

# Advance Information and Asset Prices\*

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## Abstract

This paper provides a dynamic rational expectations equilibrium model in which investors have heterogeneous information and investment opportunities. Informed investors privately receive advance information that is useful for predicting future earnings but is unrelated to current earnings. This information is immediately partially incorporated into prices, and thus stock prices may move in ways unrelated to current fundamentals. Investors' speculative and rebalancing trades in response to advance information generate short-run momentum, mimicking an underreaction pattern. When this information materializes, the stock price reverts back to its long-run mean, mimicking an overreaction pattern. Our model also helps explain the phenomenon of post-earnings announcement drift.

*Key words:* advance information, momentum and reversal effects, underreaction, overreaction, rational expectations equilibrium

*JEL Classification:* G11, G12, G14.

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# 1. Introduction

Many empirical studies have documented evidence of momentum and reversal effects in aggregate and cross-sectional stock returns. The momentum effect refers to the phenomenon of stock returns tending to exhibit unconditional positive serial correlation in the short to medium run. A related phenomenon is that, conditional on observable public events, stocks tend to experience post-event drift in the same direction as the initial event impact. The reversal effect refers to the phenomena that stock returns are negatively correlated in the long run, and that stock returns are positively related to price-scaled variables, such as the book-to-market ratio.<sup>1</sup>

The momentum and reversal effects provide a serious challenge to the efficient markets hypothesis and to standard risk-based models. Many researchers have recently shifted attention to behavioral models, where investors are boundedly rational and arbitrage is limited. A common behavioral interpretation is that investors underreact to current news and overreact to a succession of news pointing in the same direction.

In this paper, we provide a rational, heterogeneous-investor model with asymmetric information that can deliver both the momentum and reversal effects. Our model builds on the model in Wang's (1994) seminal paper. In this model, two investor types, informed and uninformed, trade in the financial market. Informed investors can invest in both publicly traded assets (a stock and a risk-free bond) and in a private investment opportunity. Informed investors have private information about the persistent and transitory components of dividends or earnings as well as about the return on the private investment. Uninformed investors can invest in the stock and bond only.<sup>2</sup> Their information consists of past earnings and stock price realizations. Based on this public information, they infer informed investors' private information. As discussed in Wang (1994), informed investors trade for speculative and rebalancing reasons. Uninformed investors trade for noninformational reasons only, because they cannot distinguish the informed investors' trading motives. Uninformed investors trade to accommodate what they perceive to be rebalancing trades by informed investors.

When informed investors trade for rebalancing reasons due to an increase in the current expected return on the private investment, they sell stock to invest in the private opportunity. The current stock price falls to induce uninformed investors to buy, who in turn expect high

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<sup>1</sup>See Cutler et al. (1991), Jegadeesh and Titman (1993), Chan et al. (1997) and Rouwenhorst (1998) for time-series and cross-sectional evidence on short term momentum, and Bernard (1992) for evidence on price continuation after public news events. For evidence on long term return reversals see DeBondt and Thaler (1985), Fama and French (1988), Poterba and Summers (1988), and Jegadeesh and Titman (1995). For evidence on the negative association between stock returns and past price-scaled variables see DeBondt and Thaler (1987), Fama and French (1992) and La Porta et al. (1997).

<sup>2</sup>That informed investors have a larger investment opportunity set is in the spirit of Merton's (1987) investor recognition hypothesis that some investors are better at identifying investment opportunities than others. This assumption is in line with having private equity, real estate, foreign exchange, and derivatives markets accessible to only a subset of investors.

stock returns in the next period. This trading pattern decreases the current stock return and raises next period's stock return if the private investment returns are independent over time. Thus, stock returns generated by rebalancing trades tend to reverse themselves. If the private investment returns are persistent, future private investment returns will change when the current private investment return changes; therefore, future stock prices will also change due to future rebalancing trades. This effect may dominate making stock returns continue themselves forever.

When informed investors trade for speculative reasons, the stock price changes to reflect these investors' expectations of the stock's future payoffs. These expectations are fulfilled later as private information becomes public. Consequently, a price change generated by a speculative trade implies future returns of the same sign, and stock returns generated by speculative trades tend to continue themselves. We show that, in this benchmark model, the effect of speculative trading is always dominated by the effect of rebalancing trading, and thus this model cannot generate short-run momentum and long-run reversals simultaneously.

In order to explain these phenomena in a unified way, we extend this model by introducing a novel mechanism that may generate momentum from both speculative and rebalancing trades. The novel ingredient is the assumption that informed investors possess private advance information about a firm's future performance, such as future shocks to earnings, that is unrelated to current performance. In this case, the stock price immediately partially incorporates the advance information. In addition, informed investors' speculative trading may be generated by private advance information, reinforcing the return continuation effect discussed earlier.

In contrast to the model without advance information, rebalancing trades in the presence of advance information may also help generate return continuation, even though the persistence of private investment returns is low. We explain this intuition by considering the effects of a good piece of one-period-ahead advance information. In response to this information shock, the current stock price rises, partially incorporating the good information. As a result, the current excess stock return also rises. When the stock and the private investment opportunity are substitute assets, good advance information about a firm's future performance also constitutes good news about the private investment return. Therefore, as informed investors buy into the private investment, they sell the stock for rebalancing reasons, even though they expect high future returns. Uninformed investors buy the stock, expecting high returns in the next period. Informed investors are contrarian and uninformed investors are trend chasers in response to good advance information. This trading behavior generates short-run momentum. Unlike a representative agent model, our heterogeneous-investor model shows that momentum is related to investor trading behavior.

In addition to generating short-run momentum in excess stock returns, private advance information may also generate long-run excess stock return reversals. This is because stock

prices may move in response to the advance information, even though fundamentals (earnings) do not change. Once the advance information materializes, the stock price reverts to its long-run mean, giving rise to what appears to be an overreaction effect.

We show that with a single piece of advance information, the short-run momentum and long-run reversal effects can occur when this information is about next period's earnings innovations. An undesirable prediction of this case is that momentum lasts only for a few periods, which seems inconsistent with empirical evidence. When the single piece of advance information is about earnings innovations many periods into the future, the model may generate counterfactual cyclic return dynamics. We thus extend this model by assuming that informed investors receive increasingly precise signals about earnings innovations as these get closer to materialize. In this case, stale information is useful for forecasting, and informed investors trade on this information.<sup>3</sup> As a result, the effects of advance information can last for a long period, causing long-lived momentum. Moreover, after a sustained streak of good news, the stock price will appear to have overshot its fundamental value and ultimately reverts itself.

Our model is also able to explain the post-earnings announcement drift phenomenon. With symmetric information, an earnings surprise reveals the same information to all investors. Positive earnings surprises are immediately incorporated into the current stock price. Afterwards, the surprise does not affect the stock price because it has already been realized. This implies that price drift cannot occur under symmetric information. With asymmetric information, a positive earnings surprise is incorporated into the price, though interpreted differently by informed and uninformed investors. To the extent that some good information about the stock remains private after the earnings surprise, the model generates a price drift.

Rational models are generally unable to explain momentum and reversal effects in a unified way.<sup>4</sup> First, some models have the mechanisms to explain momentum but not reversals. Berk, Green and Naik (1999) show that a rich variety of return patterns, including momentum effects, result from the variation of risk exposures over the life-cycle of a firm's endogenously chosen projects.<sup>5</sup> Johnson (2002) provides a standard model of firm cash flows discounted by an ordinary pricing kernel that are able to deliver the momentum effect. His key idea is that expected dividend growth rates vary over time and growth rate risk varies with the growth rates. Both models, however, cannot deliver the long-horizon reversal effect. Second, some models have the mechanisms to explain reversals but not momentum. Lewellen and Shanken (2002) predict return reversals due to correction of past forecast errors, but do not predict momentum. Fama and French (1993, 1996) show that many of the long-horizon results—such as

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<sup>3</sup>See Bernhardt and Miao (2005) for a strategic model of stock prices with stale information. See Tetlock (2007) for empirical evidence of the importance of stale information.

<sup>4</sup>One exception is Vayanos and Wolley (2008) who combine the assumptions of asymmetric information with gradual portfolio adjustment.

<sup>5</sup>See Gomes, Kogan and Zhang (2003) for an extension in a general equilibrium framework.

return reversals, the book-to-market effect, and the earnings-to-price-ratio effect—can be largely subsumed within their three-factor model. However, Fama and French (1996) point out that the momentum result of Jegadeesh and Titman (1993) constitutes the “main embarrassment” for their three-factor model (see also Fama and French (2008)). Finally, some models can predict either momentum or reversals but not both (see Wang (1993) and our benchmark model discussed above).

A common behavioral interpretation of the momentum and reversal effects is based on under- and over-reaction to news. Daniel et al. (1998) present a model based on investor overconfidence and on self-attribution. In their model, investors are overconfident about the precision of their private signals. In addition, investors update their confidence in a biased manner as they observe the outcomes of their actions. Thus, they overreact to private information and underreact to public information. The Barberis et al. (1998) model is based on different psychology phenomena, i.e., conservatism and the representativeness heuristic. Both of the preceding models study a representative agent framework. Hong and Stein (1999) analyze a model with two types of investors—news watchers and momentum traders. Each news watcher observes some private information, but fails to extract other newswatchers’ information from prices. Momentum traders can profit by trend chasing, but they use simple strategies, leading to overreaction at long horizons.

Daniel and Titman (2006) dispute both behavioral and risk-based interpretations that the reversal and book-to-market effects are a result of high expected returns on stocks of distressed firms with poor past performance. They decompose individual firm returns into two components. One, which they call tangible information, is associated with past performance as measured by accounting performance variables. The other, which they call intangible information, is the component of returns that is orthogonal to the tangible information. They show that future returns are unrelated to tangible information, but are strongly negatively related to intangible information. We may interpret the advance information in our model as intangible information. This information is unrelated to past performance, but impacts prices. Using a rational-expectations model with asymmetric information, we show that the presence of intangible information is important for generating the momentum and reversal effects.

Our paper is also related to the macroeconomics literature on the implications of news about future fundamentals for business cycles (e.g., Beaudry and Portier (2004, 2006) and Jaimovich and Rebelo (2008)). Our paper is the first to explore the implications of such news information for stock returns and investor trading behavior.

We organize the rest of the paper as follows. Section 2 presents the model. Section 3 studies the equilibrium properties of the model with and without advance information. Section 4 analyzes the predictions of the model for stock returns and trading behavior. Section 5 extends the model to incorporate multiple-pieces of advance information. Section 6 concludes

the paper. Proofs are relegated to the appendix.

## 2. The Model

Time is discrete and indexed by  $t = 1, 2, \dots$ . Consider an economy with a single good that can be either consumed or invested. There are two types of infinitely-lived investors in the economy, informed and uninformed. Informed and uninformed investors differ in their information structure and investment opportunities. The fraction of informed investors is  $\lambda \in (0, 1)$  and the fraction of uninformed investors is  $1 - \lambda$ .

### 2.1. Preferences

All investors have expected exponential utility with an identical constant absolute risk aversion parameter  $\gamma$ . We assume that investors are myopic and derive utility from next period's accumulated wealth. Thus, time  $t$  preferences are represented by

$$E_t \{ -e^{-\gamma W_{t+1}} \}, \quad (1)$$

where  $E_t$  is the expectation operator conditional on an investor's information at time  $t$  and  $W_{t+1}$  is the wealth level at time  $t+1$ . The assumption of myopic preferences rules out dynamic hedging demands and simplifies our analysis significantly. Introducing a dynamic hedging demand, however, does not change our key insights.<sup>6</sup>

### 2.2. Investment opportunities

Any investor can trade a riskless bond and a risky stock. The riskless bond is assumed to have an infinitely elastic supply at a positive constant interest rate  $r$ . Let  $R = 1 + r$  denote the gross interest rate in the economy.

The stock generates earnings  $D_t$  at time  $t$ . The earnings process is described by,

$$D_t = F_t + \varepsilon_t^D, \quad (2)$$

where  $F_t$  follows an AR(1) process,

$$F_t = a_F F_{t-1} + \varepsilon_t^F, \quad 0 < a_F < 1. \quad (3)$$

Earnings have both a persistent component  $F_t$ , with persistence given by  $a_F$ , and a temporary component  $\varepsilon_t^D$ . We assume that shocks to both components,  $\varepsilon_t^D$  and  $\varepsilon_t^F$  are independently and

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<sup>6</sup>Other papers also adopt myopic preferences for tractability (see, e.g., Bacchetta and van Wincoop (2004, 2006), Campbell et al. (1993), and Llorente et al. (2002)).

identically distributed (i.i.d.) normal random variables with means of zero and variances  $\sigma_D^2$  and  $\sigma_F^2$ , respectively. The firm is assumed to distribute 100 percent of its earnings as dividends. We therefore use the terms earnings and dividends interchangeably. The stock is in unit supply.

In addition to the publicly traded assets, a risky investment opportunity is available only to the informed investors.<sup>7</sup> This investment opportunity has constant returns to scale. Its return between period  $t$  and  $t + 1$  is  $R + q_{t+1}$ , where the excess return  $q_{t+1}$  satisfies

$$q_{t+1} = Z_t + \varepsilon_{t+1}^q. \quad (4)$$

Here,  $Z_t$  is the expected excess return to the private investment opportunity and follows:

$$Z_t = a_Z Z_{t-1} + \varepsilon_t^Z, \quad 0 < a_Z < 1, \quad (5)$$

The expected return to the private investment opportunity has a persistent component given by  $Z_t$ , with persistence given by  $a_Z$ , and a transitory component given by  $\varepsilon_t^q$ . We assume that shocks to both components,  $\varepsilon_t^q$  and  $\varepsilon_t^Z$ , are i.i.d. normal random variables with means of zero and variances  $\sigma_q^2$  and  $\sigma_Z^2$ , respectively. All shocks are uncorrelated except for  $\varepsilon_t^D$  and  $\varepsilon_t^q$ . Specifically,  $E[\varepsilon_t^D \varepsilon_t^q] = \sigma_{Dq} > 0$  so that the stock and the private investment are substitutes.

### 2.3. Information structure

All investors observe the past and current realizations of earnings and of stock prices. As in Wang (1994), informed investors have private information about the persistent component  $F_t$  of the stock and the expected return  $Z_t$  on the private investment, while uninformed investors do not. In addition to these pieces of private information, informed investors receive news about future earnings announcements that we label of advance information.<sup>8</sup> The advance information is modeled as a noisy, single piece of information about future earnings innovations:

$$S_t = \varepsilon_{t+k}^D + \varepsilon_t^S, \quad (6)$$

where  $k > 0$  and  $\varepsilon_t^S$  is an i.i.d. normal variable with mean zero and variance  $\sigma_S^2$ . The shock  $\varepsilon_t^S$  is assumed independent of all other shocks. Informed investors' information set is

$$\mathcal{F}_t^i = \{D_s, F_s, P_s, Z_s, S_s : s \leq t\}. \quad (7)$$

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<sup>7</sup>This nontraded asset makes the financial market dynamically incomplete, which prevents the existence of a fully revealing equilibrium. An alternative approach in the noisy rational expectations equilibrium literature is to introduce exogenous noise trading or liquidity trading. As Wang (1994) points out, since the noninformational component is exogenous, this approach is not preferable in studying trading behavior. In addition, this approach becomes problematic in welfare analysis.

<sup>8</sup>Bacchetta and van Wincoop (2004, 2006) construct models of investor heterogeneity with advance information to study the role of higher order expectations. However, they do not address the questions we study here.

The information set of uninformed investors is given by

$$\mathcal{F}_t^u = \{D_s, P_s : s \leq t\}. \quad (8)$$

The modeling of advance information in (6) is stylized in many respects and can be generalized to deliver better quantitative predictions. However, we think that the qualitative mechanisms we describe survive these generalizations. First, advance information news at time  $t$  refers to a known date into the future  $t + k$ . This need not be the case and, in fact, the relevant future date associated with advance information is usually uncertain, perhaps a date that can be influenced by the firm. Second, investors need not obtain advance information in every period. While this is a critical abstraction for stationarity of the model and tractability, it gains more realism when the period considered is 1 quarter or 1 year, as opposed to 1 day. Finally, informed investors may receive multiple-pieces of advance information. In addition, the informativeness of the advance information need not be constant and may vary over time or with the news itself. We will discuss some of the implications of this possibility in Section 5.

## 2.4. Equilibrium

A rational expectations equilibrium is defined in the usual way. We focus on a stationary equilibrium in which the stock price is a stationary function of state variables. The key step in defining an equilibrium is formulating the investors' portfolio choice problems. We start with an informed investor's decision problem. We use the superscript  $i$  to index a variable associated with an informed investor. Let  $P_t$  denote the time  $t$  stock price and  $Q_t = P_t + D_t - RP_{t-1}$  the stock's excess return at time  $t$ . An informed investor solves the following problem:

$$\max -E_t^i [\exp(-\gamma W_{t+1}^i)], \quad (9)$$

subject to the budget constraint:

$$\begin{aligned} W_{t+1}^i &= \theta_t^i (P_{t+1} + D_{t+1}) + \alpha_t^i (R + q_{t+1}) + (W_t^i - (\theta_t^i P_t + \alpha_t^i)) R \\ &= \theta_t^i Q_{t+1} + \alpha_t^i q_{t+1} + W_t^i R, \end{aligned} \quad (10)$$

where  $E_t^i$  denotes the conditional expectation operator given the information set  $\mathcal{F}_t^i$ ,  $W_t^i$  denotes the wealth level,  $\theta_t^i$  denotes the fraction of the stock held by  $i$ , and  $\alpha_t^i$  denotes the invested amount in the private investment opportunity. Similarly, we write an uninformed investor's decision problem as follows:

$$\max -E_t^u [\exp(-\gamma W_{t+1}^u)], \quad (11)$$

subject to

$$W_{t+1}^u = \theta_t^u Q_{t+1} + RW_t^u, \quad (12)$$

where  $E_t^u$  denotes the conditional expectation operator given the information set  $\mathcal{F}_t^u$ ,  $W_t^u$  denotes the wealth level, and  $\theta_t^u$  denotes the fraction of the stock held by  $u$ . Note that an uninformed investor does not have any private investment opportunities.

