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Lumpy Investment and Corporate Tax Policy

In the presence of both convex and nonconvex capital adjustment costs in a dynamic general equilibrium model, corporate tax policy generates both intensive and extensive margin effects via the channel of marginal Q. Its impact is determined largely by the strength of the extensive margin effect, which, in turn, depends on the cross-sectional distribution of firms. Depending on the initial distribution of firms, the economy displays asymmetric responses to tax changes. Moreover, an anticipated increase in the future investment tax credit reduces investment and adjustment rate initially.

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CORPORATE TAX POLICY IS an important instrument to influence firms' capital investment decisions and hence to stimulate the economy. Its transmission channel is through either the user cost of capital according to the neoclassical theory of investment or Tobin's marginal Q according to the q-theory of investment.¹ The neoclassical theory assumes that firms do not face any investment frictions, while the q-theory takes into account convex capital adjustment costs.

Recent empirical evidence documents that investment at the plant level is lumpy and infrequent, indicating the existence of nonconvex capital adjustment costs.² Motivated by this evidence, we address two central questions: (i) How does corporate

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1. See Jorgenson (1963), Hall and Jorgenson (1967), Tobin (1969), and Hayashi (1982). See Abel (1990) for a discussion on the relation between the two theories. See Hassett and Hubbard (1997) for a survey.

2. See, for example, Doms and Dunne (1998), Caballero, Engel, and Haltiwanger (1995), Caballero and Engel (1999), and Cooper and Haltiwanger (2006).

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tax policy affect investment at both the macro and micro levels in the presence of nonconvex capital adjustment costs? (ii) Is lumpy investment important quantitatively in determining the impact of tax policy on the economy in the short and long run?

To answer these two questions, one has to overcome two major difficulties. First, in the presence of nonconvex adjustment costs, the standard q-theory widely used in the analysis of tax policy fails in the sense that investment may not be monotonically related to marginal Q (Caballero and Leahy 1996). Even though a modified q-theory may work, the relationship between investment and marginal Q is nonlinear (Abel and Eberly 1994). Second, investment at the micro level is nonlinear, making aggregation difficult. This problem is especially severe in a dynamic general equilibrium framework, because one has to deal with the cross-sectional distribution of firms when solving equilibrium prices.

Our solution to these two difficulties is based on the generalized (S,s) approach proposed by Caballero and Engel (1999). The key idea of this approach is to introduce stochastic fixed adjustment costs, which makes aggregation tractable. We combine this approach with the Abel and Eberly (1994) approach by assuming firms face both flow convex and fixed adjustment costs. As a result, at the micro level, a modified *q*-theory applies in that investment is a nondecreasing function of marginal *Q*. In particular, there is a region for the stochastic fixed adjustment costs in which investment is zero, independent of marginal *Q*. Outside of this region, investment increases with marginal *Q*.

To resolve the curse of dimensionality issue encountered when numerically solving general equilibrium models with lumpy investment, Khan and Thomas (2003, 2008) and Bachmann, Caballero, and Engel (2008) use the Krusell and Smith (1998) algorithm, which approximates the distribution of firms by a finite set of moments.³ Typically, the first moment is sufficient. We deal with the dimensionality problem by making two assumptions as in Miao and Wang (2009): (i) firms' production technology has constant returns to scale, and (ii) fixed capital adjustment costs are proportional to the capital stock. We show that under these two assumptions, we obtain exact aggregation in the sense that the cross-sectional distribution of capital matters only to the extent of its mean. We then characterize equilibrium by a system of nonlinear difference equations, which can be tractably solved numerically. This system also makes the analysis and intuition transparent.

We abstract from aggregate uncertainty and extend Miao and Wang's (2009) model by incorporating tax policy. We study the short- and long-run effects of temporary and permanent changes in the corporate tax rate and the investment tax credit (ITC). We also analyze anticipation effects of tax changes. We find that in the presence of fixed adjustment costs, corporate tax policy affects a firm's decision on the timing and size of investment at the micro level. At the macro level, it affects the size of investment for each adjusting firm and the number of total adjusting firms or the adjustment rate. Thus, corporate tax policy has both intensive and extensive margin effects. By

^{3.} Thomas (2002) and Gourio and Kashyap (2007) apply the numerical method developed by Dotsey, King, and Wolman (1999).

contrast, it has an intensive margin effect only according to the neoclassical theory or the q-theory of investment.

We numerically show that the extensive margin effect plays an important role in determining the impact of the corporate tax changes. The strength of this effect depends crucially on the cross-sectional distribution of firms. As Miao and Wang (2009) point out, the larger the steady-state elasticity of the adjustment rate with respect to the investment trigger, the larger is the extensive margin effect. In addition, if the distribution is such that most firms have made capital adjustments prior to tax changes, then expansionary tax policy is less effective in stimulating the economy because the additional number of firms that decide to invest is constrained, while contractionary tax policy is more effective. The opposite is true if the distribution is such that most firms have not made capital adjustments prior to tax changes.

We show that the extensive margin effect is much larger in partial equilibrium than in general equilibrium. In a calibrated experiment, we find that the increase in the investment rate in partial equilibrium in response to an unanticipated temporary 10 percentage point decrease in the corporate income tax rate can be twice as large as that in general equilibrium. The intuition is that the general equilibrium price movements dampen the increase in firm profitability and marginal Q.

Furthermore, we show that under constant returns-to-scale technology and perfect competition, a temporary change in the corporate tax rate can have a permanent effect on capital accumulation in partial equilibrium, while it has a temporary effect in general equilibrium. In addition, an unanticipated temporary increase in the ITC has a larger short-run stimulative impact than a permanent increase in the ITC, in contrast to Abel's (1982) result in a partial equilibrium model. The preceding results suggest that a partial equilibrium analysis of tax policy can be quite misleading both quantitatively and qualitatively.

Our paper is related to a large literature on the impact of tax policy on investment beginning from Hall and Jorgenson (1967) and Hall (1971). Most studies use a partial equilibrium *q*-theory framework with convex adjustment costs (e.g., Summers 1981, Abel 1982, Hayashi 1982, Auerbach 1989). Instead, we use a general equilibrium deterministic growth model framework as in Chamley (1981), Judd (1985, 1987), Prescott (2002), McGrattan and Prescott (2005), House and Shapiro (2006), and Gourio and Miao (2010, 2011).⁴ Unlike these papers, this paper incorporates heterogeneous firms subject to both convex and nonconvex capital adjustment costs. Miao (2008) also incorporates these costs in a continuous time model, but he focuses on a steady-state analysis. We extend Judd's (1987) and Auerbach's (1989) results on anticipation effects of tax policy to a lumpy investment model. We show that an anticipated decrease in the future corporate income tax rate raises investment and adjustment rate immediately, while an anticipated increase in the future ITC reduces investment and adjustment rate initially.

4. See Ljungqvist and Sargent (2004, Chap. 11) for a textbook treatment. See Auerbach and Kotlikoff (1987) for the study of tax policy in the general equilibrium overlapping generations framework.

The remainder of the paper proceeds as follows. Section 1 presents the model. Section 2 analyzes equilibrium properties. Section 3 provides numerical results. Section 4 concludes. Proofs are relegated to the Appendix.

1. THE MODEL

We consider an infinite-horizon economy that consists of a representative household, a continuum of production units with a unit mass, and a government. Time is discrete and indexed by t = 1, 2, 3, ... We identify a production unit with a firm or a plant. Firms are subject to idiosyncratic shocks to fixed capital adjustment costs. To focus on the dynamic effects of permanent and temporary tax changes, we abstract away from aggregate uncertainty and long-run economic growth. By a law of large numbers, all aggregate quantities and prices are deterministic over time.

1.1 Firms

All firms have identical production technology that combines labor and capital to produce output. Specifically, if firm *j* owns capital K_t^j and hires labor N_t^j , it produces output Y_t^j according to the production function

$$Y_t^J = F(K_t^J, N_t^J). \tag{1}$$

Assume that F is strictly increasing, strictly concave, continuously differentiable, and satisfies the usual Inada conditions. In addition, it has constant returns to scale.

