Asset Bubbles and Foreign Interest Rate Shocks*

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Abstract

We provide a DSGE model of a small open economy with both domestic and international financial market frictions. Firms face credit constraints and trade an intrinsically useless asset. Low foreign interest rates are conducive to bubble formation. An asset bubble provides liquidity and relaxes credit constraints. It provides a powerful amplification and propagation mechanism. Our estimated model based on Bayesian methods explains high volatilities of consumption and stock prices relative to output, countercyclical trade balance, and procyclical stock prices observed in the Mexican data over the period 1990Q1-2011Q4.

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1 Introduction

After the 2008 global financial crisis, strong expansionary monetary policies implemented by major advanced economies stimulated recovery in capital inflows into many emerging market economies. In 2013, following Bernanke's congressional testimony about the Federal Reserve's potential goal to bring the expansionary monetary policy to its normalcy, many emerging economies experienced remarkable capital flow reversals. From the experiences of several emerging markets in the past two decades and recent capital flow reversals,[1] we have learned the stylized facts of "Sudden Stops:" the reversal of international capital flows, the sudden increase in net exports and the corresponding increase from large current account deficits to smaller deficits or smaller surpluses, declines in production and consumption, real exchange rate depreciation, and a collapse in asset prices.[2] Furthermore, Uribe and Yue (2006) and Maćkowiak (2007) document empirical evidence that US interest rate shocks are an important driver of business cycles in emerging economies. There are also discussions that a low foreign interest rate can cause capital inflows and fuel asset bubbles in housing markets. An increase in the foreign interest rate can cause asset bubbles to burst and have a large adverse impact on the domestic economy.

The goal of our paper is to develop a dynamic stochastic general equilibrium (DSGE) model to understand the impact of foreign interest rate shocks on emerging economies. We incorporate asset bubbles in a small open economy model with frictions in both the domestic credit market and the international financial market. Domestic firms use capital, imported materials, and labor to produce output. Domestic and foreign goods can be exchanged at a real exchange rate (the price of foreign goods in terms of domestic consumption goods). Following Aoki, Benigno, and Kiyotaki (2016) and Chang, Liu, and Spiegel (2015), we specify an exogenous function of foreign demand for the domestic country's exported goods, which is positively related to the real exchange rate. Firms face idiosyncratic investment efficiency shocks and credit constraints.

Firms borrow from domestic and international financial markets using capital as collateral. International financial transactions are intermediated by financial institutions or banks subject to portfolio adjustment costs. Portfolio adjustment costs represent frictions in the international financial markets and cause the interest rate parity condition to fail. The main ingredient of our model is to introduce an intrinsically useless asset for firms to trade. When the foreign interest

[1] Examples include southeast Asia and Russia in the late 1990s, South America in the early 2000s, and peripheral Europe in the late 2000s.

[2] See Mendoza (2010) and Korinek and Mendoza (2014) for a summary of this evidence.

rate is lower than the domestic interest rate, there is capital inflow. The demand for domestic bonds generates a low domestic interest rate and fuels a bubble. If all agents believe that the intrinsically useless asset has value, it can provide liquidity for firms to finance real investment. This belief can be self-fulfilling and results in a bubbly equilibrium. In particular, efficient firms sell the bubble asset to inefficient firms and use the proceeds to finance real investment. This is the crowd-in effect of the bubble. On the other hand, inefficient firms buy the bubble asset from efficient firms and do not make real investment. This is the crowd-out effect. The net effect on aggregate investment is ambiguous depending on parameter values. We show that there can also exist another type of equilibrium in which a bubble does not emerge. When no one believes that the intrinsically useless asset has value, this belief can also be self-fulfilling and results in a bubbleless equilibrium.

When the foreign interest rate rises, capital begins to flow out of the domestic country. This generates a current account surplus or a shrink of the current account deficit. The increased supply of domestic goods in foreign markets leads to real depreciation, causing imports to decline. Decreased imported inputs lower output and hence investment and consumption. Meanwhile, investing in the international financial market crowds out resources for domestic firms to buy the bubble asset. Moreover, payoffs from bonds can also help provide liquidity to finance real investment. Thus the demand for the bubble asset is weakened by capital outflows, thereby dampening the asset bubble. When the foreign interest rate is sufficiently high, the asset bubble can burst.

Our first contribution is to provide a theoretical result to characterize the condition for the emergence of an asset bubble in an infinite-horizon small open economy. This result extends the existing results of Tirole (1985), Santos and Woodford (1997), Miao, Wang, and Zhou (2015), and Miao and Wang (2018) for closed economies. Our second contribution is to provide a steady-state analysis of the impact of the foreign interest rate. We prove that a low foreign interest rate facilitates a domestic asset bubble. The size of the bubble decreases with the foreign interest rate. When the foreign interest rate is sufficiently high, the asset bubble can burst.

Our third and most important contribution is to quantitatively evaluate the impact of foreign interest rate shocks by taking our model with asset bubbles to the Mexican data over the period 1990Q1-2011Q4 using Bayesian estimation.[3] We also include four other types of exogenous

[3] See An and Schorfheide (2007) for a survey of Bayesian methods to estimate DSGE models.

shocks often used in the literature (long- and short-run productivity shocks, preference shocks, and foreign demand shocks) to fit five time series of the demeaned foreign interest rate, the sum of the US and Canada real GDP, the Mexican real GDP, and the Mexican real investment and real consumption. We find that our estimated model matches the Mexican data of business cycles and stock prices reasonably well. In particular, our estimated model matches the salient features of high volatilities of consumption and stock prices relative to output, countercyclical trade balance, and procyclical stock prices. Our key insight is that asset bubble provides a powerful amplification and propagation mechanism. To see the importance of this mechanism, we also estimate a model without asset bubble. We find that this model performs much worse. In particular, it cannot match the high volatility of stock prices and the countercyclicality of trade balance. The data also favors our bubbly model based on the marginal data density.

Our paper is related to three strands of the literature. First, our paper builds on the literature on international real business cycles (RBC).[4] Aguiar and Gopinath (2007) argue that the long-run productivity shock in a standard RBC model is important to explain the high consumption volatility and the countercyclical trade balance in emerging markets. By contrast, García-Cicco, Pancrazi, and Uribe (2010) find that when estimated over the long sample, the RBC model driven by permanent and transitory productivity shocks does a poor job at explaining observed business cycles in Argentina and Mexico along a number of dimensions. Neumeyer and Perri (2005) and Uribe and Yue (2006) argue instead that the introduction of foreign interest rate shocks coupled with financial frictions is important to explain the empirical regularities of emerging economies. Chang and Fernández (2013) estimate a model with financial frictions that includes all these three types of shocks using Bayesian methods. They find a dominant role played by financial frictions in amplifying conventional productivity shocks and, less markedly, interest rate shocks; trend shocks, in contrast, play a very minor role.

This literature typically ignores asset prices and does not study the high stock market volatility and the comovement of stock prices and the real economy. We contribute to this literature by showing that asset bubbles are important to explain these facts. To the best of our knowledge, our paper is the first one to use Bayesian methods to estimate a DSGE model of a small open economy with asset bubbles.[5]

Second, our paper is related to the recent literature on Sudden Stops (e.g, Calvo (1998),

[4] This literature is too large for us to discuss all of them. Important related early contributions include Mendoza (1991) and Backus, Kehoe, and Kydland (1992).

[5] Miao, Wang, and Xu (2015) provide a Bayesian DSGE model of a closed economy with stock price bubbles.

Gopinath (2004), Martin and Rey (2006), Gertler, Gilchrist, and Natalucci (2007), and Mendoza (2010), Fernández and Gulan (2015), and Aoki, Benigno, and Kiyotaki (2016)).[6] This literature views credit frictions as the central feature of the transmission mechanism that drives Sudden Stops. Mendoza (2010) also emphasizes the amplification and asymmetry of macroeconomic fluctuations that result from the debt-deflation transmission mechanism. Asset prices in this literature typically refer to capital prices or Tobin's (marginal) Q. By contrast, firms can own both capital and bubble assets in our model. The stock market value of the firm contains a fundamental component, equal to Tobin's Q multiplied by the capital stock, and a bubble component, equal to the value of the bubble asset. The movement of both Tobin's Q and asset bubbles contributes to the stock market fluctuations.

Third, our paper is related to the recent literature on asset bubbles in open economies (e.g., Caballero and Krishnamurthy (2006), Ventura (2012), Basco (2014), and Martin and Ventura (2015a,b)). This literature typically adopts the overlapping-generations (OLG) framework. Like our paper, this literature emphasizes the importance of credit constraints. Martin and Ventura (2012, 2015a,b) also discuss the crowd-in and crowd-out effects of asset bubbles, similar to those in our paper. Our paper differs from this literature in the addressed questions and modeling details. More importantly, unlike the OLG models, our model is in the DSGE framework, which can confront with the data using Bayesian estimation. In our infinite-horizon framework credit constraints are essential for the emergence of asset bubbles as in Kiyotaki and Moore (2008) in the sense that a bubble could not emerge without credit constraints. By contrast, a bubble can still emerge in OLG models without credit constraints and their presence allows bubbles to emerge in dynamically efficient economies (Farhi and Tirole (2012) and Martin and Ventura (2012)). Our infinite-horizon DSGE model complements the existing OLG models.

2 The Model

Consider a discrete-time (real) DSGE model of a small open economy populated by a representative household, a continuum of identical capital goods producers with a unit measure, and a continuum of ex ante identical but ex post heterogeneous firms with a unit measure. There is no government or monetary authority. The household consists of two types of members: workers and bankers. Workers supply labor to firms and trade firm shares. Financial transactions between domestic and foreign residents are intermediated by domestic financial

[6] See Korinek and Mendoza (2014) for a survey of this literature.

institutions or simply bankers. Firms buy capital goods from capital producers. Each firm is subject to idiosyncratic investment efficiency shocks. Suppose that a law of large numbers holds for idiosyncratic shocks.

2.1 Firms

There are three types of goods: domestic consumption goods, domestic capital goods, and foreign goods. Firms in the small open economy use foreign goods as an input factor to produce domestic consumption goods. As Mendoza (2010) emphasizes, imported inputs are important for the initial drop of output during a Sudden Stop. In each period $t = 0, 1, ...$, one unit of foreign goods can be exchanged for e_t units of domestic consumption goods (e_t is called the real exchange rate). The total demand of the rest of the world for the domestic consumption goods is exogenously given by

$$X_t = e_t^{\sigma} Y_t^*, \tag{1}$$

where $\sigma > 0$ and Y_t^* denotes an exogenous component of foreign demand. When e_t is larger, the domestic consumption goods are cheaper and hence foreign demand is larger.

A domestic firm indexed by $j \in [0, 1]$ uses a constant-return-to-scale technology to produce output Y_{jt} according to

$$Y_{jt} = K_{jt-1}^{\alpha}(A_t N_{jt})^{1-\alpha-\gamma} M_{jt}^{\gamma}, \quad \alpha \in (0,1), \quad \gamma \in (0,1), \quad \alpha + \gamma \in (0,1), \tag{2}$$

where A_t, K_{jt-1}, N_{jt} and M_{jt}, represent aggregate productivity, capital input, labor input, and imported material input, respectively. Let $A_t = A_t^g \exp(a_t)$ where A_t^g is the trend productivity and a_t is the transitory productivity (Aguiar and Gopinath (2007) and García-Cicco, Pancrazi, and Uribe (2010)). Assume that $A_t^g = A_{t-1}^g \exp(g_t)$,

$$\begin{aligned} g_t &= \left(1 - \rho_g\right) g + \rho_g g_{t-1} + \sigma_g \varepsilon_{gt}, \\ a_t &= \rho_a a_{t-1} + \sigma_a \varepsilon_{at}, \end{aligned}$$

where $g > 0$, $\rho_a, \rho_g \in (-1, 1)$, and $\sigma_a, \sigma_g > 0$. The positive growth rate g ensures a positive steady-state net interest rate. For balanced growth, suppose that Y_t^* grows at the same rate g on average. Let $Y_t^* = A_t y_t^*$, where y_t^* follows an AR(1) process

$$\ln(y_t^*) = \rho_{y^*} \ln\left(y_{t-1}^*\right) + \sigma_{y^*} \varepsilon_{y^* t}.$$

Here $\rho_{y^*} \in (-1, 1)$ and $\sigma_{y^*} > 0$. Assume that all innovations ε_{at}, ε_{gt}, and $\varepsilon_{y^* t}$ are IID standard normal random variables and independent of each other.

Firm j solves the following static labor and material input choice problem:

$$\max_{N_{jt},M_{jt}} \; K_{jt-1}^{\alpha}(A_tN_{jt})^{1-\alpha-\gamma}M_{jt}^{\gamma} - W_tN_{jt} - e_tM_{jt}, \tag{3}$$

where W_t denotes the wage rate. It is straightforward to show that the maximized objective is equal to $R_{kt}K_{jt-1}$, where R_{kt} satisfies

$$R_{kt} = \alpha A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}}. \tag{4}$$

We will show later that R_{kt} is equal to the marginal product of capital.

To make investment in period t, firm j purchases I_{jt} units of new capital goods from domestic capital producers at price P_{kt}. One unit of newly purchased capital is transformed into ε_{jt} units of installed capital so that the law of the motion for capital follows

$$K_{jt} = (1-\delta)K_{jt-1} + \varepsilon_{jt}I_{jt}, \tag{5}$$

where $\delta \in (0,1)$ represents the depreciation rate. Here ε_{jt} represents a firm specific investment efficiency shock that is assumed to be drawn independently and identically across firms and over time from the cumulative distribution function F (the density function is f) on $[\varepsilon_{\min}, \varepsilon_{\max}] \subset [0,\infty)$. Assume that there is no insurance market against the idiosyncratic investment efficiency shock and that investment is irreversible at the firm level so that $I_{jt} \geq 0$.

Firms can trade two types of assets: a one-period risk-free bond and a bubble asset. One unit of the bond delivers one unit of domestic consumption goods in the next period. Let R_{ft} denote the domestic market interest rate between periods t and $t+1$. When firm j's bond holdings B_{jt} in period t satisfy $B_{jt} < (\geq) 0$, B_{jt} is interpreted as borrowing (saving). Firms can borrow or lend abroad, but this must be intermediated by the bankers only. Firms face borrowing constraints and use their physical capital as collateral. The credit constraint is given by

$$\frac{B_{jt}}{R_{ft}} \geq -\mu K_{jt-1}, \tag{6}$$

where $\mu \in (0,1)$ is pledgeability parameter and reflect frictions in the domestic financial market.[7]

[7] We do not use the bubble asset as collateral and use the current value of capital as collateral only to simplify algebra. This can be justified by a particular debt contract form. As Caballero and Krishnamurthy (2006), Miao, Wang, Zhou (2015), and Miao and Wang (2018) show, using future capital value as collateral as in Kiyotaki and Moore (1997) will complicate algebra without changing any key insights.