The market clearing condition is given by

$$\lambda \theta_t^i + (1 - \lambda) \theta_t^u = 1. \quad (13)$$

### 3. Equilibrium Properties

The key to solve for the equilibrium is to construct suitable state variables that informed and uninformed investors may use in making their conditional forecasts of future returns. We define the state vector as

$$\mathbf{x}_t = (F_t, Z_t, \varepsilon_{t+k}^D, \dots, \varepsilon_t^D, \varepsilon_{t+k}^q, \dots, \varepsilon_{t+1}^q)^\top, \quad (14)$$

and the unforecastable (based on period  $t - 1$  information) shock vector as  $\boldsymbol{\varepsilon}_t = (\varepsilon_{t+k}^D, \varepsilon_t^F, \varepsilon_t^Z, \varepsilon_{t+k}^q, \varepsilon_t^S)^\top$ . Note that  $\boldsymbol{\varepsilon}_t \sim N(0, \Sigma)$ , where  $\Sigma = E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^\top]$  is the covariance matrix, with the only nonzero covariance being  $\sigma_{Dq} = E[\varepsilon_{t+k}^D \varepsilon_{t+k}^q] > 0$ . The state vector includes all future realizations of the transitory shocks to earnings and returns on private investment up to  $t + k$ . This is because informed investors can use their private information to forecast these values. For technical reasons, we also include  $\varepsilon_t^D$  as part of the state vector. We show below that  $\varepsilon_t^D$  is not priced in equilibrium and does not appear in the asset demand functions because  $\varepsilon_t^D$  has already been paid out in the form of earnings at time  $t$  and has no value for forecasting future earnings.

Throughout the paper, we benchmark our model against a model without advance information, which is a simplified version of Wang (1994). In the absence of advance information, i.e.,  $\sigma_S^2 = \infty$ , we write the state vector as  $\mathbf{x}_t = (F_t, Z_t)^\top$ , and the unforecastable shock vector as  $\boldsymbol{\varepsilon}_t = (\varepsilon_t^D, \varepsilon_t^F, \varepsilon_t^Z, \varepsilon_t^q)^\top$ .

#### 3.1. Stock price

We conjecture that the equilibrium stock price function takes the following form:

$$P_t = -p_0 + \mathbf{p}_i \hat{\mathbf{x}}_t^i + \mathbf{p}_u \hat{\mathbf{x}}_t^u, \quad (15)$$

where  $\hat{\mathbf{x}}_t^i = E_t^i[\mathbf{x}_t]$ ,  $\hat{\mathbf{x}}_t^u = E_t^u[\mathbf{x}_t]$ ,  $p_0$  is a constant, and  $\mathbf{p}_i$  and  $\mathbf{p}_u$  are row vectors of constants to be determined in equilibrium. We set  $p_{u2} = 0$ . In general, one may include  $\hat{Z}_t^u$  in the price

function in that  $p_{u2} \neq 0$ . However, from the current price,  $P_t$ , the uninformed investors can infer the sum

$$P_t + p_0 - \mathbf{p}_u \hat{\mathbf{x}}_t^u = \mathbf{p}_i \hat{\mathbf{x}}_t^i \equiv \Pi_t, \quad (16)$$

since  $\hat{\mathbf{x}}_t^u$  is observable by the uninformed investors. Thus,  $\Pi_t$  represents the information content of the equilibrium price. This implies that the equality  $\mathbf{p}_i \hat{\mathbf{x}}_t^i = \mathbf{p}_i \hat{\mathbf{x}}_t^u$  holds in equilibrium because  $\mathbf{p}_i \hat{\mathbf{x}}_t^i$  is in the information set of uninformed investors. Therefore,

$$\hat{Z}_t^u = \frac{1}{p_{i2}} \mathbf{p}_i \hat{\mathbf{x}}_t^i - \frac{\mathbf{p}_i \mathbf{I}_{-2}}{p_{i2}} \hat{\mathbf{x}}_t^u, \quad (17)$$

where  $\mathbf{I}_{-2}$  conforms with the state vector and denotes the matrix that is the same as the identity matrix, except that the (2,2) element equals zero. Thus, we can eliminate  $\hat{Z}_t^u$  in the price function (15), and set  $p_{u2} = 0$ . Equation (17) further indicates that uninformed investors' forecast errors are perfectly linearly correlated.

**Proposition 1** *If there is a solution to the system of equations given in Appendix A, then the economy has a stationary rational expectations equilibrium in which the equilibrium stock price is given by*

$$\begin{aligned} P_t = & -p_0 + \frac{a_F}{R - a_F} F_t + p_{i2} Z_t - p_{u1} \left( F_t - \hat{F}_t^u \right) \\ & + \sum_{j=1}^k \left\{ \frac{1}{R^j} E_t^i [\varepsilon_{t+j}^D] - p_{u,k+3-j} \left( E_t^i [\varepsilon_{t+j}^D] - E_t^u [\varepsilon_{t+j}^D] \right) \right\} \\ & + \sum_{j=1}^k \left\{ -\frac{e_{i2}}{R^j} E_t^i [\varepsilon_{t+j}^q] - p_{u,2k+4-j} \left( E_t^i [\varepsilon_{t+j}^q] - E_t^u [\varepsilon_{t+j}^q] \right) \right\}, \end{aligned} \quad (18)$$

where  $p_0 > 0$ , and  $e_{i2} > 0$  iff  $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$ .

The negative constant (or  $p_0 > 0$ ) in the price function reflects the discount on the price to compensate for the risk in future earnings. The second term in the price function reflects the present discounted value of dividends conditional on knowing  $F_t$ . The third term in the price function reveals that the stock and the private investment opportunity are substitutes whenever  $p_{i2} < 0$ : When the expected return on the private investment opportunity is high (i.e.,  $Z_t$  is high), informed investors invest in the private investment and rebalance their portfolios by selling the stock. This causes the stock price to drop. We are only able to show that  $p_{i2} < 0$  in the absence of advance information. In the general model, we are unable to prove  $p_{i2} < 0$  because we are unable to show  $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$  analytically. As in the model without advance information, however, this positive covariance is intuitive as it reflects the fact that unexpected earnings and the unexpected private investment return are positively correlated,

i.e.,  $\sigma_{Dq} > 0$ . We verify this result numerically in all our examples below.

The three effects discussed above would be present in a full information setup. The fourth term in the first row of the price function gives rise to speculative trading by informed investors. When uninformed investors underestimate the persistence component of dividends and  $F_t - \hat{F}_t^u > 0$ , the stock price does not immediately reflect the value of expected future dividends. When  $p_{u1} > 0$ , this causes the stock price to be lower than the full information price.<sup>9</sup>

The second and third lines of equation (18) reflect the effects of advance information. In the presence of advance information about future earnings innovations, informed investors forecast these innovations, and their forecasts are incorporated into the stock price. Specifically, earnings expected next period are discounted at rate  $R$ , earnings expected two periods later are discounted at rate  $R^2$ , and so on up to  $t + k$ , after which time no more advance information is available. Uninformed investors do not have any advance information and make forecasts based on current and past stock price and earnings data. Consequently, they make forecast errors relative to informed investors' information. These forecast errors lead to speculative trades and to price fluctuations. This discussion explains the second line of (18).

The terms on the price function that appear on the third line of (18) arise because earnings innovations are correlated with innovations to the private investment return. As informed investors learn about  $\varepsilon_{t+j}^D$  based on date  $t + j - k$  advance information for  $1 \leq j \leq k$ , they also improve their forecast about  $\varepsilon_{t+j}^q$  and can better forecast private investment returns. When the private investment and the stock are substitutes,  $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$ , a good piece of advance information at date  $t$  raises  $E_t^i[\varepsilon_{t+k}^q]$ , which induces informed investors to sell the stock at time  $t + k - 1$ , causing the current stock price to decline. As before, the stock price also incorporates the uninformed investors' errors in estimating  $\varepsilon_{t+j}^q$ , which generate further speculative trades.

Equation (18) demonstrates that the equilibrium price does not reveal the informed investors' private information. That is, uninformed investors cannot distinguish between persistent shocks to earnings and persistent shocks to private investment expected returns: Good news about future earnings (high  $F_t$  or high  $\varepsilon_{t+k}^D$ ) or bad private investment opportunities (low  $Z_t$ ) can both cause informed investors to buy the stock and the stock's price to rise. Therefore, observing price and earnings is insufficient for uninformed investors to identify these shocks. This implies that information asymmetry persists in the equilibrium.

### 3.2. Informed investors' forecast

The presence of advance information implies that both the informed and uninformed investors must solve forecasting problems.<sup>10</sup> Informed investors use their private signals on  $k$ -period-

<sup>9</sup>The expression in the first line of (18) gives the price function in the model without advance information. For this model, we are able to show sharper results that  $p_{i2} < 0$  and  $0 < p_{u1} < a_F / (R - a_F)$  in Appendix B.

<sup>10</sup>Albuquerque et al. (2007) generalize Wang (1994) to an international economy and also require both informed and uninformed agents to solve forecasting problems.

ahead earnings innovations to learn about the growth potential in both the stock and private investment. Their information processing problem is greatly simplified for two reasons. First, their information set includes knowledge of all past values of the persistent components of earnings and private investment returns. Second, their information set includes that of uninformed investors, which means that informed investors do not learn from the price level.

**Proposition 2** *Informed investors' forecasts of future noises are given by*

$$\begin{aligned} E_t^i [\varepsilon_{t+k-j}^D] &= \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2} S_{t-j}, & 0 \leq j \leq k-1 \\ E_t^i [\varepsilon_{t+k-j}^q] &= \frac{\sigma_{Dq}}{\sigma_S^2 + \sigma_D^2} S_{t-j}, & 0 \leq j \leq k-1. \end{aligned}$$

The information content of a signal  $S_t$  about earnings  $\varepsilon_{t+k}^D$  is given by  $\sigma_D^2 / (\sigma_S^2 + \sigma_D^2)$ . If advance information is infinitely precise and  $\sigma_S^2 = 0$ , then  $\sigma_D^2 / (\sigma_S^2 + \sigma_D^2) = 1$  and  $S_t$  reveals the true innovation value. If advance information is worthless and  $\sigma_S^2 = \infty$  then  $\sigma_D^2 / (\sigma_S^2 + \sigma_D^2) = 0$  and the informed investors do not use this information to update their expectation of future earnings. The signal  $S_t$  is also informative about shocks to the private investment return  $\varepsilon_{t+k}^q$  because  $E[\varepsilon_t^D \varepsilon_t^q] = \sigma_{Dq} \neq 0$ . The information content of the signal is given by  $\sigma_{Dq} / (\sigma_S^2 + \sigma_D^2)$ .

With i.i.d. signals, a signal  $S_t$  observed at  $t$  about earnings  $\varepsilon_{t+k}^D$  contains no new information about other innovations  $\varepsilon_{t+k-j}^D$  or  $\varepsilon_{t+k-j}^q$  for  $0 \leq j \leq k-1$ . Also, because  $\varepsilon_{t+k}^D$  is uncorrelated with every other shock, the signal  $S_t$  cannot be used to forecast any other future variable. News about  $\varepsilon_{t+k-j}^D$ , released at  $t-j$ , become stale information for informed investors until earnings are realized, but as we will show they are updated every period by uninformed investors as they learn from prices more about these shocks.

### 3.3. Uninformed investors' forecast

Because informed investors know more than uninformed investors, the most that uninformed investors can hope to learn is what informed investors know. Therefore, it is sufficient for uninformed investors to track the dynamics of the state vector  $\hat{\mathbf{x}}_t^i$  estimated by the informed investors. In Appendix A, we present the filtering equations for the uninformed investors. We emphasize that our model features a hierarchical information structure as in Wang (1994), and thus there is no infinite regress problem.

In the absence of advance information, only uninformed investors solve a filtering problem. In appendix B, we show that the filtering equations are given by:

$$\begin{bmatrix} \hat{F}_t^u \\ \hat{Z}_t^u \end{bmatrix} = \begin{bmatrix} a_F \hat{F}_{t-1}^u \\ a_Z \hat{Z}_{t-1}^u \end{bmatrix} + \mathbf{K} \begin{bmatrix} D_t - E_{t-1}^u [D_t] \\ \Pi_t - E_{t-1}^u [\Pi_t] \end{bmatrix}, \quad (19)$$

where  $\mathbf{K}$  is a  $2 \times 2$  matrix with elements  $k_{11}, k_{12}, k_{21} > 0$  and  $k_{22} < 0$ .

The first term on the right-hand side of (19) gives the expectation based on information prior to period  $t$ . The second term gives the update in expectations based on new information from unexpected fluctuations in earnings and the stock price. The sign restrictions on the elements of the Kalman gain matrix  $\mathbf{K}$  reveal several properties. First, unexpected high earnings (i.e.,  $D_t - E_{t-1}^u [D_t] > 0$ ) can be attributed to (i) a high transitory shock  $\varepsilon_t^D$ , (ii) a high persistent shock  $\varepsilon_t^F$ , or (iii) a high past forecasting error  $F_{t-1} - \hat{F}_{t-1}^u$ . Because uninformed investors cannot tell these shocks apart, they increase their estimate of  $F_t$  whenever they observe high unexpected earnings (i.e.,  $k_{11} > 0$ ). Because an unexpected positive earnings surprise may arise from a high past forecasting error  $F_{t-1} - \hat{F}_{t-1}^u$  and because forecast errors of  $Z_t$  and  $F_t$  are positively correlated as shown in (17), uninformed investors also revise upwards their expectation of  $Z_t$ . Hence,  $k_{21} > 0$ . Second, an unexpected increase in  $\Pi_t = p_{i1}F_t + p_{i2}Z_t$  may indicate an increase in  $F_t$  or a decrease in  $Z_t$ . Uninformed investors do not observe these two components separately, and thus raise their estimate of  $F_t$  and decrease their estimate of  $Z_t$  accordingly. This explains why  $k_{12} > 0$  and  $k_{22} < 0$ .

In the presence of advance information we are unable to sign the elements of the Kalman gain matrix given in Appendix A, but verify numerically that the effects described above carry through. In addition, there is a further confounding factor moving prices; advance information moves prices in much the same way that  $F_t$  does, though it does not move current dividends.

### 3.4. Excess stock returns

In Appendix A, we use Proposition 1 to derive the excess stock return:

$$Q_{t+1} = e_0 + e_{i2} (Z_t + E_t^i [\varepsilon_{t+1}^q]) + e_{i1} (F_t - \hat{F}_t^u) + \mathbf{b}_Q \hat{\boldsymbol{\varepsilon}}_{t+1}^i \quad (20)$$

$$+ \sum_{j=1}^k \left\{ e_{i,k+3-j} (E_t^i [\varepsilon_{t+j}^D] - E_t^u [\varepsilon_{t+j}^D]) - e_{u,2k+4-j} (E_t^i [\varepsilon_{t+j}^q] - E_t^u [\varepsilon_{t+j}^q]) \right\},$$

where  $e_0 = rp_0$ , is the unconditional mean excess returns,  $\mathbf{b}_Q$  is a constant vector,  $e_0, e_{i1}, \dots, e_{i,k+2}, e_{u,k+4}, \dots$ , and  $e_{u,2k+3}$  are constants. In the absence of advance information, equation (20) reduces to:

$$Q_{t+1} = e_0 + e_{i2}Z_t + e_{i1} (F_t - \hat{F}_t^u) + \mathbf{b}_Q \boldsymbol{\varepsilon}_{t+1}, \quad (21)$$

where  $e_{i1} > 0$  and  $e_{i2} > 0$ . This equation reveals that changes in the expected private investment return  $Z_t$  or changes in the uninformed investors' estimation error  $(F_t - \hat{F}_t^u)$  affect the excess stock return. In addition, exogenous shocks  $\boldsymbol{\varepsilon}_{t+1}$  to dividends and private investment returns also affect the excess stock return.

Comparing (20) with (21) we see that with advance information, informed investors' forecast

of the innovation  $\varepsilon_{t+1}^q$  to the private investment return also affects the  $t+1$  excess stock return. This reflects informed investors' increased ability to forecast private returns using the time  $t-k+1$  advance information about  $\varepsilon_{t+1}^D$ . Informed investors' estimates of  $\varepsilon_{t+1}^q$  change the expected private investment return  $Z_t + E_t^i[\varepsilon_{t+1}^q]$  and generate rebalancing trades at  $t$ . These trades affect the current stock price  $P_t$  and the excess stock return at time  $t+1$ ,  $Q_{t+1}$ .

The second line of equation (20) reveals that the uninformed investors' forecast errors in estimating  $\varepsilon_{t+1}^D$  and  $\varepsilon_{t+1}^q$  influence excess stock returns. The reason is that informed investors can profit from this estimation error in their speculative trading.

Using equation (20), we can derive investors' estimates of excess returns:

$$E_t^i[Q_{t+1}] = e_0 + e_{i2}(Z_t + E_t^i[\varepsilon_{t+1}^q]) + e_{i1}(F_t - \hat{F}_t^u) \quad (22)$$

$$+ \sum_{j=1}^k \left\{ e_{i,k+3-j}(E_t^i[\varepsilon_{t+j}^D] - E_t^u[\varepsilon_{t+j}^D]) - e_{u,2k+4-j}(E_t^i[\varepsilon_{t+j}^q] - E_t^u[\varepsilon_{t+j}^q]) \right\},$$

and

$$E_t^u[Q_{t+1}] = e_0 + e_{i2}(\hat{Z}_t^u + E_t^u[\varepsilon_{t+1}^q]). \quad (23)$$

The error term  $\mathbf{b}_Q \hat{\varepsilon}_{t+1}^i$  is unforecastable by both informed and uninformed investors and thus does not change either investors' estimates of excess stock returns.

An increase in  $Z_t$  makes informed investors want to substitute stocks for private investments when  $e_{i2} < 0$ , lowering the stock price  $P_t$  and raising the future excess stock return  $Q_{t+1}$ . If uninformed investors underestimate  $F_t$ , they are likely to revise their estimates upward in the next period as more information is revealed. As a result, the stock price is expected to rise in period  $t+1$ . Informed investors observe the size of uninformed investors' underestimation and as a result expect the excess stock return to rise if  $e_{i1} > 0$ . We also verify numerically that  $e_{i,k+3-j} > 0$ ,  $j = 1, \dots, k$ , so that as with  $F_t - \hat{F}_t^u$ , informed investors expect high future returns when they have positive advance information about the stock that is not shared by all investors. Likewise, we verify numerically that  $e_{u,2k+4-j} > 0$ ,  $j = 1, \dots, k$ , so that informed investors expect low future returns if they have information about future rebalancing –which drives future prices down– that is not shared by all investors. Uninformed investors expect high stock returns when they expect that informed investors are selling for rebalancing reasons.

