Long-Term Securities and Banking Crises

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Abstract

The US bank holdings of long-term securities have increased in recent years. As is witnessed by the recent bank failures including SVB, prices of long-term securities are sensitive to interest rate hikes and can trigger bank runs. To study the role of bank holdings of long-term bonds, we incorporate banks in a DSGE framework. We study how cost-push shocks and the associated passive or active interest rate hikes affect the macroeconomy including inflation, investment, and output. The procyclical bank balance sheets and long-term bond prices amplify adverse shocks, which can trigger a bank run that would not exist if banks otherwise hold short-term bonds. We introduce two types of macroprudential policies that can mitigate or prevent banking crises: a permanent tax on bank holdings of long-term bonds and a cyclical tax that responds to interest rate changes.

JEL Codes: E32, E44, E52, G01, G21

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1 Introduction

Since mid-2021, the US has witnessed a surge in inflation. This may be attributed to various causes, including pandemic-related economic dislocation, supply chain problems, and the fiscal and monetary stimuli provided in 2020 and 2021 by governments and central banks around the world in response to the pandemic. To curb high inflation, the Fed has raised interest rates eleven times since March 17, 2022. The current target rate range is 5.25-5.5%.

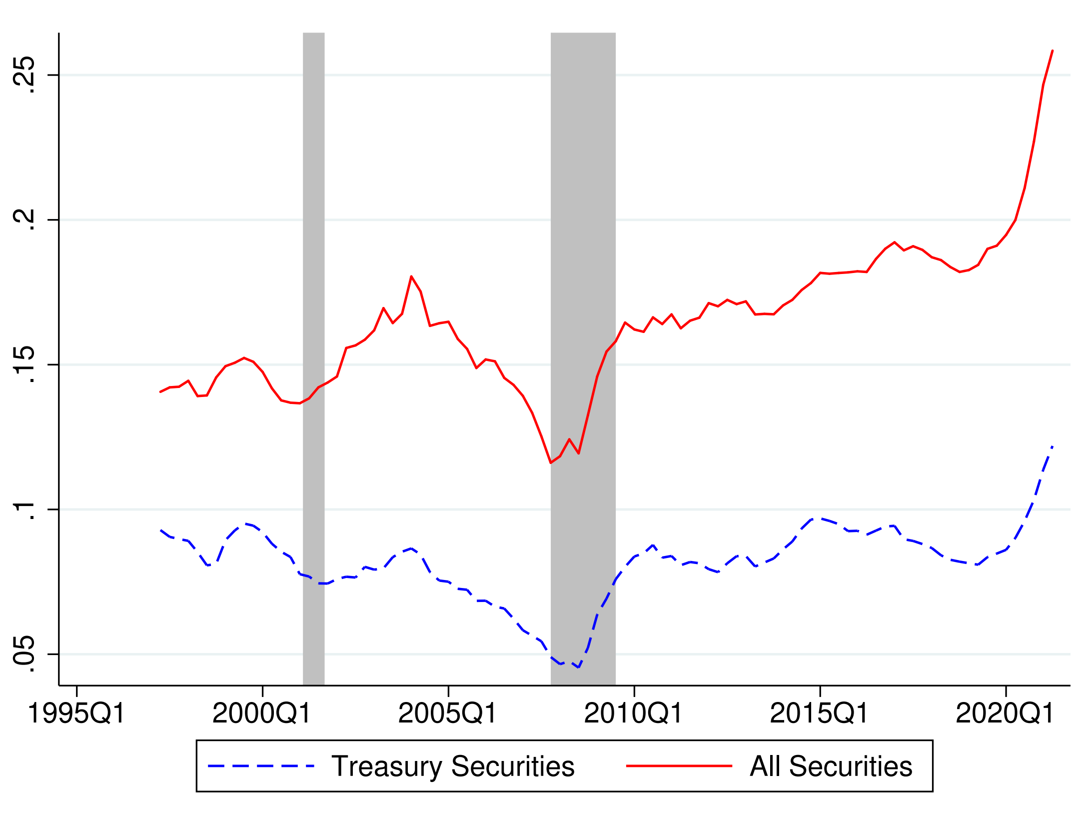
Since March 2023, we have also witnessed a few small- to mid-size US bank failures, triggering a sharp decline in global bank stock prices and swift responses by regulators to prevent potential global contagion. In particular, Silicon Valley Bank (SVB) failed after a bank run, marking the third-largest bank failure in US history and the largest since the 2007–2008 financial crisis. The cause is still under debate, but is often attributed to the fact that SVB had dramatically increased its holdings of long-term bonds since 2021. The market value of these bonds decreased significantly through 2022 and into 2023 as the Fed raised interest rates. As a result, bank profits dropped significantly and eventually could not meet large deposit withdrawals.

Besides SVB, many other US commercial banks had also increased holdings of long-term securities. Figure 1 presents the recent aggregate data of the portfolio shares of long-term securities for all commercial banks weighted by bank total assets from 1997Q2 to 2021Q2 based on the US bank-level call reports. We consider two measures of long-term securities: (i) Treasury bonds with maturities more than 1 year, and (ii) the combined Treasury bonds, MBS, and agency debt, all with maturities more than 1 year. Both measures increased in recent years and reached 12.2% and 25.8% respectively in early 2021.

Drawing lessons from the recent events, we develop a dynamic stochastic general equilibrium (DSGE) model to address the following questions: What is the role of bank holdings of long-term securities in banking crises? Can interest rate hikes cause a banking crisis? What are the underlying economic mechanisms? What policies can prevent or mitigate banking crises?

Building on Gertler and Karadi (2011, 2013) and Gertler and Kiyotaki (2010, 2015), we incorporate a banking sector into a dynamic New Keynesian (DNK) model. As in Gertler and Karadi (2011), we introduce a simple agency problem between banks and their respective depositors. The agency problem generates endogenous constraints on bank leverage ratios, which have the effect of tying overall credit flows to the equity capital in the banking sector. A deterioration of bank balance sheets will disrupt lending and borrowing in a way that raises credit costs, thereby reducing

Figure 1: US bank holdings of long-term securities



Note: This figure plots the bank portfolio share of different assets with maturities longer than 1 year. The red solid line represents bank portfolio share in all types of long-term securities (e.g. Treasury securities, agency debt, MBS, etc.). The blue dash line represents bank portfolio share in long-term Treasury securities. Assets are classified as long-term if the remaining maturity is longer than 1 year. As we do not consider interbank transactions in the model, we define bank-level total assets as the sum of loans/leases and security holdings so as to focus on the credit supply to the real economy. The aggregate time series is the average of bank portfolio shares, weighted by bank total assets. The data is from the US bank-level call reports.

firm investment.

Unlike Gertler and Karadi (2011), we allow banks to hold long-term government bonds as assets. Households can also trade these bonds. To prevent frictionless arbitrage of different types of assets, we introduce management costs. When inflation rises due to either a positive cost-push shock or a negative technology shock, interest rates rise either passively by following a Taylor rule or actively by a surprise hike. We show that long-term bond prices are sensitive to interest rate changes. Long-term bonds amplify adverse shocks to the bank balance sheets and thus generate a larger negative effect on bank lending than short-term bonds do.

Moreover, as bank profits decline following interest rate hikes, a bank run can occur when depositors decide not to roll over their deposits. We show that multiple equilibria (a no-run equilibrium and a bank-run equilibrium) can coexist. The existence of a bank-run equilibrium depends on the health of bank balance sheets. When bank balance sheets deteriorate, fears of a bank run can become self-fulfilling even in the absence of any negative fundamental shock. Bank runs, in turn, force banks to liquidate their assets at firesale prices, causing a sudden collapse in bank equity, and a deep and prolonged economic downturn. We also show that a bank run is more likely to occur if banks hold longer-term government bonds as their prices decline more in response to interest rate hikes. In a calibrated example, we find that a bank run occurs following interest rate hikes when banks hold long-term bonds, but would not exist if banks otherwise hold short-term bonds.

While we do not intend to explicitly model the recent bank failures discussed earlier, our model does capture some key elements relevant to the understanding of bank failures and to the questions we are after. Moreover, our model can be used to understand what policies can mitigate the impact of a banking crisis and reduce the likelihood of bank runs.

We introduce two types of macroprudential policies. First, we consider a permanent tax on bank holdings of long-term bonds. Such a tax reduces bank's exposure to long-term bonds and hence can mitigate adverse shocks to the economy. On the other hand, it also reduces banks' long-run holdings of long-term bonds, and thus banks' balance sheets and capital intermediated by banks. We show that there is an optimal tax rate conditional on cost-push shocks and surprise interest rate hikes.

Second, we consider a cyclical tax on bank holdings of long-term bonds in the sense that the tax rate responds to the interest rate change. We show that when the tax rate is a decreasing function of the interest rate change, the adverse cost-push shock can be mitigated and the likelihood of bank runs can be reduced. In particular, we show that the government should subsidize (tax) bank holdings of long-term assets when interest rates rise (decline).

Our paper is related to two strands of the literature. First, it is related to a large literature that studies the role of financial intermediaries in macroeconomic fluctuations. Much of this literature builds on the financial accelerator model of Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997). Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) introduce financial intermediaries (banks) to the DSGE model and show how procyclical bank balance sheets can amplify and propagate exogenous shocks. The literature along these lines typically studies credit policy or unconventional monetary policy. See Curdia and Woodford (2011), Del Negro et al. (2017), and Gertler and Karadi (2013), among others.

Our paper is closely related to Gertler and Karadi (2013). Our paper differs from theirs in three major respects. First, we study long-term bonds with a finite maturity that can better match the data, while they study infinite maturity console bonds. Second, we allow for bank runs, absent in their paper. Third, we study how macroprudential policies can prevent bank runs during interest rate hikes to curb high inflation triggered by cost-push shocks, while they examine the impact of large-scale asset purchases during a crisis triggered by capital quality shocks.

To overcome the limitation of the local linear solutions in the traditional literature,[1] Gertler and Kiyotaki (2015) and Gertler, Kiyotaki, and Prestipino (2016, 2020a, 2020b) model bank runs in the DSGE framework as rollover panics following the Calvo (1988) and Cole and Kehoe (2000) models of sovereign debt crises.[2] Solving bank-run models needs global nonlinear solution methods. Like Gertler, Kiyotaki, and Prestipino (2020a), we introduce variable capital and extend the model of Gertler and Kiyotaki (2015) to the DSGE framework. Unlike them, we focus on cost-push shocks that generate inflation and interest rate hikes. We also study new types of macroprudential policies absent from their paper.

Next, our paper is related to the literature that studies the role of macroprudential policies in preventing financial crises. Following Lorenzoni (2008), the main conceptual motive for macroprudential policies in this literature is the presence of a pecuniary externality, where individual banks fail to take account of the impact of their leverage decisions or risk exposures on the dynamics of asset prices. Recent papers in this literature include Gertler, Kiyotaki, and Queralto (2012), Angeloni and Faia (2013), Aoki, Benigno, and Kiyotaki (2016), Jeanne and Korinek (2019), Gertler, Kiyotaki, and Prestipino (2020b), and Begenau and Landvoigt (2022), among others. This literature typically focuses on the capital requirement policy or taxes/subsidies on certain variables in the bank balance sheet. Unlike this literature, we introduce a new type of macroprudential policies that tax or subsidize bank holdings of long-term securities. In our model banks fail to internalize the impact of their leverage decisions and long-term bond holding decisions on the dynamics of asset prices and the likelihood of a bank run.

1. See Mendoza (2010), He and Krishnamurthy (2019) and Brunnermeier and Sannikov (2014) for other models that use global nonlinear solution methods.

2. We do not follow the traditional literature on bank runs originating from Diamond and Dybvig (1983), in which sequential service constraints play an important role. See Allen and Gale (2009) for recent developments of this literature. See Miao and Wang (2015), Boissay, Collard, and Smets (2016), Quadrini (2017), Robatto (2019), Amador and Bianchi (2021), Elenev, Landvoigt, and Van Nieuwerburgh (2021), Boissay et al. (2021), Bianchi and Bigio (2022), and references therein for other recent macroeconomic models with banks.

2 A Baseline Model

Our baseline model builds on Gertler and Karadi (2011, 2013) and does not incorporate bank runs. Time is infinite and indexed by $t = 0, 1, 2, \cdots$. The model economy consists of households, financial intermediaries (banks), nonfinancial firms, and the fiscal/monetary authorities. Banks transfer funds from households to nonfinancial firms subject to agency costs. The government issues long-term bonds for households and banks to trade subject to management costs. The central bank conducts monetary policy following an interest rate rule. As in the standard DNK model (Woodford 2003; Galí 2015), we consider a cashless economy and model price stickiness following Calvo (1983). For simplicity, we consider only three sources of independent disturbances: an interest rate shock, a technology shock, and a cost-push shock.

