EC 403 Game Theory, Summer Term, Session 1
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Problem Set 1

It will be corrected in class on Friday, May 30.

1) THE CENTIPEDE GAME. Consider a game with two players, Albert and Bob. Albert moves first. A referee puts one dime on the table, and Albert decides whether to pass or to take the dime. In the latter case, the game is over. In the former case, the referee puts another dime on the table, and now it is Bob's turn to decide whether to pass or to take the two dimes. In the latter case, the game is over, whereas in the former case a third dime is put on the table, and now it is Albert's turn. The game goes on until either someone has not passed or one dollar is on the table. In that case, it is Bob's turn. If he passes, the dollar is split into equal parts between the two players. But if he does not, he just takes the dollar. In both cases, the game is over.

a) Draw the tree of this game. Notice that the tree could resemble a centipede. This is why this game takes such a name.
b) Identify all the subgames in this game.
c) Find the rollback equilibrium. Is this what you expect in real life? What is probably missing in the payoffs that are depicted in this game?

2) THE END OF CHESS. A scientist has created a supercomputer that is able to completely solve the game of chess. The computer has found that if both players play a perfect game, then the outcome of it will be a draw (or tie). As you know, in this game there are two players, one who conducts the White pieces and another who conducts the Black pieces. White moves first, and after everyone observes her move, Black makes her move, which is observed as well. Then White moves, and then Black, and so on. After each move, the player who has moved has the option to Offer a Draw. In such a case, the other player has to either accept the offer or continue playing (in which case that offer has been declined). A win gives a gross payoff of one unit, a draw ½ and a lose 0. Since chess is such a boring game, each player loses 0.01 utility units each time anyone makes a move. Net payoffs are calculated by subtracting utility losses to gross payoffs.

a) Draw a simplified tree involving White and Black's first two turns. In that tree, you can assume that each player moves according to what the computer advises in case she decides to move a piece. Therefore, the only strategically relevant actions are to accept or not a previous draw offer (if any) and to offer or not a draw.
b) Identify the equilibrium path of the rollback equilibrium.

3) A BEAUTIFUL MIND. Do Exercise 12 in Chapter 4 of the textbook (pages 119-120).

4) COURNOT GAME. This game receives its name from the economist who analyzed it back in the XIX century. Two firms, 1 and 2, provide exactly the same kind of good, let us say, cane sugar. They are the sole producers in the country. Each year, firm 1 produces a quantity q_1 (let us say tones) and firm 2 produces q_2. Production costs are normalized to zero for both firms. Country's market price for cane sugar is determined by the demand function $P= 1-Q$, where Q is total cane sugar production in the country, $Q=q_1+q_2$. So firm i's yearly profits ($i=1,2$) equal $q_i(1-q_i-q_j)$, where j is the other firm, and this is what firm i will try to maximize. You will be asked to predict each firm's yearly production, using several Game Theory techniques.

a) Best Response Method: for each firm i, calculate its best response for any production plan of the other firm, q_j. Take into account that production plans are nonnegative numbers. Hint: you will have to calculate the first derivative of the profit function, apply the First-Order Condition (solve the equation where the former derivative is zero), and make sure that the second derivative is negative everywhere. Draw both firms' best response functions in the same graph, where one axis contains q_1 units and the other contains q_2. Calculate the unique point where the two best response functions intersect. This yields the Nash equilibrium strategies.
b) Iterative Elimination of Never Best Responses: use the graph in part a). For firm 1, indicate in the graph which production plans are never best responses. Do the same for firm 2. Now, given the strategies than have been eliminated, think again about what other firm 1's strategies are never best responses. Do the same for firm 2. Indicate how the process converges to a unique pair of strategies, which is the Nash equilibrium.
c) Iterative Elimination of Strictly Dominated Strategies: you will only be required to follow a few steps. First, prove that for any firm i, any production plan $q_i>½$ is strictly dominated. To do so, calculate the first derivative of the payoff function, and show that it is strictly negative if $q_i>½$, given $q_j\geq 0$. This means that a slightly lower production plan is always better than q_i, thus q_i is a strictly dominated strategy. Once any production plan higher than ½ has been discarded, the next step is to prove that any production plan lower than one quarter is dominated. To do so, you have to show that for any firm i and q_i <¼, the derivative of firm i's payoff is strictly positive for any non-eliminated production plan of the other firm. This shows that q_i is strictly dominated by a slightly higher production plan. As you can see, the scope of non-eliminated strategies shrinks progressively. It can be shown (but you are not required to do so) that this iterative procedure converges to a unique strategy profile, which is the Nash equilibrium.

5) THE DUEL. Do Exercise 9 in Chapter 6 of the textbook (pages 183-184).