The bubble asset is intrinsically useless and we may think of it as uncultivated land or some toxic asset. Normalize its supply to one. Let H_{jt} denote firm j's holdings of the bubble asset chosen in period t. Assume that firms cannot short the bubble asset so that $H_{jt} \geq 0$.

The flow-of-funds constraint for firm j is given by

$$D_{jt} = R_{kt}K_{jt-1} - P_{kt}I_{jt} - \frac{B_{jt}}{R_{ft}} + B_{jt-1} + P_t\left(H_{jt-1} - H_{jt}\right), \tag{7}$$

where D_{jt} and P_t denote dividends and the price of the bubble asset, respectively. Assume that equity financing is too costly for firms to raise new funds. Then the firm faces the following equity constraint

$$D_{jt} \geq 0. \tag{8}$$

Now we describe firm j's decision problem by dynamic programming. Let $V_t(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1})$ denote firm j's value function, where we suppress aggregate state variables as arguments. The dynamic programming problem is given by

$$V_t(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1}) = \max_{H_{jt}, I_{jt} \geq 0,\ B_{jt}} D_{jt} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt}),$$

subject to (5), (6), (7), and (8). Here $\beta \in (0,1)$ denotes the subjective discount factor and $\mathbb{E}_t$ represents the conditional expectation operator. Assume that firms are owned by households so that we use the representative household's marginal utility Λ_{t+1} to discount future dividends.

2.2 Capital Producers

A representative capital goods producer creates domestic capital goods using input of domestic consumption goods subject to flow adjustment costs. They sell new capital goods to investing firms at price P_{kt} in period t. The objective function of a capital goods producer is to choose $\{I_t\}$ to solve

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} D_t^k,$$

where

$$D_t^k = P_{kt}I_t - \left[1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)^2\right] I_t \tag{9}$$

represents dividends and $\Omega_k > 0$ is an adjustment cost parameter. The adjustment cost vanishes on the deterministic balanced growth path. The optimal choice of new capital goods I_t satisfies

the first-order condition:

$$P_{kt} = 1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)^2 + \Omega_k\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)\frac{I_t}{I_{t-1}} - \beta\Omega_k\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{I_{t+1}}{I_t} - \exp(g)\right)\left(\frac{I_{t+1}}{I_t}\right)^2. \tag{10}$$

2.3 Bankers

Bankers intermediate financial transactions internationally. The international financial transactions are subject to quadratic adjustment costs that are rebated to the households in a lump-sum manner.[8] Let R^*_{ft} denote the exogenous foreign interest rate between periods t and $t+1$. Assume that R^*_{ft} follows an AR(1) process

$$\ln\left(R^*_{ft}\right) = \left(1 - \rho_{R^*_f}\right)\ln\left(R^*_f\right) + \rho_{R^*_f}\ln\left(R^*_{ft-1}\right) + \sigma_{R^*_f}\varepsilon_{R^*_f t},$$

where $\rho_{R^*_f} \in (-1, 1)$, $\sigma_{R^*_f} > 0$, and the innovation $\varepsilon_{R^*_f t}$ is an IID standard normal random variable that is independent of all other innovations in the model.

The flow-of-funds constraint of a representative banker is given by

$$D^b_t = \frac{B_{t-1}}{R_{ft-1}e_{t-1}}R^*_{ft-1}e_t - B_{t-1} - \left[\frac{\Omega}{2Y_t}\left(B_t - A_{t-1}b\right)^2 - \Omega_0 A_t\right], \tag{11}$$

where D^b_t is the profit that the banker gives to his family in period t, $B_t \geq (<) 0$ is the bond supply (demand) from the banker, and the expression in the square bracket represents the portfolio adjustment costs. Along the deterministic balanced growth path, A_t, Y_t, B_t and the adjustment costs all grow at rate g. Along this path, we let $b = B_t/A_t$ and choose Ω_0 such that the adjustment costs vanish.[9] When $\Omega = \Omega_0 = 0$, we have free capital mobility and the interest rate parity condition holds.

The interpretation of (11) is as follows. The banker sells B_{t-1} units of bonds to domestic residents in period $t-1$ at the price $1/R_{ft-1}$ and converts the proceed into units of foreign goods at the rate $1/e_{t-1}$. It then saves in a foreign bank and gets interest at the gross rate R^*_{ft-1} in period t. After converting into units of domestic consumption goods at rate e_t, the banker obtains returns $\frac{B_{t-1}}{R_{ft-1}e_{t-1}}R^*_{ft-1}e_t$. After subtracting debt repayment B_{t-1} and adjustment costs

[8] See Aoki, Benigno, and Kiyotaki (2016) for related modeling. Our modeling of bankers is similar to that described in Chapter 4 of Uribe and Schmitt-Grohe (2017). One difference is that we allow for interest rate differential in the steady state.

[9] Specifically, $\Omega_0 = \frac{\Omega}{2y}\left(\frac{\exp(g)-1}{\exp(g)}b\right)^2$ where y denotes the detrended steady state value of Y_t.

incurred for the choice of B_t in period t, we obtain the profit D_t^b given in equation (11). The banker chooses $\{B_t\}$ to maximize the expected present value of profits:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} D_t^b.$$

2.4 Households

Each household is an extended family consisting of a continuum of identical workers of unit mass and a continuum of identical bankers also of unit mass. Each worker in the family supplies labor to firms and hands in the wage income to the family. The family pools the labor income from the workers and dividends from the bankers, firms, and capital goods producers, and distributes them equally among family members. A representative household chooses shareholdings $\{\psi_{j,t+1}\}$, the family consumption and labor supply, $\{C_t\}$ and $\{N_t\}$, to maximize its lifetime utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \xi_t \beta^t \left[\ln(C_t - hC_{t-1}) - \nu \frac{N_t^{1+\varphi}}{1+\varphi} \right],$$

subject to the budget constraints

$$\begin{aligned} \int \psi_{j,t+1} \left(V_{jt} - D_{jt} \right) dj + C_t &= W_t N_t + \int \psi_{jt} V_{jt} dj \\ &\quad + D_t^b + D_t^k + \left[\frac{\Omega}{2Y_t} \left(B_t - A_{t-1} b \right)^2 - \Omega_0 A_t \right], \end{aligned} \tag{12}$$

where V_{jt} denotes firm j's stock market value. The parameter h governs the strength of consumption habit formation, $1/\varphi$ is the Frisch elasticity of labor supply, and ν is a weight on labor disutility. The variable ξ_t represents an exogenous preference shock that follows an AR(1) process

$$\ln(\xi_t) = \rho_\xi \ln\left(\xi_{t-1}\right) + \sigma_\xi \varepsilon_{\xi t},$$

where $\rho_\xi \in (-1, 1)$, $\sigma_\xi > 0$, and $\varepsilon_{\xi t}$ is an IID standard normal random variable. Given the utility function, the marginal utility is

$$\Lambda_t = \frac{\xi_t}{C_t - hC_{t-1}} - \beta h \mathbb{E}_t \frac{\xi_{t+1}}{C_{t+1} - hC_t}, \tag{13}$$

and the stochastic discount factor for asset pricing is $\beta \Lambda_{t+1} / \Lambda_t$.

Notice that we have assumed that households do not trade bonds or bubble assets. Allowing them to trade these assets will not affect our results if we assume that households cannot borrow and face a short-sales constraint on the bubble asset. In this case households will choose not to

hold any assets because their equilibrium returns are too low (see Kiyotaki and Moore (2008) and Miao, Wang, and Zhou (2015) for a similar result). We will show this point in (21) later.

2.5 Competitive Equilibrium

Denote $K_t = \int K_{jt} dj$, $M_t = \int_0^1 M_{jt} dj$, and $Y_t = \int Y_{jt} dj$. A competitive equilibrium consists of sequences of aggregate quantities $\{C_t, K_t, I_t, Y_t, B_t, H_t, M_t\}$, shareholdings $\{\psi_{j,t+1}\}$, and prices $\{W_t, P_t, R_{kt}, R_{ft}, e_t, P_{kt}\}$ such that:

(i) Households, capital goods producers, firms, workers, and bankers optimize.

(ii) The markets for labor, the bubble asset, domestic capital goods, bonds, stocks, and domestic consumption goods all clear so that

$$N_t = \int_0^1 N_{jt} dj, \; H_t = \int_0^1 H_{jt} dj = 1, \; I_t = \int_0^1 I_{jt} dj, \tag{14}$$

$$B_t = \int_0^1 B_{jt} dj, \; \psi_{j,t+1} = 1, \tag{15}$$

$$Y_t = C_t + \left[1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)^2\right] I_t + X_t. \tag{16}$$

(iii) The law of motion of aggregate capital follows

$$K_t = (1-\delta)K_{t-1} + \int_0^1 \varepsilon_{jt} I_{jt} dj.$$

3 Equilibrium System

We first derive the solution to the firm's decision problem and then characterize the equilibrium system.

3.1 Firms' Decision Problem

Define Tobin's (marginal) Q as

$$Q_t = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \frac{\partial V_{t+1}(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt})}{\partial K_{jt}}.$$

Notice that the new capital good price P_{kt} is not equal to Q_t in general due to idiosyncratic investment efficiency shocks. The following proposition characterizes firm j's optimal policies.

Proposition 1 *Denote* $\bar{\varepsilon}_t = P_{kt}/Q_t \in (\varepsilon_{\min}, \varepsilon_{\max})$.

(i) When $\varepsilon_{jt} \geq \bar{\varepsilon}_t$, firm j makes real investment,

$$I_{jt} = \frac{1}{P_{kt}} \left(R_{kt} K_{jt-1} + \mu K_{jt-1} + B_{jt-1} + P_t H_{jt-1} \right), \tag{17}$$

sells all its bubble asset, i.e., $H_{jt} = 0$, and exhausts its borrowing limit.

(ii) When $\varepsilon_{jt} < \bar{\varepsilon}_t$, firm j makes no real investment and is willing to hold any amount of bubble and bonds as long as condition (6) holds.

(iii) The Tobin's Q, the bubble price and the domestic interest rate satisfy

$$Q_t = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[R_{kt+1} + (1-\delta) Q_{t+1} + (R_{kt+1} + \mu) \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{18}$$

$$P_t = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1} \left[1 + \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{19}$$

$$\frac{1}{R_{ft}} = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[1 + \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{20}$$

and the usual transversality conditions.

Equation (17) shows that investment by efficient firms is financed by four sources: internal funds $R_{kt} K_{jt-1}$, debt collateralized by capital, μK_{jt-1}, payoffs from bond holdings B_{jt-1}, and sales of the bubble asset $P_t H_{jt-1}$. Equations (18), (19), and (20) are the asset-pricing equations for capital, the bubble asset, and the bond, respectively. The integral terms in these three equations capture the liquidity premium because capital, the bubble asset (if its price is positive), and the bond can raise the firm's net worth. We focus on the integral term in equation (19). At time $t+1$, when its investment efficiency shock $\varepsilon_{jt+1} \geq \bar{\varepsilon}_{t+1}$, firm j sells its bubble asset to finance real investment. Each dollar of the payoff can generate $(\varepsilon_{jt+1} Q_{t+1} / P_{kt+1} - 1)$ units of profits. The interpretation for the liquidity premium provided by capital and bonds are similar. The bubble asset and bonds are perfect substitutes. This insight is first developed by Kiyotaki and Moore (2008) when the bubble asset is viewed as fiat money. Notice that the investment cutoff $\bar{\varepsilon}_{t+1}$ is identical for all firms, a feature useful for aggregation.

To see the importance of the liquidity premium for the emergence of a bubble, we write the asset-pricing equation for the bubble without the liquidity premium as follows

$$P_t = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1}.$$

This equation cannot hold in a deterministic steady state with a positive bubble price for $\beta \in (0,1)$. In other words, the transversality condition will rule out bubbles.

By equations (19) and (20), we have

$$P_t > \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1} \text{ and } \frac{1}{R_{ft}} > \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t}. \tag{21}$$

These inequalities imply that households will not hold any bubble assets or bonds in equilibrium because their returns are too low.

3.2 Equilibrium System

The home country imports foreign goods and exports domestic consumption goods. Thus $NX_t = X_t - e_t M_t$ represents the trade balance or net exports. Substituting the flow-of-funds for the firms, capital goods producers and bankers, (7), (9) and (11), into the budget constraint of households, (12), and combining the resulting equation with the resource constraint (16), we obtain

$$X_t - e_t M_t = \frac{B_t}{R_{ft}} - B_{t-1} \frac{R^*_{ft-1}}{R_{ft-1}} \frac{e_t}{e_{t-1}}. \tag{22}$$

After aggregating individual decision rules and imposing market-clearing conditions, we obtain the equilibrium system shown by the following proposition. The detrended version of the equilibrium system is presented in Appendix B.

Proposition 2 *The equilibrium system is given by the following equations: (1), (10), (13), (16), (18), (19), (20), (22),* $\bar{\varepsilon}_t Q_t = P_{kt}$,

$$\begin{aligned}
0 &= \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{R^*_{ft}}{R_{ft}} \frac{e_{t+1}}{e_t} - 1 \right) - \frac{\Omega}{Y_t} \left(B_t - A_{t-1} b \right), && (23)\\
I_t &= \frac{1}{P_{kt}} \left(R_{kt} K_{t-1} + \mu K_{t-1} + B_{t-1} + P_t \right) \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} f(\varepsilon) d\varepsilon, && (24)\\
K_t &= (1-\delta) K_{t-1} + \frac{1}{P_{kt}} \left(R_{kt} K_{t-1} + \mu K_{t-1} + B_{t-1} + P_t \right) \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \varepsilon f(\varepsilon) d\varepsilon, && (25)\\
Y_t &= K_{t-1}^{\alpha} M_t^{\gamma} \left(A_t N_t \right)^{1-\alpha-\gamma}, && (26)\\
e_t M_t &= \gamma Y_t, && (27)\\
N_t &= A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t} \right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t} \right)^{\frac{\gamma}{\alpha}} K_{t-1}, && (28)\\
R_{kt} &= \alpha K_{t-1}^{\alpha-1} M_t^{\gamma} \left(A_t N_t \right)^{1-\alpha-\gamma}, && (29)\\
\nu \xi_t N_t^{\varphi} &= W_t \Lambda_t, && (30)
\end{aligned}$$

for the endogenous variables $\{Q_t, \bar{\varepsilon}_t, P_t, R_{ft}, B_t, I_t, K_t, Y_t, M_t, C_t, N_t, X_t, e_t, W_t, R_{kt}, \Lambda_t, P_{kt}\}$. *The usual transversality conditions hold.*

Equation (23) is the no-arbitrage condition for the international financial transaction derived from the banker's optimization problem. When there is no portfolio adjustment cost ($\Omega = 0$), we obtain the interest rate parity condition. This condition does not hold whenever there are adjustment costs ($\Omega > 0$). Equation (24) gives aggregate investment, which is financed by internal funds $R_{kt}K_{t-1}$, debt collateralized by physical capital μK_{t-1}, sales of the bubble asset P_t, and the repayment from bonds B_{t-1}. The integral term in this equation reflects the fact that only firms with efficiency levels higher than $\bar{\varepsilon}_t$ make investment. If there is a bubble $P_t > 0$, then each efficient firm with its efficiency level higher than $\bar{\varepsilon}_t$ will make more investment. This is the crowd-in effect of the asset bubble. On the other hand, inefficient firms with their efficient levels lower than $\bar{\varepsilon}_t$ have to buy the bubble from efficient firms and do not make investment. This is the crowd-out effect. The net effect of an asset bubble on aggregate investment depends on the relative strength of the crowd-in and crowd-out effects.