Firm *j* may make investment I_t^j to increase its existing capital stock K_t^j . Investment incurs both nonconvex and convex adjustment costs. We follow Uzawa (1969) and Hayashi (1982) and introduce convex adjustment costs into the capital accumulation equation

$$K_{t+1}^{j} = (1-\delta)K_{t}^{j} + K_{t}^{j}\Phi\left(\frac{I_{t}^{j}}{K_{t}^{j}}\right), \quad K_{1}^{j} \text{ given,}$$

$$(2)$$

where δ is the depreciation rate and $\Phi(\cdot)$ is a strictly increasing and strictly concave function.⁵ To facilitate analytical solutions, we follow Jermann (1998) and specify the convex adjustment cost function as

$$\Phi(x) = \frac{\psi}{1-\theta} x^{1-\theta} + \varsigma, \qquad (3)$$

5. Assuming convex adjustment costs that decrease profits directly as in Abel (1982) and Auerbach (1989) will not change our analysis significantly.

where $\psi > 0$ and $\theta \in (0, 1)$. Nonconvex adjustment costs are fixed costs that must be paid if and only if the firm chooses to invest. As in Cooper and Haltiwanger (2006), we measure these costs as a fraction of the firm's capital stock.⁶ That is, if firm *j* makes new investment, then it pays fixed costs $\xi_t^j K_t^j$, which is independent of the amount of investment. As will be clear later, this modeling of fixed costs is important to ensure that firm value is linearly homogenous. Following Caballero and Engel (1999), we assume that ξ_t^j is identically and independently drawn from a distribution with density ϕ over $[0, \xi_{max}]$ across firms and over time.

Each firm j pays dividends to households who are shareholders of the firm. Dividends are given by

$$D_{t}^{j} = \left(1 - \tau_{t}^{k}\right) \left(F\left(K_{t}^{j}, N_{t}^{j}\right) - w_{t}N_{t}^{j}\right) + \tau_{t}^{k}\delta K_{t}^{j} - \left(1 - \tau_{t}^{i}\right)I_{t}^{j} - \xi_{t}^{j}K_{t}^{j}\mathbf{1}_{I_{t}^{j}\neq0},$$
(4)

where w_t is the wage rate, τ_t^k is the corporate income tax rate, and τ_t^i is the ITC. Here, $\mathbf{1}_{I_t^j \neq 0}$ represents the indicator function taking the value 1 if $I_t^j \neq 0$, and 0 otherwise. We have assumed that economic depreciation is equal to physical depreciation. Allowing for more complicated depreciation allowance schemes will complicate our analysis without changing our key insights.

After observing its idiosyncratic shock ξ_t^j , firm j's objective in period t is to maximize cum-dividends market value of equity P_t^j ,

$$\max P_t^{j} \equiv E_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} D_{t+s}^{j},$$
(5)

subject to (2) and (4). Here, the expectation is taken with respect to the idiosyncratic shock distribution and $\beta^s \Lambda_{t+s} / \Lambda_t$ is the stochastic discount factor between periods *t* and *t* + *s*. We will show later that Λ_{t+s} is a household's marginal utility in period *t* + *s*.

1.2 Households

All households are identical and derive utility from the consumption and labor sequences $\{C_t, N_t\}$ according to the time-additive utility function

$$E\left[\sum_{t=1}^{\infty}\beta^{t-1}U(C_t,N_t)\right],\tag{6}$$

6. There are several ways to model fixed adjustment costs in the literature. Fixed costs may be proportional to the demand shock (Abel and Eberly 1998), profits (Caballero and Engel 1999, Cooper and Haltiwanger 2006), or labor costs (Thomas 2002, Khan and Thomas 2003, 2008).

where $\beta \in (0, 1)$ is the discount factor and U satisfies the usual assumptions. Each household chooses consumption C_t , labor supply N_t , and shareholdings α_{t+1}^j to maximize utility (6) subject to the budget constraint:

$$C_t + \int \alpha_{t+1}^j \left(P_t^j - D_t^j \right) dj = \int \alpha_t^j P_t^j dj + \left(1 - \tau_t^n \right) w_t N_t + T_t, \tag{7}$$

where τ_t^n is the labor income tax rate and T_t denotes government transfers (lump-sum taxes) if $T_t > (<) 0$. The first-order conditions are given by

$$\Lambda_t \left(P_t^j - D_t^j \right) = \beta E_t \Lambda_{t+1} P_{t+1}^j, \tag{8}$$

$$U_1(C_t, N_t) = \Lambda_t, \tag{9}$$

$$-U_2(C_t, N_t) = \Lambda_t \left(1 - \tau_t^n\right) w_t.$$
⁽¹⁰⁾

Equations (8) and (9) imply that the stock price P_t^j is given by the discounted present value of dividends as in equation (5). In addition, Λ_t is equal to the marginal utility of consumption.

1.3 Government

The government finances government spending G_t by corporate and personal taxes. We assume that lump-sum taxes (or transfers) are available so that the government budget is balanced. The government budget constraint is given by

$$G_t + T_t = \tau_t^k \int \left(F(K_t^j, N_t^j) - w_t N_t^j - \delta K_t^j \right) dj + \tau_t^n w_t N_t - \tau_t^i \int I_t^j dj, (11)$$

where T_t represents lump-sum transfers (taxes) if $T_t > 0$ ($T_t < 0$).

1.4 Competitive Equilibrium

The sequences of quantities $\{I_t^j, N_t^j, K_t^j\}_{t\geq 1}$, $\{C_t, N_t\}_{t\geq 1}$, prices $\{w_t, P_t^j\}_{t\geq 1}$ for $j \in [0, 1]$, and government policy $\{\tau_t^k, \tau_t^n, \tau_t^i, T_t, G_t\}_{t\geq 1}$ constitute a competitive equilibrium if the following conditions are satisfied:

- (i) Given prices $\{w_t\}_{t\geq 1}$, $\{I_t^j, N_t^j\}_{t\geq 1}$ solves firm j's problem (5) subject to the law of motion (2).
- (ii) Given prices $\{w_t, P_t^j\}_{t \ge 1}$, $\{C_t, N_t, \alpha_{t+1}^j\}_{t \ge 1}$ maximizes utility in (6) subject to the budget constraint (7).
- (iii) Markets clear in that

$$lpha_t^j = 1,$$

 $N_t = \int N_t^j dj,$

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$$C_t + \int I_t^j dj + \int \xi_t^j K_t^j \mathbf{1}_{I_t^j \neq 0} dj + G_t = \int F\left(K_t^j, N_t^j\right) dj.$$
(12)

(iv) The government budget constraint (11) is satisfied.

2. EQUILIBRIUM PROPERTIES

We start by analyzing a single firm's optimal investment policy, holding prices fixed. Next, we conduct aggregation and characterize equilibrium aggregate dynamics by a system of nonlinear difference equations. Finally, we analyze steady state.

2.1 Optimal Investment Policy

To simplify problem (5), we first solve a firm's static labor choice decision. Let $n_t^j = N_t^j/K_t^j$. The first-order condition with respect to labor yields

$$f'(n_t^j) = w_t, \tag{13}$$

where we define $f(\cdot) = F(1, \cdot)$. This equation reveals that all firms choose the same labor–capital ratio in that $n_t^j = n_t$ for all *j*. We can then derive firm *j*'s operating profits

$$\max_{N_t^j} F\left(K_t^j, N_t^j\right) - w_t N_t^j = R_t K_t^j,$$
(14)

where $R_t = f(n_t) - w_t n_t$ is independent of *j*. Note that R_t also represents the marginal product of capital because *F* has constant returns to scale. Let $i_t^j = I_t^j / K_t^j$ denote firm *j*'s investment rate. We can then express dividends in (4) as

$$D_t^j = \left[\left(1 - \tau_t^k \right) R_t + \tau_t^k \delta - \left(1 - \tau_t^i \right) i_t^j - \xi_t^j \mathbf{1}_{i_t^j \neq 0} \right] K_t^j,$$

and rewrite (2) as

$$K_{t+1}^{j} = [(1-\delta) + \Phi(i_{t}^{j})]K_{t}^{j}.$$
(15)

The above two equations imply that equity value or firm value are linear in capital K_t^j . We can then write firm value as $V_t^j K_t^j$ and rewrite problem (5) by dynamic programming

$$V_{t}^{j}K_{t}^{j} = \max_{i_{t}^{j}} \left[\left(1 - \tau_{t}^{k} \right) R_{t} + \tau_{t}^{k} \delta - \left(1 - \tau_{t}^{i} \right) i_{t}^{j} - \xi_{t}^{j} \mathbf{1}_{i_{t}^{j} \neq 0} \right] K_{t}^{j} + E_{t} \left[\frac{\beta \Lambda_{t+1}}{\Lambda_{t}} V_{t+1}^{j} K_{t+1}^{j} \right],$$
(16)

subject to (15). Substituting equation (15) into equation (16) yields

$$V_{t}^{j} = \max_{i_{t}^{j}} (1 - \tau_{t}^{k}) R_{t} + \tau_{t}^{k} \delta - (1 - \tau_{t}^{i}) i_{t}^{j} - \xi_{t}^{j} \mathbf{1}_{i_{t}^{j} \neq 0} + g(i_{t}^{j}) E_{t} \left[\frac{\beta \Lambda_{t+1}}{\Lambda_{t}} V_{t+1}^{j} \right],$$
(17)

where we define

$$g(i_t^j) = 1 - \delta + \Phi(i_t^j). \tag{18}$$

Since V_t^j depends on ξ_t^j , we write it as $V_t^j = V_t(\xi_t^j)$ for some function V_t and suppress its dependence on other variables. We integrate out ξ_t^j and define

$$\bar{V}_t = \int_0^{\xi_{\text{max}}} V_t(\xi) \phi(\xi) d\xi.$$
⁽¹⁹⁾

Because ξ_t^j is i.i.d. across both time and firms and there is no aggregate shock, we obtain

$$E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^j \right] = \frac{\Lambda_{t+1}}{\Lambda_t} \int_0^{\xi_{\text{max}}} V_{t+1}(\xi) \phi(\xi) d\xi = \frac{\Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1}.$$
 (20)

Define marginal Q as the discounted shadow value of a marginal unit of investment

$$Q_t = \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1}.$$
(21)

Because firm value is linearly homogeneous in capital in our model, marginal Q is equal to average Q (Hayashi 1982). Using (21), we rewrite problem (17) as

$$V_t(\xi_t^j) = \max_{i_t^j} (1 - \tau_t^k) R_t + \tau_t^k \delta - (1 - \tau_t^i) i_t^j - \xi_t^j \mathbf{1}_{i_t^j \neq 0} + g(i_t^j) Q_t.$$
(22)

From this problem, we can characterize a firm's optimal investment policy by a generalized (S, s) rule (Caballero and Engel 1999).