### 3.5. Optimal portfolios

It is straightforward to derive investors' optimal portfolios. For informed investors,

$$\theta_t^i = \frac{E_t^i[Q_{t+1}]}{\gamma(\sigma_Q^i)^2(1 - (\rho_{Qq}^i)^2)} - \frac{\rho_{Qq}^i E_t^i[q_{t+1}]}{\gamma\sigma_Q^i\sigma_q^i(1 - (\rho_{Qq}^i)^2)}, \quad (24)$$

where  $\sigma_Q^i$ , is the conditional stock return variance as perceived by informed investors,  $\sigma_q^i$ , is the conditional variance of the private investment return as perceived by informed investors, and  $\rho_{Qq}^i = Cov_t^i(Q_{t+1}, q_{t+1}) / \sigma_q^i \sigma_Q^i$ , is the conditional correlation of the private investment return and the stock return as perceived by informed investors. The preceding conditional variances and correlation are independent of time  $t$  due to the property of normal random variables. Similarly, the optimal portfolio for an uninformed investor is given by

$$\theta_t^u = \frac{1}{\gamma} \frac{E_t^u [Q_{t+1}]}{Var_t^u (Q_{t+1})}. \quad (25)$$

Equations (24) and (25) reveal that the optimal portfolios are mean-variance efficient, trading-off expected return and risk. While investors display no dynamic hedging demand, informed investors have a static hedging demand (last term on the right hand side of (24)) arising from the positive correlation between the stock return and the private investment return. Using the conditional expectations of excess stock returns derived in the previous subsection, we can provide a sharper characterization of the optimal portfolios.

**Proposition 3** *The equilibrium trading strategies satisfy*

$$\begin{aligned} \theta_t^i &= f_0^i + f_Z^i (Z_t + E_t^i [\varepsilon_{t+1}^q]) + f_F^i (F_t - \hat{F}_t^u) \\ &\quad + \sum_{j=1}^k \left\{ f_{Dj}^i (E_t^i [\varepsilon_{t+j}^D] - E_t^u [\varepsilon_{t+j}^D]) + f_{qj}^i (E_t^i [\varepsilon_{t+j}^q] - E_t^u [\varepsilon_{t+j}^q]) \right\}, \end{aligned} \quad (26)$$

and

$$\theta_t^u = f_0^u + f_Z^u (\hat{Z}_t^u + E_t^u [\varepsilon_{t+1}^q]), \quad (27)$$

where  $f_0^i, f_0^u > 0$ ,  $f_Z^i, f_F^i, f_Z^u, f_{Dj}^i$  and  $f_{qj}^i$  are constants. Also,  $f_Z^i < 0$  and  $f_Z^u > 0$  iff  $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$ .

This proposition shows that the informed investors trade for both speculative and rebalancing reasons. The expected return on the private investment determines their rebalancing trades. In addition, with advance information, informed investors' forecast of  $\varepsilon_{t+1}^q$  is not zero and hence it is included in the expected private investment return. When the assets are substitutes and  $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$ , a high expected return on the private investment leads to rebalancing selling of the stock.

Speculative trading by informed investors arises from their knowledge of uninformed investors' forecast errors about  $F_t$ ,  $E_t^i [\varepsilon_{t+j}^D]$  and  $E_t^i [\varepsilon_{t+j}^q]$ . The forecast errors  $E_t^i [\varepsilon_{t+j}^D] - E_t^u [\varepsilon_{t+j}^D]$  and  $E_t^i [\varepsilon_{t+j}^q] - E_t^u [\varepsilon_{t+j}^q]$  show up only in the model with advance information, and, together with  $F_t - \hat{F}_t^u$ , constitute the private information of informed investors. This private

information allows them to take speculative positions against expected future corrections of the uninformed investors' expectations. For example, when uninformed investors underestimate  $F_t$ , in the sense that  $F_t > \hat{F}_t^u$ , informed investors' speculative trading induces them to buy stocks in expectation of a high return in the future when high dividends are realized.

Uninformed investors trade for noninformational reasons. They are willing to trade only when they perceive to be accommodating informed investors' rebalancing trades because these trades always occur at favorable prices (i.e.  $p_{i2} < 0$  whereas  $f_Z^i < 0$  and  $f_Z^u > 0$ ). However, as in the model without advance information, their trading is subject to adverse selection because they do not know whether informed investors are trading in response to a change in  $Z_t + E_t^i [\varepsilon_{t+1}^q]$  or to superior private information.

## 4. Model predictions

### 4.1. Stock return momentum and reversal effects

The next proposition focuses on excess returns, whereas some empirical studies use raw returns.

**Proposition 4** *For any  $n \geq 1$ , we have*

$$E[Q_{t+n}|Q_t] = e_0 + e_{i2} \frac{a_Z^{n-1} Cov(Z_t, Q_t) + \mathbf{1}_{\{n \leq k\}} Cov(E_t^i[\varepsilon_{t+n}^q], Q_t)}{Var(Q_t)} Q_t, \quad (28)$$

where  $e_0 > 0$  and  $e_{i2} > 0$  if and only if  $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$ . The indicator function  $\mathbf{1}_{\{n \leq k\}}$  equals 1 if  $n \leq k$  and 0 otherwise.

To make the exposition simpler we start with the case of no advance information. In this case, we show in Appendix B that

$$E[Q_{t+n}|Q_t] = e_0 + e_{i2} a_Z^{n-1} \frac{Cov(Z_t, Q_t)}{Var(Q_t)} Q_t = e_0 + (1 - Ra_Z) a_Z^{n-1} f_Q Q_t, \quad (29)$$

where  $e_0, e_{i2} > 0$  and  $f_Q < 0$  are constants. This equation shows how current excess returns  $Q_t$  can forecast  $n$ -period-ahead, single-period excess returns. Empirical studies usually use cumulative  $n$ -period returns. Let  $Q_{t,t+n} = \sum_{j=1}^n Q_{t+j}$  denote the cumulative  $n$ -period excess return. We can then derive

$$E[Q_{t,t+n}|Q_t] = n e_0 + (1 - Ra_Z) f_{Qn} Q_t,$$

where  $f_{Qn} = (1 + a_Z + \dots + a_Z^{n-1}) f_Q$ . Thus, the properties of momentum and reversals follow from the properties of  $E[Q_{t+n}|Q_t]$  given in equation (29). This equation demonstrates that the sign of the correlation between  $Q_{t+n}$  and  $Q_t$  is the same as the sign of  $(Ra_Z - 1)$  for all

$n \geq 1$ . This means that the model cannot simultaneously predict both short-run momentum and long-run reversals in excess returns.<sup>11</sup>

As we show above (see equation (21)), next period excess returns can change because either one of the following three components changes: (i) the expected private investment return, (ii) uninformed investors' errors in estimating next-period's persistent component of earnings, and (iii) cash-flow related shocks. Since the latter two components are not correlated with the current excess returns, the serial correlation of excess returns are determined by the correlation between the first component and the current excess return. To derive the sign of this correlation, we start with  $n = 1$  and consider the effects of a positive shock to private investment returns,  $Z_t$ . Rebalancing and speculative trading motives generate two effects. First, for rebalancing reasons, informed investors sell the stock and invest in the private technology, causing the current stock price  $P_t$  and excess stock returns  $Q_t$  to fall, *ceteris paribus*. Uninformed investors buy the stock in expectation of high excess stock returns  $Q_{t+1}$  in the next period. On the other hand, a high  $Z_t$  tends to follow a high  $Z_{t-1}$ , because this process is persistent. Due to rebalancing reasons, a high  $Z_{t-1}$  causes period  $t - 1$  stock price  $P_{t-1}$  to fall. This price drop raises the current excess stock returns,  $Q_t$ , *ceteris paribus*. This effect dominates when the persistence of expected private investment returns is high enough in that  $Ra_Z > 1$ , causing positive correlation between  $Z_t$  and  $Q_t$ , and hence a continuation of excess stock returns. Otherwise, excess stock returns tend to reverse themselves.

**[Insert Figure 1 Here.]**

These effects are depicted in Figure 1. The top right panel of Figure 1 reveals that the stock price falls at time 0 and then gradually reverts back to its long-run mean as the shock to  $Z_t$  decays over time. The excess return also falls at time 0, but rises at time 1 and then gradually reverts back to the long run mean. The speed of reversion depends on the persistence of  $Z_t$ . Because persistence is low in the exercise, i.e.,  $Ra_Z < 1$ , the excess return quickly reverts back to its long-run mean. The bottom left panel illustrates the estimation errors that generate speculative trades. The bottom right panel shows the trading strategies: In response to an increase in the private investment return, the informed investors sell the stock and uninformed investors buy the stock. As the shock to  $Z_t$  decays over time, both informed and uninformed investors revert their trading positions.

Consider next the effect of speculative trading. Uninformed investors do not observe the actual increase in  $Z_t$ , but can estimate this increase by observing the falling stock prices. Because in the absence of advance information the information content of the price is given by  $\Pi_t = p_{i1}F_t + p_{i2}Z_t$ , which decreases with  $Z_t$ , observing a low price or a low  $\Pi_t$  leads to an

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<sup>11</sup>A similar condition is suggested in other papers with different models (see Wang (1993)). We note that the same result applies when  $\sigma_{Dq} < 0$ . Its proof is available upon request.

increase in the estimate of  $Z_t$  ( $\hat{Z}_t^u$ ) and a decrease in the estimate of  $F_t$  ( $\hat{F}_t^u$ ) (see equation (19)). The increase in  $\hat{Z}_t^u$  induces uninformed investors to accommodate the rebalancing trades of informed investors. The decrease in  $\hat{F}_t^u$  leads to speculative buy trades by informed investors because uninformed investors underestimate  $F_t$ . Equation (29) shows that this speculative trade effect is always dominated by the rebalancing trade effect.

The correlation of stock returns at longer lags, i.e.,  $n > 1$ , follows the same sign as the correlation between successive period returns. Because changes in the return on the private investment are persistent, the effect of rebalancing trades is also persistent, causing excess stock returns in period  $t + n$  to be correlated with excess stock returns in period  $t$ . Its impact decays at the rate  $a_Z$  as  $n$  increases as shown in equation (29).

The preceding analysis allows us to conclude that the model without advance information cannot generate momentum and reversal effects simultaneously. We now turn to the model with advance information and equation (28). This equation decomposes the correlation between  $Q_{t+n}$  and  $Q_t$  into two parts. The first part is determined largely by the covariance between the stock return and the expected return on the private investment,  $Cov(Z_t, Q_t)$ . Its sign depends on the persistence of the expected private investment return, as in the model without advance information. In particular, if the persistence is sufficiently high, then this covariance is positive. Otherwise, it is negative.

The second part reflects the effect of advance information, which plays a role if and only if  $n \leq k$ , since after period  $t + k$  all advance information up to date  $t$  loses its value. From the discussion above, we deduce that our model must have two conditions to generate short-run momentum and long-run reversals simultaneously. First, the persistence  $a_Z$  must be sufficiently small. If it is too large, we cannot generate long-run reversals. Second, given a small value of  $a_Z$ , we must have  $Cov(E_t^i[\varepsilon_{t+n}^q], Q_t) > 0$ . Otherwise, we cannot generate short-run momentum. Therefore, in the discussion below we assume that  $a_Z$  is sufficiently small.

**[Insert Figure 2 Here.]**

To describe the intuition behind the mechanism that generates momentum, consider first the case of one-period-ahead advance information, i.e.,  $k = 1$ . Suppose informed investors receive a good signal about future earnings  $\varepsilon_{t+1}^D$  at time  $t = 0$ . We assume that this signal materializes fully at time  $t + 1$  in the form of high earnings, but what happens at  $t + 1$  is not known by any investor at time  $t$ . The effects of this good signal are described in Figure 2. The good news leads to an increase in price  $P_t$  and in the excess stock return  $Q_t$  by the amount  $E_t^i[\varepsilon_{t+1}^D]/R$ . In addition, two main effects partly offset the effect of the good news on the stock price. First, uninformed investors underestimate the private signal and  $E_t^i[\varepsilon_{t+1}^D] - E_t^u[\varepsilon_{t+1}^D] > 0$  as shown in the bottom left panel of Figure 2. This increases informed investors' speculative demand for the stock. Second, a good signal about  $\varepsilon_{t+1}^D$  also helps informed investors forecast a high return

in their private investments. This gives rise to rebalancing stock sales by informed investors.<sup>12</sup>

These two effects keep the price from fully incorporating the value of the advance information. When at time  $t + 1$  earnings are realized, they will on average be higher than prices anticipated them to be, leading to short run return continuation. Thus, after a shock to advance information, rebalancing trades combine with speculative trades to produce momentum, which contrasts with the model without advance information where a tension exists between rebalancing trades and speculative trades.

The bottom right panel of Figure 2 shows that uninformed investors buy the stock in expectation of high returns in the next period, based on public information. They are thus perceived as trend-chasers. Informed investors sell because of the previously discussed rebalancing trades. They are then viewed as contrarian investors in response to good advance information. Therefore, momentum occurs as contrarian investors sell in spite of their expectation of high future stock returns.<sup>13</sup> In other words, in order for momentum to occur all investors must be expecting return continuation, but a subset of them must be willing to take the opposite side of trading positions.<sup>14</sup>

We next consider the case of advance information about  $\varepsilon_{t+k}^D$  with  $k > 1$ . We argue that serial correlation in one-period returns may display a cyclical pattern in single period returns at various horizons with negative serial correlations being followed by positive serial correlations and these followed by more negative serial correlations (see Jegadeesh and Titman (1993) for such empirical evidence). To understand the intuition, let  $a_Z$  be small and consider the effects of a one time good signal about  $k = 2$  period ahead earnings at time  $t = 0$ . This signal is realized in period  $t + k$ . The effects are described in Figure 3. We find that the stock price and the stock return respond immediately by increasing up to  $E_t^i [\varepsilon_{t+k}^D] / R^k$ . In addition, the stock price and return also immediately reflect the discounted value of uninformed investors' forecast error and of the expected high private investment return. Informed investors buy the stock, speculating on uninformed investors' forecast errors, as shown in the bottom right panel of Figure 3. These speculative trades should lead to return continuation, but as in the model without advance information they may not be strong enough to overturn the return reversal phenomenon when  $a_Z$  is small.

**[Insert Figure 3 Here.]**

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<sup>12</sup>Cochrane et al. (2007) show that in the presence of multiple assets rebalancing after good news can also occur as a consequence of market clearing.

<sup>13</sup>This type of behavior is consistent with the trading pattern that seems to arise when mutual funds sell large amounts of stock for what appears to be liquidity reasons. The returns for investors that buy are positive and significant (see Coval and Stafford (2007)).

<sup>14</sup>The patterns of momentum and reversal can be obtained in a cross section of firms when the firms have cash flows that are independently and identically distributed and comove with the private investment opportunity as described here. See also the discussion in Banerjee et al. (2008) in connection to the price drift phenomenon.

Informed investors' rebalancing trades in response to the good advance information only occur at time  $t + k - 1$ : Only then do they wish to invest more in the private technology in expectation of a high subsequent return,  $Z_{t+k-1} + E_{t+k-1}^i [\varepsilon_{t+k}^q]$  (see the second term in (26)). Therefore, as shown in the bottom right panel of Figure 3, at time  $t + k - 1$  informed investors sell after a price run-up, acting as contrarian investors, but expecting high future returns when the good advance information materializes next period. Uninformed investors act as return chasers, and are willing to buy because they expect high returns due to rebalancing by informed investors. Therefore, we obtain  $Cov(Q_{t+k}, Q_t) > 0$ . After period  $t + k$ , the date  $t$  advance information has no effect on future returns and stock returns revert to the long-run mean in that  $Cov(Q_{t+n}, Q_t) < 0$  for  $n > k$ .

We also conduct some numerical experiments. Table 1 shows the slope coefficients on the forecast of single-period returns  $Q_{t+n}$ , conditional on  $Q_t$  as well as the slope coefficient of the forecast of cumulative returns  $Q_{t,t+n}$ , conditional on  $Q_t$ . We find that when  $k = 1$ , our model generates two-period momentum followed by a stock return reversal. The two-period cumulative return is positive because the first period positive return compensates for the second period negative return. When  $k = 3$ , cumulative returns are negative at all horizons.

[Insert Table 1 Here.]

## 4.2. Return continuation and trading behavior

Unlike the representative-agent risk-based models, our model can make predictions regarding investor trading behavior. Specifically, our model predicts that (i) uninformed investors follow trend chasing strategies, also called positive feedback trades, and (ii) that such trades are profitable. Taking this prediction to the data is complicated by the fact that it is not straightforward to attach the label “uninformed” to any empirically identifiable investor class.

A natural setting to look for evidence on these predictions is in international asset allocation. In international finance, the maintained hypothesis regarding investor behavior is that foreign investors are on average less well informed about domestic stocks than the aggregate of all domestic investors. This hypothesis predicts both the existence of home bias in asset allocation (Gehrig (1993)) and of trend chasing by foreign investors (Brennan and Cao (1997)). A long list of papers have found evidence of trend chasing by foreign investors (e.g., Choe et al. (1999)). Moreover, some papers demonstrate that foreign investors' trend chasing strategies are profitable (e.g., Albuquerque et al. (2008) and Froot et al. (2000)). Grinblatt and Keloharju (2000), using Finnish stock market data, find that foreign investors behaved as return chasers and that they experience positive profits. Anecdotal evidence about the Finnish stock market suggests that some of these profits derive from rebalancing by Finnish investors: local investors in Finland were dumping Nokia stock onto foreign investors to reduce their exposure

to a growing Nokia. This evidence is consistent with the mechanism we discuss above.

In a domestic context, it is perhaps harder to label a group of investors as uninformed. However, it is common practice to identify uninformed investors with individual investors. There are two main findings about the trading of individual investors. First, individual investors act as contrarian investors, buying when the price is low and selling when the price is high. This finding is consistent with a model correlation between uninformed investors' holdings and returns conditional on persistent shocks to dividends, but not with an unconditional correlation. Second, Kaniel et al. (2007) find positive (negative) excess returns following months of intense buying (selling) by individual investors. In particular, they show that return predictability arises from uninformed trading and not from past returns. In our paper too, a horse race between trades of uninformed investors and past returns reveals that return predictability arises from the former because we can show that

$$E [Q_{t+1}|\theta_t^u, Q_t] = E [E_t^u [Q_{t+1} | \theta_t^u, Q_t] | \theta_t^u, Q_t] = E [E_t^u [Q_{t+1} | \theta_t^u]] = \gamma (\sigma_Q^u)^2 \theta_t^u,$$

where we have used the law of iterated expectations and (25). Kaniel et al. (2007) interpretation of their evidence is consistent with our story that individual investors supply the liquidity demanded by institutional investors.

In addition, our model allows us to trace out the implications of advance information on the first-order serial correlation in returns and on mean volume.<sup>15</sup> The left panel in Figure 4 depicts the effects on momentum and the right panel depicts the effects on volume of different levels of noise in advance information. Volume strictly increases with the amount of noise in advance information. This is intuitive because advance information is private to informed investors. As advance information becomes noisier, the information advantage of informed investors decreases, reducing uninformed investors' adverse selection problem, and resulting in an increase in trading volume.