Equation (25) gives the law of motion for aggregate capital. Equation (26) gives aggregate output. Equation (27) shows that the imports/output ratio is equal to γ due to the Cobb-Douglas production function. Thus GDP in our model is given by

$$GDP_t = C_t + \left[1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)^2\right] I_t + X_t - e_t M_t = (1-\gamma)Y_t.$$

Equations (28) and (30) give the labor demand and supply relations. Equation (29) shows that R_{kt} is equal to the marginal product of capital or the rental rate. We can show that the stock market value of all firms is given by[10]

$$V_t = \int \mathbb{E}_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{jt+1}\right] dj = Q_t K_t + P_t + \frac{B_t}{R_{ft}}. \tag{31}$$

It consists of three components: capital value $Q_t K_t$, bubble P_t, and bond value B_t/R_{ft}. We will show later that the movements of the asset bubble P_t is important for the stock market fluctuations.

Note that $P_t = 0$ for all t always satisfies (19). We call such an equilibrium bubbleless equilibrium. If there is an equilibrium such that $P_t > 0$ for all t, we call it bubbly equilibrium. We will focus on these two types of equilibrium in this paper.[11]

[10] Use equations (A.3)-(A.6) in Appendix A and apply the market-clearing conditions.

[11] It is possible to have other types of equilibrium such as sunspot equilibria (Tirole (1985) and Weil (1987)).

4 Deterministic Steady-State Equilibria

The economy features stochastic trend. In particular, the variables N_t, e_t, $\bar{\varepsilon}_t$, R_{kt}, R_{ft}, Q_t, and P_{kt} do not have a trend, while all other equilibrium variables grow at the rate of productivity A_t except for Λ_t which declines with A_t. We thus denote $\lambda_t = \Lambda_t A_t$ and normalize any other growing variable, say Z_t, by A_t and use a lower case variable $z_t = Z_t/A_t$ to denote the detrended value. In Appendix B we present the detrended equilibrium system. In this section we study deterministic steady state, in which all detrended variables are constant over time. We use a variable without the time subscript t to denote its steady-state value. We use a subscript f to denote any variable in a bubbleless (fundamental) steady state, while we use a subscript b to denote a variable in a bubbly steady state.

We first derive some common equations in both types of steady state. We use equation (18) and $Q = P_k/\bar{\varepsilon}$ where $P_k = 1$ to derive the steady-state rental rate of capital

$$R_k = \frac{\exp(g) - \beta + \beta\delta - \beta\mu\left(\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon) d\varepsilon\right)}{\beta\left[\bar{\varepsilon} + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon) d\varepsilon\right]}. \tag{32}$$

We then use (20) to derive the steady-state domestic interest rate

$$R_f = \frac{\exp(g)}{\beta\left[1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1\right) f(\varepsilon) d\varepsilon\right]}. \tag{33}$$

It is determined by the domestic economic growth rate, subjective discount factor, and the liquidity premium. Both R_k and R_f are functions of $\bar{\varepsilon}$, which are denoted by $R_k(\bar{\varepsilon})$ and $R_f(\bar{\varepsilon})$. The two types of steady state differ in the investment threshold $\bar{\varepsilon}$. We derive this threshold and other equilibrium variables later.

The following lemma is useful for the analysis.

Lemma 1 *The asset-to-output ratio in a steady state is*

$$\frac{b}{y} = \frac{\beta\left(R_f^*/R_f(\bar{\varepsilon}) - 1\right)}{(\exp(g) - 1)\,\Omega}, \tag{34}$$

where R_f is given by (33).

This lemma says that the steady-state asset-to-output ratio is determined by the interest rate differential. When the foreign interest rate R_f^* is higher (lower) than the domestic interest rate R_f, there is a capital account surplus (deficit) or capital outflow (inflow). When the foreign interest rate rises, capital flow reverses, that is, capital inflow decreases and capital outflow increases.

4.1 Bubbleless Steady State

Equation (34) shows that b/y depends on $\bar{\varepsilon}$. We thus simply write it as a function of $\bar{\varepsilon}$, $b/y\left(\bar{\varepsilon}\right)$. We impose the following assumption to ensure the existence of a bubbleless steady state.

Assumption 1 *Assume that*

$$\delta+\exp(g)-1<\left(R_k\left(\varepsilon_{\min}\right)+\mu+\frac{b}{y}\left(\varepsilon_{\min}\right)\frac{R_k\left(\varepsilon_{\min}\right)}{\alpha\exp\left(g\right)}\right)\mathbb{E}\left[\varepsilon\right],$$

where R_k is given by (32) and b/y is given by (34).

The following proposition characterizes the bubbleless steady state.

Proposition 3 *Let assumption 1 hold. If μ is sufficiently small, then the equation*

$$\delta+\exp(g)-1=\left(R_k\left(\bar{\varepsilon}\right)+\mu+\frac{b}{y}\left(\bar{\varepsilon}\right)\frac{R_k\left(\bar{\varepsilon}\right)}{\alpha\exp\left(g\right)}\right)\int_{\bar{\varepsilon}}^{\varepsilon_{max}}\varepsilon f(\varepsilon)d\varepsilon \tag{35}$$

has a unique solution for $\bar{\varepsilon}\in\left(\varepsilon_{\min},\varepsilon_{\max}\right)$, denoted by $\bar{\varepsilon}_f$. If further

$$1-\left(\alpha+\frac{\alpha\mu}{R_k\left(\bar{\varepsilon}_f\right)}+\frac{1}{\exp(g)}\frac{b}{y}\left(\bar{\varepsilon}_f\right)\right)\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}}f(\varepsilon)d\varepsilon>\gamma+\frac{1}{\exp(g)}\frac{b}{y}\left(\bar{\varepsilon}_f\right)\frac{\exp(g)-R_f^*}{R_f\left(\bar{\varepsilon}_f\right)}>0, \tag{36}$$

then there is a unique bubbleless steady state.

Assumption 1 allows us to use the intermediate value theorem to derive a solution to equation (35). A sufficiently small μ ensures that $R_k\left(\bar{\varepsilon}\right)$ is a decreasing function of $\bar{\varepsilon}$ (see Lemma 2 in the appendix), which ensures the uniqueness of the solution to (35). After obtaining $\bar{\varepsilon}$, we then solve for the equilibrium real exchange rate e_f using equations (1), (22), (26) and (27). Once $\bar{\varepsilon}_f$ and e_f are derived, other equilibrium variables can be easily determined using Proposition 2. Since export demand by foreigners is exogenously given by equation (1), we have to impose the two inequalities in (36) to ensure exports and consumption are positive in the bubbleless steady state.

4.2 Bubbly Steady State

The following proposition characterizes the existence of the bubbly steady state.

Proposition 4 *Suppose that the assumptions in Proposition 3 hold so that there exists a unique bubbleless steady state with the investment threshold given by $\bar{\varepsilon}_f$. If there is a bubbly steady state, then the following condition holds*

$$1<\beta\left[1+\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\bar{\varepsilon}_f}-1\right)f(\varepsilon)d\varepsilon\right]. \tag{37}$$

Conversely, if this condition holds, then the equation

$$1 = \beta \left[1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1 \right) f(\varepsilon) d\varepsilon \right] \tag{38}$$

has a unique solution for $\bar{\varepsilon} \in (\bar{\varepsilon}_f, \varepsilon_{\max})$, *denoted by* $\bar{\varepsilon}_b$, *and if further*

$$\begin{aligned} & 1 - \left(\alpha + \frac{\alpha\mu}{R_k(\bar{\varepsilon}_b)} + \frac{1}{\exp(g)} \frac{b}{y}(\bar{\varepsilon}_b) + \frac{p}{y_b} \right) \int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} f(\varepsilon) d\varepsilon \\ > \quad & \gamma + \frac{1}{\exp(g)} \frac{b}{y}(\bar{\varepsilon}_b) \frac{\exp(g) - R_f^*}{R_f(\bar{\varepsilon}_b)} > 0, \end{aligned} \tag{39}$$

where

$$\frac{p}{y_b} = \frac{\alpha}{R_k(\bar{\varepsilon}_b)} \left(\frac{\delta + \exp(g) - 1}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} \varepsilon f(\varepsilon) d\varepsilon} - \mu \right) - \alpha - \frac{1}{\exp(g)} \frac{b}{y}(\bar{\varepsilon}_b), \tag{40}$$

then there exists a unique bubbly steady state with the investment threshold given by $\bar{\varepsilon}_b$ *and the bubble-to-output ratio* p/y_b *given above.*

The interpretation for (39) is similar to that for (36) and is omitted here. Condition (37) is the key bubble existence condition, similar to that in Miao, Wang, and Zhou (2015). The main difference is that here the foreign interest rate R_f^* affects (37) because it affects the bubbleless steady-state investment threshold $\bar{\varepsilon}_f$. We interpret condition (37) as follows: Suppose that the economy is initially in the bubbleless steady state. If there is an asset bubble, i.e., land has a positive price, then the right-hand side of (37) represents the steady-state benefit of purchasing one unit of bubble, while the left-hand side is the cost of purchasing one unit of bubble. The benefit comes from the liquidity role of land discussed earlier for (19). If condition (37) holds, then the benefit is larger than the associated cost. Thus the firm is willing to pay a positive price to buy bubble.

How is condition (37) related to the traditional bubble existence condition that the interest rate in the bubbleless economy must be lower than the rate of economic growth (Tirole (1985))? We have shown before that the steady-state interest rate satisfies (33). Thus the interest rate in the bubbleless steady state is $R_{ff} = R_f(\bar{\varepsilon}_f)$. It follows that condition (37) is equivalent to $R_{ff} < \exp(g)$, which is consistent with Tirole's (1985) condition. However, unlike his OLG model, our infinite-horizon economy is not dynamically inefficient since there is no overaccumulation of capital.

Notice that condition (37) is necessary but not sufficient for the emergence of a bubble. We need additional conditions in (39) to ensure the existence of positive consumption and real exchange rate so that a bubbly equilibrium can exist.

We can easily show that the bubbly steady state interest rate R_{fb} is higher than R_{ff}, when the bubbly and bubbleless steady states coexist. The intuition is that the existence of a bubble crowds out the demand for bonds and hence raises the interest rate.

4.3 Impacts of Foreign Interest Rate

Now we discuss how an increase in the foreign interest rate affects the bubbly and bubbleless steady states.

Proposition 5 *The smaller the foreign interest rate R_f^*, the more likely a domestic bubble can emerge in the sense that condition (37) is more likely to hold. When R_f^* is sufficiently high, a bubble cannot emerge. When a bubbly steady state exists, the capital rental rate R_{kb}, the domestic interest rate R_{fb}, and the investment threshold $\bar{\varepsilon}_b$ do not change with the foreign interest rate R_f^*, but the bubble-to-output ratio $\frac{p}{y_b}$ decreases with R_f^*.*

For a smaller foreign interest rate R_f^*, it is more likely to have capital inflows (or smaller capital outflows) by Lemma 1. This tends to increase the demand for domestic bonds and decrease the domestic interest rate, and hence fuel a bubble. More formally, condition (37) is more likely to hold when R_f^* is smaller. But when the foreign interest rate is sufficiently high, this condition is violated and hence a bubble cannot emerge.

As Proposition 4 shows, the investment threshold in the bubbly steady state is determined by the steady-state asset-pricing equation for the bubble (38), which does not depend on the foreign interest rate R_f^*. Thus R_{kb} and R_{fb} do not depend on R_f^* by equations (32) and (33). A rise in R_f^* leads to a decreased size of the bubble. Intuitively, the bubble and foreign financial investments are substitutes for providing liquidity to the firms. When R_f^* is sufficiently high, the net international investment position is sufficiently high so that firms have enough liquidity to finance investment. Thus firms have no demand for the bubble asset.

5 Quantitative Results

In this section analyze the quantitative effects of foreign interest rate shocks on the business cycle properties of a small open economy. We consider emerging economies rather than developed economies because emerging economies face larger frictions in domestic and international financial markets. This feature makes emerging economies better suit our theory than developed economies. We take the bubbly equilibrium as a benchmark. We first estimate the log-linearized

version of the detrended bubbly equilibrium.[12] Using the calibrated and estimated parameters we simulate the model and compare the model-generated business cycle moments with those in the data. We then examine the variance decomposition of all shocks and study impulse response functions to illustrate the model mechanism.

5.1 Parameter Estimates

Following the literature on business cycles of emerging markets (e.g., Aguiar and Gopinath (2007) and García-Cicco, Pancrazi, and Uribe (2010)), we choose Mexico as the domestic country and take Mexican data except for stock prices from Fernández and Gulan (2015).[13] We construct the real stock price data using the stock price index taken from the CEIC database divided by the GDP deflator taken from International Financial Statistics of IMF. The sample period is from 1990Q1 to 2011Q4 except for foreign interest rates which are available only from 1994Q1 to 2011Q4. One period in the model corresponds to a quarter. We group all model parameters into two categories.