PROPOSITION 1. Firm *j*'s optimal investment policy is characterized by the (S, s) policy in that there is a unique trigger value $\bar{\xi}_t > 0$ such that the firm invests if and only if $\xi_t^j \leq \bar{\xi}_t \equiv \min\{\xi_t^*, \xi_{\max}\}$, where the cutoff value ξ_t^* satisfies the equation

$$\frac{\theta}{1-\theta} \left(\psi Q_t\right)^{\frac{1}{\theta}} \left(1-\tau_t^i\right)^{\frac{\theta-1}{\theta}} = \xi_t^*.$$
(23)

The optimal target investment rate is given by

$$i_t^j = \left(\frac{\psi Q_t}{1 - \tau_t^i}\right)^{\frac{1}{\theta}}.$$
(24)

Marginal Q satisfies

$$Q_{t} = \frac{\beta \Lambda_{t+1}}{\Lambda_{t}} \left\{ \left(1 - \tau_{t+1}^{k}\right) R_{t+1} + \tau_{t+1}^{k} \delta + (1 - \delta + \varsigma) Q_{t+1} + \int_{0}^{\tilde{\xi}_{t+1}} \left[\xi_{t+1}^{*} - \xi\right] \phi(\xi) d\xi \right\}.$$
(25)

Equation (23) says that at the value ξ_t^* , the benefit from investment per unit of capital is equal to the fixed cost of investment per unit of capital so that the firm is indifferent between investing and not investing. It is possible that ξ_t^* exceeds the upper support of the fixed costs. In this case, we set the investment trigger $\bar{\xi}_t = \xi_{\text{max}}$.

We should emphasize that the investment trigger $\bar{\xi}_i$ depends on the aggregate capital stock and corporate tax policy. Thus, corporate tax policy affects the timing of investment at the micro level in that it affects the *adjustment hazard*, $\int_0^{\bar{\xi}_i} \phi(\xi) d\xi$. Under the *q*-theory of investment, corporate tax policy has only an intensive margin effect on investment in the sense that it can affect how much a firm may invest, but cannot affect how likely a firm may invest.

Equation (24) shows that the optimal investment level is independent of a firm's characteristics such as its capital or idiosyncratic shock. It is positively related to marginal Q if and only if the firm's idiosyncratic fixed cost shock ξ_t^j is lower than the trigger value $\bar{\xi}_t$, conditioned on the aggregate state and tax policy in the economy. When $\xi_t^j > \bar{\xi}_t$, firm j chooses not to invest. This zero investment is unrelated to marginal Q. As a result, optimal investment may not be related to marginal Q in the presence of fixed adjustment costs, a point made by Caballero and Leahy (1996). But it is still a nondecreasing function of marginal Q. Thus, a modified q-theory still works (Abel and Eberly 1994).

Equation (25) is an asset-pricing equation that states that the aggregate marginal Q is equal to the present value of marginal product of capital, plus an option value of waiting because of the fixed adjustment costs. When the shock $\xi_t^j > \bar{\xi}_t$, it is not optimal to pay the fixed costs to make investment. Firms will wait to invest until $\xi_t^j \leq \bar{\xi}_t$ and there is an option value of waiting.

We may interpret (25) as the capital demand equation and (24) as the capital supply equation. Define $\tilde{Q}_t = Q_t/(1 - \tau_t^i)$ as in Abel (1982) and Goolsbee (1998). It represents the tax-adjusted price of capital.

2.2 Aggregation and Equilibrium Characterization

Given the linear homogeneity feature of firm value, we can conduct aggregation tractably. We define aggregate capital $K_t = \int K_t^j dj$, aggregate labor demand

 $N_t = \int N_t^j dj$, aggregate output $Y_t = \int Y_t^j dj$, and aggregate investment expenditure in consumption units $I_t = \int I_t^j dj$.

PROPOSITION 2. The aggregate equilibrium sequences $\{Y_t, N_t, C_t, I_t, K_{t+1}, Q_t, \bar{\xi}_t\}_{t \ge 1}$ are characterized by the following system of difference equations:

$$I_t/K_t = \left(\frac{\psi Q_t}{1 - \tau_t^i}\right)^{\frac{1}{\theta}} \int_0^{\frac{z}{\xi_t}} \phi(\xi) d\xi,$$
(26)

$$K_{t+1} = (1 - \delta + \varsigma)K_t + \frac{\psi}{1 - \theta}K_t (I_t/K_t)^{1-\theta} \left[\int_0^{\xi_t} \phi(\xi)d\xi\right]^{\theta},$$
(27)

$$Y_{t} = F(K_{t}, N_{t}) = G_{t} + I_{t} + C_{t} + K_{t} \int_{0}^{\xi_{t}} \xi \phi(\xi) d\xi, \qquad (28)$$

$$\frac{-U_2(C_t, N_t)}{U_1(C_t, N_t)} = \left(1 - \tau_t^n\right) F_2(K_t, N_t),$$
(29)

$$Q_{t} = \frac{\beta U_{1}(C_{t+1}, N_{t+1})}{U_{1}(C_{t}, N_{t})} \left[\left(1 - \tau_{t+1}^{k}\right) F_{1}(K_{t+1}, N_{t+1}) + \tau_{t+1}^{k} \delta + \left(1 - \delta + \varsigma\right) Q_{t+1} + \int_{0}^{\xi_{t+1}} \left(\xi_{t+1}^{*} - \xi\right) \phi(\xi) d\xi \right],$$
(30)

where $\bar{\xi}_t$ and ξ_t^* are given in Proposition 1 and K_1 is given.⁷

We derive equations (26) and (27) by aggregating equations (24) and (2), respectively. Equation (26) reveals that the aggregate investment rate is equal to a firm's target investment rate multiplied by the fraction of adjusting firms (or the *adjustment rate*), $\int_{0}^{\bar{\xi}_{t}} \phi(\xi) d\xi$. Thus, corporate tax policy has both intensive and extensive margin effects on investment in the presence of fixed adjustment costs. To see the magnitude of these effects intuitively, we log-linearize equation (26) for an interior solution to obtain

$$\hat{I}_{t} - \hat{K}_{t} = \underbrace{\frac{1}{\theta} \left(\hat{Q}_{t} + \frac{\tau^{i}}{1 - \tau^{i}} \hat{\tau}_{t}^{i} \right)}_{\text{intensive}} + \underbrace{\frac{\bar{\xi}\phi(\bar{\xi})}{\int_{0}^{\bar{\xi}} \phi(\xi) d\xi}}_{\text{extensive}} \hat{\xi}_{t}, \tag{31}$$

7. The usual transversality condition must be satisfied.

where for any variable X_t , X denotes its steady-state value and $\hat{X}_t = \ln X_t - \ln X$. The first term on the right-side equation (31) captures the usual intensive marginal effect in the *q*-theory of investment without nonconvex adjustment costs. A change in τ_t^k or τ_t^i affects marginal Q as revealed by asset-pricing equation (30), and thus induces firms to decide how much to invest.

The second term on the right side of equation (31) is the percentage deviation in the adjustment rate $\int_0^{\bar{\xi}_t} \phi(\xi) d\xi$, which represents the extensive margin effect. A change in τ_t^k or τ_t^i and the resulting change in marginal Q also affect a firm's decision on when to invest. Thus, tax changes affect the adjustment trigger $\bar{\xi}_t$ and hence the number of firms to make investment. The extent of this extensive margin effect is measured by the coefficient of $\hat{\xi}_t$, $\bar{\xi}\phi(\bar{\xi})/\int_0^{\bar{\xi}}\phi(\xi)d\xi$, which is the steadystate elasticity of the adjustment rate with respect to the adjustment trigger $\bar{\xi}$. The larger the elasticity, the larger is the extensive margin effect. For the power function distribution $\phi(\xi) = \eta \xi^{\eta-1}/(\xi_{\max})^{\eta}$, $\eta > 0$, we can explicitly compute the elasticity to be η .⁸ In this case, the component due to the extensive margin effect is equal to $\eta \hat{\xi}_t$. Thus, even if the investment trigger $\bar{\xi}_t$ does not change much in response to tax changes, a larger elasticity η can raise the impact on the adjustment rate.