2.1 Government

The government issues a portfolio of nominal zero-coupon bonds of different maturities. Because the Ricardian equivalence does not hold in our model due to financial and real frictions, the maturity structure matters for the macroeconomy. To model the maturity structure in a simple way, we assume that the maturities of government bonds follow a geometric structure (Cochrane 2001; Woodford 2001): one unit of the portfolio of government bonds purchased in period t pays ρ^{j-1} dollars in period $t + j$ for $j \geq 1$, $0 < \rho < 1$.

Denote by Q_t^l the nominal price of one unit of the portfolio of government bonds in period t. The real return of holding the portfolio of government bonds from period t to period $t + 1$ is

$$R_{t+1}^l = \frac{1 + \rho Q_{t+1}^l}{Q_t^l} \Pi_{t+1}^{-1}, \tag{1}$$

where $\Pi_{t+1} = P_{t+1}/P_t$ is the gross inflation and P_t is the price level.

Let B_t^n denote the face value of nominal government bonds outstanding in period t. The government's budget constraint is given by

$$(1 + \rho Q_t^l) B_{t-1}^n = S_t^g P_t + Q_t^l B_t^n,$$

where S_t^g is the real primary surplus. Define the real face value of government bonds as $B_t =$

B_t^n/P_t. We can rewrite the government's budget constraint in real terms as

$$Q_{t-1}^l B_{t-1} R_t^l = S_t^g + Q_t^l B_t. \tag{2}$$

2.2 Households

There is a continuum of identical households of measure one. A fraction of the household members are workers and the remaining fraction are bankers. The survival probability of each banker is σ. The bankers who exit become workers. Then the same amount of workers become new bankers and receive startup funds from households. Within each family, there is perfect consumption insurance.

In each period t, households can make real deposits D_t into banks in the form of one-period bonds with the nominal interest rate R_t^n from period t and $t+1$. Then the real interest rate from period t to period $t+1$ is $R_{t+1} = R_t^n/\Pi_{t+1}$. Households can also purchase government bonds and firm equity directly. Denote by B_{ht} and S_{ht} the household holdings of long-term government bonds and firm equity. Following Gertler and Karadi (2013) and Gertler, Kiyotaki, and Prestipino (2020a), we assume that households pay asset management costs for directly holding firm equity and long-term government bonds. This assumption is important to generate firesale prices when banks sell bonds and equities to households. For tractability, we specify the costs as

$$\frac{1}{2}\kappa\left(\frac{S_{ht}}{S_t}-\eta_S\right)^2 Q_t^k S_t \quad \text{and} \quad \frac{1}{2}\kappa\left(\frac{B_{ht}}{B_t}-\eta_B\right)^2 Q_t^l B_t,$$

where Q_t^k is the real price of firm equity, S_t is the aggregate quantity of firm equity claims, and κ, η_S, and η_B are positive parameters.

A representative household chooses processes of consumption $\{C_t\}$, labor supply $\{L_t\}$, long-term bond holdings $\{B_{ht}\}$, and equity holdings $\{S_{ht}\}$ to solve the following problem:

$$\max\ \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\ln\left(C_t-hC_{t-1}\right)-\frac{\chi}{1+\varphi}L_t^{1+\varphi}\right] \tag{3}$$

where $\mathbb{E}$ denotes an expectation operator, and $h, \chi, \varphi > 0$ are parameters, subject to a sequence of budget constraints

$$C_t + D_t + Q_t^l B_{ht} + Q_t^k S_{ht} + \frac{1}{2}\kappa\left(\frac{S_{ht}}{S_t} - \eta_S\right)^2 Q_t^k S_t + \frac{1}{2}\kappa\left(\frac{B_{ht}}{B_t} - \eta_B\right)^2 Q_t^l B_t$$
$$= R_t D_{t-1} + R_t^l Q_{t-1}^l B_{h,t-1} + R_t^k Q_{t-1}^k S_{h,t-1} + W_t L_t + \Phi_t - \Psi_t - T_t,$$

for $t \geq 0$, where W_t is the real wage rate, R_t^k is the real return on firm equity, Φ_t is the profits from banks and nonfinancial firms, Ψ_t is the total transfer to new bankers, and T_t is the lump-sum tax net of transfer or rebate.

By the first-order conditions, we have the following pricing equations:

$$\mathbb{E}_t \Lambda_{t,t+1} R_{t+1} = 1, \tag{4}$$
$$\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l = 1 + \kappa\left(\frac{B_{ht}}{B_t} - \eta_B\right), \tag{5}$$
$$\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^k = 1 + \kappa\left(\frac{S_{ht}}{S_t} - \eta_S\right), \tag{6}$$

where the household stochastic discount factor (SDF) between t and $t+1$ is given by

$$\Lambda_{t,t+1} \equiv \beta \frac{u_{C_{t+1}}}{u_{C_t}},\ u_{C_t} = \frac{1}{C_t - hC_{t-1}}.$$

The management costs generate differences in returns on the deposits, the long-term bonds, and capital even in the absence of risk.

W require that parameter values satisfy

$$\frac{B_h}{B} - \eta_B > \frac{S_h}{S} - \eta_S > 0,$$

so that $R^k > R^l > R$ in the deterministic steady state, where a variable without a time subscript denotes its steady-state value. In this case, we have $B_h > \eta_B B > 0$ and $S_h > \eta_S S > 0$. The preceding ranking of returns also holds in a neighborhood of the steady state, which is consistent with empirical evidence.

2.3 Banks

Banks take deposits from households, lend funds to firms, buy government bonds, and retain earnings. Let n_t denote the amount of equity capital—or net worth—that a banker has at the end of period t, d_t the deposits the bank obtains from households, s_t the quantity of financial claims on nonfinancial firms that the bank holds, and b_t the quantity of long-term government bonds. To prevent frictionless arbitrage between firm equity and long-term government bonds by banks, we assume that each bank pays a management cost for holding firm equity, given by $\frac{\psi}{2}\left(Q_t^k s_t / n_t\right)^2 n_t$, where $\psi > 0$ is a parameter. Assuming that managing bonds is less costly for banks, we simply set the cost to zero.

Banks are subject to the regulation of macroprudential policies. Assume that the macroprudential policy authority charges a tax on a bank's holdings of the long-term bonds and rebates the tax revenue to households. Let τ_t^l be the tax rate. It is possible that the macroprudential authority taxes households to subsidize bank holdings of long-term bonds when $\tau_t^l < 0$. We will determine the optimal macroprudential policy in Section 5. In either case, a representative bank's balance sheet is given by

$$Q_t^k s_t + \frac{\psi}{2}\frac{\left(Q_t^k s_t\right)^2}{n_t} + Q_t^l b_t (1 + \tau_t^l) = n_t + d_t. \tag{7}$$

The bank's real net worth is accumulated through retained earnings and thus satisfies

$$n_{t+1} = R_{t+1}^k Q_t^k s_t + R_{t+1}^l Q_t^l b_t - R_{t+1} d_t. \tag{8}$$

When a bank exits with probability $1 - \sigma$, it pays out its net worth as dividends. Each banker is risk-neutral and consumes dividends. Thus a continuing banker's objective is to maximize the expected present value of net worth when exiting:

$$V_t = \mathbb{E}_t \sum_{i=1}^{\infty} (1 - \sigma)\sigma^{i-1} \Lambda_{t,t+i} n_{t+i}, \tag{9}$$

where $\Lambda_{t,t+i}$ is the household stochastic discount factor between t and $t + i$.

To motivate a limit on the bank's ability to obtain deposits, we introduce the following costly enforcement problem: At the beginning of each period, the banker can choose to divert funds from the assets it holds and transfers the proceeds to the household of which he or she is a member. The cost to the banker is that the depositors can force the intermediary into bankruptcy and recover the remaining fraction of assets. However, it is too costly for the depositors to recover the funds that the banker diverted. For the banker not to divert assets, it faces the following incentive constraint

$$V_t \geq \theta \left(Q_t^k s_t + Q_t^l b_t \right), \tag{10}$$

where $\theta \in (0,1)$ is the diverted fraction of assets.[3] The banker chooses sequences of s_t, b_t, and d_t to maximize (9) subject to its balance sheet (7), the law of motion of net worth (8), and the incentive constraint (10).

Using the bank's balance sheet (7) to replace d_t in (8), we rewrite the net worth of a surviving bank as

$$n_t = \left(R_t^k - R_t \left(1 + \frac{\psi}{2} \frac{Q_{t-1}^k s_{t-1}}{n_{t-1}} \right) \right) Q_{t-1}^k s_{t-1} + \left(R_t^l - R_t \left(1 + \tau_{t-1}^l \right) \right) Q_{t-1}^l b_{t-1} + R_t n_{t-1}.$$

Let S_{bt} be the total quantity of loans that banks intermediate, B_{bt} the total number of government bonds they hold, and N_t the total net worth. In Appendix A, we show that $Q_t^k s_t / n_t$ is the same for each bank, making aggregation tractable. Then the law of motion of net worth of the whole banking sector is given by

$$\begin{aligned} N_t =& \sigma \left[\left(R_t^k - R_t \left(1 + \frac{\psi}{2} \frac{Q_{t-1}^k S_{b,t-1}}{N_{t-1}} \right) \right) Q_{t-1}^k S_{b,t-1} + \left(R_t^l - R_t \left(1 + \tau_{t-1}^l \right) \right) Q_{t-1}^l B_{b,t-1} \right. \\ & \left. + R_t N_{t-1} \right] + \omega N_{t-1}, \end{aligned} \tag{11}$$

where we assume the transfer to each new banker as startup funds is proportional to its previous net worth, i.e., $\Psi_t = \omega N_{t-1}$ for $\omega \in (0,1)$.[4]

In Appendix A, we derive the first-order conditions:

$$\mathbb{E}_t \Omega_{t+1} \left(R_{t+1}^k - R_{t+1} \left(1 + \psi \frac{Q_t^k s_t}{n_t} \right) \right) = \lambda_t \theta, \tag{12}$$

$$\mathbb{E}_t \Omega_{t+1} \left(R_{t+1}^l - R_{t+1} \left(1 + \tau_t^l \right) \right) = \lambda_t \theta, \tag{13}$$

where Ω_{t+1} is the bank SDF and $\lambda_t \geq 0$ is the Lagrange multiplier associated with the incentive constraint (10). Thus, as long as the cost and tax adjusted conditional expected discounted excess returns are positive, we have $\lambda_t > 0$ and the incentive constraint binds. The above two equations

3. Another way to generate the spread between equity returns and bond returns for banks is to remove the bank loan management cost and assume that the diverted fraction of loans is higher than that of long-term bonds as in Gertler and Karadi (2013).

4. The literature assumes several different forms of the transfer. For example, Gertler and Karadi (2011) take a fraction of the value of the bank assets as the transfer, while Gertler and Karadi (2013) assume a constant transfer. These alternative approaches will not change our key insights.

also imply the following steady-state relation:

$$R^k - R^l = R\left(\psi \frac{Q^k S_b}{N} - \tau^l\right).$$

We focus on equilibria with $R^k > R^l$. This means $S_b > 0$ for $\tau^l \geq 0$ and $\psi > 0$. It follows from the analysis in Section 2.2 that both banks and households provide funds to nonfinancial firms in the steady state, i.e., $S_h > 0$ and $S_b > 0$, if $R^k > R^l > R$ and $0 \leq \tau^l < \psi Q^k S_b / N$. This is also true in the neighborhood of the steady state. Clearly, the assumption of management costs with $\kappa > 0$ and $\psi > 0$ plays an important role.

2.4 Retailers

The production sector of the model is standard. There are three types of nonfinancial firms: monopolistically competitive retailers, competitive intermediate goods producers, and competitive capital producers. We introduce nominal price rigidities through retailers. We describe each in turn.