The parameters in the first category is either calibrated or taken from the literature. We set the unconditional mean of the domestic growth rate $g = 1.0\%$, which is the point estimate of long-run growth rate of Mexico. As in the literature (e.g., Aguiar and Gopinath (2007) and Chang and Fernández (2013)), set the subjective discount factor $\beta = 0.99$, the capital share $\alpha = 0.32$, and the capital depreciation rate $\delta = 0.05$. We set $\varphi = 2$ so that the Frisch elasticity is 0.5 and set the labor weight parameter ν so that the steady state labor hours equal to 1/3. Set the share of imported goods $\gamma = 0.27$, which is consistent with the data in our sample period. Set the price elasticity of exported domestic goods $\sigma = 1.0$, which lies in the range of the empirical estimates by Feenstra et al. (2014). Suppose that the idiosyncratic investment efficiency shock is drawn from a Pareto distribution with distribution function $F(\varepsilon) = 1 - \left(\frac{\varepsilon_{min}}{\varepsilon}\right)^{\eta}$. Set $\eta = 5$ and $\varepsilon_{\min} = (\eta - 1)/\eta$ so that the mean of ε is 1 and the investment-to-GDP ratio is 0.2 in the bubbly steady state. This value is in line with the average investment-to-GDP ratio of Mexico in our sample period.

Set the steady-state quarterly (gross) foreign interest rate $R_f^* = 1.006$, which is the median of foreign interest rates faced by Mexican corporations. These interest rates are constructed using the Corporate Emerging Market Bond Index (CEMBI) spreads over the period 2001Q4-

[12] We present the detrended equilibrium system and its log-linearized version in Appendices B and C. We find that both the bubbly and bubbleless equilibria are locally unique for our numerical solutions.

[13] We also confront our model with data of Argentina and the results are quantitatively similar and available upon request.

2011Q4 (Fernández and Gulan (2015)). Set the parameter for the adjustment cost of capital flow $\Omega = 0.20$ so that the net exports/GDP ratio in the bubbly steady state is -1.41%, as observed on average in Mexico over our sample period.

[Insert Table 1 Here]

The calibrated parameters are listed in Table 1. All the other parameters are estimated by Bayesian methods. They include parameters governing all of the shock processes, the habit formation parameter h, the aggregate capital adjustment cost parameter Ω_k and the financial friction parameter μ.

Shocks and Data Our model includes five exogenous shocks. The parameters governing these shocks are $\rho_g, \sigma_g, \rho_a, \sigma_a, \rho_\xi, \sigma_\xi, \rho_{y^*}, \sigma_{y^*}, \rho_{R_f^*}$ and $\sigma_{R_f^*}$. We use five quarterly time series of data in our estimation: the demeaned foreign interest rate, the sum of the US and Canada real GDP, the Mexican real GDP, and the Mexican real investment and real consumption. We use the Emerging Markets Bond Index Plus (EMBI+) spreads over the period 1994Q1-2011Q4 to construct foreign interest rates to estimate the underlying shocks, as frequently used in literature (Uribe and Yue (2006), Fernández-Villaverde et al (2011), and Fernández and Gulan (2015)). This index provides secondary market prices of emerging market bonds which are actively traded and denominated in US dollars in a few emerging markets. Fernández and Gulan (2015) show that the EMBI index and the CEMBI index are highly correlated. Thus we use the EMBI index, instead of the CEMBI index, in our Bayesian estimation because the former has a much larger sample size.

Following Adolfson et al (2007), Lubik and Schorfheide (2007), and Justiniano and Preston (2010), we use the sum of the US and Canada real GDP to estimate the foreign demand shocks, as exports to US and Canada constitute roughly 80% of Mexican total exports. The Mexican real GDP, real investment, and real consumption data are informative to estimate the underlying long- and short-run productivity shocks as well as preference shocks. In particular, Aguiar and Gopinath (2007) argue that the long-run productivity shock is the driving force to generate the high consumption volatility relative to output, which is one of the defining characteristics of emerging markets.

Priors and Posteriors We choose the standard prior distributions for h and Ω_k as in literature. The prior of the collateral parameter μ follows Beta distribution with mean 0.15 and

standard deviation 0.05. Such a prior mean is at the lower end of the estimates suggested by Covas and Den Hann (2011) for the US data. We expect a lower value of μ for the Mexican data because financial markets in emerging economies are usually less developed than in developed economies.

Assume that the prior distribution for the persistence parameter for the foreign interest rate shock follows a Beta distribution with mean 0.85 and standard deviation 0.05. The 90 percent interval of this prior distribution covers the values used in the related literature (e.g. Neumeyer and Perri (2005), Uribe and Yue (2006), and Chang and Fernández (2013)). Following Smets and Wouters (2007) and Liu, Wang and Zha (2013), we assume the prior distributions of the persistence parameters for all other shocks follow a Beta distribution with mean 0.5 and standard deviation 0.2. The prior distributions of the standard deviation of innovations for all shocks follow an inverse Gamma distribution with mean 1 percent and standard deviation ∞. We find that our estimates of these parameters are quite robust and not sensitive to the prior distribution.

The information of the prior and posterior distributions of the estimated parameters is listed in Table 2. The modes, means, 5th and 95th percentiles of the posterior distributions for the estimated parameters are computed using the Metropolis-Hastings algorithm with 200,000 draws.[14]

[Insert Table 2 Here]

We take the posterior modes as the parameter estimates. We find the estimate for h is 0.62, close to the value 0.50 estimated by Liu, Wang and Zha (2013) and the value 0.608 estimated by Jermann and Quadrini (2012). Our estimate for Ω_k is 2.26, which is in line with the values estimated in other studies (e.g. 2.48 in Christiano, Eichenbaum, and Evans (2005) and 2.0 in Liu, Waggoner and Zha (2011)). Our estimate for the financial market friction parameter μ is 0.17, lying in the range (0.1-0.4) estimated by Covas and Den Hann (2011) using the US data.

Our estimated foreign interest rate shock is similar to those reported in Neumeyer and Perri (2005) ($\rho_{R_f^*} = 0.81, \sigma_{R_f^*} = 0.63\%$), Uribe and Yue (2006) ($\rho_{R_f^*} = 0.83, \sigma_{R_f^*} = 0.7\%$), and Chang and Fernández (2013) ($\rho_{R_f^*} = 0.81, \sigma_{R_f^*} = 0.42\%$). The estimated foreign demand shock is quite persistent (0.997) and not volatile (0.79%). This is not surprising because Mexico's two largest trade partners – the US and Canada – are two advanced economies and have relatively stable

[14] The Markov chain Monte Carlo (MCMC) univariate convergence diagnostic shows that our posterior distribution of each parameter constructed from random draws converges to a stationary distribution.

business cycles. As in Liu, Wang and Zha (2013) and García-Cicco, Pancrazi, and Uribe (2010), the estimated preference shock is quite volatile.

The estimates for the long- and short-run productivity shocks are debated in the literature. For example, Aguiar and Gopinath (2007) use the GMM method based on the Mexican quarterly data during 1980-2003 to estimate a major role of trend productivity shocks in business cycles. They find that there is little persistence, but high volatility in the long-run productivity shock. Using Mexican annual data during 1900-2005, García-Cicco, Pancrazi, and Uribe (2010) find that the quarterly persistence of the long-run productivity shock is 0.71 and the quarterly volatility is 0.84%. Using Bayesian methods based on the same sample in Aguiar and Gopinath (2007), Chang and Fernández (2013) find that the persistence of long-run and short-run productivity shocks are 0.72 and 0.89, and the standard deviations of innovations are 0.12% and 0.66%. Compared to these studies, our estimates for the long-run productivity shock give similar unconditional volatility, but the short-run productivity shock is more volatile.

5.2 Estimation of the Bubbleless Model

To compare with the bubbleless economy, we fix the calibrated parameter values as in the previous subsection and re-estimate the other parameters based on the bubbleless equilibrium. The estimation results are also given in Table 2. We find that the log marginal densities of data for the baseline bubbly model and the bubbleless model are 1113.3 and 1103.1, respectively and the log posterior likelihood at the posterior modes for the baseline bubbly model and the bubbleless model are 1161.7 and 1152.9, respectively. Both measures suggest that the data favor the bubbly model given the same prior information.

Now we compare the posterior modes for the shocks in the two economies. We find that the estimates for the volatilities and persistence of the foreign demand shock and the foreign interest rate shock are similar across the two economies, indicating that these two shocks are primarily pinned down by the exogenous data series instead of model properties. Notably, the estimates of the two types of productivity shocks have large differences: the estimated long-run productivity shock in the bubbleless model is much larger than that in the bubbly model, while the short-run productivity shock in the bubbleless model is much smaller than that in the bubbly model. Specifically, the unconditional volatility of the long-run productivity shock in the bubbleless model is 4.3 times larger than that in the bubbly model (4.08% vs. 0.93%). But the unconditional volatility of the short-run productivity shock in the bubbleless model is

only 22% of that in the bubbly model (0.97% vs. 4.47%).

The key intuition is that the bubbleless model does not have a large amplification mechanism generated by asset bubbles. Thus it needs large long-run productivity and preference shocks, together with weaker habit formation (0.14 in the bubbleless model vs. 0.62 in the bubbly model) and smaller capital adjustment costs (0.03 in the bubbleless model vs. 2.26 in the bubbly model), to match the high volatilities of consumption and investment relative to output in the data.

5.3 Business Cycle Moments

In this subsection we simulate the benchmark bubbly model with the calibrated and estimated parameters, and then compare the model-generated real business cycle moments with those observed in the data. We also compare our benchmark model with two bubbleless models. Bubbleless model 1 uses the same parameter values as in the benchmark model, but focuses on the bubbleless equilibrium. Bubbleless model 2 uses the re-estimated parameter values as in Section 5.2. We report the results in Table 3. All variables except for the net exports/GDP ratio are in logs and Hodrick-Prescott (HP) filtered. The net exports/GDP ratio is directly HP filtered.

[Insert Table 3 Here]

We find that our benchmark bubbly model matches the business cycle volatility and co-movement in the data quite well. One weakness is that our bubbly model overpredicts the net exports/GDP ratio volatility (3.06% vs. 1.61%) and underpredicts the stock price volatility relative to GDP (4.69 vs. 6.39) given that these data are out of sample for our estimation. By contrast, both bubbleless models perform much worse, especially along the dimension related to stock prices. The key reason is that both of these models lack the propagation and amplification mechanisms generated by the bubble asset. To understand the intuition, we first consider bubbleless model 1 by shutting down the bubble channel in the benchmark model using the same parameter values. We find that bubbleless model 1 explains about 47% of investment volatility and 31% of stock market volatility in the data. Bubbleless model 1 also implies weak correlations between investment and GDP and between stock prices and GDP, which are inconsistent with the data. Moreover, bubbleless model 1 shows a wrong sign of correlation between the net exports/GDP ratio and GDP.

To give the bubbleless model a fairer chance to match the data, we consider bubbleless model 2 with re-estimated parameter values. Because of the lack of the amplification and propagation mechanism generated by the asset bubble, the model implied consumption, investment, and stock market volatilities are too low, compared to the data. Moreover, the model implied correlations between investment and GDP and between consumption and GDP are too low, and the model implied correlation between the net exports/GDP ratio and GDP has the wrong sign.

Strong countercyclicality of the trade balance is a striking feature that distinguishes emerging economies from developed economies, which show mild countercyclical or procyclical trade balance. For example, according to Neumeyer and Perri (2005), the correlation between the net exports/GDP ratio and GDP is -0.61 on average in emerging markets, while the correlation is -0.23 on average in developed economies. Similarly, Aguiar and Gopinath (2007) report that the average correlation for emerging markets is -0.51, while the average correlation for developed economies is only -0.17. Furthermore, Fernández and Gulan (2015) document that the average correlation between the net exports/GDP ratio and GDP is -0.40 in emerging economies and 0.33 in developed economies.

We will present impulse response functions to understand why our model with asset bubbles can explain countercyclical trade balance in Section 5.5.

5.4 Variance Decomposition

In this section we study the relative importance of the five shocks in the bubbly economy using variance decomposition. Specifically, we calculate the fractions of the movements in the impulse responses generated by each shock, with one standard deviation each. The results are displayed in Table 4.

[Insert Table 4 Here]

Consistent with the literature (e.g., Aguiar and Gopinath (2007), Chang and Fernández (2013), and Fernández and Gulan (2015)), long-run productivity shocks play an important role in explaining variations of output, investment, consumption, the net exports/GDP ratio, and stock prices. Short-run productivity shocks are also important to explain variations of output, investment, and stock prices, but not so important for variations of consumption and the net exports/GDP ratio. Preference shocks explain a large fraction of consumption movements in

the short- and medium-run. Since the estimated foreign demand shocks are small, they do not contribute much to Mexican business cycles, generally less than 4%.

Notably, foreign interest rate shocks play a nontrivial role, particularly in explaining the variations of the net exports/GDP ratio (26.7% in the impact period), stock prices (20.2% in the impact period), and investment (18.9% in the impact period). Foreign interest rate shocks are transmitted through the asset bubble channel given financial frictions in our model. Our finding is consistent with the previous studies on the importance of foreign interest rate shocks. For example, Neumeyer and Perri (2005) report that the output volatility declines by about 30% when eliminating foreign interest rate shocks. By applying a VAR analysis to seven emerging markets, Martin and Yue (2006) document that foreign interest rate shocks explain about one-third of movements in aggregate activity and about 43% of movements in trade balance/output ratio. Based on a Bayesian estimation to an annual sample of Argentine data, García-Cicco, Pancrazi, and Uribe (2010) find that country premium shocks are responsible for substantial fractions of investment growth (62.4%) and variations in trade balance/GDP ratio (78.9%) but only small fractions of output growth (2.9%) and consumption growth (5.2%). Chang and Fernández (2013) report that in Mexico foreign interest rate shocks are responsible for 5.1% of output growth variation, 9.2% of consumption growth variation, 22.2% of investment growth variation and 41.2% of trade balance/GDP ratio change.

5.5 Impulse Responses

We now present impulse response functions for the long- and short-run productivity shocks and foreign interest rate shocks to understand our model mechanism because these are the most important shocks to explain Mexican business cycles as shown in the previous subsection. Figures 1 through 3 present the results for the benchmark bubbly model (solid lines) and for bubbleless model 1 (dash lines).[15] All variables are expressed as the percent deviation from their deterministic balanced growth values, except for the interest rate and the net exports/GDP ratio which are in level deviation.