We can also explicitly compute the response of the investment trigger $\bar{\xi}_t$ to changes in the tax rates. Log-linearizing equation (23) yields

$$\widehat{\xi}_t = \frac{1}{\theta} \left(\hat{Q}_t + \frac{(1-\theta)\,\tau^i}{1-\tau^i} \hat{\tau}_t^i \right)$$

Plugging this expression into (31), we can compare the relative importance of the two components due to the intensive and extensive margin effects. For the power function distribution, we can see that the investment response due to the extensive margin effect is η times as large as that due to the intensive margin effect when only capital tax is changed but the ITC is fixed (i.e., $\hat{\tau}_i^i = 0$). Thus, for $\eta > 1$, the extensive margin effect is larger than the intensive margin effect.

2.3 Steady State

We consider a deterministic steady state in which government policy variables $(\tau_t^k, \tau_t^n, \tau_t^i, T_t, G_t)$ stay constant over time. In addition, all aggregate equilibrium quantities, prices, and the investment trigger are constant over time, while at the firm level, firms still face idiosyncratic fixed-costs shocks. The following proposition characterizes the steady-state aggregate variables $(Y, C, N, K, I, Q, \bar{\xi})$, when $\bar{\xi}$ is an interior solution.

^{8.} Gourio and Kashyap (2007) use a "compressed" distribution for the fixed cost to get a large extensive margin effect. From Figure 3 in their paper, we can see that the distribution is very steep at large values of fixed costs implying that the steady-state elasticity of the adjustment rate is large.

PROPOSITION 3. Suppose that

$$0 < \delta - \varsigma < \frac{\psi}{(1-\theta)^{\theta} \, \theta^{(1-\theta)}} \left(\frac{\xi_{\max}}{1-\tau^{i}}\right)^{1-\theta} \int_{0}^{\xi_{\max}} \phi(\xi) d\xi.$$
(32)

Then the steady-state investment trigger $\overline{\xi} \in (0, \xi_{max})$ is the unique solution to the equation

$$\delta - \varsigma = \frac{\psi}{(1-\theta)^{\theta} \,\theta^{(1-\theta)}} \left(\frac{\bar{\xi}}{1-\tau^{i}}\right)^{1-\theta} \int_{0}^{\bar{\xi}} \phi(\xi) d\xi.$$
(33)

Given this value $\overline{\xi}$, the steady-state value of Q is given by

$$Q = \frac{1}{\psi} \left(\frac{\bar{\xi}\left(1-\theta\right)}{\theta}\right)^{\theta} \left(1-\tau^{i}\right)^{1-\theta}.$$
(34)

The other steady-state values (I, K, C, N) satisfy

$$\frac{I}{K} = \frac{Q}{1 - \tau^i} \left(\delta - \varsigma\right) \left(1 - \theta\right),\tag{35}$$

$$F(K, N) = I + C + K \int_0^{\bar{\xi}} \xi \phi(\xi) d\xi + G,$$
(36)

$$\frac{-U_2(C,N)}{U_1(C,N)} = (1-\tau^n)F_2(K,N),$$
(37)

$$Q = \frac{\beta}{1 - \beta (1 - \delta + \varsigma)} \Biggl\{ \left(1 - \tau^k \right) F_1(K, N) + \tau^k \delta + \int_0^{\bar{\xi}} \left(\bar{\xi} - \xi \right) \phi(\xi) d\xi \Biggr\}.$$
(38)

Condition (32) ensures that equation (33) has a unique interior solution by the intermediate value theorem. It is satisfied for the parameterization analyzed in Section 3. We can derive some interesting theoretical results from the above proposition. First, equation (33) shows that the steady-state investment trigger is independent of the corporate tax rate τ^k , but decreases with the ITC τ^i . Thus, the steady-state adjustment rate $\int_0^{\xi} \phi(\xi) d\xi$ has the same property. Second, it follows from equation (34) that the steady-state marginal Q is independent of τ^k , but decreases with τ^i . We can also show that the tax-adjusted producer price of capital $\tilde{Q} = Q/(1 - \tau^i)$

increases with τ^{i} .⁹ Third, it is straightforward from (35) to show that the steady-state investment rate is independent of τ^{k} , but increases with τ^{i} . The intuition behind the latter result is as follows. An increase in τ^{i} has negative an extensive margin effect because it reduces the adjustment rate, but has a positive intensive margin effect because it raises \tilde{Q} . Overall, the negative extensive margin effect is dominated by the positive intensive margin effect following an increase in τ^{i} . Except for the predictions regarding the adjustment rate, the preceding results are still valid in a model with convex adjustment costs only or in a standard growth model without adjustment costs.

3. NUMERICAL RESULTS

We evaluate our lumpy investment model quantitatively and compare this model with two benchmark models. The first one is a standard growth model with distortionary taxes, obtained by removing all adjustment costs in the model presented in Section 1. The second one is obtained by removing fixed adjustment costs only. We call this model *partial adjustment model*. In each of the benchmark models, all firms make identical decisions, and thus they give the same aggregate equilibrium allocations as that in a standard representative-agent and a representative-firm growth model. Because we can characterize the equilibria for all three models by systems of nonlinear difference equations, we can solve them numerically using standard methods.¹⁰ To do so, we need first to calibrate parameter values.

3.1 Baseline Parameterization

For all model economies, we take the Cobb–Douglas production function, $F(K, N) = K^{\alpha}N^{1-\alpha}$, and the period utility function, $U(C, N) = \ln(C) - \varphi N$, where $\varphi > 0$ is a parameter. We fix the length of period to correspond to 1 year, as in Thomas (2002), and Khan and Thomas (2003, 2008). Annual frequency allows us to use empirical evidence on establishment-level investment in selecting parameters for the fixed adjustment costs.

We first choose parameter values for preferences and technology to ensure that the steady state of the standard growth model with taxes is consistent with the long-run values of key postwar U.S. aggregates. Specifically, we set the subjective discount factor to $\beta = 0.9615$ so that the implied real annual interest rate is 4%. We choose the value of φ in the utility function so that the steady-state hours are about one-third of available time spent in market work. We set the capital share $\alpha = 0.36$, implying

^{9.} See Goolsbee (1998) for empirical evidence.

^{10.} We use the Dynare package version 3.0 to solve our models numerically (see Juillard 2005). The solution uses a nonlinear method based on a Newton-type algorithm described in Juillard (1996). The idea is to first solve the two steady states prior to and after a tax reform. Then use these two steady states as the initial and terminal conditions for the equilibrium system of nonlinear difference equations. Finally, solve this system using a Newton-type algorithm.

TABLE 1 Baseline Parameter Values										
0.9615	2.5843	0.36	0.1	0.35	0.32	0	0.105	1/5.98	0.0242	1

a labor share of 0.64, which is close to the labor income share in the NIPA. We take the depreciate rate $\delta = 0.1$, as in the literature on business cycles.

In a steady state, all tax rates are constant over time. By the estimates from McGrattan and Prescott (2005) and Prescott (2002), we set capital tax rate $\tau^k = 0.35$ and labor tax rate $\tau^n = 0.32$. Because the ITC is typically used as a short-run stimulative policy, we set $\tau^i = 0$ in the steady state. We assume that the government spending G_t is constant over time and equal to 20% of the steady-state output in the standard growth model. We fix this level of government spending for all experiments below.

It is often argued that convex adjustment costs are not observable directly and hence cannot be calibrated based on average data over the long run. Thus, we impose the two restrictions, $\psi = \delta^{\theta}$ and $\varsigma = -\theta \delta/(1-\theta)$, so that the partial adjustment model and the standard growth model give identical steady-state allocations (e.g., Q = 1 and $I/K = \delta$). Following Thomas (2002) and Khan and Thomas (2003), we set $\theta = 1/5.98$, implying that the *Q*-elasticity of the investment rate is 5.98.

