the case for all the other factors.

[*a* to *b*] [*c* to *d*] {
    ....
    ....
    return expr
} specifies the relationship in detail between the pairs specified by { a to b } X { c to d } . The ellipses in the previous block represent a piece of code written using one or more of the following constructs. The cost incurred due to the subjective factor is given by the expression *expr* .

Assignments: assignment of expressions to user defined variables. e.g. score = 15/(DISTANCE^0.5)

We have two built-in variables FLOW and DISTANCE which give the material flow and the distance between the two departments for which the code is being executed. As the layout changes during the process of solving the problem so does the distance, but material flow for that pair remains the same.

Expressions: expressions are constructed by the addition, subtraction, multiplication, division of two or more expressions, one expression raised to the power by another expression, parenthesized expressions and function calls with expressions as arguments. Although the program does not provide the facility for defining functions, it has some built in functions. They are : sin(), cos(), log(), log 10(), exp(), sqrt() and abs().

If-else: They help in taking a decision in the course of execution of the program. The if else construct is as shown below.

I f (*expr*) {
    ....
    ....
}

if the expression *expr* produces a non-zero value then the statements within the curly braces are executed.

If (*expr*) {
    ....
    ....
} else {
    ....
    ....
}

if the expression *expr* produces a non-zero value then the statements within the curly braces following the *if*( ) are executed, else, the statements within the curly braces following the else are executed.

Relational operators:

The following relational operators are provided:

>	greater than
<	less than
==	equal to
>=	greater than equal to
<=	less than equal to

They help in determining the relationship between two expressions.

Logical operators:

The following logical operators are provided.

& &	logical and
| |	logical or

They enable the user to logically connect two expressions forming a new expression.

Arithmetic Operator:

The following arithmetic operators are provided.

+	add
-	subtract
*	multiply
/	divide
^	exponent

while:

It enables the user to write a loop.

whil e (*expr*) {
    ....
    ....
}

The statements within the *while* block are executed as long as the value of *expr* is non-zero.

5. An Example

As observed in section (2), there is a need of a system for specifying the complex relationship between two departments in a facility layout problem. The utility of the presented system will be illustrated by an example.

There are 12 facilities with the following distance and flow matrices.

Distance Matrix:

From \ To	2	3	4	5	6	7	8	9	10	11	12
1	5	2	4	1	0	0	6	2	1	1	1
2		3	0	2	2	2	0	4	5	0	0
3			0	0	0	0	5	5	2	2	2
4				5	2	2	10	0	0	5	5
5					10	0	0	0	5	1	1
6						5	1	1	5	4	0
7							10	5	2	3	3
8								0	0	5	0
9									0	10	10
10										5	0
11											2

Flow Matrix:

From \ To	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	1	2	3	4	2	3	4	5
2		1	2	2	1	2	3	3	2	3	4
3			1	3	2	1	2	4	3	2	3
4				4	3	2	1	5	4	3	2
5					1	2	3	1	2	3	4
6						1	2	2	1	2	3
7							1	3	2	1	2
8								4	3	2	1
9									1	2	3
10										1	2
11											1

Let us assume in addition to the material handling cost, the long term cost incurred due to the noise and the heat produced by some departments are significant. The departments 1,2 and 3 are especially noise producing and the departments 6 to 12 are especially sensitive to noise. Also the departments 4 to 7 produce excessive heat and the departments 8 to 12 are especially sensitive to temperature.

Let us further assume that

The loss due to noise $\propto 1/(\text{distance})^a$
The loss due to heat $\propto 1/(\text{distance})^b$
Where $a = ½$
and $b = 2$

And the proportionality constants have been determined to be 145\$-m² and 120 \$-m$^{1/2}$ respectively. Existing methods cannot handle such relationship between departments.

This information can be coded into the input as (the *italicized* words are not a part of the code and are provided to explain the program):

```
NDEP   =  12   number of departments = 12
NLOC   =  12   number of locations = 12
NFAC   =  2         number of subjective factors = 2

starting solution
start {8 11 5 3 2 4 12 10 9 1 6 7}

distances {
        read d12d           read distance matrix file
}

flows {
        read d12 f          read flow matrix file
}

costs incurred due to subjective factors
factor 1 { noise
        [ 1 to 12 ] [ 1 to 12 ] return 0
        [ 1 to 3 ] [ 6 to 12 ] {
        a = 0.5
        return 120/(DISTANCE^a)}
}

factor 2 { temperature
        [ 1 to 12 ] [ 1 to 12 ] return 0
        [ 4 to 7 ] [ 8 to 12 ] {
          b = 2
          return 145/(DISTANCE^b)
  }
}
```

The code

```
        [ 1 to 3 ] [ 6 to 12 ] {
                a = 0.5
                return 120 / (DISTANCE^a) }
```

in the block labeled factor 1, instructs the program to store the instruction within the curly braces for the department pairs (1,6), (1,7),..., (1,12), (2,6), (2,7), ..., (2,12), (3,6), (3,7), ..., (3,12). This code is executed to calculate the effect of factor 1 (sound) on layout cost. Similar is the case for the factor 2.

The code inside the factor block is executed sequentially. The first lines in both the factor 1 and factor 2 block are provided to ensure that the costs incurred for all the department pairs other than those especially sensitive to these subjective factors are zero.

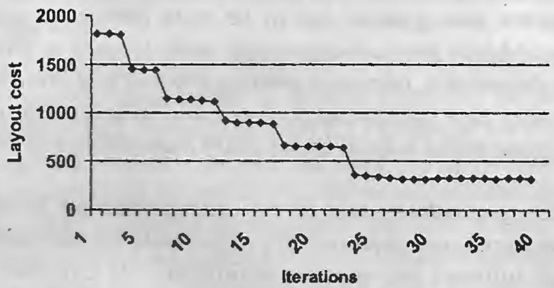
The initial layout is
8 11 5 3 2 4 12 10 9 1 6 7

meaning the department 8 is assigned to location 1, department 11 is assigned to location 2 and so on.

This layout has an initial total cost of 1841. When this data is fed into the program with tabu size = 4 and *max_iter* = 10, it produces a layout with total cost = 315 after 29 iterations. Then it iterates for 10 more times and stops thereafter as no cost reduction occurs. The final layout is:

12 9 11 10 8 4 7 6 3 1 2 5

The cost reduces in the following manner:



6. Conclusion

This method can be extended to handle the three dimensional facility layout problems by appropriately modifying and formulating the QAP as suggested by Univer Cinar (1971).

A poor layout can have a significant impact on the performance of an organization. Hence while planning the layout all the important qualitative and quantitative factors should be taken into account. The behaviors of the factors in the model should be as close to the actual behavior of the factor as possible. In this paper we present a language using which the analyst can specify the behavior of a factor, as he perceives it to be. Thus he can exactly specify the effect of a particular factor for any pair of departments. This will help the program to generate a better layout. At the heart of the program we have a tabu search engine which uses the above mentioned information to calculate the cost of the layout and tries to minimize it. It uses a fixed tabu size and intensification strategy and a diversification strategy. This enables the program to find good solutions relatively quickly.

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AUTHORS

Mr. Nachiket Sahoo : Mr. N. Sahoo has done his B. Tech in Industrial Engineering at the Department of Industrial Engineering and Management, IIT Kharagpur and is currently working as associate Consultant at i2 technologies Limited, Bangalore.

Mr. Tushar Shekhar : Mr. T. Shekhar has done his B. Tech in Industrial Engineering at the Department of Industrial Engineering and Management, IIT Kharagpur and is currently working as Research Consultant with the sponsored Research and Industrial Consultancy, IIT Kharagpur.