[Insert Figures 1-3 Here]

Figures 1 and 2 show that the long- and short-run productivity shocks have qualitatively similar impulse response properties. The main difference is that there is a trend effect for the

[15] The results for bubbleless model 2 with re-estimated parameters are similar to those for bubbleless model 1 and available upon request.

long-run productivity shock. In response to a positive shock to the long-run productivity, the asset bubble expands on impact, which raises firms' net worth and allows them to make more investment.[16] This generates a large amplification effect, in contrast to the bubbleless economy, which displays much weaker responses of consumption, investment, output, and stock prices. More importantly, the real exchange rate and net exports exhibit opposite responses for the bubbly and bubbleless economies. In particular, for the bubbleless economy, in response to a positive productivity shock, output rises on impact, thereby raising the supply of domestic goods in the international market and leading to a real depreciation. Thus the net exports increase, generating a positive correlation between the net exports and output. By contrast, for the bubbly economy, due to the amplification and propagation effects of the asset bubble, both domestic investment and consumption increase much more than those in the bubbleless economy in response to a positive shock to long- or short-run productivity (Figure 1 and 2), leading to more domestic demand for the foreign goods and hence a real appreciation. Thus the net exports decrease while the domestic output increases, generating a negative correlation between the net exports and output.

Figure 3 presents the impulse responses to a positive foreign interest shock. In response to this shock, bankers tend to borrow from domestic firms by selling bonds to invest in the international financial market so that capital begins to flow out of the domestic economy. Capital outflows reduce resources for domestic firms to invest in the bubble asset. As a result, the asset bubble dampens substantially, leading to a decline in net worth, investment, consumption, output, and stock prices. Meanwhile, the rise of the foreign interest rate also reduces the demand for the domestic bond, leading to domestic interest rate to rise, which further reduces the demand for the bubble asset. Capital outflows also increase the supply of domestic consumption goods to foreign markets, causing a real depreciation. The real depreciation increases exports and decreases imports. As a result, the trade balance increases.

We emphasize that the responses of the domestic interest rate and the asset bubble reinforce each other. Since output and consumption decline,the domestic interest rate has to rise further to induce savings. Such a further rise in the domestic interest rate causes the price of the bubble asset to drop further, leading to investment and output to decline further. This loop continues and is exaggerated by the real exchange rate depreciation, which interacts with the amplification

[16] The expansion of the asset bubble also has a crowd-out effect in that inefficient firms must buy asset from efficient firms and do not make investment. Our numerical results show that this effect is dominated by the crowd-in effect.

effect of bubbles in an open economy, and explains why consumption falls more than output in the bubbly economy. Such strong interactions among foreign interest rate, real exchange rate, and domestic macroeconomy are also found in empirical studies. Calvo, Leiderman and Reinhard (1993) show that foreign interest rate shocks are a major source of capital inflows and real exchange rate appreciation in Latin America. They also find that during a period of low US interest rates, an important part of capital inflows to Mexico has financed increased private investment. Canova (2005) documents that a US contractionary monetary shock significantly and simultaneously increases Latin American interest rates and US monetary policy shocks cause strong fluctuations in Latin American output. Maćkowiak (2007) reports that domestic interest rate increases and real exchange rate depreciates in response to a US monetary policy contraction, which is an economically significant and robust response across multiple emerging markets.

Consistent with the stylized fact of "Sudden Stop" in emerging markets, Tobin's Q and the stock price fall on impact in response to a positive foreign interest rate shock. This pattern can hardly be generated in typical real business cycle models, in which capital stock falls in recession periods, yielding higher marginal product of capital (rental rate). Sine marginal Q reflects the present value of the rental rate, it must rise. By contrast, in our model the rental rate of capital falls due to real depreciation as imported foreign goods are an input for production (see equation (4)). Mendoza (2010) also generates this result in a small open economy with occasional binding borrowing constraint. Unlike in his model, the stock market value of the firm in our model includes the market value of capital and the bubble asset, as shown in equation (31). It decreases sharply on impact due to the fall of Tobin's Q and the shrink of the asset bubble.

By contrast, Figure 3 shows counterfactual impulse responses for the bubbleless economy. In particular, the responses of the bubbleless economy are much weaker than those of the bubbly economy in terms of magnitudes, and may move in directions opposite to those of the bubbly economy. Without an asset bubble, the negative net worth effect due to falling asset prices is absent. Thus the increase of the domestic interest rate on impact is much smaller in the bubbleless economy than in the bubbly economy. Without the negative net wroth effect of the asset bubble, the increased domestic interest rate allows financially constrained firms to increase saving to finance more investment, causing consumption and output to rise. The effect is, however, quite weak.

The impact on the real exchange rate is mixed in the bubbleless economy. On the one hand, capital outflows and the increased demand for imported input in production imply real depreciation of domestic consumption goods. On the other hand, increased investment and consumption cause real appreciation. The net effect in our numerical results is that domestic consumption goods appreciate slightly. Thus, the trade balance decreases rather than increases. The impact on Tobin's Q and stock prices is quite weak. All these predictions are inconsistent with the data.

6 Conclusion

We have provided a DSGE model of a small open economy with asset bubbles. We find that asset bubbles provide a powerful amplification and propagation mechanism. They are important to explain the high stock market volatility and the comovement between the stock market and the real economy. In addition to the long-run productivity shock found to be important in the literature, our Bayesian estimation shows that the foreign interest rate shock is important to drive the movements of asset bubbles and stock prices. The foreign interest rate shock explains about 20% of the variations of the stock market for the Mexican data over 1990Q1-2011Q4.

Appendix

A Proofs

Proof of Proposition 1 We first consider firm j's decision problem. Solving the static labor and imported material choice problem in (3) gives demand for labor and imported material

$$N_{jt} = A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} K_{jt-1}, \tag{A.1}$$

$$M_{jt} = A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\alpha+\gamma}{\alpha}} K_{jt-1}. \tag{A.2}$$

Substituting these equations into (3) yields (4).

Now we solve the firm's dynamic problem. Conjecture that the value function takes the following form

$$V_t(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1}) = \phi_t^K(\varepsilon_{jt})K_{jt-1} + \phi_t^H(\varepsilon_{jt})H_{jt-1} + \phi_t^B(\varepsilon_{jt})B_{jt-1}, \tag{A.3}$$

where $\phi_t^K(\varepsilon_{jt})$, $\phi_t^H(\varepsilon_{jt})$ and $\phi_t^B(\varepsilon_{jt})$ are to be determined. In this case Tobin's Q satisfies

$$Q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^K(\varepsilon) dF(\varepsilon). \tag{A.4}$$

Conjecture that

$$P_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^H(\varepsilon) dF(\varepsilon), \tag{A.5}$$

$$\frac{1}{R_{ft}} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^B(\varepsilon) dF(\varepsilon). \tag{A.6}$$

Substituting the flow-of-funds constraint and the conjectured value function into the right-hand side of the Bellman equation, we obtain

$$\begin{aligned} & D_{jt} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}\left(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt}\right) \\ = \; & R_{kt}K_{jt-1} - P_{kt}I_{jt} - \frac{B_{jt}}{R_{ft}} + B_{jt-1} + P_t\left(H_{jt-1} - H_{jt}\right) + (1-\delta)Q_tK_{jt-1} + \varepsilon_{jt}Q_tI_{jt} + \frac{B_{jt}}{R_{ft}} + P_tH_{jt} \\ = \; & R_{kt}K_{jt-1} + (1-\delta)Q_tK_{jt-1} + B_{jt-1} + P_tH_{jt-1} + \left(\varepsilon_{jt}Q_t - P_{kt}\right)I_{jt}. \end{aligned} \tag{A.7}$$

If $\varepsilon_{jt} < P_{kt}/Q_t$, the firm will not invest, i.e. $I_{jt} = 0$. And the firm is indifferent between saving and borrowing, and is indifferent between purchasing and selling bubble. If $\varepsilon_{jt} \geq P_{kt}/Q_t$, the firm makes real investment as much as possible. Thus it exhausts the borrowing limit and sells bubble to finance investment. By (6), (7), (8) and $H_{jt} \geq 0$, we have

$$\begin{aligned} P_{kt}I_{jt} &\leq R_{kt}K_{jt-1} - \frac{B_{jt}}{R_{ft}} + B_{jt-1} + P_t\left(H_{jt-1} - H_{jt}\right) \\ &\leq R_{kt}K_{jt-1} + \mu K_{jt-1} + B_{jt-1} + P_tH_{jt-1}. \end{aligned}$$

We then obtain (17).

Plugging the decision rules in the Bellman equation, we obtain

$$V_t(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1}) = \begin{cases} R_{kt}K_{jt-1} + (1-\delta)K_{jt-1} + B_{jt-1} + P_tH_{jt-1} \\ +(\varepsilon_{jt}Q_t/P_{kt} - 1)\left(R_{kt}K_{jt-1} + \mu K_{jt-1} + B_{jt-1} + P_tH_{jt-1}\right), & \text{if } \varepsilon_{jt} \geq \bar{\varepsilon}_t; \\ R_{kt}K_{jt-1} + (1-\delta)K_{jt-1} + B_{jt-1} + P_tH_{jt-1}, & \text{if } \varepsilon_{jt} < \bar{\varepsilon}_t. \end{cases}$$

Matching coefficients in the preceding equation and equation (A.3) and making use of equation (A.4), (A.5) and (A.6) yield the equations in Proposition 1. Q.E.D.

Proof of Proposition 2 Equation (23) follows from the first-order condition for the banker's decision problem. We use the decision rule in Proposition 1 and the Law of Large Numbers to derive aggregate investment in equation (24)

$$\begin{aligned} I_t &= \int_{\varepsilon_{jt}<\bar{\varepsilon}_t} 0 \cdot dj + \int_{\varepsilon_{jt}\geq\bar{\varepsilon}_t} \frac{1}{P_{kt}}\left(R_{kt}K_{jt-1} + \mu K_{jt-1} + B_{jt-1} + P_tH_{jt-1}\right) dj \\ &= \frac{1}{P_{kt}}\left(R_{kt}K_{t-1} + \mu K_{t-1} + B_{t-1} + P_t\right)\left(\int_{\bar{\varepsilon}_t}^{\varepsilon_{max}} f(\varepsilon)d\varepsilon\right), \end{aligned}$$

where the last equality is due to the fact that ε is IID. Equation (25) follows from aggregating equation (5).

Substituting (A.1) into the expression of aggregate labor, we have

$$\begin{aligned} N_t &= \int_0^1 N_{jt}dj = \int_0^1 A_t^{\frac{1-\alpha-\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}}\left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} K_{jt-1}dj \\ &= A_t^{\frac{1-\alpha-\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}}\left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} K_{t-1}. \end{aligned}$$

Equation (28) follows from the preceding equation.

Substituting (A.2) into the expression of aggregate imported material, we have

$$\begin{aligned} M_t &= \int_0^1 M_{jt}dj = \int_0^1 A_t^{\frac{1-\alpha-\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}}\left(\frac{\gamma}{e_t}\right)^{\frac{\alpha+\gamma}{\alpha}} K_{jt-1}dj \\ &= A_t^{\frac{1-\alpha-\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}}\left(\frac{\gamma}{e_t}\right)^{\frac{\alpha+\gamma}{\alpha}} K_{t-1}. \end{aligned} \tag{A.8}$$

Substituting (A.1) and (A.2) into the expression of the aggregate output Y_t, we can derive

$$\begin{aligned}
Y_t &= \int_0^1 K_{jt-1}^{\alpha}(A_t N_{jt})^{1-\alpha-\gamma} M_{jt}^{\gamma} dj \\
&= \int_0^1 K_{jt-1}^{\alpha} A_t^{1-\alpha-\gamma} A_t^{\frac{1-\alpha-\gamma}{\alpha}(1-\alpha-\gamma)} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}(1-\alpha-\gamma)} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}(1-\alpha-\gamma)} K_{jt-1}^{1-\alpha-\gamma} \\
&\quad A_t^{\frac{1-\alpha-\gamma}{\alpha}\gamma} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}\gamma} \left(\frac{\gamma}{e_t}\right)^{\frac{\alpha+\gamma}{\alpha}\gamma} K_{jt-1}^{\gamma} dj \\
&= A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} K_{t-1}.
\end{aligned}$$

Using the last equation and the preceding expressions of N_t and M_t, we can derive equations (26) and (27).

We substitute the flow-of-funds constraints for the firms, capital goods producers and bankers, (7), (9) and (11), into the budget constraint of the households (12), and obtain

$$C_t = Y_t - e_t M_t - \left[1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)^2\right] I_t - \frac{B_t}{R_{ft}} + B_{t-1}\frac{R_{ft-1}^*}{R_{ft-1}}\frac{e_t}{e_{t-1}}.$$

Then comparing it with the resource constraint (16) will directly lead to (22). Finally, we use (4), (28), and (A.8) to derive (29). Q.E.D.

Proof of Lemma 1 We use the detrended deterministic equilibrium system presented in Appendix B to study steady state. By (23), we derive the steady-state condition

$$0 = \frac{\beta}{\exp(g)}\left(\frac{R_f^*}{R_f} - 1\right) - \frac{\Omega}{y}\left(b - \frac{b}{\exp(g)}\right).$$

We then obtain (34). Q.E.D.

Lemma 2 *When $\mu > 0$ is sufficiently small, $\frac{\partial R_k(\bar{\varepsilon})}{\partial \bar{\varepsilon}} < 0$.*

Proof. Substituting $Q = 1/\bar{\varepsilon}$ into the steady-state version of (B.1), we can derive

$$\begin{aligned}
R_k &= \frac{\left[\exp(g) - \beta + \beta\delta\right]/\bar{\varepsilon} - \beta\mu \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1\right) f(\varepsilon) d\varepsilon}{\beta\left[1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1\right) f(\varepsilon) d\varepsilon\right]} \\
&= \frac{\left[\exp(g) - \beta + \beta\delta\right] - \beta\mu \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon) d\varepsilon}{\beta\left[\bar{\varepsilon} + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon) d\varepsilon\right]}, \qquad \text{(A.9)}
\end{aligned}$$

which is equation (32).