The retailers purchase intermediate goods $j \in [0, 1]$ at the wholesale price P_{wt} from intermediate goods firms and sell it as the retail goods at price P_{jt} to final goods producers. The final output is a CES composite of a continuum of retail goods of measure unity:

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The demand for retail goods j is given by

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} Y_t, \tag{14}$$

where P_{jt} is the intermediate good j's price and the price index is given by

$$P_t \equiv \left[\int_0^1 P_{jt}^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}.$$

We follow Adam and Woodford (2012) to incorporate cost-push shocks by adding a random tax/subsidy τ_t^c on each retailer's sale revenue. The tax revenue is transferred to households. Each retailer can adjust its prices with probability $1 - \gamma$ in each period. Following Erceg, Henderson, and Levin (2000), we assume that whenever each retailer does not reset its price with probability

γ, its price is automatically increased at the steady-state inflation rate. The retailer selling good j chooses the nominal price P_{jt}^* in period t to maximize the discounted present value of real profits

$$\max_{P_t^*} \sum_{k=0}^{\infty} \gamma^k \mathbb{E}_t \left[\Lambda_{t,t+k} \left(\frac{(1-\tau_{t+k}^c)\Pi^k P_{jt}^*}{P_{t+k}} - P_{w,t+k} \right) Y_{jt+k}^* \right],$$

subject to the demand curve

$$Y_{jt+k}^* = \left(\frac{\Pi^k P_{jt}^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}.$$

From the optimality conditions, we can derive the linearized new Keynesian Phillips curve (NKPC)

$$\widehat{\Pi}_t = \beta \mathbb{E}_t \widehat{\Pi}_{t+1} + \kappa_\pi \widehat{P}_{wt} + \mu_t,$$

where a variable with a hat denotes the log-linear deviation from its steady state, $\kappa_\pi = (1 - \beta\gamma)(1-\gamma)/\gamma$, and

$$\mu_t = -\kappa_\pi \log \left(\frac{1-\tau_t^c}{1-\tau^c} \right).$$

The variable P_{wt} represents the marginal cost in the standard DNK model. The variable μ_t captures a cost-push shock that moves the NKPC. Following Gelain and Ilbas (2017), we assume that μ_t follows an ARMA(1,1) process

$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu + \rho_{ma} \varepsilon_{t-1}^\mu,$$

where ε_t^μ is an independently and identically distributed (iid) normal random variable with mean zero and variance σ_μ^2. Notice that while we use the linear NKPC to motivate μ_t as cost-push shocks, we still adopt the same interpretation for our nonlinear solutions studied later.

2.5 Intermediate Goods Producers

There exists a continuum of intermediate goods producers indexed by $j \in [0,1]$. Intermediate goods producers acquire capital and hire labor to produce intermediate goods, which are then sold to retailers at the real price of P_{wt}. They finance their capital acquisition each period by obtaining funds from intermediaries and households. They issues S_t claims equal to the number of total units of capital acquired K_{t+1} and price each claim at the price of a unit of capital Q_t^k.

The production function of each firm j is given by

$$Y_{jt} = A_t K_{jt}^{\alpha} L_{jt}^{1-\alpha},\ \alpha \in (0,1)\,, \tag{15}$$

where K_{jt} is capital, L_{jt} is labor, and A_t follows an AR(1) process

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t^a,$$

and ε_t^a is an iid normal random variable with mean zero and variance σ_a^2.

Solving the optimal labor hiring decision yields

$$W_t = P_{wt}(1-\alpha)\frac{Y_{jt}}{L_{jt}}, \tag{16}$$

suggesting all firms share the same ratio of K_{jt}/L_{jt}. It follows from (14) that aggregate output satisfies

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}/\Delta_t,$$

where L_t is aggregate labor and Δ_t denotes the price dispersion

$$\Delta_t = \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} dj.$$

By (16), the gross profits for each unit of capital is given by

$$Z_t = \frac{P_{wt}Y_{jt} - W_t L_{jt}}{K_{ji}} = P_{wt}\frac{\alpha Y_{jt}}{K_{jt}}. \tag{17}$$

Integrating (17) over j and using (14), we have

$$Z_t = \frac{\Delta_t P_{wt} \alpha Y_t}{K_t}.$$

Given that each intermediate good firm earns zero profits, it simply pays out the ex post return to capital Z_{t+1} and the undepreciated capital value to households and intermediaries. Thus the real return on firm equity is defined as

$$R_{t+1}^k = \frac{Z_{t+1} + (1-\delta)Q_{t+1}^k}{Q_t^k}, \tag{18}$$

where δ is the depreciation rate of capital.

2.6 Capital Goods Producers

Capital goods producers make new capital using input of final output subject to adjustment costs. They buy undepreciated capital from intermediate goods firms and then sell new capital to them. The objective of a capital producer is to choose a sequence of investment $\{I_t\}$ to solve

$$\max \mathbb{E} \sum_{t=0}^{\infty} \Lambda_{0,t} \left\{ Q_t^k K_{t+1} - I_t - Q_t^k (1-\delta) K_t \right\},$$

subject to the law of motion for capital

$$K_{\tau+1} = (1-\delta)K_t + I_t - f(I_t, K_t),$$

where $\Lambda_{0,t} \equiv \beta u_{C_t} / u_{C_0}$ and f denotes the adjustment cost function. Following Chari, Kehoe, and McGrattan (2007), we take the following functional form:

$$f(I_t, K_t) = \frac{\Omega_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t,$$

where $\Omega_k > 0$ is a parameter.

2.7 Monetary and Macroprudential Policies

For simplicity, we assume that the monetary authority is committed to the following interest rate rule:

$$R_t^n = \rho_r R_{t-1}^n + (1-\rho_r) \left(R^n + \phi_\pi \log \frac{\Pi_t}{\Pi} + \phi_y \log \frac{Y_t}{Y} \right) + v_t^m, \tag{19}$$

where R_t^n is the nominal interest rate from t to $t+1$ and R^n denotes its steady-state value. Here $\rho_r \in (0,1)$ is an interest-rate smoothing parameter. The response coefficients ϕ_π and ϕ_y are important for equilibrium determinacy. As in the NK literature, we focus on the monetary regime by assuming that these parameters satisfy a generalized Taylor principle.

Assume that the monetary policy shock v_t^m follows an AR(1) process

$$v_t^m = \rho_m v_{t-1}^m + \varepsilon_t^m,$$

where ε_t^m is an iid normal random variable with mean zero and variance σ_m^2.

We consider two types of macroprudential policy. First, a permanent policy specifies a constant tax rate $\tau_t^l = \tau^l$ on the holdings of long-term bonds for all t. Second, a cyclical policy allows the tax rate τ_t^l to respond to the nominal interest rate according to the following rule

$$\tau_t^l = \phi_l (R_t^n - R^n), \tag{20}$$

where ϕ_l is a parameter.

2.8 Equilibrium

An equilibrium with sticky prices is defined in the usual way. A full list of equilibrium conditions can be found in Appendix B. Here we describe the resource constraint and market-clearing conditions. The resource constraint is given by

$$Y_t = C_t + I_t + G_t,$$

where $G_t = T_t - S_t^g$ is the government spending. Following Gertler and Karadi (2011, 2013) and Gertler, Kiyotaki, and Prestipino (2020a), we assume that the government fixes its spending $G_t = G$ for all t and fixes its supply of government bonds $B_t = B$ for all t. Unlike them, we do not consider credit policy or unconventional monetary policy. The government adjusts lump-sum taxes to satisfy its budget constraints (2). We also assume that capital and bond management costs are rebated to households so that they do not enter the resource constraint. Assuming that they are a waste of resources does not change our key insights.

As each unit of firm equity claim finances one unit of capital, we have $K_{t+1} = S_t$. The market-clearing conditions for government bonds and firm equity are given by

$$B_t = B_{ht} + B_{bt},$$

$$S_t = S_{ht} + S_{bt}.$$

3 Model Analysis

Because of the complexity of our baseline model, there is no analytical equilibrium solution. To derive numerical solutions, we need to calibrate the model first. We then study impulse responses of the economy to shocks when the economy initially stays at the deterministic steady state. We use Dynare ((Adjemian et al. 2022)) to derive nonlinear perfect foresight solutions and find they are also close to the log-linear solutions.

3.1 Calibration

We calibrate the model without the macroprudential policy and one period in the model corresponds to a quarter. There are 31 parameters in total listed in Table 1. We categorize all parameters into two groups. The parameters in the group $\{\sigma, \theta, \psi, \omega, \eta_S, \eta_B, \kappa, \rho\}$ are specific to our model. The rest are conventional. We first discuss the conventional parameters. For capital share α, discount factor β, price adjustment probability γ, elasticity of substitution ε, and the capital depreciation rate δ, we choose the standard values in the business cycle literature.

Table 1: Calibrated Parameters at Quarterly Frequency

Parameter	Value	Description	Target
Banks			
σ	0.94	Survival rate of bankers	Annual bank dividend payout ratio of 24%
θ	0.4988	Seizure rate of bank assets	Annual excess return 4% on firm equity
ψ	0.0011	Bank's management cost for firm equity	Annual excess return 2% on long-term bonds
ω	0.0189	Transfers to new bankers	Bank leverage of 6
Households			
η_S	0.4667	Reference household share of firm equity claims	50% of firm equity held by banks
η_B	0.6690	Reference household share of long-term bonds	Long-term bonds accounts for 22% of bank assets
κ	0.3	Households' management cost parameter	Responses of long-term yield to monetary policy shocks
Government bonds			
ρ	0.96	Decay rate of maturity structure	Average government bond maturity of 5 years
$Q^l B/Y$	4	Long-term gov bonds to GDP ratio	Public debt-to-GDP ratio of 100%
Conventional			
α	0.33	Capital elasticity	Capital income share
β	0.998	Discount factor	Annual nominal rate of 2.81%
γ	0.75	Price adjustment probability	Ave. duration of price adjustment of 1 year
ε	11	Elasticity of Substitution	Price markup of 10%
τ^c	1/(1-ε)	Revenue tax/subsidy	No monopoly distortion $P_w = 1$
χ	8.2976	Labor disutility	Numbers of hours worked of 0.33
δ	0.025	Capital depreciation rate	Annual capital depreciation rate of 10%
Ω_k	20	Investment adjustment cost	Elasticity of capital price to investment capital ratio
G/Y	0.2	Government spending share	Average government spending share
Π	1.005	Trend inflation	Annual inflation of 2%
h	0.27	Habit parameter	Gelain and Ilbas (2017)
φ	0.49	Inverse Frisch elasticity	Gelain and Ilbas (2017)
ϕ_π	1.98	Taylor rule parameter on inflation	Gelain and Ilbas (2017)
ϕ_y	0.08	Taylor rule parameter on output gap	Gelain and Ilbas (2017)
ρ_r	0.85	Monetary policy smoothing	Gelain and Ilbas (2017)
ρ_a	0.9	AR parameter of productivity shocks	Gelain and Ilbas (2017)
ρ_m	0.18	AR parameter of monetary policy shocks	Gelain and Ilbas (2017)
ρ_μ	0.79	AR parameter of cost-push shocks	Gelain and Ilbas (2017)
ρ_{ma}	0.54	MA parameter of cost-push shocks	Gelain and Ilbas (2017)
$100 \cdot \sigma_a$	0.37	Standard deviation of productivity shocks	Gelain and Ilbas (2017)
$100 \cdot \sigma_\mu$	0.10	Standard deviation of cost-push shocks	Gelain and Ilbas (2017)
$100 \cdot \sigma_m$	0.12	Standard deviation of monetary shocks	Gelain and Ilbas (2017)

We set the annual trend inflation rate $(\Pi - 1) * 4 = 2\%$, consistent with the Fed's inflation target. We choose $\beta = 0.998$ so that the implied steady-state nominal interest rate per annum is $4(\Pi/\beta - 1) = 2.81\%$, which roughly aligns with the Fed's expectation of the policy rate in the longer run.[5]

We set the labor disutility parameter χ such that the steady-state hours worked is 1/3. We also set the revenue tax/subsidy for intermediate-good firms $\tau^c = 1/(1-\varepsilon)$ such that $P_w = 1$ in the steady state to remove the distortion due to monopolistic competition. We set the investment adjustment cost parameter $\Omega_k = 20$ so that the elasticity of capital price with respect to the investment-capital ratio equals 0.5 (Chari, Kehoe, and McGrattan 2007).

We set $G/Y = 0.2$ so that the government spending to GDP ratio is roughly consistent with the average of the US data from 1950Q1 to 2020Q1. We set $Q^l B/Y = 4$ so that the annualized public debt to GDP ratio is 100%. There has been a rising trend of government indebtedness in the US since WWII, especially during recent times after the 2008 Great Recession. We find that the average public debt to GDP ratio is 100% in the US data from 2010Q1 to 2020Q1. According to the CBO's projections, the Federal debt held by the public will increase in each year and reach 118% of GDP in 2033 and will reach 195% of GDP in 2053.[6] Our target of $Q^l B/Y = 4$ therefore reflects a balance between the past and the future values of the public debt to GDP ratio.

For other conventional parameters, we use the estimates from Gelain and Ilbas (2017) based on a model similar to Gertler and Karadi (2011). These parameters include the inverse Frisch elasticity φ, the habit formation parameter h, the parameters describing the monetary policy rule ρ_r, ϕ_π, ϕ_y, and the parameters that characterize the exogenous shock processes.