Following the literature (e.g., Khan and Thomas 2003, 2008, Fiori 2012), we assume the uniform distribution for the fixed cost with the support $[0, \xi_{max}]$. We calibrate the parameter ξ_{max} to match the steady-state inaction rate $(1 - \xi/\xi_{max})$ of 0.081 reported by Cooper and Haltiwanger (2006). This gives $\xi_{max} = 0.0242$. In this case, we can compute that in the steady state, total fixed adjustment costs account for 2.0% of output, 10.0% of total investment spending, and 1.0% of the aggregate capital stock, which are reasonable according to the estimation by Cooper and Haltiwanger. Cooper and Haltiwanger also report that the positive spike rate is about 0.186. Our model can capture positive investment spikes. However, because our model implies that the target investment rate is identical for all firms, our model cannot match the spike rate observed in the data.¹¹

We list the baseline parameter values in Table 1. Suppose that the economy in period 1 is in the initial steady state with parameter values given in Table 1. We then consider the economy's responses to changes in corporate tax policy by changing sequences of tax rates $\{\tau_t^k\}$ and ITC $\{\tau_t^i\}$. We hold labor income tax rates τ^n constant at the value in Table 1 and allow lump-sum taxes to adjust to balance government budget. For all tax policy experiments studied below, we assume that the tax policy is announced

^{11.} By incorporating plant-level productivity shocks and maintenance investment, Khan and Thomas (2008) and Bachmann, Caballero, and Engel (2009) can match both the spike rate and the inaction rate observed in the data.

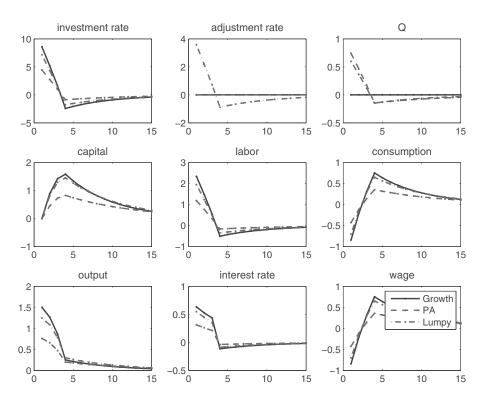


Fig. 1. Response to Unanticipated Temporary Decrease in τ^k .

Notes: The 10 percentage point tax cut lasts from periods 1 to 4. This policy is announced and implemented in period 1. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time. For any variable X_t , each panel plots $\ln X_t - \ln X$, where X is the steady-state value of X_t .

in period 1 and agents have perfect foresight about future tax rates. Following the public finance literature (e.g., Judd 1985, 1987, Mertens and Ravn 2011, 2012), we say that a tax policy is *unanticipated (anticipated)* when it is actually implemented at the same time of the announcement date (in a future date).

3.2 Temporary Changes in the Corporate Tax Rate

We start with the first policy experiment in which the corporate tax rate τ_t^k decreases by 10 percentage points temporarily and this decrease lasts for only four periods. After this decrease, τ_t^k reverts to its previous level. Suppose this tax policy is unanticipated initially, but once it occurs in period 1, agents have perfect foresight about the future tax rates.

Figure 1 presents the transitional dynamics for the standard growth, partial adjustment, and lumpy investment models (labeled as "Growth," "PA," and "Lumpy," respectively) following this tax policy. We find that these three models display similar transitional dynamics. Because the responses in the standard growth model are well known in the literature (e.g., Ljungqvist and Sargent 2004, Chap. 11), our analysis will focus on the other two models. For these models, the decrease in τ_t^k in periods 1– 4 raises the after-tax marginal product of capital and hence marginal Q immediately by (30), leading to a jump of investment in the initial period. Note that the impact jump of marginal Q is larger for the partial adjustment model than for the lumpy investment model. Marginal Q starts to decrease until period 4 and then gradually rises to its steady-state value because τ_t^k rises to its original level permanently starting in period 5. Consequently, the investment rate follows a similar path by the intensive margin effect (see (31)). The capital stock rises until period 4, and then decreases monotonically to the original steady-state level.

We define the after-tax (gross) interest rate between periods t and t + 1 as

$$r_t = \frac{U_1(C_t, N_t)}{\beta U_1(C_{t+1}, N_{t+1})} = \frac{C_{t+1}}{\beta C_t}.$$

It is also equal to the after-tax return on capital by equation (30). In response to the temporary decrease in τ_t^k , the after-tax return on capital rises initially and decreases until period 4. After this period, it gradually rises to its original steady-state level. Due to the substitution effect, consumption drops initially and then rises until period 4. After period 4, it decreases to its original steady-state level. Note that the wealth effect is small because the decrease in τ_t^k must be accompanied by an increase in lump-sum taxes that leaves the government budget balanced.

Given our adopted utility function, the wage rate w_t satisfies $\varphi C_t = (1 - \tau^n) w_t$. Thus, wage and consumption must follow identical dynamics, causing the labor hours to follow a pattern opposite to consumption because the marginal product of labor equals the after-tax wage. Output rises in the initial period because labor rises and capital is predetermined. After period 1, output gradually decreases to its steady-state value.

Figure 1 reveals that the short-run impact on investment and output for the partial adjustment model is smaller than for the standard growth model because of the smoothing effect of convex capital adjustment costs. The presence of fixed capital adjustment costs in the lumpy investment model makes the short-run impact of tax changes larger. The reason is that tax policy in this model has an additional extensive margin effect as discussed in Section 2.2. Figure 1 shows that the investment rate along the extensive margin (i.e., the adjustment rate) rises by about 3.6% on impact for the lumpy investment model. The total impact increase in the investment rate on impact in the partial adjustment model is due to the intensive margin effect only, which is about 4.5%. Even though the lumpy investment model delivers a smaller intensive margin effect, this model generates a larger overall response of investment to tax changes than the partial adjustment model.

The fact that the intensive margin effect is smaller in the lumpy investment model than in the partial adjustment model is also consistent with a smaller response of

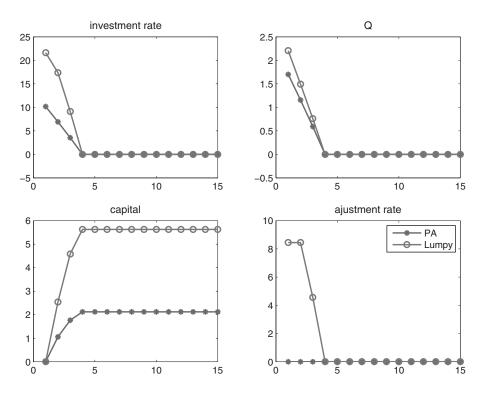


FIG. 2. Response to Unanticipated Temporary Decrease in τ^k in Partial Equilibrium.

Notes: The 10 percentage point tax cut lasts from periods 1 to 4. This policy is announced and implemented in period 1. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time. For any variable X_t , each panel plots $\ln X_t - \ln X$, where X is the steady-state value of X_t .

marginal Q in the lumpy investment model presented in Figure 1. If the lumpy investment model implied a larger intensive margin effect, this effect combined with the extensive margin effect would generate a much larger investment response for the lumpy investment model. The resulting large capital would lead to a small marginal product of capital. This would give a smaller response of marginal Q in the lumpy investment model than in the partial adjustment, contradicting Figure 1.

Despite the large extensive margin effect, the dynamic responses to the tax change for the lumpy investment model are similar to those for the standard growth model and the partial adjustment model. The reason is that the general equilibrium price movements smooth aggregate investment dynamics.¹² Figure 2 illustrates this point by presenting the transitional dynamics for the partial adjustment and lumpy investment models in partial equilibrium. In particular, we fix the interest rate and the wage rate

^{12.} Thomas (2002) first makes this point and shows that lumpy investment is quantitatively irrelevant for business cycles. Using a different numerical solution method, Khan and Thomas (2003, 2008) confirm her finding. In an analytical framework, Miao and Wang (2009) provide conditions under which Thomas's result can hold true.

at their steady-state values prior to tax changes through the transition period so that the system of equations (23), (26), (27), and

$$Q_{t} = \beta \left\{ \left(1 - \tau_{t+1}^{k} \right) R_{t+1} + \tau_{t+1}^{k} \delta + (1 - \delta + \varsigma) Q_{t+1} + \int_{0}^{\bar{\xi}_{t+1}} \left[\xi_{t+1}^{*} - \xi \right] \phi(\xi) d\xi \right\}$$
(39)

characterizes the partial equilibrium dynamics of $\{I_t, K_{t+1}, Q_t, \bar{\xi}_t\}$, where $\bar{\xi}_t = \min\{\xi_t^*, \xi_{\max}\}$. In response to the tax cut, marginal Q rises by about 1.7% and 2.2% for the partial adjustment and lumpy investment models, respectively, much higher than the corresponding values, 0.7% and 0.6% in general equilibrium. As a result, the increase in the adjustment rate in partial equilibrium is much higher than that in general equilibrium. In particular, the increase in Q in partial equilibrium is so high that all firms decide to make capital adjustments in the first two periods for the lumpy investment model. This large extensive margin effect causes the aggregate investment rate rises by about 21% in the lumpy investment model. This increase is much larger than the corresponding increase of 7.2% in general equilibrium.