The partial derivative of R_k with respect to $\bar{\varepsilon}$ is

$$\frac{\partial R_k}{\partial \bar{\varepsilon}} = \frac{\mu\beta \int_{\bar{\varepsilon}}^{\varepsilon_{max}} \varepsilon f(\varepsilon) d\varepsilon - \left[\exp(g) - \beta + \beta\delta\right] F(\bar{\varepsilon})}{\beta\left[\bar{\varepsilon} + \int_{\bar{\varepsilon}}^{\varepsilon_{max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon) d\varepsilon\right]^2}.$$

When $\mu = 0$, it is negative. So when μ is sufficiently small, it is also negative. ■

Proof of Proposition 3 We use the detrended deterministic equilibrium system presented in Appendix B to study steady state. In the bubbleless steady state with $p = 0$, we divide both sides of the steady-state version of equation (B.6) by k to derive

$$\delta + \exp(g) - 1 = \left(R_k + \mu + \frac{b}{k}\right) \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon f(\varepsilon) d\varepsilon.$$

Since $R_k = \alpha \exp(g) y/k$, we then obtain (35). In that equation, R_k is a decreasing function of $\bar{\varepsilon}$. By (33), R_f is an increasing function of $\bar{\varepsilon}$. Thus, the expression on the right-hand side of equation (35) is a decreasing function of $\bar{\varepsilon}$. This expression takes value 0 when $\bar{\varepsilon} = \varepsilon_{\max}$ and is larger than $\delta + \exp(g) - 1$ when $\bar{\varepsilon} = \varepsilon_{\min}$ by Assumption 1. Thus it follows from the intermediate value theorem that there is a unique solution for $\bar{\varepsilon} \in (\varepsilon_{\min}, \varepsilon_{\max})$, denoted by $\bar{\varepsilon}_f$, to the equation (35).

According to (B.7), $P_{kf} = 1$ in the steady state. After determining $\bar{\varepsilon}_f$, we can derive $Q_f = P_{kf}/\bar{\varepsilon}_f = 1/\bar{\varepsilon}_f$, $R_{kf} = R_k(\bar{\varepsilon}_f)$, and $R_{ff} = R_f(\bar{\varepsilon}_f)$ by definition and equations (32) and (33). By Lemma 1, $b_f/y_f = b/y(\bar{\varepsilon}_f)$. By (B.5), we can derive

$$\frac{i_f}{y_f} = \frac{1}{\exp(g)} \left(R_{kf} \frac{k_f}{y_f} + \mu \frac{k_f}{y_f} + \frac{b_f}{y_f}\right) \int_{\bar{\varepsilon}_f}^{\varepsilon_{max}} f(\varepsilon) d\varepsilon.$$

Since $R_{kf} = \alpha \exp(g) y_f/k_f$ by (B.17), we have

$$\frac{i_f}{y_f} = \left(\alpha + \frac{\alpha\mu}{R_{kf}} + \frac{1}{\exp(g)} \frac{b_f}{y_f}\right) \int_{\bar{\varepsilon}_f}^{\varepsilon_{max}} f(\varepsilon) d\varepsilon.$$

Substituting (B.9) into equation (B.15) and dividing both sides by y_t, we obtain

$$\frac{x_t}{y_t} = \gamma + \frac{b_t}{y_t R_{ft}} - \frac{1}{\exp(g)} \frac{b_{t-1}}{y_t} \frac{R^*_{ft-1}}{R_{ft-1}} \frac{e_t}{e_{t-1}}. \tag{A.10}$$

In the bubbleless steady state, the equation above implies

$$\frac{x_f}{y_f} = \gamma + \frac{1}{\exp(g)} \frac{b_f}{y_f} \frac{\exp(g) - R^*_f}{R_{ff}}. \tag{A.11}$$

Since exports are positive by equation (1), we need the expression above to be positive, which gives the second inequality in condition (36).

We use the resource constraint to derive

$$\begin{aligned} \frac{c_f}{y_f} &= 1 - \frac{i_f}{y_f} - \frac{x_f}{y_f} \\ &= 1 - \left(\alpha + \frac{\alpha\mu}{R_{kf}} + \frac{1}{\exp(g)} \frac{b_f}{y_f}\right) \left(\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}} f(\varepsilon) d\varepsilon\right) - \gamma - \frac{1}{\exp(g)} \frac{b_f}{y_f} \frac{\exp(g) - R^*_f}{R_{ff}} \end{aligned} \tag{A.12}$$

To ensure $c_f > 0$, we impose the first inequality in condition (36).

Given $\bar{\varepsilon}_f$ determined earlier, we next determine the steady-state labor. By equation (B.14),

$$\lambda = \frac{\exp(g) - \beta h}{[\exp(g) - h]\, c}. \tag{A.13}$$

According to the Proof of Proposition 2, it is easy to see that $w_t N_t = (1-\alpha-\gamma) y_t$. Using this equation, (30) and (A.13), we obtain

$$\nu N_f^{1+\varphi} = w_f N_f \lambda_f = (1-\alpha-\gamma) y_f \lambda_f = (1-\alpha-\gamma) \frac{\exp(g) - \beta h}{\exp(g) - h} \frac{y_f}{c_f},$$

which pins down N_f.

Now we determine the real exchange rate e_f. Substituting (B.17) and (B.9) into (B.8) yields

$$y_t = \left(\frac{\alpha y_t}{R_{kt}}\right)^{\alpha} \left(\frac{\gamma y_t}{e_t}\right)^{\gamma} N_t^{1-\alpha-\gamma}, \tag{A.14}$$

or

$$y_t = \left(\frac{\alpha}{R_{kt}}\right)^{\frac{\alpha}{1-\alpha-\gamma}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{1-\alpha-\gamma}} N_t. \tag{A.15}$$

Therefore, combining equation (B.8) and (B.11) generates

$$\frac{x_f}{y_f} = \frac{e_f^{\sigma} y^*}{\left(\frac{\alpha}{R_{kf}}\right)^{\frac{\alpha}{1-\alpha-\gamma}} \left(\frac{\gamma}{e_f}\right)^{\frac{\gamma}{1-\alpha-\gamma}} N_f},$$

which leads to

$$e_f = \left[\frac{x_f}{y_f} \frac{\gamma^{\frac{\gamma}{1-\alpha-\gamma}}}{y^*} \left(\frac{\alpha}{R_{kf}}\right)^{\frac{\alpha}{1-\alpha-\gamma}} N_f\right]^{\frac{1}{\sigma + \frac{\gamma}{1-\alpha-\gamma}}}. \tag{A.16}$$

This equation can be used to solve for a closed-form expression for e_f given that R_{kf}, R_{ff}, and b_f/y_f are determined by $\bar{\varepsilon}_f$.

After e_f is determined, we use equation (B.11) to solve for x_f. Then k_f, m_f and y_f can be derived directly using equations (B.17), (B.9), and (A.15). Since we have already solved b_f/y_f, i_f/y_f, c_f/y_f, x_f/y_f, and y_f, we can derive b_f, i_f, c_f, and x_f. Finally, we use (B.16) to solve for w_f and (B.14) to solve for λ_f. Q.E.D.

Proof of Proposition 4 We use the detrended deterministic equilibrium system presented in Appendix B to study steady state. Suppose that a bubbly steady state exists. We want to

derive condition (37). In the bubbly steady state with $p > 0$, we use the steady-state version of (B.2) to derive (38). We use (B.6) to derive

$$\begin{aligned}\frac{p}{y_b} &= \frac{\alpha}{R_{kb}}\left(\frac{\delta+\exp(g)-1}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon}-\mu\right)-\alpha-\frac{1}{\exp(g)}\frac{b_b}{y_b}\\ &= \frac{\alpha}{R_{kb}}\left(\frac{\delta+\exp(g)-1}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{max}}\varepsilon f(\varepsilon)d\varepsilon}-\mu\right)-\alpha-\frac{1}{\exp(g)}\frac{\beta}{(\exp(g)-1)\Omega}\left(\frac{R_f^*}{R_{fb}}-1\right)>0. \quad \text{(A.17)}\end{aligned}$$

In the bubbleless steady state, we use (B.6) to derive

$$0=\frac{\alpha}{R_{kf}}\left(\frac{\delta+\exp(g)-1}{\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon}-\mu\right)-\alpha-\frac{1}{\exp(g)}\frac{\beta}{(\exp(g)-1)\Omega}\left(\frac{R_f^*}{R_{ff}}-1\right). \quad \text{(A.18)}$$

Since R_k decreases in $\bar{\varepsilon}$ by Lemma 2 and R_f increases in $\bar{\varepsilon}$ by (33), we can deduce that the expression

$$\frac{\alpha}{R_k}\left(\frac{\delta+\exp(g)-1}{\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon}-\mu\right)-\alpha-\frac{1}{\exp(g)}\frac{\beta}{(\exp(g)-1)\Omega}\left(\frac{R_f^*}{R_f}-1\right)$$

increases in $\bar{\varepsilon}$. It follows from (A.17) and (A.18) that $\bar{\varepsilon}_b > \bar{\varepsilon}_f$. Thus we use (38) to deduce condition (37).

Now suppose that (37) holds. We want to derive a bubbly steady state. The expression on the right-hand side of equation (38) is a decreasing function of $\bar{\varepsilon}$. It takes the value $\beta < 1$ when $\bar{\varepsilon} = \varepsilon_{\max}$ and a value larger than 1 when $\bar{\varepsilon} = \bar{\varepsilon}_f$ by (37). By the intermediate value theorem, there is a unique solution, denoted by $\bar{\varepsilon}_b$, to equation (38).

By equation (10), we have $P_{kb} = 1$. After determining $\bar{\varepsilon}_b$, we can derive $Q_b = P_{kb}/\bar{\varepsilon}_b = 1/\bar{\varepsilon}_b$, $R_{kb} = R_k(\bar{\varepsilon}_b)$, and $R_{fb} = R_f(\bar{\varepsilon}_b)$ by definition and equations (32) and (33). By Lemma 1, $b_b/y_b = b/y(\bar{\varepsilon}_b)$. We then use (B.6) to derive

$$\begin{aligned}\frac{p}{y_b} &= \frac{\alpha}{R_{kb}}\left(\frac{\delta+\exp(g)-1}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon}-\mu\right)-\alpha-\frac{1}{\exp(g)}\frac{b}{y}(\bar{\varepsilon}_b) \quad \text{(A.19)}\\ &> \frac{\alpha}{R_{kf}}\left(\frac{\delta+\exp(g)-1}{\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon}-\mu\right)-\alpha-\frac{1}{\exp(g)}\frac{b}{y}(\bar{\varepsilon}_f)\\ &= 0,\end{aligned}$$

where the inequality is due to the fact that R_k and $\int_{\bar{\varepsilon}}^{\varepsilon_{max}}\varepsilon f(\varepsilon)d\varepsilon$ decrease in $\bar{\varepsilon}$ and R_f increases in $\bar{\varepsilon}$. The last equality follows from (35) and $R_{kf} = R_k(\bar{\varepsilon}_f)$. Thus $p > 0$.

We use a similar procedure in the proof of Proposition 3 to solve for the bubbly steady-state labor:

$$\nu N_b^{1+\varphi} = (1-\alpha-\gamma)\frac{\exp(g)-\beta h}{\exp(g)-h}\frac{y_b}{c_b}.$$

When the second inequality in (39) holds, the expression on the right-hand side of the equality is positive so that the equation above gives a unique positive solution for N_b.

Next we derive the bubbly steady-state real exchange rate e_b. Again, similar to proof of Proposition 3, equations (A.10) and (A.15) imply

$$\frac{1}{N_b} e_b^{\sigma} y^* \left(\frac{R_{kb}}{\alpha}\right)^{\frac{\alpha}{1-\alpha-\gamma}} \left(\frac{e_b}{\gamma}\right)^{\frac{\gamma}{1-\alpha-\gamma}} = \frac{x_b}{y_b} = \gamma + \frac{1}{\exp(g)} \frac{b_b}{y_b} \frac{\exp(g) - R_f^*}{R_{fb}}.$$

Given that the second inequality in (39) holds and steady-state labor N_b is solved, the expression on the right-hand side is positive so that the equation above gives a unique solution for $e_b > 0$.

We can then solve for x_b using equation (B.11), and solve for k_b, m_b and y_b using equations (B.17), (B.9), and (A.15). Since we have solved for b_b/y_b and y_b, we can derive b_b. We solve for p using (A.19) and solve for i_b and w_b using equations (B.5) and (B.16). Finally, we solve for c_b using equation (B.10) and λ_b using equation (B.14). We need the first inequality in (39) to ensure $c_b > 0$. Q.E.D.

Proof of Proposition 5 It follows from Lemma 1 that b/y increases with R_f^*. We deduce that the expression on the right-hand side of equation (35) increases in R_f^* and decreases in $\overline{\varepsilon}_f$. Thus the solution $\overline{\varepsilon}_f$ to this equation must increase with R_f^*. When R_f^* is small, $\overline{\varepsilon}_f$ is small and hence the expression on the right-hand side of equation (37) is likely to hold so that an asset bubble is likely to emerge. But when R_f^* is sufficiently high, $\overline{\varepsilon}_f$ approaches $\varepsilon_{\max}$. In this case condition (37) cannot hold and hence a bubble cannot emerge.

When a bubble emerges, $\overline{\varepsilon}_b$ is determined by equation (38) which does not depend on R_f^*. Thus Q_b, R_{kb} and R_{fb} do not depend on R_f^* either. They all remain constant when R_f^* varies.

By Lemma 1, b_b/y_b increases with R_f^*. It follows from (40) that p/y_b decreases with R_f^*. Q.E.D.