We now discuss the calibration of the parameters in the first group. According to U.S. Department of the Treasury (2020), the historical weighted average maturity of Treasury marketable securities outstanding is around 5 years between 1980 and 2020. The weighted average maturity (Macaulay duration) of the government bond portfolio in the frictionless steady state is $R^n / (R^n - \rho)$. As $R^n = \Pi/\beta$, the weighted average maturity is $1/(1 - \beta\Pi^{-1}\rho)$. Although our model features frictions, we use this formula to calibrate ρ for simplicity. We set $\rho = 0.96$ such that the weighted average maturity is around 5 years.

Following Aoki, Benigno, and Kiyotaki (2016), we choose σ such that the annualized dividend payout to bank net worth ratio is $4 * (1 - \sigma) = 24\%$. As in Gertler and Karadi (2013), we choose

5. The average target level for the federal funds rate in the longer run is 2.76% in the FOMC's dot plot in Sept 2023.
6. See the Budget and Economic Outlook (2023-2033) published by the CBO.

the values of θ, ψ, and ω such that the steady-state bank leverage is 6 and the annualized excess returns on firm equity and long-term government bonds over the short (deposit) rate are 4% and 2% . The bank leverage of 6 is in the middle range of the literature.[7]

The two reference shares η_B and η_S affect the steady-state asset holdings by the households, which in turn affect the asset holdings by the banks given fixed supply. Hence, we calibrate η_B and η_S to match the asset structure of the banking sector. The ratio of bank holdings of Treasury securities (all maturities) over their assets is targeted at 22%, roughly matching the latest data of 2021Q2 we obtained from the US bank-level call reports.[8] We set η_S so that 50% of all firm equity is intermediated by the banking sector, which is consistent with the calibration chosen by Gertler and Karadi (2013) and Gertler, Kiyotaki, and Prestipino (2020b).

Because the management cost parameter κ affects the long-term bond yield, we choose $\kappa = 0.3$ to match the responsiveness of long-term bond yield to monetary policy shocks. Nakamura and Steinsson (2018) show that a monetary policy shock that raises the short-term nominal interest rate by 67 basis points (bps) would increase the 5-year interest rate by 73 bps. Our choice of κ roughly matches this target.

3.2 Amplification Effects

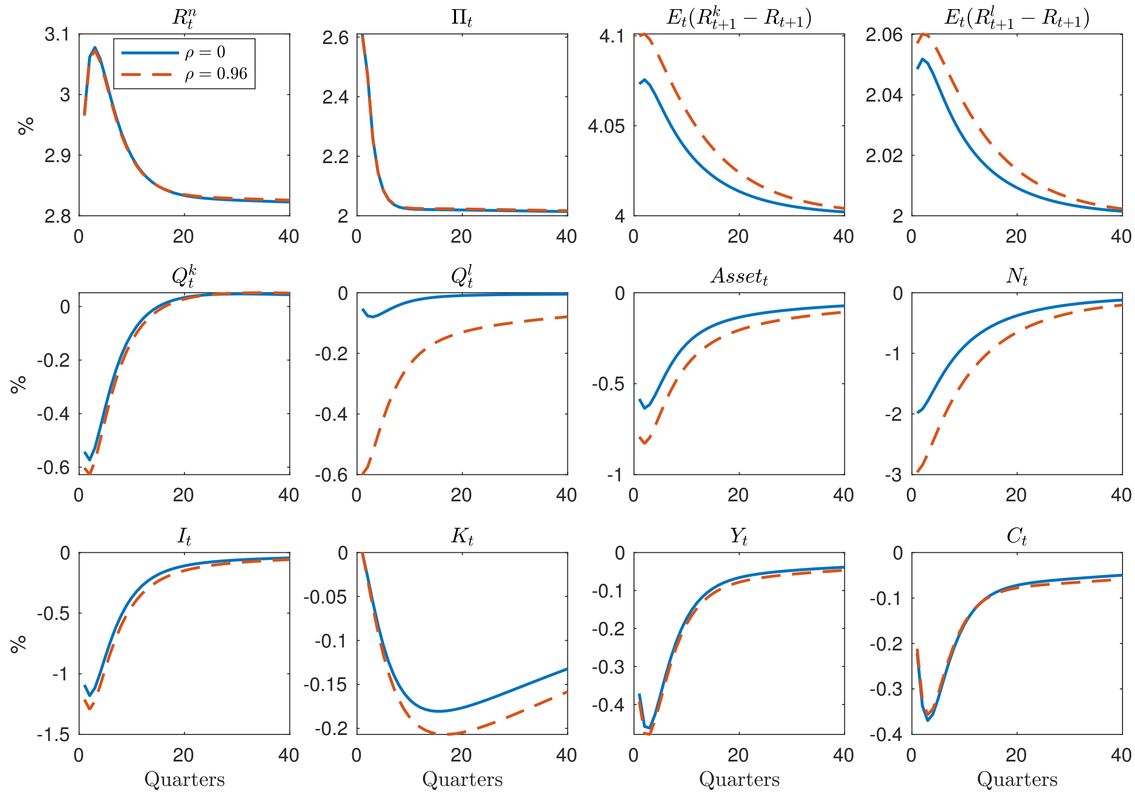
To understand our model mechanism, we shut down the macroprudential policy by setting $\tau_t^l = 0$ for all t and study impulse responses of the economy to contractionary shocks to R_t^n, A_t, and μ_t. We illustrate how bank holdings of long-term bonds can amplify contractionary shocks, compared to holdings of short-term bonds.

Figure 2 plots the impulse response functions of the model economy to a positive one-standard-deviation cost-push shock. It compares the case of long-term bonds ($\rho = 0.96$) with that of short-term bonds ($\rho = 0$). Figure 2 shows that the positive cost-push shock pushes up inflation, leading the central bank to raise interest rates according to the monetary policy rule (19). The rise of interest rates causes the price of government bonds Q^l to decline. This causes bank assets to decline and the balance sheet to shrink. Both the conditional expected excess returns on capital $\mathbb{E}_t\left[R_{t+1}^k - R_{t+1}\right]$ and on long-term bonds $\mathbb{E}_t\left[R_{t+1}^l - R_{t+1}\right]$ over the short-term interest rate rise

7. Aoki, Benigno, and Kiyotaki (2016), Gertler and Karadi (2011), and Gertler, Kiyotaki, and Queralto (2012) target at 4. Gertler and Karadi (2013) target at 6. Gertler, Kiyotaki, and Prestipino (2020a) and Gertler, Kiyotaki, and Prestipino (2020b) target at 10. See Gertler and Karadi (2011) and Gertler and Karadi (2013) for the empirical evidence for the leverage ratios of investment banks and commercial banks in the US.

8. We compute this ratio in the data as the average of the ratios of bank Treasury holdings (all maturities) over the sum of Treasury holdings (all maturities) and their total loans and leases, weighted by the bank assets.

Figure 2: Long-term bonds amplify cost-push shocks



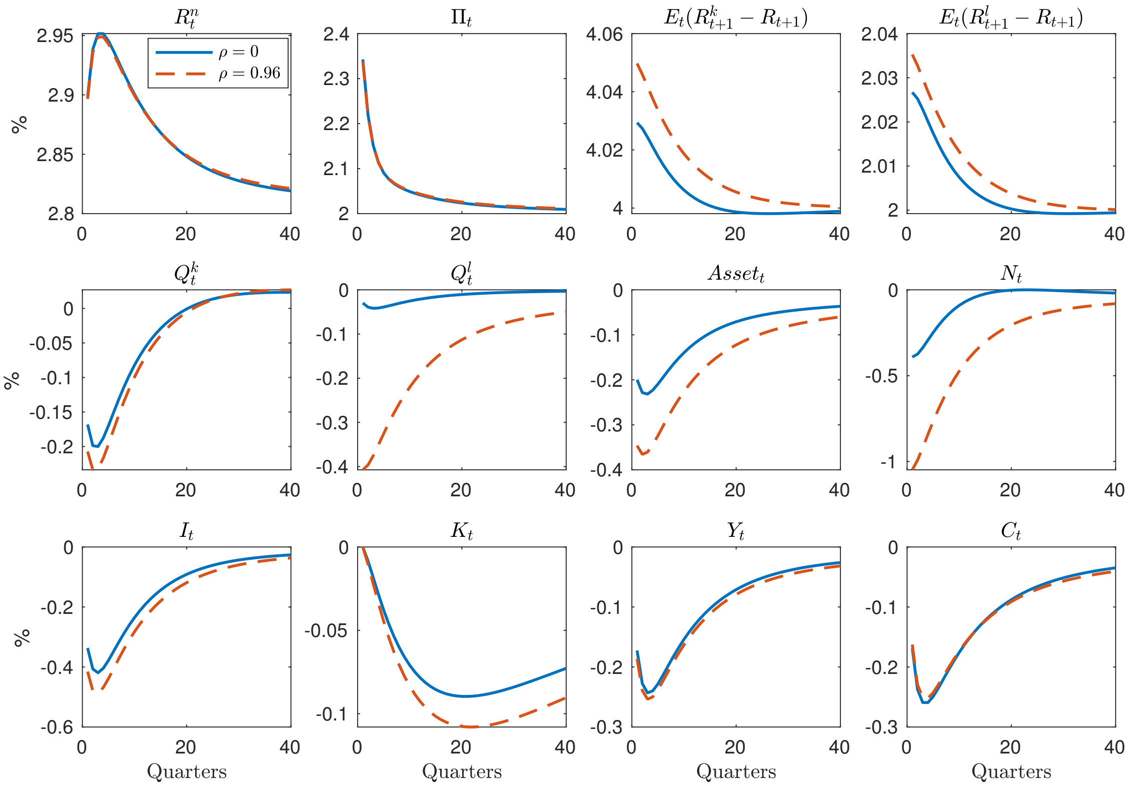
Note: This figure plots the impulse responses of selected variables to a one-standard deviation positive cost-push shock for $\rho = 0$ and $\rho = 0.96$. The total bank assets are denoted by $Asset_t = Q_t^k S_{bt} + Q_t^l B_{bt}$. Nominal interest rate R_t^n, inflation Π_t, excess return on capital $\mathbb{E}_t(R_{t+1}^k - R_{t+1})$, and excess return on long-term bond $\mathbb{E}_t(R_{t+1}^l - R_{t+1})$ are in annualized percentage points. The rest of variables are in percentage deviation from steady states.

sharply on impact. As the cost of capital rises, investment, output, and consumption all decline. As the cost-push shock is quite persistent ($\rho_\mu = 0.79$), inflation is above the target 2% for a long time. It also takes a long time for the economy to recover from a recession because the bank has to re-accumulate net worth slowly.

The key observation is that the decline of the long-term bond price is much larger than that of the short-term bond price. The larger decline in the bond price generates a larger loss to the bank assets, causing lower bank net worth and a smaller bank balance sheet. As a result, investment and capital decline more than in the case of short-term bonds. The same amplification effect applies to a negative technology shock as illustrated in Figure 3. We will not provide a discussion as it is similar to that for Figure 2.

Figure (4) shows the impulse responses to a 25-bp positive interest rate shock to the Taylor rule (19). A rise in the interest rate reduces aggregate demand and hence reduces inflation, in

Figure 3: Long-term bonds amplify technology shocks



Note: This figure plots the impulse responses of selected variables to a one-standard deviation negative technology shock for $\rho = 0$ and $\rho = 0.96$. The total bank assets are denoted by $Asset_t = Q^k_t S_{bt} + Q^l_t B_{bt}$. Nominal interest rate R^n_t, inflation Π_t, excess return on capital $\mathbb{E}_t(R^k_{t+1} - R_{t+1})$, and excess return on long-term bond $\mathbb{E}_t(R^l_{t+1} - R_{t+1})$ are in annualized percentage points. The rest of variables are in percentage deviation from steady states.

contrast to the previous cases of a positive cost-push shock and a negative technology shock. But the contractionary responses for other variables in Figure 4 are qualitatively the same as before. Thus we omit a detailed discussion here.