**[Insert Figure 4 Here.]**

The effect of noise in advance information on momentum is non-monotonic. When advance information is very noisy, it cannot be used by informed investors to forecast future investment opportunities, and the model reduces to the model without advance information. When  $a_Z$  is small, as assumed in the construction of the figure, the model generates reversals. When advance information is very precise, there is too much adverse selection and too little trading. Hence, momentum also disappears. The effect of rebalancing and of speculative trades on momentum is highest for intermediate levels of noise in advance information.

Figure 4 suggests that the relationship between momentum and volume may not be monotonic, especially if this relationship is driven by the variation in  $\sigma_S$ . When advance information is

<sup>15</sup>Let volume be  $vol_t = (1 - \lambda) |\Delta\theta_t^u|$ . Then  $E [vol_t] = (1 - \lambda) \sqrt{2var(\Delta\theta_t^u)}/\pi$ .

very precise, the two variables are positively correlated, but when advance information is very noisy, volume and momentum are negatively correlated. The relationship between volume and momentum when advance information is very precise is consistent with the evidence in Connolly and Stivers (2003).

### 4.3. Post-earnings announcement drift

The empirical literature on earnings announcements documents a persistent price drift after earnings surprises. In our model with heterogeneous information, we assume that earnings surprises are computed with respect to all available public information. Thus, we define earnings surprises as  $D_t - E_{t-1}^u(D_t)$ . We show the following result:

**Proposition 5** *For any  $n \geq 1$ , we have*

$$E [Q_{t+n}|D_t - E_{t-1}^u(D_t)] = e_0 + e_{i2}a_Z^{n-1}E [Z_t|D_t - E_{t-1}^u(D_t)] \quad (30)$$

$$+ e_{i2}d_1 \frac{\mathbf{1}_{\{n \leq k-1\}} \text{Cov}(E_t^u[\varepsilon_{t+n}^D], D_t - E_{t-1}^u(D_t))}{\text{Var}(D_t - E_{t-1}^u(D_t))} [D_t - E_{t-1}^u(D_t)],$$

where  $e_0$  is a constant,  $d_1 > 0$ , and  $e_{i2} > 0$  if and only if  $\text{Cov}_t^i(Q_{t+1}, q_{t+1}) > 0$ . The indicator function  $\mathbf{1}_{\{n \leq k-1\}}$  equals 1 if  $n \leq k-1$  and 0 otherwise.

Again, consider first the model without advance information. In this case, Appendix B shows that

$$E [Q_{t+n}|D_t - E_{t-1}^u(D_t)] = e_0 + a_Z^{n-1}d_1 [D_t - E_{t-1}^u(D_t)], \quad (31)$$

where  $e_0$  and  $e_{i2}, d_1 > 0$  are constants. Because empirical studies on the post-earnings announcement drift consider the cumulative stock return after an earnings surprise, we compute the conditional cumulative return as:

$$E [Q_{t,t+n}|D_t - E_{t-1}^u(D_t)] = ne_0 + d_{1n} [D_t - E_{t-1}^u(D_t)],$$

where  $d_{1n} = (1 + a_Z + \dots + a_Z^{n-1})d_1$ . This equation reveals that positive surprises lead to positive returns because  $d_{1n} > 0$ . In addition, cumulative stock returns increase with the investment horizon since  $\partial(d_{1n})/\partial n > 0$ , but at a decreasing rate since  $\partial^2(d_{1n})/\partial n < 0$ .

Intuitively, if information is symmetric, all investors share the same earnings surprise in the current period, as they do all other information. Therefore, all information is immediately incorporated in the current stock price. From next period on, the surprise is already realized and does not affect the stock price. This implies that under symmetric information a price drift cannot occur. In the model with asymmetric information but no advance information, uninformed investors may interpret a positive earnings surprise as an increase in the persistent

component of earnings, and may consequently raise their estimate of this component ( $\hat{F}_t^u$ ) and their estimate of the private investment return ( $\hat{Z}_t^u$ ) (see Section 3.3). As uninformed investors overestimate the persistent component of earnings, informed investors take speculative sale positions. Uninformed investors do not know informed investors' trading motives and may think they sell for rebalancing reasons because uninformed investors believe that the private investment return goes up. Thus, uninformed investors buy the stock to accommodate informed investors' speculative trades, in anticipation of a high return in the next period. This causes a positive correlation between excess return and earnings announcements. Because both  $\hat{F}_t^u$  and  $\hat{Z}_t^u$  are persistent, the above trading patterns are also persistent, leading return continuation.

Note that in the model, the phenomenon of post-earnings announcement drift does not depend on the existence of momentum. While post-earnings announcement drift arises from uninformed investors' estimation errors, we have shown in the previous subsection that informed investor's speculative trades based on these estimation errors are not sufficient to overcome the strength of the rebalancing trades in driving the serial correlation in returns.

When we allow for advance information a new effect arises. A positive earnings surprise at time  $t$  may also occur because uninformed investors underestimate  $F_{t-1}$ . Observing past prices, but not advance information signals, they may believe informed investors receive good advance information about  $\varepsilon_{t+1}^D, \dots, \varepsilon_{t+k-1}^D$ , which materializes at any time from  $t+1$  to  $t+k-1$ .<sup>16</sup> In response to a positive earnings surprise at date  $t$ , uninformed investors raise their estimate of  $F_t$  and revise the forecasts  $E_t^u[\varepsilon_{t+n}^D]$  down, for  $n \leq k-1$ . As a result, low expected future dividends are then associated with low expected returns.

Despite this dampening effect, Table 2 illustrates numerically that our model with advance information can still generate the post-earnings announcement drift phenomenon. The table displays the slope coefficients of the forecast of single-period returns  $Q_{t+n}$ , conditional on the earnings announcement  $D_t - E_{t-1}^u[D_t]$  as well as the slope coefficients of cumulative returns  $Q_{t,t+n}$ , conditional on  $D_t - E_{t-1}^u[D_t]$  for various values of  $n$  and  $k$ .

[Insert Table 2 Here.]

#### 4.4. Stock market profits

An interesting question is whether our previously discussed trading strategies cause informed or uninformed investors to systematically lose money. If the stock has zero net supply, then stock market trading is a zero sum game. When some investors make money, investors taking the other side of the trading positions will lose money. Our model does not result in a zero sum game, because the stock has positive net supply. In our model, both informed and uninformed

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<sup>16</sup>Note that advance information about  $\varepsilon_{t+k}^D$  was just released, which explains why the second term only appears for  $n \leq k-1$ .

investors can make positive profits from the stock market because there is a return to risk sharing and all investors are risk averse.

We first consider uninformed investors' trading profits. Uninformed investors lose money when informed investors trade for speculative reasons. However, when informed investors trade for rebalancing reasons, uninformed investors trade at favorable prices and earn profits. Formally, using (23) and (27), we show that uninformed investors's average profits are equal to

$$E[\theta_t^u Q_{t+1}] = f_0^u e_0 + e_{i2} f_Z^u Var\left(\hat{Z}_t^u + E_t^u[\varepsilon_{t+1}^q]\right),$$

where  $f_Z^u > 0$  if  $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$ . Thus, a sufficient condition for  $E[\theta_t^u Q_{t+1}] > 0$  is that  $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$ . The first term on the right hand side of the above equation derives from a positive risk premium and the second term represents profits from accommodating rebalancing trades.

We can similarly compute informed investors' trading profits in the stock market. We are unable to prove analytically that informed investors make profits from the stock market. We have verified this fact numerically. The intuition is the following. Informed investors make profits when they trade for speculative reasons. When they trade for rebalancing reasons, they lose money from the stock market, but make money from private investment opportunities. Because of risk aversion, informed investors also earn an equity risk premium by holding stocks. Overall, they can still make money from the stock market.

## 5. Extension: multiple pieces of advance information

The analysis thus far has shown that the model without advance information cannot generate momentum and reversal effects simultaneously. In addition, the model with one-period-ahead advance information can generate short-run momentum followed by reversals in stock returns. However, momentum only lasts for few periods. Finally, when informed investors receive a single piece of long-horizon advance information, the model generates a seemingly counterfactual cyclical behavior of serial correlation in one-period stock returns.

In order to generate long-lived momentum followed by long-run reversals, we extend the model to incorporate multiple pieces of advance information. Specifically, we assume that at time  $t$ , informed investors receive a vector of signals  $(S_t^k, \dots, S_t^1)$  about earnings innovations at  $t + 1$  through  $t + k$ :

$$S_t^k = \varepsilon_{t+k}^D + \varepsilon_t^{S_k}, \dots, S_t^1 = \varepsilon_{t+1}^D + \varepsilon_t^{S_1}. \quad (32)$$

Each  $\varepsilon_t^{S_n}$  is assumed to be an i.i.d. normal random variable with mean zero, variance  $\sigma_{S_n}^2$ , and

independent of any other shock. The informed investors' information set is thus given by

$$\mathcal{F}_t^i = \left\{ D_s, F_s, P_s, Z_s, (S_s^n)_{n=1, \dots, k} : s \leq t \right\}. \quad (33)$$

This assumption is quite natural as new information, say about end-of-quarter earnings, is likely to arrive at intermediate periods as the quarter nears its end. In addition, stale information is still useful for forecasting, and hence affects stock prices.

The intuition behind this modeling device is that the successive advance information news about the same future earnings can generate long-lived, large speculative trading effects and momentum. An important modeling issue is how to specify the quality of signals. Because up to period  $t$  informed investors will have received  $k - 1$  signals on  $\varepsilon_{t+1}^D$  already, the stock price increasingly reveals  $\varepsilon_{t+1}^D$  to the uninformed investors, reducing the motive for speculative trading by informed investors. It is therefore possible that, with too much information in prior periods, only the rebalancing trade motive is at work, generating negative serial correlation in returns. In order to obtain long-lived momentum when  $k > 1$ , the advance information needs to increase in quality as we approach the earnings realization, i.e.,  $\sigma_{S_k}^2 > \dots > \sigma_{S_1}^2$ . In this case, return reversals occur after at least  $k$  periods as the advance information effect dissipates and the stock price overshoots its long-run mean. This gives rise to the same overreaction patterns as described in behavioral theories.

In Appendix C, we show that after casting the problem into vector and matrix forms, we can use the previous solution method to show that the equilibrium in this section displays the same form as in Section 3. The only difference is that the informed investors' forecasting problem is different because they now have multiple pieces of advance information. We omit the detailed derivation here and turn to a numerical analysis.

For ease of exposition, we focus on the case with  $k = 2$ . We begin with a discussion of two limiting results. First, when the signal about two-period-ahead earnings is completely uninformative (i.e.,  $\sigma_{S_2} = \infty$ ), the model becomes that in Section 3 with  $k = 1$ . Consequently, our previous results in Section 3 apply here. Second, when the signal about the two-period-ahead earnings innovation is extremely precise (i.e.,  $\sigma_{S_2} \rightarrow 0$ ), we find numerically that asymmetric information increases so much that there is no trading in equilibrium. As a result, stock returns are serially uncorrelated. The intuition is as follows. The stock price incorporates the information about the persistent and transitory components of earnings as well as the expected return on the private investment. Uninformed investors use the earnings realizations and the stock price to infer the value of these variables, but may attribute changes in earnings innovations to changes in the various components of earnings or to changes in the private investment return. When informed investors receive very precise information about earnings innovations, they trade on this information more aggressively, and uninformed investors believe that informed

investors' trading is generated by a speculative motivate rather than a rebalancing motive. Hence, uninformed investors refrain from trading.

We now turn to intermediate values of  $\sigma_{S_2}$ . Table 3 displays the slope coefficients of the forecast of single-period returns  $Q_{t+n}$ , conditional on  $Q_t$  as well as the slope coefficients of the forecast of cumulative returns  $Q_{t,t+n}$ , conditional on  $Q_t$  for  $\sigma_{S_2} = 1$  and various values of  $\sigma_{S_1}$ . This table reveals that our model with advance information about earnings innovations over two successive periods can generate momentum and reversal effects simultaneously. In addition, the duration of momentum depends on the precision of the advance information signals. In particular, when the signal about one-period-ahead earnings becomes more precise relative to the signal about two-period-ahead earnings, the momentum effect lasts longer. On the other hand, when this signal is sufficiently imprecise, the momentum effect disappears.

**[Insert Table 3 Here.]**

Table 4 displays the slope coefficients of the forecast of single-period (cumulative) returns  $Q_{t+n}$  ( $Q_{t,t+n}$ ) conditional on the earnings announcement  $D_t - E_{t-1}^u [D_t]$ , for  $\sigma_{S_2} = 1$  and various values of  $\sigma_{S_1}$ . The table shows that the price drift effect is stronger as the signal about one-period-ahead earnings innovations becomes less precise relative to the signal about two-period-ahead earnings innovations.

**[Insert Table 4 Here.]**

## 6. Conclusion

In this paper, we present a heterogeneous-investor rational expectations equilibrium model with asymmetric information that can deliver the momentum and reversal effects in a unified way. Our key insight is to assume that informed investors possess advance information in the form of signals about future earnings that are uncorrelated with current earnings and that this news is informative about their other (private) investment opportunities. The stock price underreacts to this information and uninformed investors can profit by following trend-chasing strategies. Simultaneously, advance information makes prices move in ways that are unrelated to current fundamentals. These price movements, which Daniel and Titman (2006) call the intangible component of returns, can predict future return reversals. After a sustained streak of good news, the advance information materializes and the price starts reverting back to its long-run mean, giving the appearance of having overshoot its fundamental value. Thus, our model provides a rational account of the underreaction and overreaction phenomena.

We may extend our model in several directions. First, our model focuses on the implications of advance information for momentum and reversal effects. It would be interesting to further

study the implications for trading volume, as in Wang (1994) and Llorente et al. (2002). Second, we follow Wang (1994) and assume a hierarchical information structure. Considering the case where information is symmetrically dispersed might be worthwhile (see, e.g., Bacchetta and van Wincoop (2004, 2006)). In this case, higher-order expectations play an important role. Finally, we assume that advance information is private to informed investors only. In work in progress, we consider the case where advance information is public to all investors in the economy.



and

$$\mathbf{y}_t^i = \mathbf{A}_{yi} \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^i + \hat{\boldsymbol{\varepsilon}}_t^i, \quad (\text{A.2})$$

where the innovation  $\hat{\boldsymbol{\varepsilon}}_t^i = \mathbf{y}_t^i - E_{t-1}^i [\mathbf{y}_t^i]$  is normally distributed with mean zero and variance

$$\boldsymbol{\Sigma}_i = E [\hat{\boldsymbol{\varepsilon}}_t^i (\hat{\boldsymbol{\varepsilon}}_t^i)^\top] = \mathbf{A}_{yi} \mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top \mathbf{A}_{yi}^\top + (\mathbf{A}_{yi} \mathbf{B}_x + \mathbf{B}_{yi}) \boldsymbol{\Sigma} (\mathbf{A}_{yi} \mathbf{B}_x + \mathbf{B}_{yi})^\top. \quad (\text{A.3})$$

We focus on the steady state Kalman filtering as in Wang (1994). The covariance matrix  $\boldsymbol{\Omega}_i = E_t^i [(\mathbf{x}_t - \hat{\mathbf{x}}_t^i) (\mathbf{x}_t - \hat{\mathbf{x}}_t^i)^\top]$  and the Kalman gain matrix  $\mathbf{K}_i$  satisfy

$$\boldsymbol{\Omega}_i = (\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) - \mathbf{K}_i \mathbf{A}_{yi} (\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}), \quad (\text{A.4})$$

and

$$\mathbf{K}_i = (\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_{yi}^\top \left[ \mathbf{A}_{yi} (\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_{yi}^\top + \boldsymbol{\Sigma}_{yy} \right]^{-1}, \quad (\text{A.5})$$

where we define

$$\boldsymbol{\Sigma}_{xx} = \mathbf{B}_x \boldsymbol{\Sigma} \mathbf{B}_x^\top, \quad \boldsymbol{\Sigma}_{yy} = \mathbf{B}_{yi} \boldsymbol{\Sigma} \mathbf{B}_{yi}^\top. \quad (\text{A.6})$$

We have  $\boldsymbol{\Omega}_i \mathbf{A}_{yi}^\top = \mathbf{K}_i \boldsymbol{\Sigma}_{yy}$  and  $\boldsymbol{\Omega}_i (\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx})^{-1} = \mathbf{I} - \mathbf{K}_i \mathbf{A}_{yi}$ . We will use these two equations to simplify the informed investors' forecast problem. For expositional convenience, we consider the case where  $k = 1$ . The solution for general  $k > 1$  is similar. Let  $\boldsymbol{\Omega}_i = (\omega_{ij})_{5 \times 5}$ . Using the first equation yields

$$\begin{bmatrix} \omega_{11} + \omega_{14} & \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} + \omega_{24} & \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} + \omega_{34} & \omega_{31} & \omega_{32} & \omega_{33} \\ \omega_{41} + \omega_{44} & \omega_{41} & \omega_{42} & \omega_{43} \\ \omega_{51} + \omega_{54} & \omega_{51} & \omega_{52} & \omega_{53} \end{bmatrix} = \sigma_S^2 \begin{bmatrix} 0 & 0 & 0 & k_{14} \\ 0 & 0 & 0 & k_{24} \\ 0 & 0 & 0 & k_{34} \\ 0 & 0 & 0 & k_{44} \\ 0 & 0 & 0 & k_{54} \end{bmatrix}.$$

We thus obtain

$$\begin{aligned} \omega_{1j} &= \omega_{j1} = 0, \\ \omega_{2j} &= \omega_{j2} = 0, \\ \omega_{4j} &= \omega_{j4} = 0, \\ \sigma_S^2 k_{j4} &= \omega_{j3}. \end{aligned}$$

That is,

$$\mathbf{\Omega}_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_S^2 k_{34} & 0 & \sigma_S^2 k_{54} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_S^2 k_{54} & 0 & \omega_{55} \end{bmatrix}.$$

Using the second equation, we can derive

$$\begin{aligned} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{54} \sigma_S^2 \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} - k_{34} \sigma_S^2 \frac{\sigma_q^2}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} & 0 & k_{34} \sigma_S^2 \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} - k_{54} \sigma_S^2 \frac{\sigma_D^2}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_{55} \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} - k_{54} \sigma_S^2 \frac{\sigma_q^2}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} & 0 & -\omega_{55} \frac{\sigma_D^2}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} + k_{54} \sigma_S^2 \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} \end{bmatrix} \\ & = \begin{bmatrix} 1 - (k_{11} + k_{12}) & -k_{13} & -k_{14} & -k_{11} & 0 \\ -(k_{21} + k_{22}) & 1 - k_{23} & -k_{24} & -k_{21} & 0 \\ -(k_{31} + k_{32}) & -k_{33} & 1 - k_{34} & -k_{31} & 0 \\ -(k_{41} + k_{42}) & -k_{43} & -k_{44} & 1 - k_{41} & 0 \\ -(k_{51} + k_{52}) & -k_{53} & -k_{54} & -k_{51} & 1 \end{bmatrix}. \end{aligned}$$

Equating terms gives

$$\begin{aligned} k_{34} &= \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2}, k_{54} = \frac{\sigma_{qD}}{\sigma_S^2 + \sigma_D^2}, \\ \omega_{55} &= \sigma_q^2 \left( 1 - \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2} \rho_{qD}^2 \right). \end{aligned}$$

And generalizing to  $k \geq 1$  yields

$$\mathbf{K}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2} \\ \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sigma_{qD}}{\sigma_S^2 + \sigma_D^2} \\ \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} \end{bmatrix}.$$

Substituting this equation into equation (A.1) yields the desired result. Q.E.D.