B Detrended Equilibrium System

The variables Q_t, $\overline{\varepsilon}_t$, e_t, R_{ft}, P_{kt}, R_{kt} and N_t do not have trend. All other equilibrium variables in Proposition 2 grow at the rate of technical progress except for Λ_t. Let $\Lambda_t = \lambda_t/A_t$ and any other growing variable $Z_t = z_t A_t$. We then obtain the detrended equilibrium system:

$$Q_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\exp(g_{at+1})} \left[R_{kt+1} + (1-\delta) Q_{t+1} + (R_{kt+1} + \mu) \int_{\overline{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{B.1}$$

$$p_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} p_{t+1} \left[1 + \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{B.2}$$

$$\frac{1}{R_{ft}} = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\exp(g_{at+1})} \left[1 + \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{B.3}$$

$$0 = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\exp(g_{at+1})} \left(\frac{R^*_{ft}}{R_{ft}} \frac{e_{t+1}}{e_t} - 1 \right) - \frac{\Omega}{y_t} \left(b_t - \frac{b}{\exp(g_{at})} \right), \tag{B.4}$$

$$i_t = \frac{1}{P_{kt}} \left[R_{kt} \frac{k_{t-1}}{\exp(g_{at})} + \mu \frac{k_{t-1}}{\exp(g_{at})} + \frac{b_{t-1}}{\exp(g_{at})} + p_t \right] \left(\int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} f(\varepsilon) d\varepsilon \right), \tag{B.5}$$

$$k_t = \frac{1-\delta}{\exp(g_{at})} k_{t-1} + \frac{1}{P_{kt}} \left[R_{kt} \frac{k_{t-1}}{\exp(g_{at})} + \mu \frac{k_{t-1}}{\exp(g_{at})} + \frac{b_{t-1}}{\exp(g_{at})} + p_t \right] \left(\int_{\bar{\varepsilon}_t}^{\varepsilon_{max}} \varepsilon f(\varepsilon) d\varepsilon \right), \tag{B.6}$$

$$\begin{aligned} P_{kt} = {} & 1 + \frac{\Omega_k}{2} \left(\exp(g_{at}) \frac{i_t}{i_{t-1}} - \exp(g) \right)^2 + \Omega_k \left(\exp(g_{at}) \frac{i_t}{i_{t-1}} - \exp(g) \right) \exp(g_{at}) \frac{i_t}{i_{t-1}} \\ & - \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_k \left(\exp(g_{at+1}) \frac{i_{t+1}}{i_t} - \exp(g) \right) \exp(g_{at+1}) \left(\frac{i_{t+1}}{i_t} \right)^2 \end{aligned} \tag{B.7}$$

$$y_t = \left(\frac{k_{t-1}}{\exp(g_{at})} \right)^{\alpha} m_t^{\gamma} N_t^{1-\alpha-\gamma}, \tag{B.8}$$

$$e_t m_t = \gamma y_t, \tag{B.9}$$

$$y_t = c_t + \left[1 + \frac{\Omega_k}{2} \left(\exp(g_{at}) \frac{i_t}{i_{t-1}} - \exp(g) \right)^2 \right] i_t + x_t, \tag{B.10}$$

$$x_t = e_t^{\sigma} y_t^*, \tag{B.11}$$

$$\bar{\varepsilon}_t Q_t = P_{kt}, \tag{B.12}$$

$$\nu \xi_t N_t^{\varphi} = w_t \lambda_t, \tag{B.13}$$

$$\lambda_t = \frac{\xi_t}{c_t - h \frac{c_{t-1}}{\exp(g_{at})}} - \beta h \mathbb{E}_t \frac{\xi_{t+1}}{\exp(g_{at+1}) c_{t+1} - h c_t} \tag{B.14}$$

$$x_t - e_t m_t = \frac{b_t}{R_{ft}} - \frac{b_{t-1}}{\exp(g_{at})} \frac{R^*_{ft-1}}{R_{ft-1}} \frac{e_t}{e_{t-1}}, \tag{B.15}$$

$$N_t = \left(\frac{1-\alpha-\gamma}{w_t} \right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t} \right)^{\frac{\gamma}{\alpha}} \frac{k_{t-1}}{\exp(g_{at})}, \tag{B.16}$$

$$R_{kt} = \alpha \exp(g_{at}) \frac{y_t}{k_{t-1}}, \tag{B.17}$$

for the endogenous variables $\{Q_t, \bar{\varepsilon}_t, p_t, R_{ft}, b_t, i_t, k_t, y_t, m_t, c_t, x_t, e_t, w_t, R_{kt}, P_{kt}, N_t, \lambda_t\}$. The growth of the aggregate productivity is

$$A_t / A_{t-1} \equiv \exp(g_{at}) = \exp(g_t) \exp(a_t) / \exp(a_{t-1}). \tag{B.18}$$

In the deterministic steady state $g_{at} = g$.

C Log-linearized System

We present the log-linearized system for the bubbly equilibrium. The log-linearized system for the bubbleless equilibrium system is similar except that $P_t = 0$. Let $\hat{b}_t$, $\hat{g}_{at}$, and $\hat{g}_t$ denote level deviations from their steady state values, and $\hat{z}_t$ denotes the log deviation from its steady state value for any other variable z_t.

(1) The log-linearized version of equation (B.1):

$$\begin{aligned}\hat{Q}_t &= \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \mathbb{E}_t\hat{g}_{at+1} + \frac{\beta}{\exp(g)}\frac{R_k}{Q}\mathbb{E}_t\hat{R}_{kt+1} + \frac{\beta}{\exp(g)}(1-\delta)\mathbb{E}_t\hat{Q}_{t+1} \\ &\quad + \frac{\beta}{\exp(g)}\frac{R_k\frac{1}{\eta-1}\left(\frac{\varepsilon_{min}}{\overline{\varepsilon}}\right)^{\eta}}{Q}\mathbb{E}_t\hat{R}_{kt+1} - \eta\frac{\beta}{\exp(g)}\frac{(R_k+\mu)\frac{1}{\eta-1}\left(\frac{\varepsilon_{min}}{\overline{\varepsilon}}\right)^{\eta}}{Q}\mathbb{E}_t\hat{\overline{\varepsilon}}_{t+1}. \end{aligned} \tag{C.1}$$

(2) The log-linearized version of equation (B.2):

$$\hat{p}_t = \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t + \mathbb{E}_t\hat{p}_{t+1} - \eta(1-\beta)\mathbb{E}_t\hat{\overline{\varepsilon}}_{t+1}. \tag{C.2}$$

(3) The log-linearized version of equation (B.3):

$$-\hat{R}_{ft} = \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \mathbb{E}_t\hat{g}_{at+1} - \eta(1-\beta)\mathbb{E}_t\hat{\overline{\varepsilon}}_{t+1}. \tag{C.3}$$

(4) The log-linearized version of equation (B.4):

$$\begin{aligned}0 &= \frac{\beta}{\exp(g)}\left(\frac{R_f^*}{R_f}-1\right)\left(\mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \mathbb{E}_t\hat{g}_{at+1}\right) + \frac{\beta}{\exp(g)}\frac{R_f^*}{R_f}\left(\hat{R}_{ft}^* - \hat{R}_{ft} + \mathbb{E}_t\hat{e}_{t+1} - \hat{e}_t\right) \\ &\quad -\frac{\Omega}{y}\hat{b}_t + \Omega\frac{\exp(g)-1}{\exp(g)}\frac{b}{y}\hat{y}_t - \frac{\Omega}{\exp(g)}\frac{b}{y}\hat{g}_{at}. \end{aligned} \tag{C.4}$$

(5) The log-linearized version of equation (B.5):

$$\begin{aligned}\hat{\imath}_t + \hat{P}_{kt} &= \left(\frac{\varepsilon_{min}}{\overline{\varepsilon}}\right)^{\eta}\frac{R_k k}{\exp(g)i}\left(\hat{R}_{kt} + \hat{k}_{t-1} - \hat{g}_{at}\right) + \left(\frac{\varepsilon_{min}}{\overline{\varepsilon}}\right)^{\eta}\frac{\mu k}{\exp(g)i}\left(\hat{k}_{t-1} - \hat{g}_{at}\right) \\ &\quad + \left(\frac{\varepsilon_{min}}{\overline{\varepsilon}}\right)^{\eta}\frac{1}{\exp(g)i}\hat{b}_{t-1} - \left(\frac{\varepsilon_{min}}{\overline{\varepsilon}}\right)^{\eta}\frac{b}{\exp(g)i}\hat{g}_{at} \\ &\quad + \left(\frac{\varepsilon_{min}}{\overline{\varepsilon}}\right)^{\eta}\frac{p}{i}\hat{p}_t - \eta\hat{\overline{\varepsilon}}_t. \end{aligned} \tag{C.5}$$

(6) The log-linearized version of equation (B.6):

$$\hat{k}_t = \frac{1-\delta}{\exp(g)}\left(\hat{k}_{t-1} - \hat{g}_{at}\right) + \frac{\delta + \exp(g) - 1}{\exp(g)}\left(\hat{\imath}_t + \hat{\overline{\varepsilon}}_t\right). \tag{C.6}$$

(7) The log-linearized version of equation (B.7):

$$\hat{P}_{kt} = \Omega_k\exp(2g)\left[\hat{g}_{at} + (1+\beta)\hat{\imath}_t - \hat{\imath}_{t-1} - \beta\mathbb{E}_t\hat{g}_{at+1} - \beta\mathbb{E}_t\hat{\imath}_{t+1}\right]. \tag{C.7}$$

(8) The log-linearized version of equation (B.8):

$$\hat{y}_t = \alpha \hat{k}_{t-1} - \alpha \hat{g}_{at} + \gamma \hat{m}_t + (1 - \alpha - \gamma)\hat{N}_t. \tag{C.8}$$

(9) The log-linearized version of equation (B.9):

$$\hat{y}_t = \hat{e}_t + \hat{m}_t. \tag{C.9}$$

(10) Log-linearized version of equation (B.10):

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{\imath}_t + \frac{x}{y}\hat{x}_t. \tag{C.10}$$

(11) The log-linearized version of equation (B.11):

$$\hat{x}_t = \sigma \hat{e}_t + \hat{y}_t^*. \tag{C.11}$$

(12) The log-linearized version of equation (B.12):

$$\hat{\bar{\varepsilon}}_t + \hat{Q}_t = \hat{P}_{kt}. \tag{C.12}$$

(13) The log-linearized version of equation (B.13):

$$\hat{\xi}_t + \varphi \hat{N}_t = \hat{w}_t + \hat{\lambda}_t. \tag{C.13}$$

(14) The log-linearized version of equation (B.14):

$$\begin{aligned} \hat{\lambda}_t \quad = \quad & -\frac{\exp(g)}{\exp(g) - \beta h}\left[\frac{\exp(g)}{\exp(g) - h}\hat{c}_t - \frac{h}{\exp(g) - h}\left(\hat{c}_{t-1} - \hat{g}_{at}\right) - \hat{\xi}_t\right] \\ & + \frac{\beta h}{\exp(g) - \beta h}\left[\frac{\exp(g)}{\exp(g) - h}\left(\mathbb{E}_t \hat{g}_{at+1} + \mathbb{E}_t \hat{c}_{t+1}\right) - \frac{h}{\exp(g) - h}\hat{c}_t - \mathbb{E}_t \hat{\xi}_{t+1}\right]. \end{aligned} \tag{C.14}$$

(15) The log-linearized version of equation (B.15):

$$\begin{aligned} 0 \quad = \quad & \frac{x}{y}\hat{x}_t - \gamma\left(\hat{e}_t + \hat{m}_t\right) - \frac{1}{R_f y}\hat{b}_t + \frac{b}{R_f y}\hat{R}_{ft} + \frac{R_f^*}{\exp(g) R_f y}\hat{b}_{t-1} \\ & + \frac{b}{\exp(g) y}\frac{R_f^*}{R_f}\left(-\hat{g}_{at} + \hat{R}_{ft-1}^* - \hat{R}_{ft-1} + \hat{e}_t - \hat{e}_{t-1}\right). \end{aligned} \tag{C.15}$$

(16) The log-linearized version of equation (B.16):

$$\hat{N}_t = -\frac{1-\gamma}{\alpha}\hat{w}_t - \frac{\gamma}{\alpha}\hat{e}_t + \hat{k}_{t-1} - \hat{g}_{at}. \tag{C.16}$$

(17) The log-linearized version of equation (B.17):

$$\hat{R}_{kt} = \hat{g}_{at} + \hat{y}_t - \hat{k}_{t-1}. \tag{C.17}$$

(18) The log-linearized version of equation (B.18):

$$\hat{g}_{at} = \hat{g}_t + \hat{a}_t - \hat{a}_{t-1}. \tag{C.18}$$

The log-linearized shock processes are given by

$$\begin{aligned}
\hat{g}_t &= \rho_g \hat{g}_{t-1} + \sigma_g \varepsilon_{gt}, \\
\hat{a}_t &= \rho_a \hat{a}_{t-1} + \sigma_a \varepsilon_{at}, \\
\hat{\xi}_t &= \rho_\xi \hat{\xi}_{t-1} + \sigma_\xi \varepsilon_{\xi t}, \\
\hat{y}^*_t &= \rho_{y^*} \hat{y}^*_{t-1} + \sigma_{y^*} \varepsilon_{y^* t}, \\
\hat{R}^*_{ft} &= \rho_{R^*_f} \hat{R}^*_{ft-1} + \sigma_{R^*_f} \varepsilon_{R^*_f t}.
\end{aligned}$$

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Table 1: Calibrated Parameters

Parameter	Value	Description
g	0.01	Mean of quarterly growth rate of long-run productivity
β	0.99	Discount factor
α	0.32	Capital share in output
δ	0.05	Depreciation rate
φ	2	Inverse Frisch elasticity
γ	0.27	Imported good share in output
σ	1.0	Price elasticity of export
η	5	Shape parameter of Pareto distribution
R_f^*	1.006	Average quarterly gross foreign interest rate
Ω	0.20	Capital flow adjustment cost

Table 2: Estimated Parameters

Parameter	Prior Distribution			Posterior Distribution			
	Distr.	Mean	S.D.	Mode	Mean	5%	95%
			Bubbly model				
h	Beta	0.5	0.2	0.62	0.63	0.54	0.71
Ω_k	Gamma	3	2	2.26	2.51	1.64	3.33
μ	Beta	0.15	0.05	0.17	0.15	0.07	0.23
ρ_g	Beta	0.5	0.2	0.95	0.92	0.86	0.98
ρ_a	Beta	0.5	0.2	0.65	0.64	0.56	0.74
ρ_ξ	Beta	0.5	0.2	0.32	0.33	0.16	0.49
ρ_{y^*}	Beta	0.5	0.2	0.997	0.996	0.992	0.9995
$\rho_{R_f^*}$	Beta	0.85	0.05	0.76	0.76	0.71	0.81
σ_g (%)	Inv. Gamma	0.01	Inf	0.29	0.35	0.23	0.47
σ_a (%)	Inv. Gamma	0.01	Inf	3.40	3.61	2.98	4.23
σ_ξ (%)	Inv. Gamma	0.01	Inf	9.37	9.97	8.02	11.99
σ_{y^*} (%)	Inv. Gamma	0.01	Inf	0.79	0.81	0.71	0.91
$\sigma_{R_f^*}$ (%)	Inv. Gamma	0.01	Inf	0.47	0.49	0.42	0.56
			Bubbleless model				
h	Beta	0.5	0.2	0.14	0.20	0.05	0.35
Ω_k	Gamma	3	2	0.03	0.06	0.01	0.11
μ	Beta	0.15	0.05	0.06	0.08	0.02	0.15
ρ_g	Beta	0.5	0.2	0.13	0.14	0.04	0.24
ρ_a	Beta	0.5	0.2	0.44	0.43	0.28	0.58
ρ_ξ	Beta	0.5	0.2	0.84	0.81	0.71	0.93
ρ_{y^*}	Beta	0.5	0.2	0.997	0.995	0.991	0.9995
$\rho_{R_f^*}$	Beta	0.85	0.05	0.89	0.89	0.84	0.93
σ_g (%)	Inv. Gamma	0.01	Inf	4.05	4.19	3.66	4.72
σ_a (%)	Inv. Gamma	0.01	Inf	0.87	0.90	0.79	1.02
σ_ξ (%)	Inv. Gamma	0.01	Inf	4.12	4.58	3.54	5.61
σ_{y^*} (%)	Inv. Gamma	0.01	Inf	0.80	0.82	0.72	0.92
$\sigma_{R_f^*}$ (%)	Inv. Gamma	0.01	Inf	0.43	0.44	0.38	0.50

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table 3: Real Business Cycle Moments

Moment	Data	Bubbly	Bubbleless 1	Bubbleless 2
		Standard Deviation		
$\text{std}(\text{gdp}_t)$ (%)	2.68	3.16	1.89	2.82
$\text{std}(\text{c}_t)/\text{std}(\text{gdp}_t)$	1.32	1.54	1.82	0.98
$\text{std}(\text{i}_t)/\text{std}(\text{gdp}_t)$	2.89	2.86	1.35	1.84
$\text{std}(\text{nx}_t/\text{gdp}_t)$ (%)	1.61	3.06	1.92	2.30
$\text{std}(\text{sp}_t)/\text{std}(\text{gdp}_t)$	6.39	4.69	1.98	0.62
		Correlation with GDP		
$\text{corr}(\text{c}_t, \text{gdp}_t)$	0.73	0.73	0.33	0.19
$\text{corr}(\text{i}_t, \text{gdp}_t)$	0.79	0.75	0.15	0.41
$\text{corr}(\text{nx}_t/\text{gdp}_t, \text{gdp}_t)$	-0.50	-0.49	0.52	0.77
$\text{corr}(\text{sp}_t, \text{gdp}_t)$	0.64	0.48	0.11	0.64

Note: Column 3 and 4 display the real business cycle moments for the bubbly and bubbleless economies based on the same parameter values estimated using the bubbly model. Column 5 displays the real business cycle moments for bubbleless model 2 using re-estimated parameter values.