To understand the intuition behind why long-term bonds generate larger responses to interest rate changes, we use the bank's first-order condition to derive

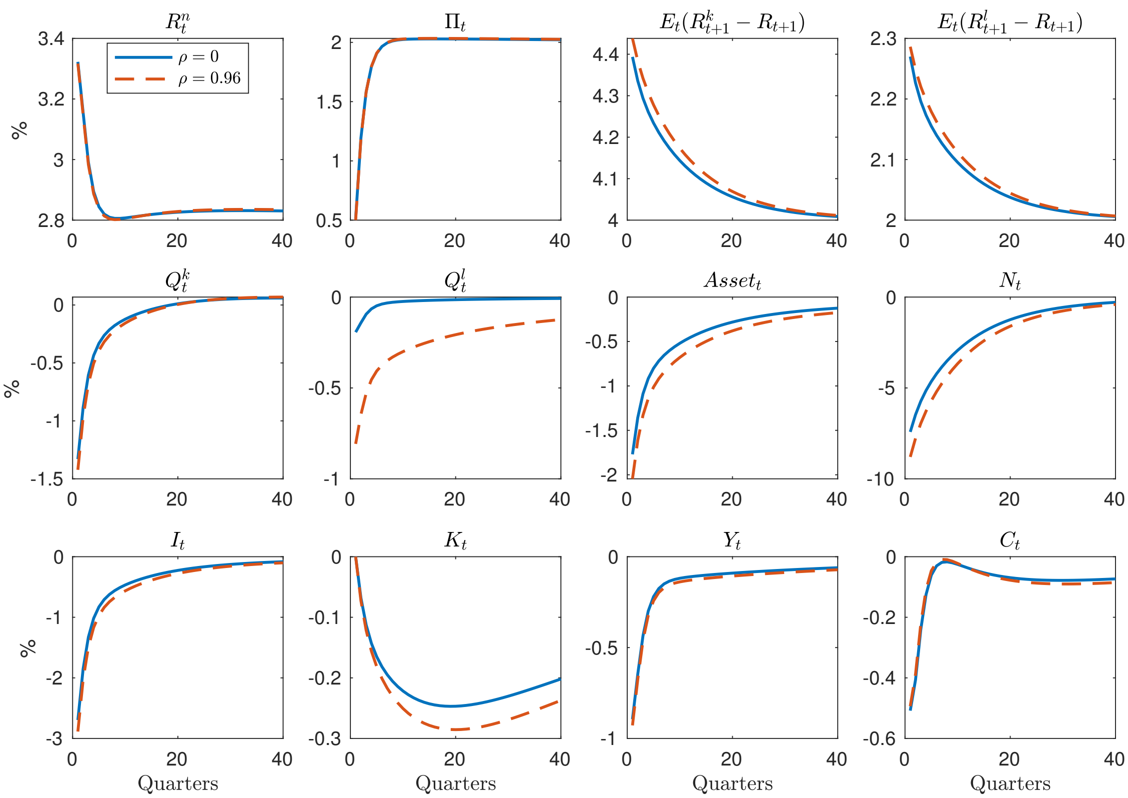
$$Q^l_t = \mathbb{E}_t \tilde{\Omega}_{t,t+1}(1 + \rho Q^l_{t+1}),$$

where $\tilde{\Omega}_{t,t+1}$ is the adjusted SDF for banks. The derivation is in Appendix A. We can rewrite the above pricing equation as

$$Q^l_t = \sum_{j=1}^{\infty} \rho^{j-1} \tilde{\Omega}_{t,t+j}. \tag{21}$$

Due to the financial friction of the banking sector, a contractionary shock worsens the bank's net

Figure 4: Long-term bonds amplify monetary policy shocks



Note: This figure plots the impulse responses of selected variables to a 25-bp positive monetary policy shock for $\rho = 0$ and $\rho = 0.96$. The total bank assets are denoted by $Asset_t = Q_t^k S_{bt} + Q_t^l B_{bt}$. Nominal interest rate R_t^n, inflation Π_t, excess return on capital $\mathbb{E}_t(R_{t+1}^k - R_{t+1})$, and excess return on long-term bond $\mathbb{E}_t(R_{t+1}^l - R_{t+1})$ are in annualized percentage points. The rest of variables are in percentage deviation from steady states.

worth and reduces the bank's SDF $\tilde{\Omega}_{t,t+1}$ below its steady-state level for a considerable period of time. From (21), we see that the longer the maturity (the larger ρ), the more reduction in the bank's future SDF is priced in and the lower the current bond price.

Moreover, there is feedback between the price of long-term bonds and the bank net worth. A worsening of bank net worth reduces the bank's SDF, reducing the bond price especially when the maturity is long. A low bond price in turn deteriorates the bank's net worth.

An alternative way to understand the intuition is to use the households' pricing equations (4) and (5) and the definition (1). For simplicity, we consider the deterministic case and derive

$$Q_t^l = \sum_{j=0}^{\infty} \frac{\rho^j}{R_t^n z_t \cdots R_{t+j}^n z_{t+j}}, \tag{22}$$

where we define

$$z_{t+j} \equiv 1 + \kappa \left(\frac{B_{h,t+j}}{B_{t+j}} - \eta_B \right). \tag{23}$$

It follows that, for a larger ρ, a change in the nominal interest rate R_t^n has a larger long-lasting effect on the bond price Q_t^l. In particular, for the one-period bond with $\rho = 0$, the impact lasts only one period as $Q_t^l = 1/(R_t^n z_t)$. Thus the price of longer-term bonds is more sensitive to interest rate changes. Moreover, when the household share of long-term government bonds $\left(B_{h,t+j}/B_{t+j}\right)$ increases when interest rates rise, the bond price Q_t^l declines even more.

4 Bank Runs

In the previous section, we showed that contractionary shocks reduce bank net worth, making the banking sector prone to bank runs. The impact on the net worth is larger if banks hold longer-term assets. As a result, we expect that a bank run is more likely to occur for banks holding longer-term assets. In this section, we adopt the approach of Gertler and Kiyotaki (2015) and extend our baseline model to incorporate bank runs.

For simplicity, we only study the possibility of an unexpected bank run. In particular, we assume that when households make deposits in period $t-1$ that mature in period t, they attach zero probability to the possibility of a run in period t. However, we allow for the chance of a run ex post as deposits mature in period t and households must decide whether to roll them over for another period.

Two types of equilibria may coexist. First, in a normal no-run equilibrium, households roll over their deposits in banks and bank net worth is positive (i.e., $n_t > 0$). Bank incentive constraints are also satisfied. We have studied such an equilibrium in Section 2. Second, in a bank-run equilibrium, each household decides not to roll over its deposits and perceives that other households will do the same, forcing banks into liquidation and this forced liquidation makes the banks insolvent (i.e., $n_t = 0$). Then households use their funds to acquire bank assets directly.

Since all banks in our model are identical, the conditions for a run on the banking system will be the same for the depositors at each individual bank. As a result, runs in our model are on the entire banking system, not on individual banks. We next derive the conditions for a bank run.

4.1 Bank Run Conditions

At the beginning of period t, after the realization of shocks, depositors decide whether to roll over their deposits with the bank. If they choose to "run", the bank liquidates its assets (capital and government bonds) and turns the proceeds over to households. Let Z_t^* be the firm profits, P_t^* the aggregate price level, and Q_t^{k*} and Q_t^{l*} the liquidation prices of capital and bonds in the event of a forced liquidation of the banking system. Then a run on the system is possible if the nominal liquidation value of bank assets

$$P_t^* \left(Z_t^* + (1-\delta) Q_t^{k*} \right) S_{b,t-1} + \left(1 + \rho Q_t^{l*} \right) P_{t-1} B_{b,t-1}$$

is smaller than its outstanding nominal liability to the depositors, $R_{t-1}^n P_{t-1} D_{t-1}$, in which case the bank's net worth would be wiped out.

Define the recovery rate in the event of a bank run x_t as

$$x_t = \frac{P_t^* \left(Z_t^* + (1-\delta) Q_t^{k*} \right) S_{b,t-1} + \left(1 + \rho Q_t^{l*} \right) P_{t-1} B_{b,t-1}}{R_{t-1}^n P_{t-1} D_{t-1}},$$

which can be rewritten as

$$x_t = \frac{R_t^{k*} Q_{t-1}^k S_{b,t-1} + R_t^{l*} Q_{t-1}^l B_{b,t-1}}{R_t^* D_{t-1}},$$

where we define

$$R_t^{k*} = \frac{Z_t^* + (1-\delta) Q_t^{k*}}{Q_{t-1}^k},\ R_t^{l*} = \frac{1 + \rho Q_t^{l*}}{Q_{t-1}^l \Pi_t^*},\ R_t^* = \frac{R_{t-1}^n}{\Pi_t^*},\ \Pi_t^* = \frac{P_t^*}{P_{t-1}}.$$

Then a sufficient condition for a bank run equilibrium is $0 \leq x_t < 1$. If this condition is satisfied, then a bank-run equilibrium and a no-run equilibrium may coexist. Of course, we also need other usual conditions for the existence of a no-run equilibrium.

4.2 Liquidation Prices

To understand a bank run equilibrium, it is important to determine the liquidation prices Q_t^{k*} and Q_t^{l*}. In the event of a bank run in period t, banks assets are liquidated and households acquire them so that $S_{ht} = S_t$ and $B_{ht} = B_t$. As banks are insolvent at t, they have zero net worth, $N_t = 0$. For simplicity we assume that bank runs happen only once in some period t if possible.

The banking system then rebuilds itself over time as new banks enter. Following Gertler and

Kiyotaki (2015), we assume that new banks cannot begin operating until the period after the panic run. The banking system rebuilds its equity and assets as new banks enter at $t+1$ onwards. We also assume that bankers receive startup funds N^{new} from households in period $t+1$, $N_{t+1} = N^{new}$. From $t+2$ on, bank net worth follows the dynamics in equation (11).

We now use (1), (18), and the household Euler equations (5)-(6) to derive the liquidation prices. In the event of a bank run at t, we have

$$1 + \kappa\left(1 - \eta_B\right) = \mathbb{E}_t \Lambda_{t,t+1} \frac{1 + \rho Q^l_{t+1}}{Q^{l*}_t \Pi_{t+1}},$$
$$1 + \kappa\left(1 - \eta_S\right) = \mathbb{E}_t \Lambda_{t,t+1} \frac{Z_{t+1} + \left(1 - \delta\right) Q^k_{t+1}}{Q^{k*}_t}.$$

From $t+1$ on, we have the usual Euler equations to determine the asset prices at normal times Q^k_{t+j} and Q^l_{t+j} for $j \geq 1$. As $0 < B_{ht} < B_t$ and $0 < S_{ht} < S_t$ at normal times, the liquidation prices Q^{k*}_t and Q^{l*}_t of assets fall below their normal levels at t. In Appendix C, we provide the complete system of equations for a bank run equilibrium and describe an algorithm to compute such an equilibrium.

4.3 Simulations

In this subsection, we use simulations to study how cost-push shocks and the associated interest rate hikes can generate a bank run. We compute global nonlinear perfect foresight solutions, assuming that μ_t and v^m_t follow deterministic dynamics after the adverse shock. We then check the condition $0 \leq x_t < 1$. If this condition is satisfied, then multiple equilibria may emerge, i.e., a bank run equilibrium may coexist with a no-run equilibrium. In this case, we allow for a sunspot which can shift the economy from the no-run to the run equilibrium.

To calculate the lead-up to the bank run, we compute the perfect foresight path up to the point where the run occurs. For simplicity, we also assume that a bank run can occur only in one period. After the run, we then compute a new perfect foresight path back to the no-run equilibrium steady state, given the values of the state variables in the wake of the run. For multiple equilibria to exist, we also need to check the Blanchard-Khan conditions such that the no-run equilibrium steady state is a saddle point. In the exercises here, we only consider an unanticipated bank run by assuming that individuals perceive zero probability of a run. We describe the detailed algorithm in Appendix C.

To compute numerical solutions, we need to assign a value to the new net worth N^{new} for the banks to restart one-period later after the bank run in some period t, $N_{t+1} = N^{new}$. We assume that it is a fraction of the bank net worth before the period of the bank run: $N^{new} = \xi N_{t-1}$. We set ξ so that the additional output loss caused by the bank run relative to the case without bank run roughly matches the estimation of Baron, Verner, and Xiong (2021). Using data for 46 countries over the period of 1870 to 2016, they estimate that the additional reduction in the real GDP due to a banking panic is averaged around 2.3% over the three-year period after the panic (Figure III Panel A in their paper). In our model, the additional output loss caused by the bank run is 2.19% averaging over 12 quarters after the run.