We also emphasize that in a partial equilibrium model with competitive firms and constant-returns-to-scale technology, Q can be determined independently of capital. This can be seen from equation (39), in which the marginal product of capital R_t is constant when the wage rate is fixed.¹³ A temporary cut in τ_t^k from periods 1 to 4 induces Q to rise immediately and then decrease monotonically to its steady-state value. The investment rate also follows the same pattern. Because the investment rate never falls below its steady-state level, a temporary change in τ_t^k has a permanent effect on the capital stock, as revealed in Figure 2. This result is in sharp contrast to that in general equilibrium, suggesting that a partial equilibrium analysis of tax policy can be quite misleading.

We next turn to the case in which the temporary decrease in τ_t^k is anticipated initially to be effective in period 2 and lasts for four periods. Figure 3 presents the transitional dynamics. The future tax cut raises the future after-tax marginal product of capital, *ceteris paribus*. Because marginal Q reflects the present value of the future after-tax marginal product of capital by (30), it rises immediately. Thus, anticipating the tax cut in the future, adjusting firms react by raising investment immediately. In addition, the adjustment rate also rises immediately in the lumpy investment model. The investment rate and the adjustment rate continue to rise until period 4. The household reacts by reducing consumption and raising labor supply immediately. From periods 1 to 4, consumption and hours gradually rise. Output also rises from periods 1 to 4. Starting from period 5, the economy's responses are similar to those in the case of unexpected tax change presented in Figure 1.

13. Using equation (14), we can show that $R_t = \alpha \left(\left(1 - \alpha\right) / w_t \right)^{\frac{1-\alpha}{\alpha}}$. It is constant over time when wage w_t is fixed at the initial steady-state value prior to tax changes.

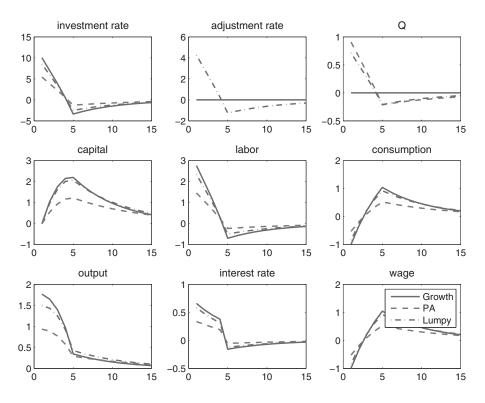


FIG. 3. Response to Anticipated Temporary Decrease in τ^k .

Notes: The 10 percentage point tax cut lasts from periods 2 to 5. This policy is announced in period 1 and implemented in period 2. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time. For any variable X_t , each panel plots $\ln X_t - \ln X$, where X is the steady-state value of X_t .

3.3 Permanent Changes in the Corporate Tax Rate

Figure 4 presents transitional dynamics following an unanticipated permanent 10 percentage point cut in the corporate tax rate τ_t^k in period 1. This policy is announced and implemented in period 1 and agents have perfect foresight about future tax rates. The economy's immediate response in this case is qualitatively similar to that in the case of an announced but initially unanticipated temporary corporate tax cut via the same channel of marginal Q (see Figure 1). However, the short-run impact is larger because the response of Q is larger. In particular, in response to a permanent corporate tax cut, the investment rate and the adjustment rate rise by about 14% and 7% on impact, respectively. In the long run, the economy moves to a new steady state after a permanent tax cut, while the steady state does not change after a temporary tax cut. In addition, the transition paths following an unanticipated permanent tax cut are monotonic, rather than nonmonotonic.

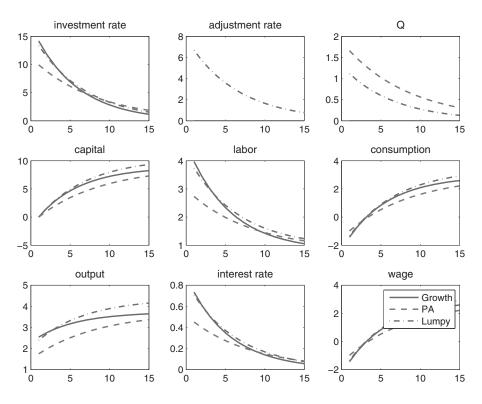


FIG. 4. Response to Unanticipated Permanent Decrease in τ^k .

Notes: The 10 percentage point tax cut lasts forever starting from period 1. This policy is announced and implemented in period 1. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time. For any variable X_t , each panel plots $\ln X_t - \ln X$, where X is the steady-state value of X_t .

We emphasize that the impact of tax policy depends on the initial cross-sectional distribution of firms. In our baseline calibration, 91.9% of firms have made capital adjustments in the initial steady state. This high adjustment rate leaves less room for more firms to make adjustments in response to a decrease in τ^k . If the initial adjustment rate is small, more firms will respond to a capital tax cut, making the extensive margin effect large. To illustrate this point, we conduct a hypothetical experiment with the initial steady-state adjustment rate equal to 0.2.¹⁴ Figure 5 plots the economy's responses to an announced permanent 10 percentage point decrease in τ^k enacted in period 1. We find that both the short- and long-run effects in this case are much larger than the case in Figure 4. The main reason is due to the larger extensive margin effect. In particular, the adjustment rate rises on impact by 10% in

^{14.} We recalibrate ξ_{max} in the lumpy investment model to match this adjustment rate, 0.2.

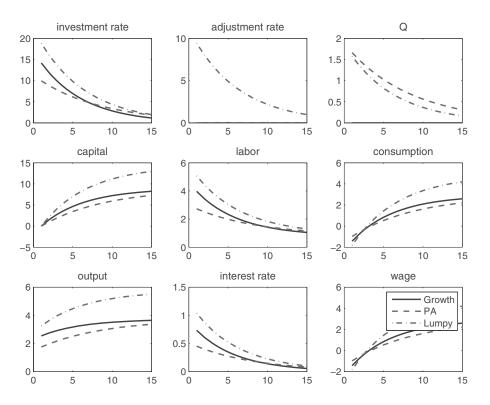


FIG. 5. Response to Unanticipated Permanent Decrease in τ^k When the Initial Adjustment Rate Is 0.2.

Notes: The 10 percentage point tax cut lasts forever starting from period 1. This policy is announced and implemented in period 1. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time. For any variable X_t , each panel plots $\ln X_t - \ln X$, where X is the steady-state value of X_t .

the model with the low initial adjustment rate as opposed to 7% in the model with the high initial adjustment rate.

While the high initial adjustment rate constrains the effectiveness of an expansionary tax policy, it makes a contractionary tax policy more effective. To illustrative this point, Figure 6 presents the economy's response to an announced permanent 10 percentage point increase in τ^k enacted in period 1 for the parameter values with the adjustment rate equal to 0.919. This figure reveals that the adjustment rate decreases by about 8% immediately, causing the aggregate investment rate to fall by about 15% immediately. Comparing Figure 6 with Figure 4, we find that the responses to the tax increase and decrease are roughly symmetric for the standard growth and partial adjustment models, but are highly asymmetric for the lumpy investment model. In particular, the investment rate and the adjustment rate rise by about 13.4% and 6.7%, respectively, in response to a 10 percentage point cut in τ^k , but they decrease by about 15.1% and 7.6% , respectively, in response to a 10 percentage point raise in τ^k .

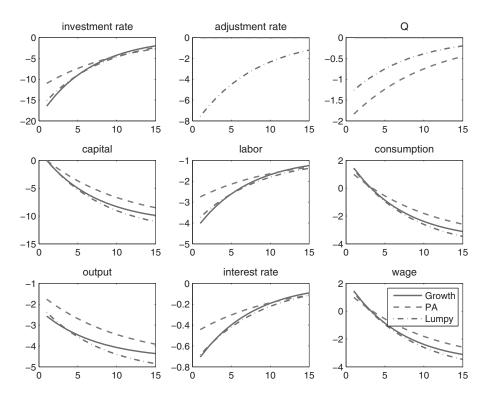


FIG. 6. Response to Unanticipated Permanent Increase in τ^k .

NOTES: The 10 percentage point tax increase lasts forever starting from period 1. This policy is announced and implemented in period 1. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time. For any variable X_t , each panel plots $\ln X_t - \ln X$, where X is the steady-state value of X_t .

The intuition behind the asymmetry is the following. When the distribution of fixed costs is such that the initial steady-state adjustment rate (equal to 0.919) is very large, the cutoff $\bar{\xi}$ triggering investment is very large. This implies that most firms have already made investment when their fixed costs are below the cutoff. In response to an unanticipated permanent cut in τ^k , the cutoff rises inducing firms with fixed costs lying between the initial and new cutoffs to make investment. However, since the initial cutoff is already large, the increase in the cutoff is limited and may make the new cutoff reach the upper bound of the fixed cost. This may happen when the tax cut is large or when the steady-state elasticity of the adjustment rate with respect to the investment trigger is large. By contrast, in response to an unanticipated permanent increase in τ^k , the cutoff falls, inducing firms with fixed costs lying between the initial and new cutoffs not to invest. Since the initial cutoff is large, its fall can be large and hence the fall in the adjustment rate and the investment rate can also be large.