**Uninformed investors' filtering problem.** Uninformed investors face state dynamics given in (A.1) and their observation system is given by

$$\mathbf{y}_t^u = \mathbf{A}_{yu} \hat{\mathbf{x}}_t^i,$$

where  $\mathbf{A}_{yu} = [\mathbf{c}_1^\top + \mathbf{c}_{k+3}^\top, \mathbf{p}_i^\top]^\top$ . By the Kalman filtering theory (see, e.g., Wang (1994)), uninformed investors' conditional forecast of the state vector is given by the steady-state Kalman filters:

$$\hat{\mathbf{x}}_t^u = \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \mathbf{K}_u \hat{\mathbf{e}}_t^u, \quad (\text{A.7})$$

and

$$\hat{\mathbf{e}}_t^u \equiv \mathbf{y}_t^u - E_{t-1}^u[\mathbf{y}_t^u], \quad (\text{A.8})$$

where  $\hat{\mathbf{e}}_t^u$  is normally distributed with mean zero and covariance matrix

$$\Sigma_u = \mathbf{A}_{yu} \mathbf{A}_x \Omega_u \mathbf{A}_x^\top \mathbf{A}_{yu}^\top + \mathbf{A}_{yu} \mathbf{K}_i \Sigma_i \mathbf{K}_i^\top \mathbf{A}_{yu}^\top.$$

Moreover, the covariance matrix  $\Omega_u = E_t^u [(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u)(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u)^\top]$  and the Kalman gain matrix  $\mathbf{K}_u$  satisfy

$$\Omega_u = (\mathbf{A}_x \Omega_u \mathbf{A}_x^\top + \Sigma_{xx}) - \mathbf{K}_u \mathbf{A}_{yu} (\mathbf{A}_x \Omega_u \mathbf{A}_x^\top + \Sigma_{xx}), \quad (\text{A.9})$$

where

$$\mathbf{K}_u = (\mathbf{A}_x \Omega_u \mathbf{A}_x^\top + \Sigma_{xx}) \mathbf{A}_{yu}^\top [\mathbf{A}_{yu} (\mathbf{A}_x \Omega_u \mathbf{A}_x^\top + \Sigma_{xx}) \mathbf{A}_{yu}^\top]^{-1}, \quad (\text{A.10})$$

and  $\Sigma_{xx} = \mathbf{K}_i \Sigma_i \mathbf{K}_i^\top$ .

**Proof of Proposition 1:** We first use the conjectured price function (15) to derive

$$\begin{aligned} Q_{t+1} &= P_{t+1} + D_{t+1} - RP_t \\ &= -p_0 + \mathbf{p}_i \hat{\mathbf{x}}_{t+1}^i + \mathbf{p}_u \mathbf{I}_{-2} \hat{\mathbf{x}}_{t+1}^u + F_{t+1} + \varepsilon_{t+1}^D - R(-p_0 + \mathbf{p}_i \hat{\mathbf{x}}_t^i + \mathbf{p}_u \mathbf{I}_{-2} \hat{\mathbf{x}}_t^u) \\ &= e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u + \mathbf{b}_Q \hat{\mathbf{e}}_{t+1}^i, \end{aligned} \quad (\text{A.11})$$

where

$$e_0 = rp_0, \quad (\text{A.12})$$

$$\mathbf{e}_i = (\mathbf{p}_i + \mathbf{c}_1 + \mathbf{c}_{k+3}) \mathbf{A}_x - R\mathbf{p}_i + \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{A}_x \left( \mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} \right), \quad (\text{A.13})$$

$$\mathbf{e}_u = \mathbf{p}_u \mathbf{I}_{-2} \mathbf{A}_x - R\mathbf{p}_u \mathbf{I}_{-2} - \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{A}_x \left( \mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} \right), \quad (\text{A.14})$$

$$\mathbf{b}_Q = (\mathbf{p}_i + \mathbf{c}_1 + \mathbf{c}_{k+3}) \mathbf{K}_i + \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{K}_i. \quad (\text{A.15})$$

and where we have used

$$\begin{aligned}
\hat{\boldsymbol{\varepsilon}}_{t+1}^u &= \mathbf{y}_{t+1}^u - \mathbf{A}_{yu} \mathbf{A}_x \hat{\mathbf{x}}_t^u \\
&= \mathbf{A}_{yu} \hat{\mathbf{x}}_{t+1}^i - \mathbf{A}_{yu} \mathbf{A}_x \hat{\mathbf{x}}_t^u \\
&= \mathbf{A}_{yu} \mathbf{A}_x (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) + \mathbf{A}_{yu} \mathbf{K}_i \hat{\boldsymbol{\varepsilon}}_{t+1}^i \\
&= \mathbf{A}_{yu} \mathbf{A}_x \left( \mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} \right) (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) + \mathbf{A}_{yu} \mathbf{K}_i \hat{\boldsymbol{\varepsilon}}_{t+1}^i,
\end{aligned}$$

noting that  $\varepsilon_{t+1}^D$  –and not its expectation– is in  $\hat{\mathbf{x}}_{t+1}^i$  at time  $t + 1$ . The last equality follows from (17),

$$\hat{Z}_t^i - \hat{Z}_t^u = \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} (\hat{\mathbf{x}}_t^u - \hat{\mathbf{x}}_t^i),$$

and from

$$\begin{aligned}
\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u &= \mathbf{I}_{-2} (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) + \mathbf{c}_2^\top (\hat{Z}_t^i - \hat{Z}_t^u) \\
&= \mathbf{I}_{-2} (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) \\
&= \left( \mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} \right) (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u).
\end{aligned}$$

We next derive the conditional expectations:

$$E_t^i [Q_{t+1}] = e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u, \quad (\text{A.16})$$

$$E_t^u [Q_{t+1}] = e_0 + (\mathbf{e}_i + \mathbf{e}_u) \hat{\mathbf{x}}_t^u. \quad (\text{A.17})$$

We use (17) to substitute out  $\hat{Z}_t^u$  in  $E_t^u [Q_{t+1}]$  (because likely  $e_{i2} \neq 0$ ) and derive

$$\begin{aligned}
E_t^u [Q_{t+1}] &= e_0 + (\mathbf{e}_i + \mathbf{e}_u) \hat{\mathbf{x}}_t^u \\
&= e_0 + \mathbf{e}_i \left[ \mathbf{I}_{-2} \hat{\mathbf{x}}_t^u + \mathbf{c}_2^\top \hat{Z}_t^u \right] + \mathbf{e}_u \hat{\mathbf{x}}_t^u \\
&= e_0 + \mathbf{e}_i \left[ \mathbf{I}_{-2} \hat{\mathbf{x}}_t^u + \mathbf{c}_2^\top \hat{Z}_t^i - \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2} (\hat{\mathbf{x}}_t^u - \hat{\mathbf{x}}_t^i) \right] + \mathbf{e}_u \hat{\mathbf{x}}_t^u \\
&= e_0 + \mathbf{e}_i \left( \mathbf{c}_2^\top \mathbf{c}_2 + \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2} \right) \hat{\mathbf{x}}_t^i + \left[ \mathbf{e}_i \left( \mathbf{I}_{-2} - \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2} \right) + \mathbf{e}_u \right] \hat{\mathbf{x}}_t^u \\
&= e_0 + \tilde{\mathbf{e}}_i \hat{\mathbf{x}}_t^i + \tilde{\mathbf{e}}_u \hat{\mathbf{x}}_t^u.
\end{aligned}$$

It is easy to derive

$$E_t^i [q_{t+1}] = E_t^i [Z_t + \varepsilon_{t+1}^q] = (\mathbf{c}_2 + \mathbf{c}_{2k+3}) \hat{\mathbf{x}}_t^i.$$

We can also derive the conditional variances

$$Var_t^i(Q_{t+1}) = \mathbf{b}_Q \boldsymbol{\Sigma}_i \mathbf{b}_Q^\top, \quad Var_t^i(q_{t+1}) = \sigma_q^2,$$

$$Cov_t^i(Q_{t+1}, q_{t+1}) = \mathbf{b}_Q E_t^i [\hat{\boldsymbol{\varepsilon}}_{t+1}^i \varepsilon_{t+1}^q] = \mathbf{b}_Q E_t^i [\hat{\boldsymbol{\varepsilon}}_{t+1}^i \mathbf{x}_t^\top] \mathbf{c}_{2k+3}^\top = \mathbf{b}_Q \mathbf{A}_{yi} \mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{c}_{2k+3}^\top,$$

where we have used the following result to derive the last equation,

$$\begin{aligned} E_t^i [\hat{\boldsymbol{\varepsilon}}_{t+1}^i \mathbf{x}_t^\top] &= E_t^i [\mathbf{A}_{yi} \mathbf{A}_x (\mathbf{x}_t - \hat{\mathbf{x}}_t^i) \mathbf{x}_t^\top + (\mathbf{A}_{yi} \mathbf{B}_x + \mathbf{B}_{yi}) \varepsilon_{t+1} \mathbf{x}_t^\top] \\ &= E_t^i [\mathbf{A}_{yi} \mathbf{A}_x (\mathbf{x}_t - \hat{\mathbf{x}}_t^i) \mathbf{x}_t^\top] = E_t^i [\mathbf{A}_{yi} \mathbf{A}_x (\mathbf{x}_t - \hat{\mathbf{x}}_t^i) (\mathbf{x}_t - \hat{\mathbf{x}}_t^i)^\top] \\ &= \mathbf{A}_{yi} \mathbf{A}_x \boldsymbol{\Omega}_i. \end{aligned}$$

Furthermore, we can derive the uninformed investors' conditional variance

$$\begin{aligned} Var_t^u(Q_{t+1}) &= E_t^u [(\mathbf{e}_i (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) + \mathbf{b}_Q \hat{\boldsymbol{\varepsilon}}_{t+1}^i) (\mathbf{e}_i (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) + \mathbf{b}_Q \hat{\boldsymbol{\varepsilon}}_{t+1}^i)^\top] \\ &= \mathbf{e}_i \boldsymbol{\Omega}_u \mathbf{e}_i^\top + \mathbf{b}_Q \boldsymbol{\Sigma}_i \mathbf{b}_Q^\top. \end{aligned}$$

where we have used the following result:

$$E_t^u [(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) \hat{\boldsymbol{\varepsilon}}_{t+1}^{i\top}] = E_t^u [E_t^i [(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) \hat{\boldsymbol{\varepsilon}}_{t+1}^{i\top}]] = E_t^u [(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) E_t^i [\hat{\boldsymbol{\varepsilon}}_{t+1}^{i\top}]] = 0.$$

Now, we use (24) and (25) to show that optimal stock holdings are linear functions of  $\hat{\mathbf{x}}_t^i$  and  $\hat{\mathbf{x}}_t^u$ . Using the preceding conditional expectations we get

$$\theta_t^i = \frac{e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u}{\gamma (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} - \frac{\rho_{Qq}^i (\mathbf{c}_2 + \mathbf{c}_{2k+3}) \hat{\mathbf{x}}_t^i}{\gamma \sigma_Q^i \sigma_q^i (1 - (\rho_{Qq}^i)^2)}, \quad (\text{A.18})$$

$$\theta_t^u = \frac{e_0 + \tilde{\mathbf{e}}_i \hat{\mathbf{x}}_t^i + \tilde{\mathbf{e}}_u \hat{\mathbf{x}}_t^u}{\gamma (\sigma_Q^u)^2}. \quad (\text{A.19})$$

We use the market clearing condition (13) to determine the coefficients in the price function. Substituting (A.18) and (A.19) into (13) yields

$$1 = \frac{\lambda}{\gamma} \left[ \frac{e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u}{(\sigma_Q^i)^2 (1 - \rho_{Qq}^2)} - \frac{\rho_{Qq} (\mathbf{c}_2 + \mathbf{c}_{2k+3}) \hat{\mathbf{x}}_t^i}{\sigma_Q^i \sigma_q^i (1 - \rho_{Qq}^2)} \right] + (1 - \lambda) \frac{e_0 + \tilde{\mathbf{e}}_i \hat{\mathbf{x}}_t^i + \tilde{\mathbf{e}}_u \hat{\mathbf{x}}_t^u}{\gamma (\sigma_Q^u)^2}.$$

Matching coefficients on the constant,  $\hat{\mathbf{x}}_t^i$ , and  $\hat{\mathbf{x}}_t^u$  we obtain

$$e_0 = \gamma \frac{(\sigma_Q^i)^2 (\sigma_Q^u)^2 (1 - \rho_{Qq}^2)}{\lambda (\sigma_Q^u)^2 + (1 - \lambda) (\sigma_Q^i)^2 (1 - \rho_{Qq}^2)} > 0, \quad (\text{A.20})$$

$$\frac{\lambda}{\gamma} \left[ \frac{\mathbf{e}_i}{(\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} - \frac{\rho_{Qq}^i (\mathbf{c}_2 + \mathbf{c}_{2k+3})}{\sigma_Q^i \sigma_q^i (1 - (\rho_{Qq}^i)^2)} \right] + \frac{1 - \lambda}{\gamma} \frac{\mathbf{e}_i (\mathbf{c}_2^\top \mathbf{c}_2 + \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2})}{(\sigma_Q^u)^2} = 0, \quad (\text{A.21})$$

$$0 = \frac{\lambda}{\gamma} \frac{\mathbf{e}_u}{(\sigma_Q^i)^2 (1 - \rho_{Qq}^2)} + (1 - \lambda) \frac{\mathbf{e}_i (\mathbf{I}_{-2} - \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2}) + \mathbf{e}_u}{\gamma (\sigma_Q^u)^2}. \quad (\text{A.22})$$

Equations (A.12) and (A.20) imply that  $p_0 > 0$ . Using

$$\mathbf{e}_i \left( \mathbf{c}_2^\top \mathbf{c}_2 + \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2} \right) = e_{i2} \left[ \begin{array}{cccc} \frac{p_{i1}}{p_{i2}} & 1 & \frac{p_{i3}}{p_{i2}} & \dots & \frac{p_{i,2k+3}}{p_{i2}} \end{array} \right],$$

together with (A.21), we have

$$\begin{aligned} \frac{\lambda}{\gamma} \frac{e_{i1}}{(\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} + \frac{1 - \lambda}{\gamma} \frac{e_{i2} \frac{p_{i1}}{p_{i2}}}{(\sigma_Q^u)^2} &= 0, \\ \frac{\lambda}{\gamma} \left[ \frac{e_{i2}}{(\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} - \frac{\rho_{Qq}^i}{\sigma_Q^i \sigma_q^i (1 - (\rho_{Qq}^i)^2)} \right] + \frac{1 - \lambda}{\gamma} \frac{e_{i2}}{(\sigma_Q^u)^2} &= 0, \\ \frac{\lambda}{\gamma} \frac{e_{i3}}{(\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} + \frac{1 - \lambda}{\gamma} \frac{e_{i2} \frac{p_{i3}}{p_{i2}}}{(\sigma_Q^u)^2} &= 0, \\ &\dots \\ \frac{\lambda}{\gamma} \left[ \frac{e_{i,2k+3}}{(\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} - \frac{\rho_{Qq}^i}{\sigma_Q^i \sigma_q^i (1 - (\rho_{Qq}^i)^2)} \right] + \frac{1 - \lambda}{\gamma} \frac{e_{i2} \frac{p_{i,2k+3}}{p_{i2}}}{(\sigma_Q^u)^2} &= 0. \end{aligned}$$

Solving these equation gives

$$e_{i2} = \frac{\lambda (\sigma_Q^u)^2}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \quad (\text{A.23})$$

$$e_{i1} = \frac{p_{i1}}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \quad (\text{A.24})$$

$$e_{i3} = \frac{p_{i3}}{p_{i1}} e_{i1},$$

...

$$e_{i,2k+2} = \frac{p_{i,2k+2}}{p_{i1}} e_{i1},$$

$$e_{i,2k+3} = \left[ 1 - \frac{p_{i,2k+3}}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} \right] \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}.$$

Turning now to (A.22), we use

$$\mathbf{e}_i \left( \mathbf{I}_{-2} - \frac{1}{p_{i2}} \mathbf{c}_2^T \mathbf{p}_i \mathbf{I}_{-2} \right) = \left[ e_{i1} - e_{i2} \frac{p_{i1}}{p_{i2}} \quad 0 \quad e_{i3} - e_{i2} \frac{p_{i3}}{p_{i2}} \quad \dots \quad e_{i,2k+3} - e_{i2} \frac{p_{i,2k+3}}{p_{i2}} \right],$$

to derive

$$\begin{aligned} 0 &= \frac{\lambda}{\gamma} \frac{e_{u1}}{(\sigma_Q^i)^2 (1 - \rho_{Qq}^2)} + (1-\lambda) \frac{e_{i1} - e_{i2} \frac{p_{i1}}{p_{i2}} + e_{u1}}{\gamma (\sigma_Q^u)^2}, \\ 0 &= \frac{\lambda}{\gamma} \frac{e_{u2}}{(\sigma_Q^i)^2 (1 - \rho_{Qq}^2)} + (1-\lambda) \frac{e_{u2}}{\gamma (\sigma_Q^u)^2}, \\ 0 &= \frac{\lambda}{\gamma} \frac{e_{u3}}{(\sigma_Q^i)^2 (1 - \rho_{Qq}^2)} + (1-\lambda) \frac{e_{i3} - e_{i2} \frac{p_{i3}}{p_{i2}} + e_{u3}}{\gamma (\sigma_Q^u)^2}, \\ &\dots \\ 0 &= \frac{\lambda}{\gamma} \frac{e_{u,2k+3}}{(\sigma_Q^i)^2 (1 - \rho_{Qq}^2)} + (1-\lambda) \frac{e_{i,2k+3} - e_{i2} \frac{p_{i,2k+3}}{p_{i2}} + e_{u,2k+3}}{\gamma (\sigma_Q^u)^2}. \end{aligned}$$

Solving these equations yields

$$\begin{aligned}
e_{u1} &= \frac{p_{i1}}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \\
e_{u2} &= 0, \\
e_{u3} &= \frac{p_{i3}}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \\
&\dots \\
e_{u,2k+3} &= \frac{p_{i,2k+3} - p_{i2}}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}.
\end{aligned} \tag{A.25}$$

To solve an equilibrium, we need to determine  $1 + (2k + 3) + (2k + 2)$  price coefficients  $p_0$ ,  $\mathbf{p}_i$ , and  $\mathbf{p}_u$  (note that  $p_{u2} = 0$ ). Equations (A.12) and (A.20) determine  $p_0$ . Equating equations (A.13) with equations in (A.23), we obtain  $(2k + 3)$  equations. Equating equations (A.14) with equations in (A.25), we obtain  $(2k + 3)$  equations. Note that the second equation  $e_{u2} = 0$  is a redundant identity. Therefore, we essentially have  $(2k + 2)$  equations. In summary, we obtain  $(2k + 3) + (2k + 2)$  equations to solve for  $(2k + 3) + (2k + 2)$  unknowns of  $\mathbf{p}_i$ , and  $\mathbf{p}_u$ . When solving these equations, we need to substitute in the preceding variances and covariances. If there is a solution, then we obtain a stationary equilibrium.