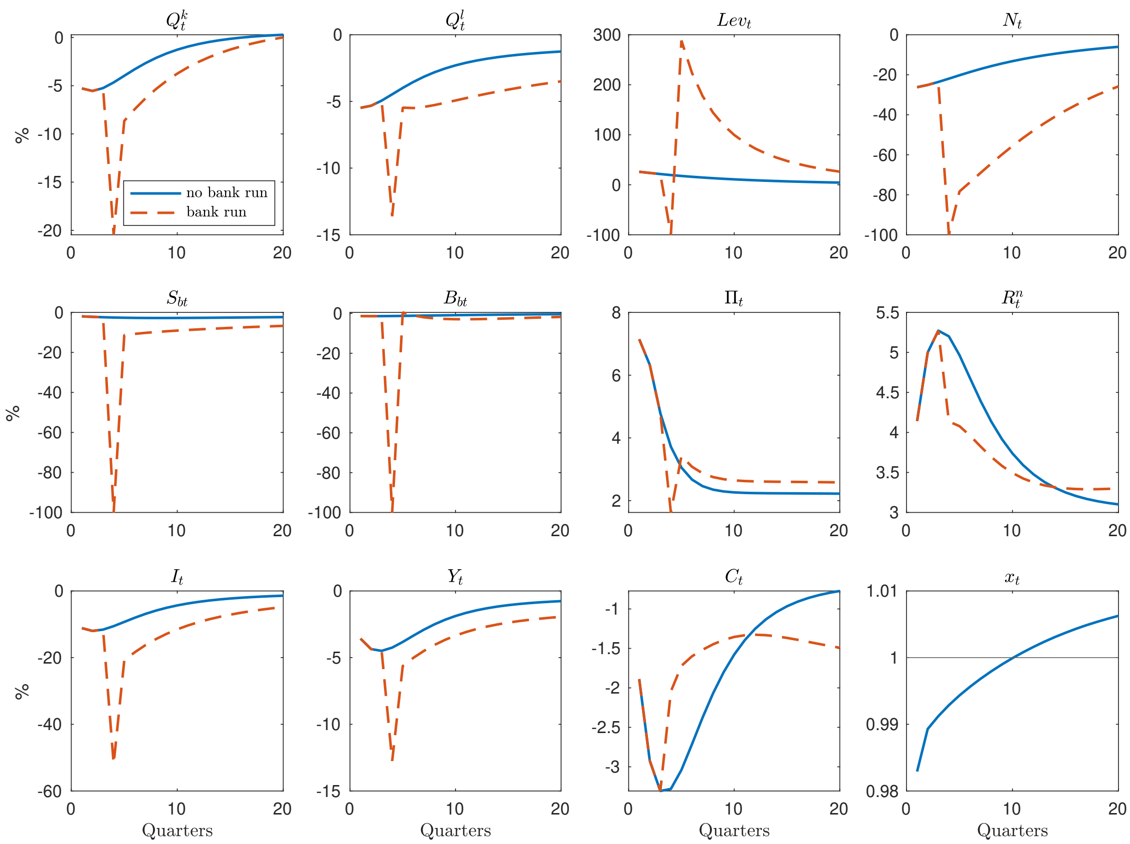
Suppose that the economy in period zero is at the no-run equilibrium steady state. Figure 5 plots the responses of the economy to a one-time positive cost-push shock in period 1 only. We set the size of the shock to 10 standard deviations ($\varepsilon_1^{\mu} = 0.01$) so that the annualized inflation rate rises to 7% on impact, which roughly matches the recent increase in the inflation rate in the US. Following the monetary policy rule (19), the central bank passively responds to the rise of inflation by raising the annual interest rate to 4.2%, which leads to a decline in the asset prices and bank net worth. The impaired bank net worth makes bank run possible, as indicated by the recovery rate x_t less than one. As shown in the last panel of Figure 5, $x_t < 1$ for $1 \leq t \leq 10$. Thus a bank run can happen in any period between periods 1 and 10.

We simply assume that a bank run occurs ex post in period 4. During the bank run, banks liquidate all their assets and their net worth is zero. The capital price falls by 20% and the price of long-term bonds falls by 14% in period 4. The bank run leads to a sharp decline in investment, causing persistent low levels of capital and output. Starting from period 5 on, banks rebuild their net worth and the economy gradually moves to the no-run equilibrium steady state.

Next, we assume that the central bank actively raises interest rates to control high inflation. Figure 6 plots the responses of the economy to the same cost-push shock plus a sequence of 2 consecutive 25-bp monetary policy shocks. This figure shows that inflation rises to 4.8% on impact, lower than 7% in Figure 5. Inflation drops to 1% during the bank run in period 4 and then rises to 3.2%. It stays above the inflation rate in the no-run equilibrium for a long time and slowly transition to the target rate of 2%.

The nominal interest rate rises to 4.3% on impact and continues to rise all the way to 5.8% in period 4 and then starts to decline gradually to its steady state level. This more aggressive monetary tightening generates a similar recession measured by investment, output, and consumption

Figure 5: Ex post bank run after a cost-push shock

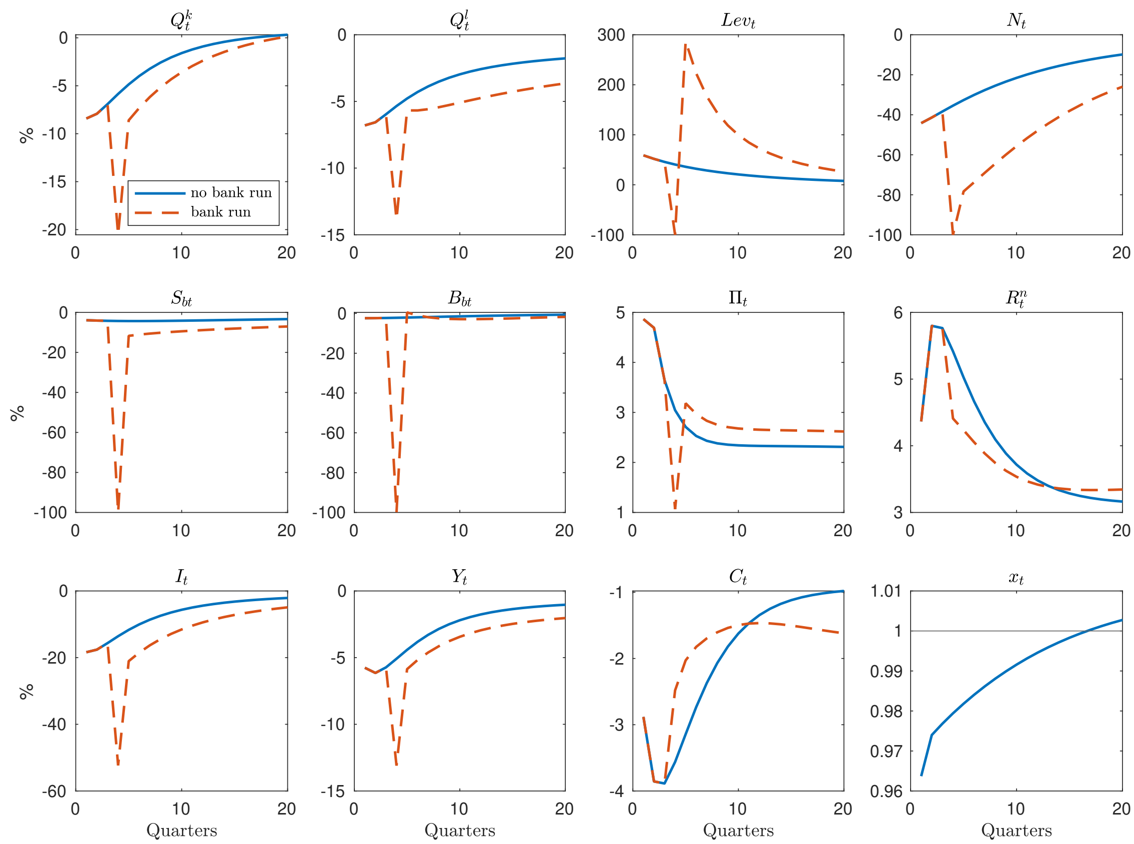


Note: This figure plots compares the transition paths with and without a bank run. A cost-push shock hits the economy in period 1 ($\varepsilon_1^{\mu} = 0.01$). The bank run occurs in period 4. The bank leverage is defined as $Lev_t = Asset_t/N_t$. Nominal interest rate R_t^n and inflation Π_t are in annualized percentage points. Bank recovery rate x_t is in level. The rest of variables are in percentage deviation from steady states.

to the case without monetary policy shocks. However, the larger interest rate hike significantly reduces the asset prices and bank net worth by more. The capital price, bond price, and bank net worth fall by 8%, 7%, and 45%, respectively, compared to 5%, 6%, and 25% in the case without monetary policy shocks in Figure 5. As a result, the recovery rate is lower and remains less than 1 longer. The last panel of Figure 6 shows that x_t is less than 1 until period 16. Thus bank runs are more likely to occur when there are unexpected interest rate hikes.

What is the role of the maturity structure of bank assets? Figure 7 compares the recovery rates and the impulse responses of selected variables in the no-run equilibrium to the same set of shocks (a 10-standard-deviation cost-push shock followed by 2 consecutive 25-bp monetary policy shocks) under different maturities ρ of government bonds. When all government bonds are one-period with a maturity of 3 months ($\rho = 0$), the decline in the bond price is much smaller. Thus the

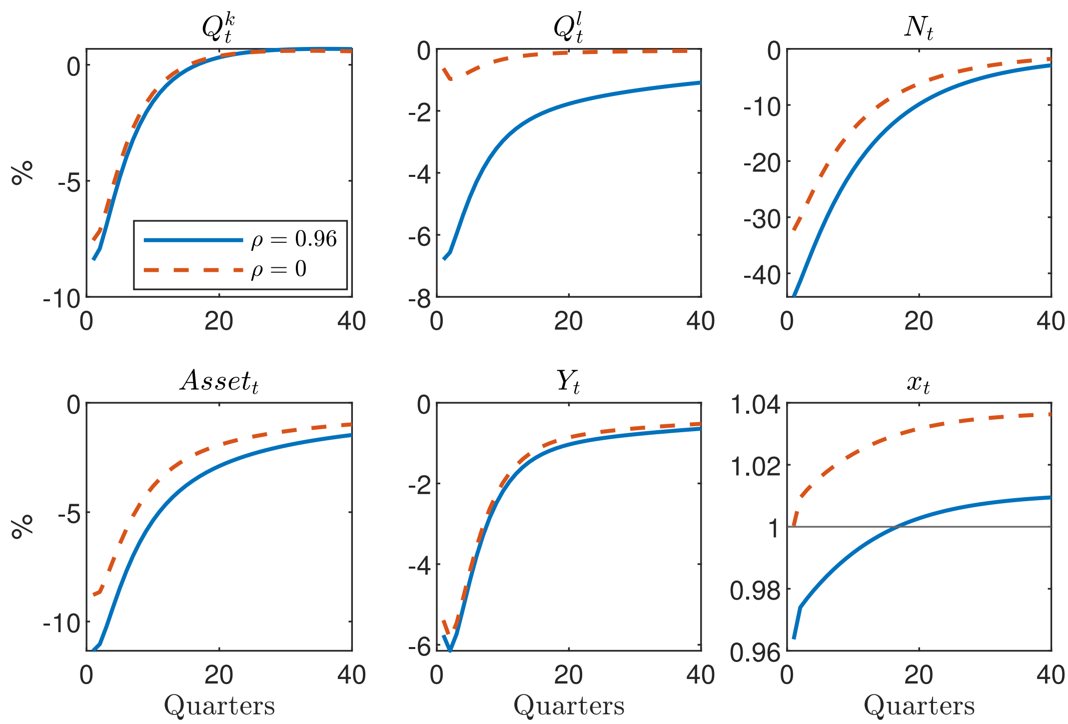
Figure 6: Ex post bank run after cost-push shocks and monetary policy shocks



Note: This figure plots compares the transition paths with and without a bank run. A cost-push shock hits the economy in period 1 ($\varepsilon_1^{\mu} = 0.01$) followed by 2 consecutive 25-bp monetary policy shocks. The bank run occurs in period 4.The bank leverage is defined as $Lev_t = Asset_t / N_t$. Nominal interest rate R_t^n and inflation Π_t are in annualized percentage points. Bank recovery rate x_t is in level. The rest of variables are in percentage deviation from steady states.

losses of bank asset values and net worth are smaller. As a result, a bank run never happens when bonds held by banks are short-term, as the recovery rate x_t stays above 1 for all t. By contrast, in the case of long-term bonds with a maturity of 5 years ($\rho = 0.96$), the decline of long-term bond prices is much larger, although the impact on output is similar in the no-run equilibrium. We find that $x_t < 1$ until period 16, implying that a bank run may occur in any period between $t = 1$ and $t = 16$. When a bank run occurs, the adverse impact on the macroeconomy is significant as we discussed earlier.

Figure 7: Longer maturity increases bank run likelihood



Note: This figure plots the recovery rates and the impulse responses of selected variables in the non-run equilibrium for $\rho = 0$ and $\rho = 0.96$. A cost-push shock hits the economy in period 1 ($\varepsilon_1^{\mu} = 0.01$) followed by 2 consecutive 25-bp monetary policy shocks. Total bank assets are denoted by $Asset_t = Q_t^k S_{bt} + Q_t^l B_{bt}$. Bank recovery rate x_t is in level. The rest of variables are in percentage deviation from steady states.