Table 4: Variance Decomposition of Bubbly Economy (%)

Period	Long-run Prod.	Short-run Prod.	Preference	Foreign Demand	Foreign Interest Rate
			Output		
Q1	10.1	85.5	0.2	1.7	2.4
Q4	43.7	45.0	0.4	3.1	7.8
Q8	65.7	24.5	1.0	2.9	5.8
Q16	85.4	9.4	0.7	1.9	2.6
			Consumption		
Q1	28.1	11.5	49.7	1.5	9.2
Q4	50.5	10.5	27.0	1.8	10.2
Q8	66.5	6.8	17.2	1.6	7.8
Q16	82.0	3.5	8.9	1.2	4.4
			Investment		
Q1	34.2	26.2	17.8	2.9	18.9
Q4	40.6	20.0	18.6	2.9	17.8
Q8	49.5	15.2	16.9	2.9	15.6
Q16	63.7	10.8	12.1	2.3	11.1
			Net exports/GDP		
Q1	54.2	0.1	16.9	2.1	26.7
Q4	63.0	1.7	6.7	2.1	26.6
Q8	65.9	1.9	6.0	1.8	24.4
Q16	67.9	2.0	5.6	1.7	22.8
			Stock price		
Q1	39.8	24.9	11.9	3.1	20.2
Q4	41.5	24.2	13.0	2.9	18.5
Q8	44.3	22.7	12.6	2.8	17.6
Q16	51.9	19.3	11.0	2.5	15.2

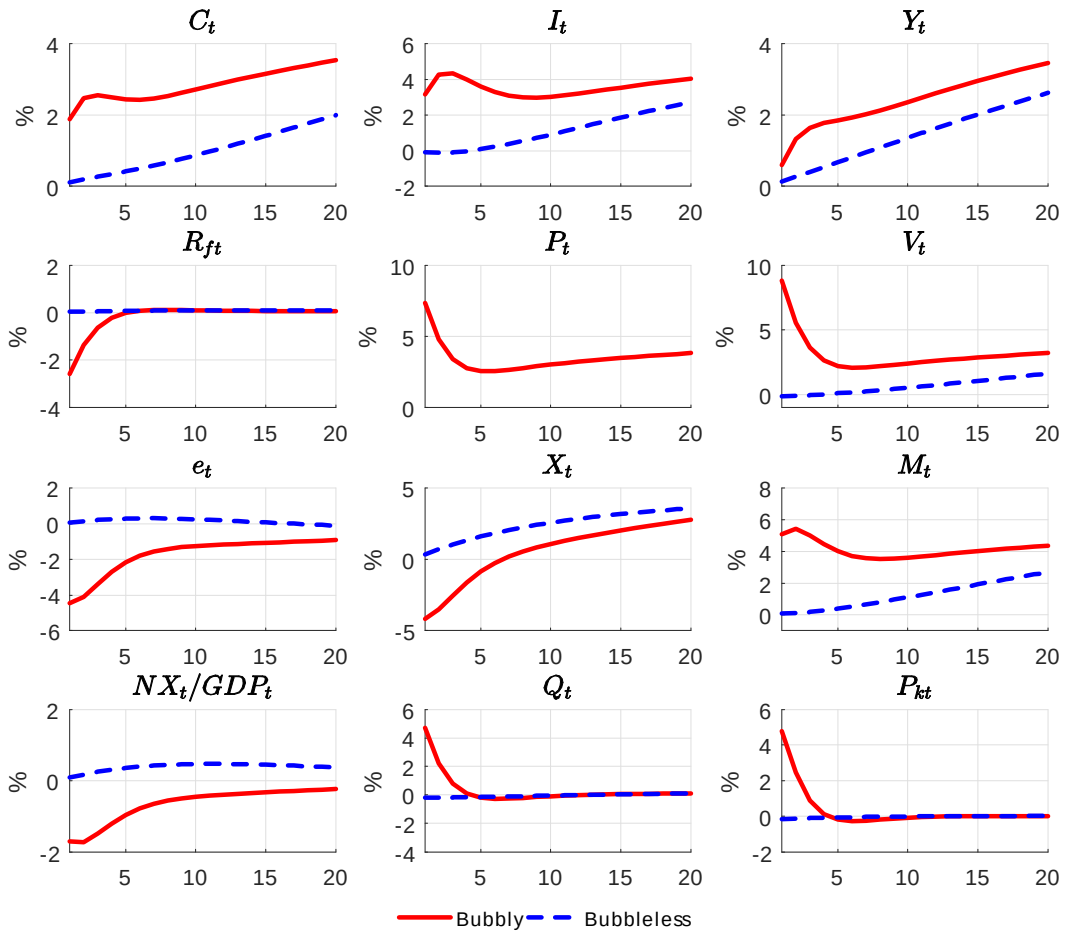

Figure 1: This figure plots the impulse response to a positive one-standard-deviation shock to the long-run productivity for the domestic economy. The solid lines describe the bubbly economy based on the bubbly model estimation. The dashed lines describe the bubbleless economy with the same parameters. R_{ft} and NX_t/GDP_t are expressed as level deviations from their nonstochastic steady state values. e_t, Q_t and P_{kt} are expressed as percentage deviations from their nonstochastic steady state values. Other variables are expressed as percentage deviations from their corresponding nonstochastic steady state values with trend along balanced growth paths.

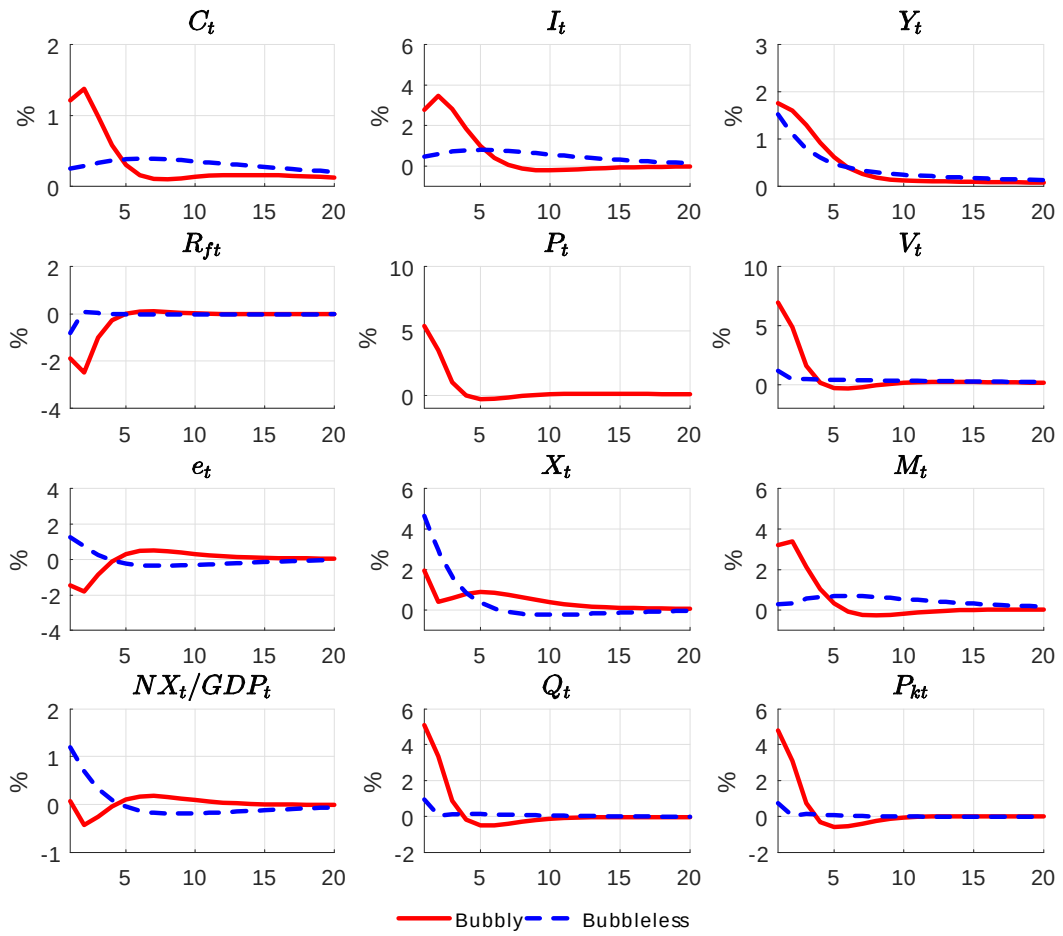

Figure 2: This figure plots the impulse response to a positive one-standard-deviation shock to the short-run productivity for the domestic economy. The solid lines describe the bubbly economy based on the bubbly model estimation. The dashed lines describe the bubbleless economy with the same parameters. R_{ft} and NX_t/GDP_t are expressed as level deviations from their nonstochastic steady state values. e_t, Q_t and P_{kt} are expressed as percentage deviations from their nonstochastic steady state values. Other variables are expressed as percentage deviations from their corresponding nonstochastic steady state values with trend along balanced growth paths.

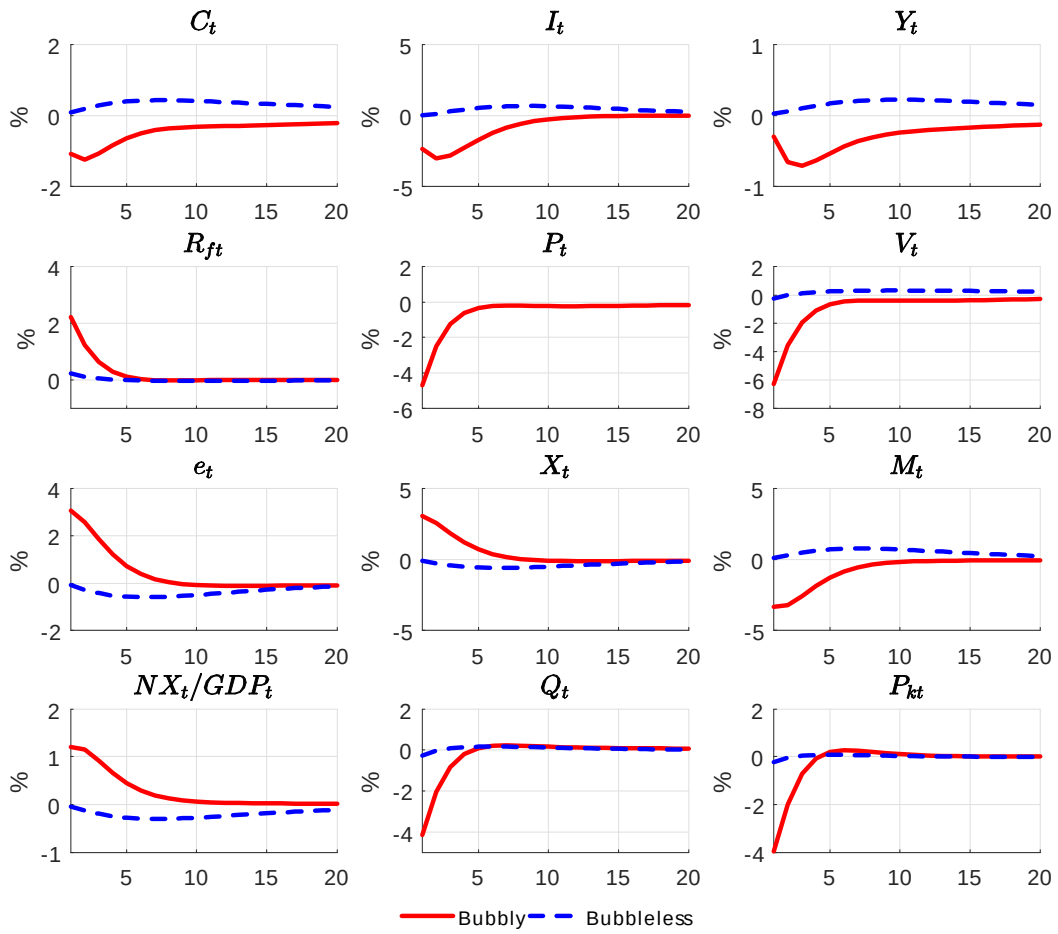

Figure 3: This figure plots the impulse response to a positive one-standard-deviation shock to the foreign interest rate for the domestic economy. The solid lines describe the bubbly economy based on the bubbly model estimation. The dashed lines describe the bubbleless economy with the same parameters. R_{ft} and NX_t/GDP_t are expressed as level deviations from their nonstochastic steady state values. e_t, Q_t and P_{kt} are expressed as percentage deviations from their nonstochastic steady state values. Other variables are expressed as percentage deviations from their corresponding nonstochastic steady state values with trend along balanced growth paths.