Now we derive restrictions on the coefficients in the price function. Adding equations in (A.23) and (A.25) yields

$$e_{i,j} + e_{u,j} = 0, \text{ for all } j \neq 2, 2k + 3, \tag{A.26}$$

$$e_{i2} + e_{u2} = e_{i2}, \tag{A.27}$$

$$e_{i,2k+3} + e_{u,2k+3} = e_{i2}. \tag{A.28}$$

Adding equations (A.13)-(A.14), we get

$$\mathbf{e}_i + \mathbf{e}_u = (\mathbf{p}_i + \mathbf{c}_1 + \mathbf{c}_{k+3}) \mathbf{A}_x - R\mathbf{p}_i + \mathbf{p}_u \mathbf{I}_{-2} \mathbf{A}_x - R\mathbf{p}_u \mathbf{I}_{-2}. \tag{A.29}$$

Simplifying yields

$$p_{i1} + p_{u1} = a_F / (R - a_F), \quad (\text{A.30})$$

$$p_{i2} = \frac{-e_{i2}}{R - a_Z}, \quad (\text{A.31})$$

$$p_{ij} + p_{uj} = \frac{1}{R^{3+k-j}}, \text{ for } 3 \leq j \leq k+2, \quad (\text{A.32})$$

$$p_{ij} + p_{uj} = -\frac{e_{i2}}{R^{2k+3-(j-1)}}, \text{ for } j \geq k+4, \quad (\text{A.33})$$

and

$$p_{i,k+3} + p_{u,k+3} = 0. \quad (\text{A.34})$$

We next show that  $p_{u,k+3} = p_{i,k+3} = 0$ . Note that from equation (A.14) we get

$$e_{u,k+3} = -R p_{u,k+3} + \mathbf{p}_u \mathbf{K}_{u,2} a_Z p_{i,k+3},$$

where  $\mathbf{K}_{u,2}$  is the second column of matrix  $\mathbf{K}_u$ . Substituting for the equation for  $e_{u,k+3}$  in (A.23) and  $p_{i,k+3} + p_{u,k+3} = 0$  yields

$$\frac{p_{i,k+3}}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2} = (R + \mathbf{p}_u \mathbf{K}_{u,2} a_Z) p_{i,k+3}.$$

If we suppose  $p_{i,k+3} \neq 0$ , then we write the above expression as

$$\frac{1}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2} = R + \mathbf{p}_u \mathbf{K}_{u,2} a_Z. \quad (\text{A.35})$$

Now use the last equation (for  $2k+3$ ) in (A.14) to derive

$$e_{u,2k+3} = -R p_{u,2k+3} + \mathbf{p}_u \mathbf{K}_{u,2} a_Z p_{i,2k+3}.$$

We substitute the equation for  $e_{u,2k+3}$  in (A.25) and  $p_{i,2k+3} + p_{u,2k+3} = -\frac{e_{i2}}{R}$  to derive

$$\frac{p_{i,2k+3} - p_{i2}}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2} - e_{i2} = p_{i,2k+3} (R + \mathbf{p}_u \mathbf{K}_{u,2} a_Z).$$

Substituting the equation for  $e_{i2}$  in (A.23) yields

$$\left( \frac{p_{i,2k+3}}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)} - 1 \right) \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2} = p_{i,2k+3} (R + \mathbf{p}_u \mathbf{K}_{u,2} a_Z).$$

If we replace the value of  $R + \mathbf{p}_u \mathbf{K}_{u,2} a_Z$  from (A.35), we get an impossibility. Therefore,  $p_{i,k+3} = 0 = p_{u,k+3}$ . Using (A.25) again we obtain  $e_{u,k+3} = e_{i,k+3} = 0$ .

Using Proposition (A.11) and the coefficients restrictions (A.26)-(A.28), we can derive equation (20). Q.E.D.

**Proof of Proposition 3:** We focus on conditional expectations of future returns in (24) and (25). Using equations (A.16) and (A.26), we can derive the expression in (26). Using equations (A.17) and (A.26)-(A.28), we can derive

$$E_t^u [Q_{t+1}] = e_0 + e_{i2} E_t^u [Z_t + \varepsilon_{t+1}^q] \quad (\text{A.36})$$

and (27). Q.E.D.

**Proof of Proposition 4:** Using (A.36) and the law of iterated expectations, we compute

$$\begin{aligned} E [Q_{t+1}|Q_t] &= E [E_t^u [Q_{t+1}] | Q_t] = E [E_t^i [Q_{t+1}] | Q_t] \\ &= e_0 + e_{i2} E [E_t^i (Z_t + \varepsilon_{t+1}^q) | Q_t] \\ &= e_0 + e_{i2} \frac{Cov (Z_t, Q_t) + Cov (E_t^i [\varepsilon_{t+1}^q], Q_t)}{Var (Q_t)} Q_t. \end{aligned}$$

Also for general  $n \geq 1$ ,

$$\begin{aligned} E [Q_{t+n}|Q_t] &= e_0 + e_{i2} E [Z_{t+n-1} + \varepsilon_{t+n}^q | Q_t] \\ &= e_0 + e_{i2} \frac{(\mathbf{c}_2 + \mathbf{c}_{2k+3}) \mathbf{A}_x^{n-1} Cov (\hat{\mathbf{x}}_t^i, Q_t)}{Var (Q_t)} Q_t \\ &= e_0 + e_{i2} \frac{a_Z^{n-1} Cov (Z_t, Q_t) + \mathbf{1}_{\{n \leq k\}} Cov (E_t^i [\varepsilon_{t+n}^q], Q_t)}{Var (Q_t)} Q_t, \end{aligned}$$

noting that  $(\mathbf{c}_2 + \mathbf{c}_{2k+3}) \mathbf{A}_x^{n-1} = a_Z^{n-1} \mathbf{c}_2 + \mathbf{1}_{\{n \leq k\}} \mathbf{c}_{2k+3-(n-1)}$ , where  $\mathbf{1}_{\{n \leq k\}}$  is an indicator function equal to 1 if  $n \leq k$  and 0 otherwise. Q.E.D.

**Proof of Proposition 5:** We prove the case with  $n = 1$  and  $k = 2$ . The proof for general  $n$  and  $k$  is similar. By the law of iterated expectations, we have

$$\begin{aligned}
& E [Q_{t+1}|D_t - E_{t-1}^u (D_t)] \\
&= E [E_t^u [Q_{t+1}] | D_t - E_{t-1}^u (D_t)] = e_0 + e_{i2} E [E_t^u [Z_t + \varepsilon_{t+1}^q] | D_t - E_{t-1}^u (D_t)] \\
&= e_0 + e_{i2} E [E_t^i [Z_t + \varepsilon_{t+1}^q] | D_t - E_{t-1}^u (D_t)] \\
&= e_0 + e_{i2} E [Z_t | D_t - E_{t-1}^u (D_t)] + e_{i2} E [E_t^i [\varepsilon_{t+1}^q] | D_t - E_{t-1}^u (D_t)] \\
&= e_0 + e_{i2} E [Z_t | D_t - E_{t-1}^u (D_t)] + e_{i2} \frac{\sigma_{Dq}}{\sigma_D^2} E [E_t^i [\varepsilon_{t+1}^D] | D_t - E_{t-1}^u (D_t)] \\
&= e_0 + e_{i2} E [Z_t | D_t - E_{t-1}^u (D_t)] + e_{i2} \frac{\sigma_{Dq}}{\sigma_D^2} E [E_t^i [\varepsilon_{t+1}^D] | D_t - E_{t-1}^u (D_t)] \\
&= e_0 + e_{i2} E [Z_t | D_t - E_{t-1}^u (D_t)] + e_{i2} d_1 \frac{Cov (E_t^i [\varepsilon_{t+1}^D], D_t - E_{t-1}^u (D_t))}{Var (D_t - E_{t-1}^u (D_t))} [D_t - E_{t-1}^u (D_t)]
\end{aligned}$$

where we have used Proposition 2 and defined  $d_1 = \sigma_{Dq}/\sigma_D^2 > 0$ . Q.E.D.

## B Solution for the model without advance information

The solution for the model without advance information can be obtained as a special case of that for our general model. In addition, this model is a simplified version of Wang (1994). However, because of its simplicity, we are able to derive sharper results by signing many coefficients in the equilibrium equations. These results are absent in Wang (1994). Thus, we will focus on these extra results, without repeating the detailed derivations of equilibrium.

**Uninformed investors' filtering problem.** The state vector and the unforecastable shock vector are given by  $\mathbf{x}_t = (F_t, Z_t)^\top$  and  $\boldsymbol{\varepsilon}_t = (\varepsilon_t^D, \varepsilon_t^F, \varepsilon_t^Z, \varepsilon_t^q)^\top$ , respectively. Let  $\Sigma = E [\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^\top]$ . We then have

$$\mathbf{x}_t = \mathbf{A}_x \mathbf{x}_{t-1} + \mathbf{B}_x \boldsymbol{\varepsilon}_t. \quad (\text{B.1})$$

where

$$\mathbf{A}_x = \begin{bmatrix} a_F & 0 \\ 0 & a_Z \end{bmatrix}, \quad \mathbf{B}_x = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \quad (\text{B.2})$$

The uninformed investor has signals  $\mathbf{y}_t = (D_t, \Pi_t)^\top$ , which satisfies

$$\mathbf{y}_t = \mathbf{A}_y \mathbf{x}_t + \mathbf{B}_y \boldsymbol{\varepsilon}_t, \quad (\text{B.3})$$

where

$$\mathbf{A}_y = \begin{bmatrix} 1 & 0 \\ p_{i1} & p_{i2} \end{bmatrix}, \quad \mathbf{B}_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (\text{B.4})$$

Define

$$\boldsymbol{\Sigma}_{xx} = \mathbf{B}_x \boldsymbol{\Sigma} \mathbf{B}_x^\top = \begin{bmatrix} \sigma_F^2 & 0 \\ 0 & \sigma_Z^2 \end{bmatrix}, \quad \boldsymbol{\Sigma}_{yy} = \mathbf{B}_y \boldsymbol{\Sigma} \mathbf{B}_y^\top = \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{B.5})$$

and  $\boldsymbol{\Omega}_t = E_t^u [(\mathbf{x}_t - \hat{\mathbf{x}}_t^u)(\mathbf{x}_t - \hat{\mathbf{x}}_t^u)^\top]$ . As in Wang (1994), we focus on the steady-state Kalman filtering. Let  $\boldsymbol{\Omega}$  be the solution to the Riccati equation:

$$\boldsymbol{\Omega} = (\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) - \mathbf{K} \mathbf{A}_y (\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}), \quad (\text{B.6})$$

where

$$\mathbf{K} = (\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_y^\top [\mathbf{A}_y (\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_y^\top + \boldsymbol{\Sigma}_{yy}]^{-1}. \quad (\text{B.7})$$

We then obtain the following steady-state filters:

$$\hat{\mathbf{x}}_t^u = \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \mathbf{K} \hat{\boldsymbol{\varepsilon}}_t^u, \quad (\text{B.8})$$

and

$$\mathbf{y}_t = \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \hat{\boldsymbol{\varepsilon}}_t^u. \quad (\text{B.9})$$

where  $\hat{\boldsymbol{\varepsilon}}_t^u = \mathbf{y}_t - E_{t-1}^u [\mathbf{y}_t]$  is the innovation, which is normally distributed with mean of zero and covariance matrix

$$\text{Var}(\hat{\boldsymbol{\varepsilon}}_t^u) = \mathbf{A}_y \mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top \mathbf{A}_y^\top + (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y) \boldsymbol{\Sigma} (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y)^\top. \quad (\text{B.10})$$

Post-multiplying both sides of (B.6) by  $\mathbf{A}_y^\top$  and subtracting  $\mathbf{K} \boldsymbol{\Sigma}_{yy}$  from both sides yields

$$\boldsymbol{\Omega} \mathbf{A}_y^\top - \mathbf{K} \boldsymbol{\Sigma}_{yy} = (\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_y^\top - \mathbf{K} [\mathbf{A}_y (\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_y^\top + \boldsymbol{\Sigma}_{yy}] = 0.$$

This equality can be written as

$$\begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix} \begin{bmatrix} 1 & p_{i1} \\ 0 & p_{i2} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & 0 \end{bmatrix},$$

where  $\boldsymbol{\Omega} = (\omega_{ij})$  with  $\omega_{12} = \omega_{21}$ . Therefore, we get the following 4 equations

$$\begin{bmatrix} \omega_{11} & \omega_{11} p_{i1} + \omega_{12} p_{i2} \\ \omega_{21} & \omega_{21} p_{i1} + \omega_{22} p_{i2} \end{bmatrix} = \sigma_D^2 \begin{bmatrix} k_{11} & 0 \\ k_{21} & 0 \end{bmatrix}.$$

Solving yields

$$k_{11} = \omega_{11} / \sigma_D^2 > 0, \quad k_{21} = \omega_{21} / \sigma_D^2 > 0, \quad (\text{B.11})$$

$$\omega_{12} = -p_{i1} / p_{i2} \omega_{11} > 0, \quad \omega_{21} = -p_{i2} / p_{i1} \omega_{22} > 0, \quad (\text{B.12})$$

where we have used the sign restrictions  $p_{i1} > 0$  and  $p_{i2} < 0$  proved below and the fact that the variances  $\omega_{11}, \omega_{22} > 0$ .

Now post-multiply both sides of (B.6) by  $(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx})^{-1}$  to get

$$\boldsymbol{\Omega} (\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx})^{-1} = \mathbf{I} - \mathbf{K} \mathbf{A}_y$$

or

$$\begin{aligned} & \omega_{11} \begin{bmatrix} 1 & -\frac{p_{i1}}{p_{i2}} \\ -\frac{p_{i1}}{p_{i2}} & \left(\frac{p_{i1}}{p_{i2}}\right)^2 \end{bmatrix} \begin{bmatrix} a_F^2 \omega_{11} + \sigma_F^2 & -a_Z a_F \frac{p_{i1}}{p_{i2}} \omega_{11} \\ -a_F a_Z \frac{p_{i1}}{p_{i2}} \omega_{11} & a_Z^2 \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 1 - k_{11} - k_{12} p_{i1} & -k_{12} p_{i2} \\ -k_{21} - k_{22} p_{i1} & 1 - k_{22} p_{i2} \end{bmatrix} \end{aligned}$$

Simplify this equation to obtain

$$\begin{aligned} & \frac{\omega_{11}}{\Delta} \begin{bmatrix} a_Z (a_Z - a_F) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 & \frac{p_{i1}}{p_{i2}} [a_F (a_Z - a_F) \omega_{11} - \sigma_F^2] \\ -\frac{p_{i1}}{p_{i2}} [a_Z (a_Z - a_F) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2] & -\left(\frac{p_{i1}}{p_{i2}}\right)^2 [a_F (a_Z - a_F) \omega_{11} - \sigma_F^2] \end{bmatrix} \\ &= \begin{bmatrix} 1 - k_{11} - k_{12} p_{i1} & -k_{12} p_{i2} \\ -k_{21} - k_{22} p_{i1} & 1 - k_{22} p_{i2} \end{bmatrix}, \end{aligned} \quad (\text{B.13})$$

with

$$\begin{aligned} \Delta &= (a_F^2 \omega_{11} + \sigma_F^2) \left( a_Z^2 \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 \right) - \left( a_F a_Z \frac{p_{i1}}{p_{i2}} \omega_{11} \right)^2 \\ &= \sigma_F^2 a_Z^2 \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + a_F^2 \sigma_Z^2 \omega_{11} + \sigma_F^2 \sigma_Z^2 > 0 \end{aligned} \quad (\text{B.14})$$

Now using the top right hand corner equation:

$$\frac{\omega_{11} p_{i1}}{\Delta p_{i2}^2} [\sigma_F^2 - (1 - a_F^2) \omega_{11} + (1 - a_F a_Z) \omega_{11}] = k_{12}, \quad (\text{B.15})$$

with  $\sigma_F^2 - (1 - a_F^2) \omega_{11} > 0$  and  $a_F, a_Z \in (0, 1)$ , we obtain  $k_{12} > 0$ . Similarly, we have

$$-\frac{\omega_{11} p_{i1}}{\Delta p_{i2}} \left[ a_Z (a_Z - a_F) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 \right] = -k_{21} - k_{22} p_{i1} > 0. \quad (\text{B.16})$$

Since

$$a_Z (a_Z - a_F) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 = \sigma_Z^2 - (1 - a_Z^2) \omega_{22} + (1 - a_F a_Z) \omega_{22} > 0,$$

we obtain

$$-p_{i1}k_{22} = k_{21} - \frac{\omega_{11}}{\Delta} \frac{p_{i1}}{p_{i2}} \left[ a_Z (a_Z - a_F) \left( \frac{p_{i1}}{p_{i2}} \right)^2 \omega_{11} + \sigma_Z^2 \right] > 0 \quad (\text{B.17})$$

implying  $k_{22} < 0$ . Note using the top left hand corner equation in (B.13),

$$1 - k_{11} - k_{12}p_{i1} = \frac{\omega_{11}}{\Delta} \left[ a_Z (a_Z - a_F) \left( \frac{p_{i1}}{p_{i2}} \right)^2 \omega_{11} + \sigma_Z^2 \right] > 0.$$

Having shown that the right hand side of this expression is positive and that, under our guess,  $p_{i1} > 0$ , we obtain  $k_{11} < 1$ . Q.E.D.