5 Macroprudential Policies

When the economy is vulnerable to shocks, especially to shocks that generate interest rate hikes, policies may be useful to stabilize the economy. It is critical to stabilize banks' holdings of long-term assets because their values are sensitive to changes of interest rates, causing bank balance sheets to be sensitive too. Weakened bank balance sheets can cause a recession or bank runs. In this section, we analyze two types of macroprudential policies introduced in Section 2.

5.1 Permanent Policy

Suppose that the government imposes a constant tax $\tau_t^l = \tau^l$ for all t on bank holdings of long-term government bonds and transfers the tax revenue to households. We hold the steady-state debt/GDP ratio $Q^l B/Y = 4$ fixed when we change τ^l. There are two major effects. First, there is a steady-state effect in the sense that the permanent tax changes the steady state of the economy.

The tax lowers a bank's long-run holdings of long-term bonds and hence lowers its net worth. As a result, it also lowers capital intermediated by the banking system.

Second, the tax on long-term bonds has a dynamic effect in the sense that it changes the dynamic responses of the economy to shocks. Figure 8 illustrates this point for $\tau^l = 0$ and $\tau^l = 0.07$ and for a cost-push shock ($\varepsilon_1^\mu = 0.01$) followed by two consecutive unexpected interest rate hikes ($\varepsilon_1^m = \varepsilon_2^m = 0.0025$). The top-left panel plots the paths of the bond price in the no-run equilibrium. The top-right panel plots the bond price in the bank-run equilibrium when a bank run happens in period 4.

The tax on bank holdings of long-term bonds causes these bonds to shift from banks to households. In particular, the steady-state share of long-term bonds held by households (B_h/B) increases from 70% to 90% when the tax rate τ^l increases from 0 to 0.07. In response to interest rate hikes, long-terms bonds flow from banks to households, but the percentage increase in the household share of long-term bonds relative to its larger steady-state base for $\tau^l = 0.07$ is smaller than that for $\tau^l = 0$, as shown in the bottom-left panel in Figure 8. It follows from the household pricing equation (5) or (22)-(23) that the decline in the price of long-term bonds for $\tau^l = 0.07$ is also smaller.

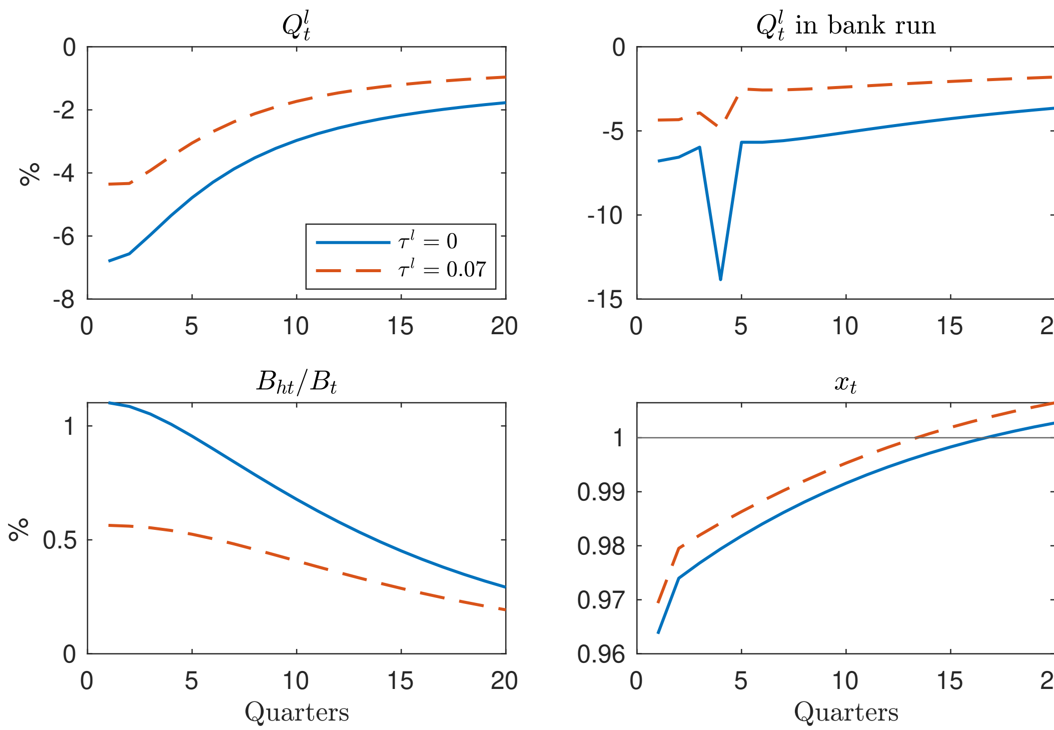
More importantly, since banks have fewer government bonds to liquidate during the bank run in period 4, the liquidation price of long-term bonds falls only by 5% when $\tau^l = 0.07$ instead of 14% when $\tau^l = 0$. As a result, the recovery rate is higher and becomes greater than 1 sooner. In particular, when $\tau^l = 0.07$, a bank run is possible ($x_t < 1$) in any period until $t = 13$ and is not possible $(x_t > 1)$ for $t > 13$. But when $\tau^l = 0$, a bank run is possible ($x_t < 1$) in any period until $t = 16$ and is not possible $(x_t > 1)$ for $t > 16$.

Next we study the welfare implications. Let V and V^* denote the equilibrium life-time utilities before and after a macroprudential policy conditional on realized shocks. Let Θ denote the consumption gain from the policy. Then given (3) we can compute

$$\Theta = \exp\left[\left(V^* - V\right)\left(1 - \beta\right)\right] - 1.$$

We also study the impact on the likelihood of a potential bank run, though we only focus on unanticipated bank run in this paper. For an anticipated bank run studied in Gertler and Kiyotaki (2015), let p_t denote the probability that households assign at t to a bank run happening at $t+1$. Following Gertler and Kiyotaki (2015), we assume that p_t is related to the economic fundamental,

Figure 8: Constant tax $\tau_t^l = \tau^l$ mitigates bond price decline



Note: This figure plots the impulse responses of selected variables under different levels of constant tax on government bonds. A cost-push shock hits the economy at period 1 ($\varepsilon_1^u = 0.01$) followed by 2 consecutive 25-bp monetary policy shocks ($\varepsilon_1^m = \varepsilon_2^m = 0.0025$). The recovery rate x_t is in level. The rest variables are in percentage deviation from steady states.

the aggregate recovery rate x_t, in the following simple form:

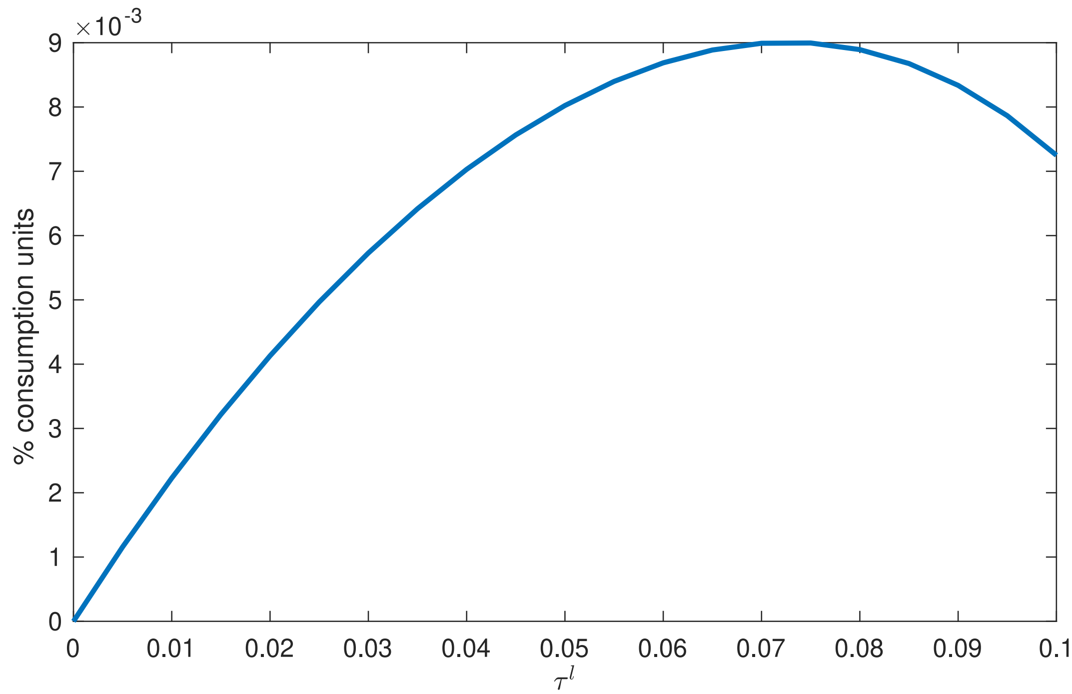
$$p_t = \max\{1 - \mathbb{E}_t x_{t+1}, 0\}.$$

Assume that the events of a bank run happen independently over time. Then the probability of bank runs happening in any period over an infinite horizon is given by

$$1 - \prod_{i=0}^{\infty}(1 - p_{t+i}).$$

Figure 9 plots the welfare gain Θ for various levels of the tax rate τ^l under the no-run equilibrium conditional on the shocks discussed above. It shows that the constant tax improves welfare when the tax rate is small. This is because the constant tax reduces the bank's exposure to long-term bonds in the steady state. As a result, the bank is less affected by the decline in the bond price upon contractionary shocks. On the other hand, the constant tax reduces the welfare when the tax rate is too high. This is because a higher constant bond tax also reduces the steady-state bank net worth and hence the size of the bank balance sheet and capital intermediated by the bank. We find that there is an optimal tax rate given by $\tau^l = 0.07$. At this tax rate, the welfare gain is 0.009% under the no-run equilibrium and the bank run probability is reduced by 4.9%. The welfare gain is very small because we only consider conditional gains given a sequence of small shocks.

Figure 9: Welfare gain under constant tax



Note: This figure plots the welfare improvements under different levels of constant tax τ^l. Welfare is measured in consumption units. Welfare is calculated conditional on the cost-push shock in period 1 ($\varepsilon_1^{\mu} = 0.01$) and 2 consecutive 25-bp monetary policy shocks in period 1 and 2.

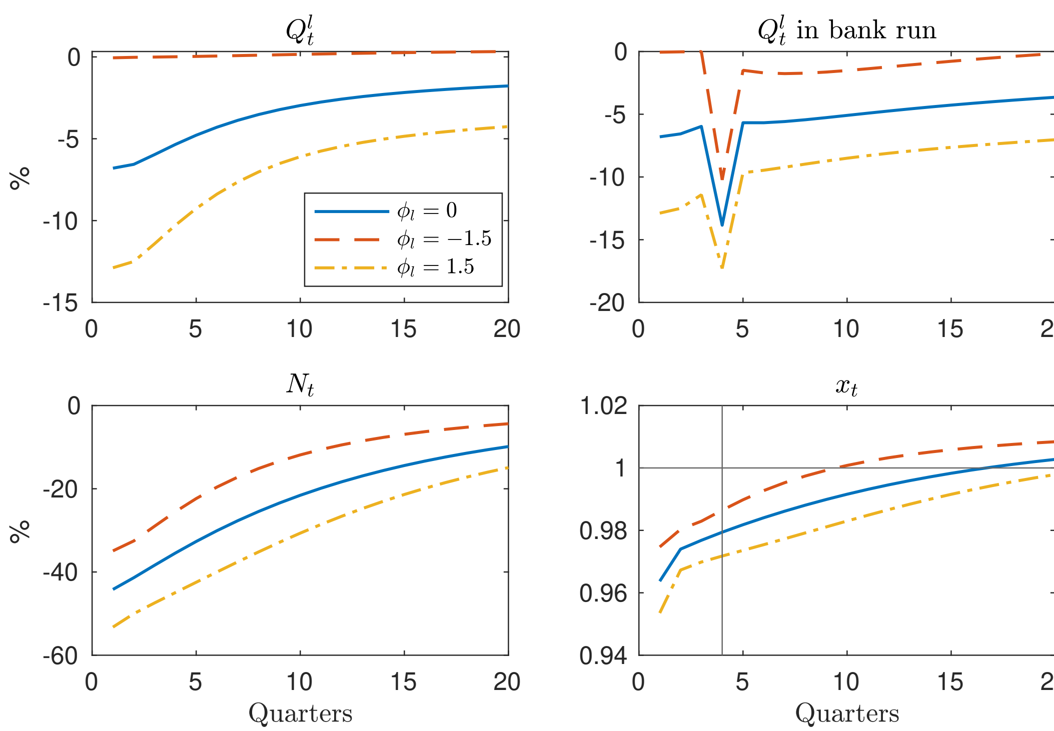
As the permanent bond tax policy can reduce the bank run probability, it is possible that a bank run occurs without the bond tax ($\tau^l = 0$), but it never occurs with the tax ($\tau^l = 0.07$). This is indeed the case at $t = 15$, because $x_{15} < 1$ when $\tau^l = 0$ but $x_{15} > 1$ when $\tau^l = 0.07$ as shown in Figure 8. For a simple illustration, we suppose that a bank run never happens in any other periods in both cases with and without the bond tax. We then compute that the welfare gain with the bond tax that preventing a bank run is 0.068% in terms of the consumption equivalent. Thus preventing a bank run raises the welfare gain by about 6 times more than that (0.009%) for the no-run case.