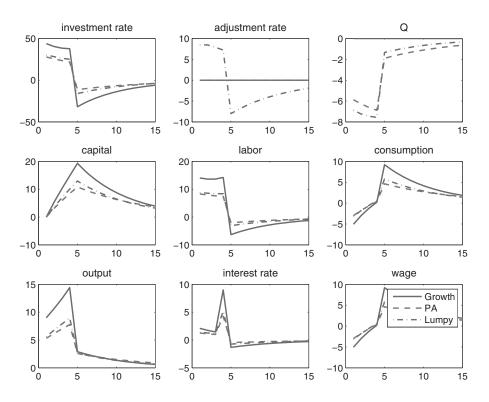


FIG. 7. Response to Unanticipated Temporary Increase in τ^i .

Notes: Let $\tau_t^i = 0.1$ for t = 1, 2, 3, 4 and $\tau_t^i = 0$ for $t \ge 5$. This policy is announced and implemented in period 1. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time. For any variable X_t , each panel plots $\ln X_t - \ln X$, where X is the steady-state value of X_t .

When the permanent tax cut is initially anticipated to be enacted in the future, marginal Q rises immediately for the same reason in the case of the anticipated temporary tax cut discussed in Section 3.2. Thus, both the investment rate and the adjustment rate must rise initially. They continue to rise until the enactment date of the tax cut. After this date, the economy transits to the new steady state as in the case presented in Figure 4. We omit a detailed discussion.

3.4 Temporary Changes in the ITC

Suppose that a 10% ITC is imposed from periods 1 to 4, that is, $\tau_t^i = 0.1$ for t = 1, ..., 4. In period 5, this policy is repealed so that $\tau_t^i = 0$ for $t \ge 5$. This tax policy is announced and implemented in period 1 and agents have perfect foresight about future tax rates. Figure 7 presents the economy's dynamic responses. By equation (26), the increase in the ITC has a direct positive impact on investment by reducing the tax-adjusted price of capital $\tilde{Q}_t = Q_t/(1 - \tau_t^i)$, ceteris paribus. But it

also has an indirect negative impact on marginal Q by (30) because the increase in the investment rate raises the capital stock and reduces the marginal product of capital, *ceteris paribus*. Our numerical results show that the positive direct effect dominates, and thus \tilde{Q}_t and the investment rate rise on impact. The investment rate then falls until period 4. In period 5, it drops sharply below its steady-state level because the ITC is removed in period 5. After period 5, investment gradually rises to its previous steady-state level. Consumption follows the opposite pattern as resources are devoted to investment.

Even though marginal Q falls immediately, the investment trigger rises on impact, because the increase in the ITC dominates the fall in Q in equation (23). This rise is so large that all firms make adjustments immediately (the investment trigger reaches the upper support of fixed costs). In addition, almost all firms in the lumpy investment model continue to make capital adjustments until period 4. The adjustment rate drops below its steady-state level in period 5. After period 5, it gradually rises to its previous steady-state level.

In contrast to the case of changes in the corporate income tax rate, the extensive margin effect accounts for less of the increase in the aggregate investment rate in the lumpy investment model in response to the increase in the ITC. In addition, the initial rise of the investment rate is smaller in the lumpy investment model than in the standard growth model. This result is also different from that in the case of corporate income tax cuts. The intuition is that the decrease in marginal Q in the short run reduces firms' incentives to make large investment. Even though the extensive margin effect in the presence of fixed costs raises firms' responses to the increase in the ITC, this effect is not large enough.

Next, we suppose that the 10% ITC is enacted in period 2 and lasts for four periods. This tax policy is anticipated initially. Figure 8 presents the economy's responses to this tax policy. Contrary to the responses presented in Figure 7, investment and output decrease immediately, but consumption increases immediately. The intuition comes from the dynamics of marginal Q characterized in Proposition 2. Anticipating the fall of Q in period 2 due to the enactment of the 10% ITC, Q falls immediately at date 1. Because the investment rate is determined by equation (26) and $\tau_t^i = 0$ for t = 1, the investment rate must decrease in period 1. In period 2, the 10% ITC makes the new investment good cheaper, but $\tilde{Q}_2 = Q_2/(1 - \tau_2^i)$ actually rises. Thus, the investment rate jumps up in period 2. Starting from period 2, the economy's responses are similar to those in the case of the announced temporary increase in the ITC.

3.5 Permanent Changes in the ITC

We finally consider two experiments in which there is a permanent 10% ITC. First, Figure 9 presents the economy's responses when this tax change is unanticipated previously. It is announced and enacted in period 1 and agents have perfect foresight about the future tax rates. For all three models, the investment rate rises immediately, but the increase is less than that if the ITC is temporary, as shown in Figure 7. This is in sharp contrast to Abel's (1982) result that a permanent ITC provides a greater

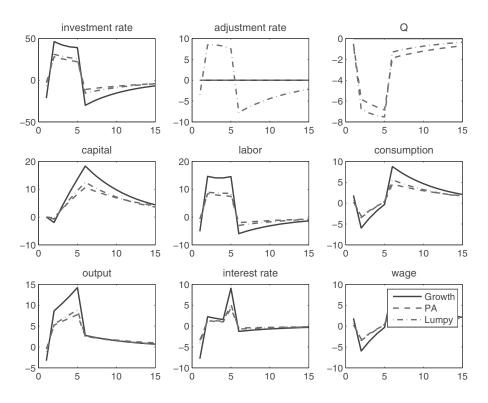


FIG. 8. Response to Anticipated Temporary Increase in τ^i .

Notes: The 10 percentage point tax increase lasts from periods 2 to 5. This policy is announced in period 1 and implemented in period 2. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time. For any variable X_t , each panel plots $\ln X_t - \ln X$, where X is the steady-state value of X_t .

stimulus to investment than a temporary ITC for a competitive firm with constant returns to scale. The reason is that Abel (1982) uses a partial equilibrium model rather than a general equilibrium model. As we point out in Section 3.2 in partial equilibrium, Q can be determined independently of capital for competitive firms with constant-returns-to-scale technology. We can then show that the initial rise of the investment rate is independent of the duration of the ITC. By contrast, in general equilibrium, Q and capital must be jointly determined as shown in Proposition 2. In particular, the interest rate and the wage rate change over time in response to tax changes. As shown in Figures 8 and 9, the initial fall in Q is larger in response to a permanent increase in the ITC than in response to a temporary increase in the ITC.

A permanent increase in the ITC changes the economy's steady state. For the three models, the capital stock, output, consumption, and labor are higher in the new steady state. But the steady-state adjustment rate in the lumpy investment model is lower than its initial steady-state value discussed in Section 2.3.

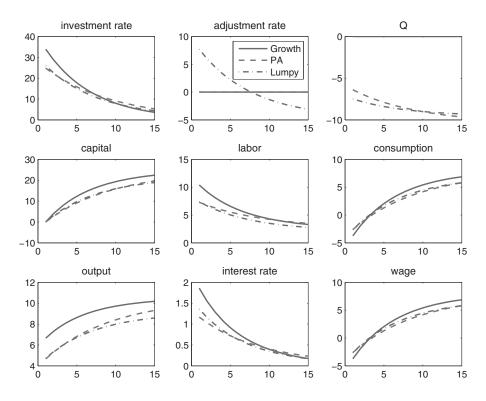


FIG. 9. Response to Unanticipated Permanent Increase in τ^i .

Notes: The 10 percentage point tax increase lasts forever starting from period 1. This policy is announced and implemented in period 1. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time. For any variable X_t , each panel plots $\ln X_t - \ln X$, where X is the steady-state value of X_t .

In the second experiment, we solve the case in which the permanent increase in the ITC is announced in period 1 and anticipated to be enacted in some future date, say in period 2. We find that both the adjustment rate and the investment rate fall on impact by the same intuition discussed for the case of the anticipated temporary increase in the ITC. These rates then jump up at the enactment date of the increase in the ITC. After this date, the transition paths are similar to those presented in Figure 9. We thus omit a detailed discussion.

4. CONCLUSIONS

In this paper, we have studied the impact of corporate tax policy on the economy in the presence of both convex and nonconvex capital adjustment costs in a dynamic general equilibrium model with firm heterogeneity. Our model permits exact aggregation and is analytically tractable. Our main results are as follows. First, corporate tax policy generates both intensive and extensive margin effects via the channel of marginal Q. At the micro level, it affects a firm's decision on the size and timing of investment. At the macro level, it affects each adjusting firm's investment size and the number of adjusting firms. Second, the impact of corporate tax policy depends on the cross-sectional distribution of firms. If most firms have not made capital adjustments initially, an expansionary corporate tax policy is more effective, while a contractionary corporate tax policy is less effective. The opposite result holds true if most firms have made capital adjustments initially. Third, introducing convex adjustment costs in the standard growth model smooths the economy's responses to tax changes. Introducing nonconvex adjustment costs on top of it raises the economy's responses to tax changes and brings the equilibrium outcome closer to that in the standard growth model. The responses can be larger than those in the standard growth model for a permanent change in the corporate tax rate, but smaller for a permanent or temporary change in the ITC.