**Price function.** As in the proof of Proposition 1, the price function is given by the first line of equation (18) but noting that the equilibrium parameters  $\mathbf{p}_i$  and  $\mathbf{p}_u$  will differ. The excess return  $Q_{t+1}$  satisfies

$$Q_{t+1} = e_0 + \mathbf{e}_i \mathbf{x}_t + \mathbf{e}_u \hat{\mathbf{x}}_t^u + \mathbf{b}_Q \boldsymbol{\varepsilon}_{t+1}, \quad (\text{B.18})$$

where  $e_{u2} = 0$ ,

$$e_{u1} = p_{u1} [a_F (1 - k_{11} - p_{i1}k_{12}) - R + a_Z p_{i1}k_{12}]. \quad (\text{B.19})$$

$$e_{i1} + e_{u1} = -(p_{i1} + p_{u1})(R - a_F) + a_F, \quad (\text{B.20})$$

$$e_{i2} = -p_{i2}(R - a_Z). \quad (\text{B.21})$$

In addition,  $e_{i1}$ ,  $e_{i2}$  and  $e_{u1}$  satisfy equations (A.23), (A.24), and (A.25). Note that the system of these three equations along with equations (B.19)-(B.21) do not admit an analytical solution for  $p_{i1}$ ,  $p_{i2}$ , and  $p_{u1}$ . An equilibrium exists if this system has a solution. A simple iterative numerical procedure can solve this system as in Appendix A.

Even though we cannot solve the equilibrium explicitly, we can derive restrictions on the coefficients in the price function,  $p_{i2} < 0$  and  $0 < p_{u1} < a_F / (R - a_F)$ , as claimed in Section 3.1. First, adding equations (A.24) and (A.25) yields  $e_{i1} + e_{u1} = 0$ . Equation (B.20) then implies

$$p_{i1} + p_{u1} = \frac{a_F}{R - a_F}. \quad (\text{B.22})$$

We then take the ratio of (A.25) and (A.24), and substitute for  $e_{i2}$ , using (B.21) to derive

$$p_{i1} = -(R - a_Z) e_{u1} \frac{(1 - \lambda) \left( \sigma_Q^i \right)^2 \left( 1 - \left( \rho_{Qq}^i \right)^2 \right)}{\lambda \left( \sigma_Q^u \right)^2}. \quad (\text{B.23})$$

We wish to show that  $p_{i1} > 0$ . Suppose to the contrary that  $p_{i1} < 0$ . Then equation (B.23)

implies that  $e_{u1} > 0$ . We next use (B.19) to show that  $p_{u1} < 0$ . To this end, we use the two equations implied in the top row of (B.13) to substitute for  $1 - k_{11} - p_{i1}k_{12}$  and  $p_{i1}k_{12}$ , respectively. We then obtain

$$\begin{aligned}
& a_F(1 - k_{11} - p_{i1}k_{12}) - R + a_Z p_{i1}k_{12} \\
= & \frac{\omega_{11}}{\Delta} a_F \left[ a_Z(a_Z - a_F) \left( \frac{p_{i1}}{p_{i2}} \right)^2 \omega_{11} + \sigma_Z^2 \right] - R \\
& - \frac{\omega_{11}}{\Delta} a_Z \left( \frac{p_{i1}}{p_{i2}} \right)^2 [a_F(a_Z - a_F)\omega_{11} - \sigma_F^2] \\
= & \frac{\omega_{11}}{\Delta} \left( a_F \sigma_Z^2 + a_Z \left( \frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \right) - R. \\
= & \frac{a_Z \left( \frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + a_F \sigma_Z^2 \omega_{11}}{a_Z^2 \left( \frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + a_F^2 \sigma_Z^2 \omega_{11} + \sigma_F^2 \sigma_Z^2} - R,
\end{aligned}$$

We now show this expression is negative and thus deduce  $p_{u1} < 0$ . It suffices to show that

$$\frac{\omega_{11}}{\Delta} \left( a_F \sigma_Z^2 + a_Z \left( \frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \right) = \frac{a_Z \left( \frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + a_F \sigma_Z^2 \omega_{11}}{a_Z^2 \left( \frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + a_F^2 \sigma_Z^2 \omega_{11} + \sigma_F^2 \sigma_Z^2} < 1,$$

where we have substituted equation (B.14). This inequality is equivalent to

$$a_Z(1 - a_Z) \left( \frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + (1 - a_F) a_F \sigma_Z^2 \omega_{11} < \sigma_F^2 \sigma_Z^2.$$

This inequality is true since we can show that

$$\begin{aligned}
& a_Z(1 - a_Z) \left( \frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + (1 - a_F) a_F \sigma_Z^2 \omega_{11} \\
= & a_Z(1 - a_Z) \sigma_F^2 \omega_{22} + (1 - a_F) a_F \sigma_Z^2 \omega_{11} \\
< & a_Z(1 - a_Z) \frac{\sigma_F^2 \sigma_Z^2}{(1 - a_Z^2)} + (1 - a_F) a_F \frac{\sigma_F^2 \sigma_Z^2}{(1 - a_F^2)} \\
= & \frac{a_Z}{1 + a_Z} \sigma_F^2 \sigma_Z^2 + \frac{a_F}{1 + a_Z} \sigma_F^2 \sigma_Z^2 \\
< & \sigma_F^2 \sigma_Z^2,
\end{aligned}$$

where we have used (B.12) and the definition of  $\Omega$  to derive

$$\begin{aligned} \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} &= \omega_{22} = \text{Var}\left(Z_t - \hat{Z}_t\right) = \frac{\sigma_Z^2}{1 - a_Z^2} - \text{Var}\left(\hat{Z}_t\right) \\ \omega_{11} &= \text{Var}\left(F_t - \hat{F}_t\right) = \frac{\sigma_F^2}{1 - a_F^2} - \text{Var}\left(\hat{F}_t\right), \end{aligned}$$

and we note

$$a_Z, a_F \in (0, 1) \text{ and } \frac{a_Z}{1 + a_Z} + \frac{a_F}{1 + a_Z} < 1.$$

Therefore, equation (B.19) and  $e_{u1} > 0$  imply that  $p_{u1} < 0$ , which contradicts (B.22) because equation (B.22) implies that  $p_{i1} + p_{u1}$  must be positive.

As a result, we must have  $p_{i1} > 0$ . We then use (B.23) to deduce that  $e_{u1} < 0$ . Since we have shown that the expression in the square bracket in equation (B.19) is negative, we conclude that  $p_{u1} > 0$ . By (B.18), we can show that

$$\text{Cov}_t^i(Q_{t+1}, q_{t+1}) = \mathbf{b}_Q E_t^i[\boldsymbol{\varepsilon}_{t+1} \boldsymbol{\varepsilon}_{t+1}^\top] \mathbf{c}_4^\top = \mathbf{b}_Q \Sigma \mathbf{c}_4^\top = (1 + p_{u1} k_{11}) \sigma_{Dq}, \quad (\text{B.24})$$

which is positive by (B.11). Use this result and equation (A.24) to obtain  $e_{i2} > 0$ . By this result, equation (B.21), and  $a_Z < R$ , we obtain  $p_{i2} < 0$ . Q.E.D.

**Proof of equation (29):** We use equation (21) to compute

$$E[Q_{t+1}|Q_t] = E[E_t^u[Q_{t+1}]|Q_t] = e_0 + e_{i2} E[E_t^u[Z_t]|Q_t] \quad (\text{B.25})$$

$$= e_0 + e_{i2} E[Z_t|Q_t] = e_0 + e_{i2} \frac{\text{Cov}(Z_t, Q_t)}{\text{Var}(Q_t)} Q_t, \quad (\text{B.26})$$

where  $e_0$  and  $e_{i2} > 0$  are constants, as shown in the proof of Proposition 1.

Now we use (15) to compute the unconditional covariance  $\text{Cov}(Z_t, Q_t)$ :

$$\begin{aligned} \text{Cov}(Z_t, Q_t) &= E[Z_t(p_{i1}F_t + p_{u1}\hat{F}_t^u + p_{i2}Z_t + F_t + \varepsilon_t^D \\ &\quad - R(p_{i1}F_{t-1} + p_{u1}\hat{F}_{t-1}^u + p_{i2}Z_{t-1}))] \\ &= E\left[p_{i2}Z_t^2 - p_{i2}RZ_tZ_{t-1} + p_{u1}Z_t\hat{F}_t^u - p_{u1}a_ZRZ_{t-1}\hat{F}_{t-1}^u\right] \\ &= p_{i2}(1 - Ra_Z)\text{Var}(Z_t) + p_{u1}(1 - Ra_Z)E\left(Z_t\hat{F}_t^u\right), \end{aligned}$$

where we use  $E[Z_tF_t] = E[Z_tF_{t-1}] = E[Z_t\varepsilon_t^D] = 0$ .

Multiplying by  $Z_t$  in both sides of (17) and taking expectations, we obtain

$$E\left[Z_t\hat{Z}_t\right] = \text{Var}(Z_t) - \frac{p_{i1}}{p_{i2}}E\left[Z_t\hat{F}_t^u\right].$$

We then derive

$$\frac{p_{i1}}{p_{i2}} E \left[ Z_t \hat{F}_t^u \right] = Var (Z_t) - Var \left( \hat{Z}_t^u \right)$$

using

$$E \left( Z_t \hat{Z}_t^u \right) = E \left[ E_t^u \left( Z_t \hat{Z}_t^u \right) \right] = E \left[ \hat{Z}_t^u \hat{Z}_t^u \right] = Var \left( \hat{Z}_t^u \right).$$

Finally, we obtain

$$Cov (Z_t, Q_t) = (1 - Ra_Z) p_{i2} \left\{ Var (Z_t) + \frac{p_{u1}}{p_{i1}} \left[ Var (Z_t) - Var \left( \hat{Z}_t^u \right) \right] \right\}.$$

Define

$$f_Q = p_{i2} e_{i2} \frac{Var (Z_t) + \frac{p_{u1}}{p_{i1}} \left[ Var (Z_t) - Var \left( \hat{Z}_t^u \right) \right]}{Var (Q_t)}.$$

To show this expression is negative, we only need to prove  $Var (Z_t) > Var \left( \hat{Z}_t^u \right)$  because Proposition 1 shows that  $p_{i2} < 0$ , and  $p_{i1}, p_{u1} > 0$ . This fact follows from the decomposition of variance. Finally, for  $n > 1$ ,

$$\begin{aligned} E [Q_{t+n}|Q_t] &= E [E_t^u [Q_{t+n}] | Q_t] = e_0 + e_{i2} E [E_t^u [Z_{t+n-1}] | Q_t] \\ &= e_0 + a_Z^{n-1} e_{i2} E [Z_t | Q_t] = e_0 + a_Z^{n-1} e_{i2} \frac{Cov (Z_t, Q_t)}{Var (Q_t)} Q_t. \end{aligned}$$

We can then use the previous analysis to obtain the desired result. Q.E.D.

**Proof of equation (31):** Noting that the earnings surprise is in the uninformed investors' information set at time  $t$ ,

$$\begin{aligned} &E [Q_{t+1}|D_t - E_{t-1}^u (D_t)] = E [E_t^u (Q_{t+1}) | D_t - E_{t-1}^u (D_t)] = e_0 + e_{i2} E [Z_t | D_t - E_{t-1}^u (D_t)] \\ &= e_0 + e_{i2} \frac{E \left[ Z_t \left( \varepsilon_t^D + \varepsilon_t^F + a_F (F_{t-1} - \hat{F}_{t-1}^u) \right) \right]}{Var (D_t - E_{t-1}^u (D_t))} [D_t - E_{t-1}^u (D_t)] \\ &= e_0 + e_{i2} a_Z \frac{a_F E \left[ Z_{t-1} (F_{t-1} - \hat{F}_{t-1}^u) \right]}{Var (D_t - E_{t-1}^u (D_t))} [D_t - E_{t-1}^u (D_t)] \\ &= e_0 + e_{i2} a_Z \frac{a_F E \left[ (Z_{t-1} - \hat{Z}_{t-1}^u) (F_{t-1} - \hat{F}_{t-1}^u) \right]}{Var (D_t - E_{t-1}^u (D_t))} [D_t - E_{t-1}^u (D_t)], \end{aligned}$$

where the second line follows from equation (21) and the last line follows from

$$E \left[ \hat{Z}_{t-1}^u (F_{t-1} - \hat{F}_{t-1}^u) \right] = E \left[ \hat{Z}_{t-1}^u E_{t-1}^u (F_{t-1} - \hat{F}_{t-1}^u) \right] = 0.$$

Finally, noting that (17) implies that

$$E \left[ \left( Z_{t-1} - \hat{Z}_{t-1}^u \right) \left( F_{t-1} - \hat{F}_{t-1}^u \right) \right] > 0,$$

we get that

$$d_1 = a_F \frac{E \left[ \left( Z_{t-1} - \hat{Z}_{t-1}^u \right) \left( F_{t-1} - \hat{F}_{t-1}^u \right) \right]}{\text{Var} \left( D_t - E_{t-1}^u \left( D_t \right) \right)} > 0.$$

Similarly, we can show that for all  $n \geq 2$ ,

$$\begin{aligned} E \left[ Q_{t+n} | D_t - E_{t-1}^u \left( D_t \right) \right] &= E \left[ E_t^u \left[ Q_{t+n} \right] | D_t - E_{t-1}^u \left( D_t \right) \right] \\ &= e_0 + e_{i2} E \left[ E_t^u \left[ Z_{t+n-1} \right] | D_t - E_{t-1}^u \left( D_t \right) \right] \\ &= e_0 + e_{i2} a_Z^{n-1} E \left[ Z_t | D_t - E_{t-1}^u \left( D_t \right) \right] \\ &= e_0 + a_Z^{n-1} d_1 \left[ D_t - E_{t-1}^u \left( D_t \right) \right], \end{aligned}$$

as desired. Q.E.D.

## C Solution for the model in Section 5

To solve the model with multiple pieces of advance information, we define the state vector in equation (14), as in Section 3. Unlike Section 3, we define the unforecastable error term based on the period  $t - 1$  information as

$$\boldsymbol{\varepsilon}_t = \left( \varepsilon_{t+k}^D, \varepsilon_t^F, \varepsilon_t^Z, \varepsilon_{t+k}^q, \varepsilon_t^{S_k}, \dots, \varepsilon_t^{S_1} \right)^\top.$$

It is normally distributed with mean zero and covariance matrix  $\Sigma = E \left[ \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^\top \right]$ . The only positive covariance that shows up in  $\Sigma$  is  $E \left( \varepsilon_{t+k}^D \varepsilon_{t+k}^q \right) = \sigma_{Dq} > 0$ . We can apply the same method in Section 3 to solve the model. The only modification lies in the inference problem for informed investors. This problem results in conditional expectations given by

$$\begin{aligned} E_t^i \left[ \varepsilon_{t+k}^D \right] &= \rho_k S_t^k, \\ E_t^i \left[ \varepsilon_{t+j}^D \right] &= E_{t-1}^i \left[ \varepsilon_{t+j}^D \right] + \rho_j S_t^j, \quad 1 \leq j \leq k-1, \end{aligned}$$

and

$$\begin{aligned} E_t^i \left[ \varepsilon_{t+k}^q \right] &= \frac{\sigma_{Dq}}{\sigma_D^2} \rho_k S_t^k, \\ E_t^i \left[ \varepsilon_{t+j}^q \right] &= E_{t-1}^i \left[ \varepsilon_{t+j}^q \right] + \frac{\sigma_{Dq}}{\sigma_D^2} \rho_j S_t^j, \quad 1 \leq j \leq k-1, \end{aligned}$$

where

$$\rho_k = \frac{\sigma_D^2}{\sigma_D^2 + \sigma_{S_k}^2}, \rho_{k-1} = \frac{(1 - \rho_k) \sigma_D^2}{(1 - \rho_k) \sigma_D^2 + \sigma_{S_{k-1}}^2}, \dots, \rho_1 = \frac{(1 - \rho_k) \dots (1 - \rho_2) \sigma_D^2}{(1 - \rho_k) \dots (1 - \rho_2) \sigma_D^2 + \sigma_{S_1}^2}.$$

Because the signals  $S_{t+j-k}^k, \dots, S_{t+j-1}^1$  arrive sequentially and are correlated, each of them contributes to lowering the conditional variance of earnings innovations  $\varepsilon_{t+j}^D$  for  $0 < j < k$ , but, incrementally, the precision of each new signal changes with  $\sigma_{S_j}^2$ . Using these expressions we can construct a new Kalman gain matrix  $\mathbf{K}_i$ . Additional details are available upon request. Q.E.D.

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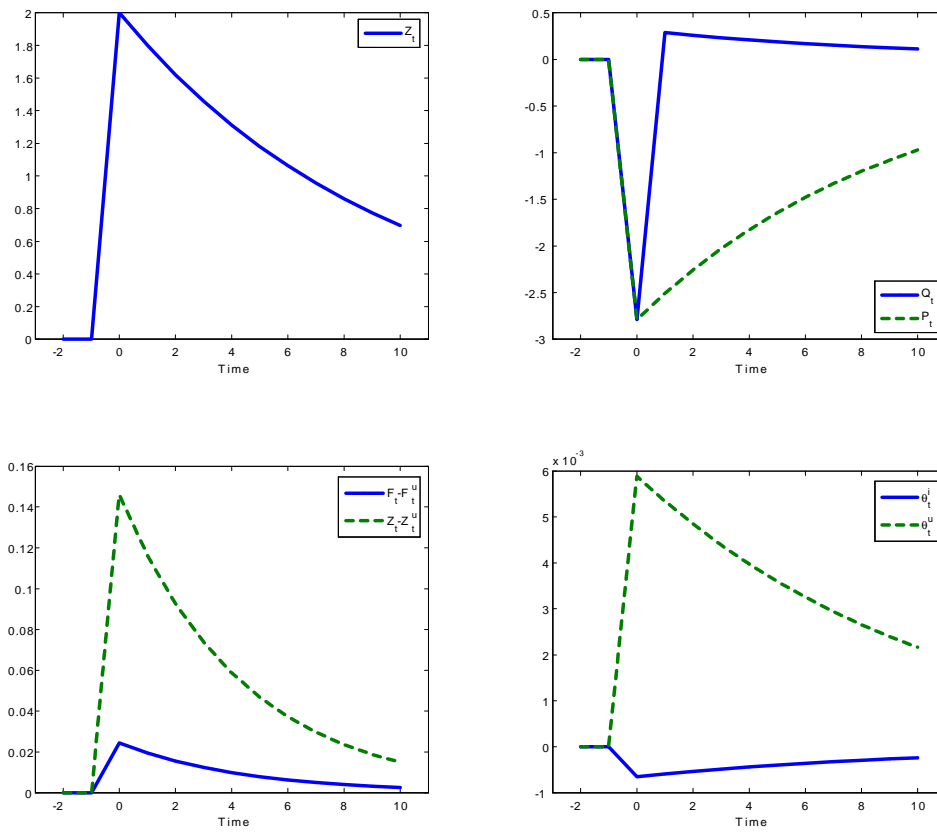


Figure 1: **Impulse response function for a shock to  $Z_t$  in the model without advance information.** Shock equals one standard deviation of  $\varepsilon_t^Z$  and occurs at  $t = 0$ . The economy starts off at the deterministic steady state. All variables are demeaned. Parameters are  $\sigma_D = .5$ ,  $\sigma_{Dq} = 1.35$ ,  $\sigma_F = .1$ ,  $\sigma_Z = 2$ ,  $\sigma_q = 3$ ,  $a_F = a_Z = .9$ ,  $\gamma = 5$ ,  $\lambda = .9$ , and  $r = .0025$ .

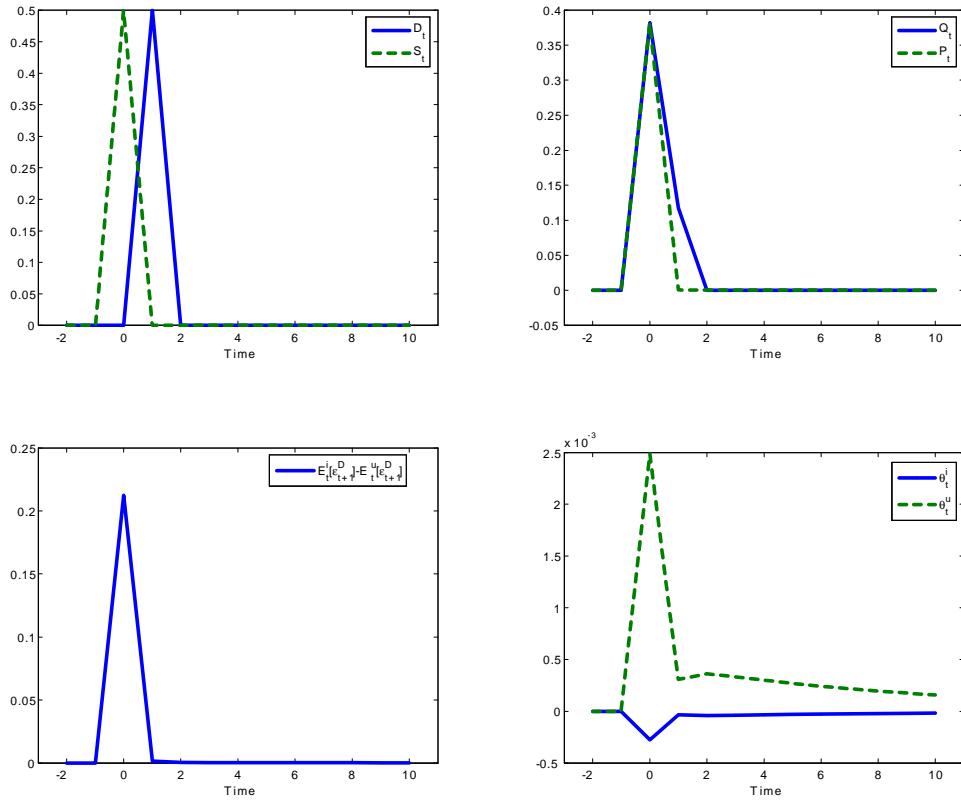


Figure 2: **Impulse response functions for a shock to  $\varepsilon_1^D$  that is first observed through a signal  $S_0$  at time 0.** Shock to  $\varepsilon_1^D$  equals one standard deviation of  $\varepsilon_t^D$ . The economy starts off at the deterministic steady state. All variables are demeaned. Parameters are  $k = 1$ ,  $\sigma_D = .5$ ,  $\sigma_F = .01$ ,  $\sigma_Z = 2$ ,  $\sigma_q = 3$ ,  $\sigma_S = .2$ ,  $\sigma_{Dq} = 1.35$ ,  $a_F = a_Z = .9$ ,  $\gamma = 5$ ,  $\lambda = .9$ , and  $r = .0025$ .

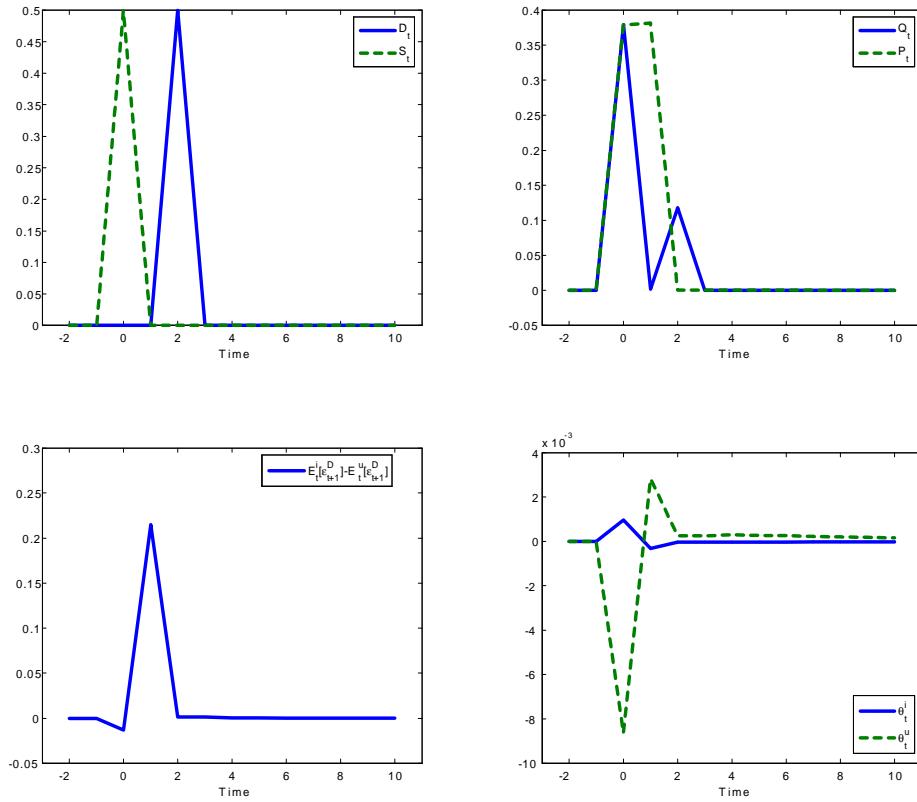


Figure 3: **Impulse response functions for a shock to  $\varepsilon_2^D$  that is first observed through a signal at  $S_0$  at time 0.** Shock to  $\varepsilon_2^D$  equals one standard deviation of  $\varepsilon_t^D$ . The economy starts off at the deterministic steady state. All variables are demeaned. Parameters are  $k = 2$ ,  $\sigma_D = .5$ ,  $\sigma_F = .01$ ,  $\sigma_Z = 2$ ,  $\sigma_q = 3$ ,  $\sigma_S = .2$ ,  $\sigma_{Dq} = 1.35$ ,  $a_F = a_Z = .9$ ,  $\gamma = 5$ ,  $\lambda = .9$ , and  $r = .0025$ .

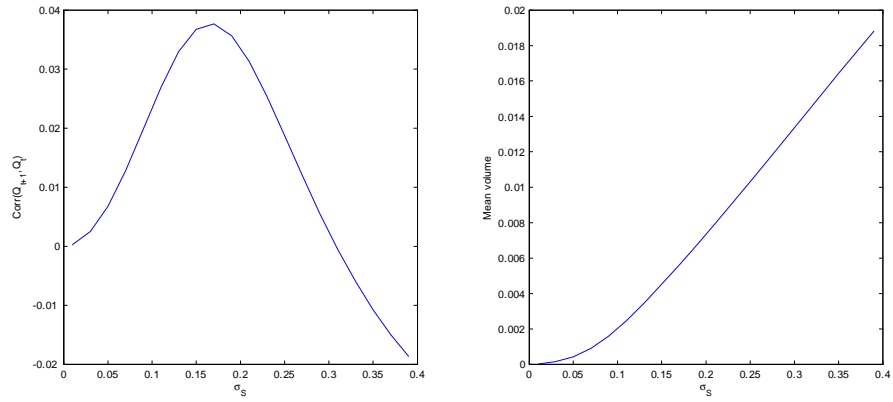


Figure 4: **One period serial correlation in returns (left panel) and trading volume (right panel) for various levels of noise in advance information,  $\sigma_S$ .** Parameters are  $k = 1$ ,  $\sigma_D = .5$ ,  $\sigma_F = .01$ ,  $\sigma_Z = 2$ ,  $\sigma_q = 3$ ,  $\sigma_{Dq} = 1.35$ ,  $a_F = a_Z = .9$ ,  $\gamma = 5$ ,  $\lambda = .9$ , and  $r = .0025$ .

**Table 1.**  
 MOMENTUM AND REVERSAL  
 IN THE MODEL WITH A SINGLE PIECE OF ADVANCE INFORMATION

$n \setminus k$	1		2		3	
	One pe- riod, $b_n$	Cumu- lative	One pe- riod, $b_n$	Cumu- lative	One pe- riod, $b_n$	Cumu- lative
1	0.0337	0.0337	-0.0197	-0.0197	-0.0198	-0.0198
2	-0.0178	0.0160	0.0353	0.0156	-0.0178	-0.0377
3	-0.0160	-0.0000	-0.0161	-0.0005	0.0366	-0.0010
4	-0.0144	-0.0144	-0.0145	-0.0150	-0.0146	-0.0157
5	-0.0130	-0.0274	-0.0131	-0.0281	-0.0132	-0.0288
6	-0.0117	-0.0391	-0.0118	-0.0399	-0.0118	-0.0407
7	-0.0105	-0.0495	-0.0106	-0.0504	-0.0107	-0.0513
8	-0.0094	-0.0590	-0.0095	-0.0600	-0.0096	-0.0609
9	-0.0085	-0.0675	-0.0086	-0.0685	-0.0086	-0.0696
10	-0.0077	-0.0752	-0.0077	-0.0762	-0.0078	-0.0773

The columns labeled “One period” display the slope coefficients of regressing single period returns,  $Q_{t+n}$ , on current returns:

$$Q_{t+n} = a_n + b_n Q_t + \varepsilon_{t,n}.$$

Likewise, the columns labeled “Cumulative” display the slope coefficients of regressing cumulative excess returns,  $Q_{t,t+n}$ , on current returns,  $Q_t$ .  $n$  is the holding period and  $k$  is the number of advance periods in the advance information. We set  $\sigma_D = 0.5$ ,  $\sigma_F = 0.01$ ,  $\sigma_Z = 2$ ,  $\sigma_q = 3$ ,  $\sigma_S = 0.2$ ,  $\sigma_{Dq} = 1.35$ ,  $a_F = a_Z = 0.9$ ,  $\lambda = 0.9$ ,  $\gamma = 5$ , and  $r = 0.0025$ .

**Table 2.**  
 POST-EARNINGS ANNOUNCEMENT DRIFT  
 IN THE MODEL WITH A SINGLE PIECE OF ADVANCE INFORMATION

$n \setminus k$	1		2		3	
	One pe- riod, $d_n$	Cummu- lative	One pe- riod, $d_n$	Cummu- lative	One pe- riod, $d_n$	Cummu- lative
1	0.0593	0.0593	0.0505	0.0505	0.0469	0.0469
2	0.0533	0.1126	0.0501	0.1006	0.0421	0.0891
3	0.0480	0.1606	0.0451	0.1457	0.0423	0.1314
4	0.0432	0.2038	0.0406	0.1863	0.0380	0.1694
5	0.0389	0.2427	0.0365	0.2228	0.0342	0.2036
6	0.0350	0.2776	0.0329	0.2557	0.0308	0.2345
7	0.0315	0.3091	0.0296	0.2852	0.0277	0.2622
8	0.0283	0.3375	0.0266	0.3119	0.0250	0.2872
9	0.0255	0.3630	0.0240	0.3358	0.0225	0.3096
10	0.0230	0.3859	0.0216	0.3574	0.0202	0.3298

The columns labeled “One period” display the slope coefficients of regressing single-period returns,  $Q_{t+n}$ , on the earnings announcement surprise,  $D_t - E_{t-1}^u [D_t]$ :

$$Q_{t+n} = a_n + d_n [D_t - E_{t-1}^u (D_t)] + \varepsilon_{t,n}.$$

Likewise, the columns labeled “Cumulative” display the slope coefficients of regressing cumulative excess returns,  $Q_{t,t+n}$ , on the earnings surprise,  $D_t - E_{t-1}^u [D_t]$ .  $n$  is the holding period and  $k$  is the number of advance periods in the advance information. We set  $\sigma_D = 0.5$ ,  $\sigma_F = 0.01$ ,  $\sigma_Z = 2$ ,  $\sigma_q = 3$ ,  $\sigma_S = 0.2$ ,  $\sigma_{Dq} = 1.35$ ,  $a_F = a_Z = 0.9$ ,  $\lambda = 0.9$ ,  $\gamma = 5$ , and  $r = 0.0025$ .

**Table 3.**  
 MOMENTUM AND REVERSAL  
 IN THE MODEL WITH MULTIPLE PIECES OF ADVANCE INFORMATION

$n \setminus \sigma_{S_1}$	0.1		0.15		0.2		0.30	
	One Period	Cumu- lative	One Period	Cumu- lative	One Period	Cumu- lative	One Period	Cumu- lative
1	0.0180	0.0180	0.0267	0.0267	0.0222	0.0222	-0.0046	-0.0046
2	0.0033	0.0213	0.0018	0.0285	-0.0048	0.0173	-0.0204	-0.0250
3	-0.0018	0.0195	-0.0071	0.0213	-0.0153	0.0020	-0.0281	-0.0530
4	-0.0016	0.0179	-0.0064	0.0149	-0.0138	-0.0118	-0.0252	-0.0783
5	-0.0014	0.0164	-0.0058	0.0091	-0.0124	-0.0242	-0.0227	-0.1010
6	-0.0013	0.0151	-0.0052	0.0039	-0.0112	-0.0354	-0.0204	-0.1214
7	-0.0012	0.0140	-0.0047	-0.0008	-0.0101	-0.0454	-0.0184	-0.1398
8	-0.0011	0.0129	-0.0042	-0.0050	-0.0091	-0.0545	-0.0166	-0.1564
9	-0.0009	0.0120	-0.0038	-0.0088	-0.0081	-0.0626	-0.0149	-0.1713
10	-0.0009	0.0111	-0.0034	-0.0122	-0.0073	-0.0700	-0.0134	-0.1847

The columns labeled “One period” display the slope coefficients of regressing single period returns,  $Q_{t+n}$ , on current returns:

$$Q_{t+n} = a_n + b_n Q_t + \varepsilon_{t,n}.$$

Likewise, the columns labeled “Cumulative” display the slope coefficients of regressing cumulative excess returns,  $Q_{t,t+n}$ , on current returns,  $Q_t$ .  $n$  is the holding period and  $\sigma_{S_1}$  is the variance of advance information about one-period ahead earnings. We set  $k = 2$ ,  $\sigma_D = 0.5$ ,  $\sigma_F = 0.01$ ,  $\sigma_Z = 2$ ,  $\sigma_q = 3$ ,  $\sigma_{S_2} = 1$ ,  $\sigma_{Dq} = 1.35$ ,  $a_F = a_Z = 0.9$ ,  $\lambda = 0.9$ ,  $\gamma = 5$ , and  $r = 0.0025$ .

**Table 4.**  
 POST-EARNINGS ANNOUNCEMENT DRIFT  
 IN THE MODEL WITH MULTIPLE PIECES OF ADVANCE INFORMATION

$n \setminus \sigma_{S_1}$	0.1		0.15		0.2		0.30	
	One period	Cumulative	One period	Cumulative	One period	Cumulative	One period	Cumulative
1	0.0100	0.0100	0.0289	0.0289	0.0407	0.0407	0.0443	0.0443
2	0.0209	0.0309	0.0396	0.0685	0.0478	0.0885	0.0455	0.0898
3	0.0188	0.0497	0.0357	0.1042	0.0430	0.1315	0.0409	0.1307
4	0.0169	0.0666	0.0321	0.1363	0.0387	0.1701	0.0368	0.1676
5	0.0152	0.0819	0.0289	0.1652	0.0348	0.2050	0.0332	0.2007
6	0.0137	0.0956	0.0260	0.1912	0.0313	0.2363	0.0298	0.2306
7	0.0123	0.1079	0.0234	0.2146	0.0282	0.2645	0.0269	0.2574
8	0.0111	0.1191	0.0211	0.2356	0.0254	0.2899	0.0242	0.2816
9	0.0100	0.1291	0.0190	0.2546	0.0228	0.3128	0.0218	0.3034
10	0.0090	0.1381	0.0171	0.2717	0.0206	0.3333	0.0196	0.3229

The columns labeled “One period” display the slope coefficients of regressing single-period returns,  $Q_{t+n}$ , on the earnings announcement surprise,  $D_t - E_{t-1}^u [D_t]$ :

$$Q_{t+n} = a_n + d_n [D_t - E_{t-1}^u (D_t)] + \varepsilon_{t,n}.$$

Likewise, the columns labeled “Cumulative” display the slope coefficients of regressing cumulative excess returns,  $Q_{t,t+n}$ , on the earnings surprise,  $D_t - E_{t-1}^u [D_t]$ .  $n$  is the holding period and  $\sigma_{S_1}$  is the variance of advance information about one-period ahead earnings. We set  $k = 2$ ,  $\sigma_D = 0.5$ ,  $\sigma_F = 0.01$ ,  $\sigma_Z = 2$ ,  $\sigma_q = 3$ ,  $\sigma_{S_2} = 1$ ,  $\sigma_{Dq} = 1.35$ ,  $a_F = a_Z = 0.9$ ,  $\lambda = 0.9$ ,  $\gamma = 5$ , and  $r = 0.0025$ .