Fourth, a permanent increase in the ITC raises the steady-state tax-adjusted price of capital, but reduces the steady-state adjustment rate. In addition, an unanticipated temporary increase in the ITC has a larger short-run stimulative impact than a permanent increase in the ITC, even for competitive firms with constant-returns-to-scale technology. This finding contrasts with Abel's (1982) result in a partial equilibrium model. Finally, we extend Judd's (1987) and Auerbach's (1989) results on anticipation effects of tax policy to a general equilibrium model with both convex and nonconvex capital adjustment costs. In particular, an anticipated decrease in the future corporate income tax rate raises investment and adjustment rate immediately, while an anticipated increase in the future ITC reduces investment and adjustment rate initially.

Our model may be useful for empirical work and some of our results have novel testable implications. For example, our predictions regarding the impact of corporate tax policy on the adjustment rate can be tested empirically. One limitation of analysis is the constant-return-to-scale assumption. Under this assumption, all firms choose to make either zero investment or identical positive investment. This assumption allows us to conduct exact aggregation and simplifies model analysis significantly. But it also comes with a cost in the sense that our model can match the inaction rate but not the spike rate as in the micro level evidence. This feature is similar to the Calvo (1983) model of the pricing decision in which all firms either do not adjust price or adjust to the same price level. Sveen and Weinke (2007) follow the Calvo approach to model lumpy investment. Unlike this approach, the probability of adjustment for a firm in our model is given by $\int_{\overline{\xi}}^{\xi_{\text{max}}} \phi(\xi) d\xi$, which is state dependent. Note that the main goal of our paper is to study the aggregate implications of tax policy, but not to fit all dimensions of the micro level evidence. To fit all dimensions of the micro level evidence, one has to assume decreasing return to scale and introduce other assumptions as in Khan and Thomas (2008) and Bachmann, Caballero, and Engel (2008). This extension will make the model complicated and increase computation burden significantly. We leave it for future research.

APPENDIX: PROOFS

PROOF OF PROPOSITION 1. From (22), we can show that the target investment level i_t^j satisfies the first-order condition

$$1 - \tau_t^i = g'(i_t^j) \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1}.$$
(A1)

By equations (3), (18), and (21), we can derive equation (24). Using this equation, we define $V_t^a(\xi_t^j)$ as firm value per unit of capital when the firm chooses to invest. It is given by

$$V_t^a(\xi_t^j) = (1 - \tau_t^k) R_t + \tau_t^k \delta - (1 - \tau_t^i) i_t^j - \xi_t^j + g(i_t^j) Q_t$$

$$= (1 - \tau_t^k) R_t + \tau_t^k \delta + (1 - \delta + \varsigma) Q_t$$

$$+ \frac{\theta}{1 - \theta} (\psi Q_t)^{\frac{1}{\theta}} (1 - \tau_t^i)^{\frac{\theta - 1}{\theta}} - \xi_t^j.$$
(A2)

Define V_t^n as firm value per unit of capital when the firm chooses not to invest. By (22), (18), and (3), it satisfies

$$V_t^n = \left(1 - \tau_t^k\right) R_t + \tau_t^k \delta + (1 - \delta + \varsigma) Q_t, \tag{A3}$$

which is independent of ξ_t^j . We can then rewrite problem (22) as

$$V_t(\xi_t^J) = \max\{V_t^a(\xi_t^J), V_t^n\}.$$
(A4)

Clearly, there is a unique cutoff value ξ_t^* given in (23) satisfying the condition

$$V_t^a\left(\xi_t^*\right) = V_t^n,\tag{A5}$$

$$V_t^a(\xi_t^j) > V_t^n \quad \text{if and only if } \xi_t^j < \xi_t^*. \tag{A6}$$

Because the support of ξ_t^j is $[0, \xi_{max}]$, the investment trigger is given by $\bar{\xi}_t \equiv$ $\min \left\{ \xi_t^*, \xi_{\max} \right\}.$ We can show that

$$\bar{V}_t = \int_0^{\xi_{\max}} V_t(\xi) \phi(\xi) d\xi$$

 \square

$$= \int_{\xi_{t}}^{\xi_{\max}} V_{t}^{n} \phi(\xi) d\xi + \int_{0}^{\xi_{t}} V_{t}^{a}(\xi) \phi(\xi) d\xi$$
$$= V_{t}^{n} + \int_{0}^{\xi_{t}} \left[V_{t}^{a}(\xi) - V_{t}^{n} \right] \phi(\xi) d\xi.$$

We use equations (A2), (A3), and (23) to derive

$$V_t^a(\xi) - V_t^n = \frac{\theta}{1 - \theta} \left(\psi Q_t \right)^{\frac{1}{\theta}} \left(1 - \tau_t^i \right)^{\frac{\theta - 1}{\theta}} - \xi$$

$$= \xi_t^* - \xi.$$
(A7)

Using the above two equations, (A3), and (21), we obtain (25).

PROOF OF PROPOSITION 2. From (13), we deduce that all firms choose the same laborcapital ratio n_t . We thus obtain $N_t = n_t K_t$. We then derive

$$Y_{t} = \int Y_{t}^{j} dj = \int F(K_{t}^{j}, N_{t}^{j}) dj = \int F(1, n_{t}^{j}) K_{t}^{j} dj$$
$$= F(1, n_{t}) \int K_{t}^{j} dj = F(1, n_{t}) K_{t} = F(K_{t}, N_{t}),$$

which gives the first equality in equation (28). The second equality in equation (28) follows from a law of large number, the market clearing condition (12), and Proposition 1. We use equation (13) and $n_t^j = n_t$ to show

$$F_2(K_t, N_t) = \left(1 - \tau_t^n\right) w_t. \tag{A8}$$

By the constant return to scale property of F, we also have

$$R_t = F_1(K_t, N_t). \tag{A9}$$

We next derive aggregate investment

$$I_t = \int I_t^j dj = \int i_t^j K_t^j dj = K_t \int_0^{\overline{\xi}_t} \left(\frac{\psi Q_t}{1 - \tau_t^i}\right)^{\frac{1}{\theta}} \phi(\xi) d\xi,$$

where the second equality uses the definition of i_t^j , and the third equality uses a law of large numbers and Proposition 1. We thus obtain (26). By definition

$$K_{t+1} = \int_0^1 \left[(1-\delta) + \Phi(i_t^j) \right] K_t^j dj.$$

Substituting the optimal investment rule in equation (24) and using equation (26), we obtain (27).

Finally, equation (30) follows from substitution of equations (9) and (A9) into equation (25). Equation (29) follows from equations (9), (10), and (A8). \Box

PROOF OF PROPOSITION 3. In an interior steady state, $\xi^* = \overline{\xi}$ and equations 26 and (23) imply that

$$\frac{I}{K} = \left(\frac{\psi Q}{1 - \tau^i}\right)^{\frac{1}{\theta}} \int_0^{\frac{\xi}{\theta}} \phi(\xi) d\xi, \tag{A10}$$

$$\bar{\xi} = \frac{\theta}{1-\theta} \left(\psi Q\right)^{\frac{1}{\theta}} \left(1-\tau^{i}\right)^{\frac{\theta-1}{\theta}}.$$
(A11)

From these two equations, we obtain

$$\frac{I}{K} = \frac{\bar{\xi}}{1 - \tau^i} \frac{1 - \theta}{\theta} \int_0^{\bar{\xi}} \phi(\xi) d\xi.$$
(A12)

In steady state, equation (27) becomes

$$\delta - \varsigma = \frac{\psi}{1 - \theta} \left(I/K \right)^{1-\theta} \left[\int_0^{\bar{\xi}} \phi(\xi) d\xi \right]^{\theta}.$$
 (A13)

Substituting equation (A12) into the above equation yields equation (33). The expression on the right-hand side of this equation increases with $\bar{\xi}$. The condition in this proposition guarantees a unique interior solution $\bar{\xi} \in (0, \xi_{\text{max}})$ exists.

Equation (34) follows from (A11). Equations (A12) and (A13) imply that

$$\delta - \varsigma = \frac{\psi}{1 - \theta} \frac{I}{K} \left(\frac{\xi^* \left(1 - \theta \right)}{\left(1 - \tau^i \right) \theta} \right)^{-\theta}.$$
(A14)

From this equation and equation (34), we obtain (35). The other equations in the proposition follow from the steady-state versions of equations (29) and (30). \Box

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