5.2 Cyclical Tax

Since banks are vulnerable to interest rate hikes, we consider a cyclical tax policy that responds to interest rate changes as in (20). Figure 10 studies the effects of this type of policy for $\phi_l = -1.5, 0, 1.5$. For $\phi_l < 0$, an increase (decrease) in the interest rate causes the government to subsidize (tax) bank holdings of long-term bonds. An opposite interpretation applies to $\phi_l > 0$.

Figure 10 shows that the policy of $\phi_l = -1.5$ can mitigate the adverse impact on the economy in response to the cost-push shock and interest rate hikes. Intuitively, as the interest rate rises in response to the shocks, subsidizing bank holdings of long-term bonds can reduce the banks' loss of long-term bonds. Thus it can mitigate the decline of bank net worth and asset prices in the no-run equilibrium as shown in the left two panels of Figure 10. Moreover, the liquidation price also declines by less in a bank run as shown in the top-right panel. The recovery rate x_t for $\phi_l = -1.5$ is higher along the transition path and returns to the steady state sooner than that without the policy ($\phi_l = 0$). A bank run for $\phi_l = -1.5$ is not possible ($x_t > 1$) after period 9. But a bank run for $\phi_l = 0$ is not possible ($x_t > 1$) after period 16.

Figure 10: Cyclical tax $\tau_t^l = \phi_l(R_t^n - R^n)$



Note: This figure plots the impulse responses of selected variables under different cyclical tax on government bonds. A cost-push shock hits the economy in period 1 ($\varepsilon_1^{\mu} = 0.01$) followed by 2 consecutive 25-bp monetary policy shocks ($\varepsilon_1^m = \varepsilon_2^m = 0.0025$). The recovery rate x_t is in level. The rest variables are in percentage deviation from steady states.

Unlike the permanent tax policy, the cyclical tax policy does not change the steady state. The cyclical tax policy with $\phi_l < 0$ is like an automatic stabilizer. To find an optimal policy, we search for ϕ_l in the interval $[-2, 0]$. We do not consider positive values because they amplify shocks to the economy and reduce welfare as shown in Figure 10. We find that the optimal value is at the corner ($\phi_l = -2$). To have an interior solution, we need to add costs of the cyclical policy. For example, we may assume that the government raises distortionary taxes on household incomes to subsidize bank bond holdings. Such an analysis is beyond the scope of this paper.

Table 2: Run probability under combinations of cyclical taxes and monetary policies

		Macroprudential policy		
		$\phi_l = -1$	$\phi_l = -1.5$	$\phi_l = -2$
Monetary policy	$\phi_\pi = 1.5$	-10.64%	-12.95%	-14.37%
	$\phi_\pi = 1.98$	-6.12%	-8.39%	-10.08%
	$\phi_\pi = 2.2$	-5.21%	-7.32%	-8.93%

Note: This table lists the bank run probability reduction for various macroprudential and monetary policy combinations relative to the baseline case ($\phi_\pi = 1.98, \phi_l = 0$). The bank run probability is conditional on a cost-push shock in period 1 ($\varepsilon_1^\mu = 0.01$) followed by 2 consecutive 25-bp monetary policy shocks ($\varepsilon_1^m = \varepsilon_2^m = 0.0025$).

We finally study the interaction between macroprudential and monetary policies. Table 2 presents the reduction in the bank run probability under different combinations of monetary policy and macroprudential policy rules relative to the baseline case ($\phi_\pi = 1.98, \phi_l = 0$). This table shows that when monetary policy is more dovish (a smaller ϕ_π), bank run probabilities are smaller for a given macroprudential policy rule. On the other hand, given a monetary policy rule, a more aggressive macroprudential policy rule (a larger $|\phi_l|$) to stabilize long-term bond prices is more effective to reduce bank-run probabilities.

6 Conclusion

In response to high inflation, central banks often raise interest rates aggressively. As values of long-term securities are sensitive to interest rate changes, bank holdings of long-term securities can amplify the impact of interest rate hikes on the macroeconomy. In this paper we have incorporated a banking sector in a DNK model to study the role of bank holdings of long-term government bonds. We show that shocks that trigger high inflation and the associated interest rate hikes can cause a recession and bank runs. We also introduce two types of macroprudential policies that can mitigate or prevent a banking crisis.

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Appendix

A Banker's Optimization Problem

In this appendix, we derive the first-order conditions from a representative banker's optimization problem. The banker chooses deposits d_t, the quantity of financial claims on nonfinancial firms s_t, and the quantity of long-term government bonds b_t to solve the following dynamic programming problem:

$$V_t = \max_{\{d_t, s_t, b_t\}} \mathbb{E}_t \Lambda_{t,t+1} \left[(1-\sigma) n_{t+1} + \sigma V_{t+1} \right],$$

subject to the following constraints

$$n_t + d_t = Q_t^k s_t \left(1 + \frac{\psi}{2} \frac{Q_t^k s_t}{n_t} \right) + Q_t^l b_t (1 + \tau_t^l), \tag{A.1}$$

$$n_{t+1} = R_{t+1}^k Q_t^k s_t + R_{t+1}^l Q_t^l b_t - R_{t+1} d_t, \tag{A.2}$$

$$V_t \geq \theta \left(Q_t^k s_t + Q_t^l b_t \right). \tag{A.3}$$

Conjecture that the bank value is given by

$$V_t = \phi_t n_t,$$

where ϕ_t is a coefficient to be determined. Substituting this conjectured value function into the above optimization problem, using (A.2) to eliminate n_{t+1}, and using (A.1) to eliminate d_t, we can rewrite the banker's problem as

$$\begin{aligned} &\max_{\{s_t, b_t\}} \mathbb{E}_t \Omega_{t+1} \left[(R_{t+1}^k - R_{t+1}(1 + \frac{\psi}{2} \frac{Q_t^k s_t}{n_t})) Q_t^k s_t + (R_{t+1}^l - R_{t+1}(1 + \tau_t^l)) Q_t^l b_t + R_{t+1} n_t \right] \\ &\quad s.t. \quad \phi_t n_t \geq \theta \left(Q_t^k s_t + Q_t^l b_t \right) \end{aligned}$$

where $\Omega_{t+1} = \Lambda_{t,t+1}(1 - \sigma + \sigma \phi_{t+1})$.

Denote by $\lambda_t \geq 0$ the Lagrangian multiplier associated with the bank's incentive constraint. The Lagrangian can be written as:

$$\begin{aligned} \mathcal{L} = &\mathbb{E}_t \Omega_{t+1} \left[(R_{t+1}^k - R_{t+1}(1 + \frac{\psi}{2} \frac{Q_t^k s_t}{n_t})) Q_t^k s_t + (R_{t+1}^l - R_{t+1}(1 + \tau_t^l)) Q_t^l b_t + R_{t+1} n_t \right] \\ &+ \lambda_t \left(\phi_t n_t - \theta Q_t^k s_t - \theta Q_t^l b_t \right). \end{aligned}$$

Then the first-order conditions for s_t and b_t are given by

$$\mathbb{E}_t\Omega_{t+1}\left(R^k_{t+1} - R_{t+1}\left(1+\psi\frac{Q^k_t s_t}{n_t}\right)\right) = \lambda_t\theta, \tag{A.4}$$

$$\mathbb{E}_t\Omega_{t+1}\left(R^l_{t+1} - R_{t+1}(1+\tau^l_t)\right) = \lambda_t\theta. \tag{A.5}$$

Eliminating λ_t yields a no-arbitrage condition

$$\mathbb{E}_t\Omega_{t+1}\left(R^k_{t+1} - R_{t+1}\left(1+\psi\frac{Q^k_t s_t}{n_t}\right)\right) = \mathbb{E}_t\Omega_{t+1}\left(R^l_{t+1} - R_{t+1}(1+\tau^l_t)\right).$$

Since $R^k_{t+1}, R^l_{t+1}, R_{t+1}$ and τ^l_t are aggregate variables, we conclude that all banks choose the same ratio of firm equity to net worth $Q^k_t s_t/n_t$.

Using $V_{t+1} = \phi_{t+1}n_{t+1}$, (A.1), (A.2), (A.4), and (A.5), we can derive that the value function satisfies

$$\begin{aligned} V_t &= \lambda_t(\theta Q^k_t s_t + \theta Q^l_t b_t) + \mathbb{E}_t\Omega_{t+1}R_{t+1}n_t + \mathbb{E}_t\Omega_{t+1}R_{t+1}\frac{\psi}{2}\left(\frac{Q^k_t s_t}{n_t}\right)^2 n_t \\ &= \lambda_t n_t \phi_t + \mathbb{E}_t\Omega_{t+1}R_{t+1}n_t + \mathbb{E}_t\Omega_{t+1}R_{t+1}\frac{\psi}{2}\left(\frac{Q^k_t s_t}{n_t}\right)^2 n_t \end{aligned}$$

where the second equality follows from the complementary slackness condition

$$\lambda_t\left(\phi_t n_t - \theta Q^k_t s_t - \theta Q^l_t b_t\right) = 0.$$

Using (A.5) to substitute for λ_t, we obtain

$$\begin{aligned} V_t \quad = \quad & \frac{\mathbb{E}_t\Omega_{t+1}(R^l_{t+1} - R_{t+1}(1+\tau^l_t))}{\theta}\phi_t n_t + \mathbb{E}_t\Omega_{t+1}R_{t+1}n_t \\ & + \mathbb{E}_t\Omega_{t+1}R_{t+1}\frac{\psi}{2}\left(\frac{Q^k_t s_t}{n_t}\right)^2 n_t. \end{aligned}$$

As $V_t = \phi_t n_t$, matching coefficients of n_t yields

$$\phi_t = \frac{\theta\mathbb{E}_t\Omega_{t+1}R_{t+1}\left(1+\frac{\psi}{2}\left(\frac{Q^k_t s_t}{n_t}\right)^2\right)}{\theta - \mathbb{E}_t\Omega_{t+1}(R^l_{t+1} - R_{t+1}(1+\tau^l_t))}.$$

We can also derive an asset pricing equation for long-term bonds. From (A.5), we have

$$Q^l_t = \mathbb{E}_t\tilde{\Omega}_{t,t+1}(1+\rho Q^l_{t+1}),$$

where

$$\tilde{\Omega}_{t,t+1} = \frac{\Omega_{t+1}\Pi_{t+1}^{-1}}{\mathbb{E}_t \Omega_{t+1} R_{t+1}(1+\tau_t^l) + \lambda_t \theta}.$$

We can thus derive that

$$Q_t^l = \sum_{j=1}^{\infty} \tilde{\Omega}_{t,t+j} \rho^{j-1}$$

where $\tilde{\Omega}_{t,t+j} = \prod_{k=t}^{t+j-1} \tilde{\Omega}_{k,k+1}$.

B No-Run Equilibrium System

The no-run equilibrium is characterized by the following 29 equations in 29 variables $\{R_t^n, R_t, R_t^l, R_t^k, Q_t^l, Q_t^k, B_t, B_{ht}, B_{bt}, S_t, S_{ht}, S_{bt}, N_t, \phi_t, W_t, Z_t, S_t^g, Y_t, C_t, K_t, L_t, I_t, p_t^*, \Pi_t, P_{wt}, \Delta_t, \Gamma_t^a, \Gamma_t^b, \tau_t^l\}$ for $t \geq 0$. Here $\{N_t, B_{bt}, B_t, S_t, S_{bt}, Q_t^l, Q_t^k, C_t, \Delta_t, R_t^n, A_t, v_t^m, \mu_t, \tau_t^l\}$ are predetermined variables and $\{N_{-1}, B_{-1}, B_{b,-1}, S_{-1}, S_{b,-1}, Q_{-1}^l, Q_{-1}^k, C_{-1}, \Delta_{-1}, R_{-1}^n, A_{-1}, v_{-1}^m, \mu_{-1}, \tau_{-1}^l\}$ are exogenously given. The shock innovations $\{\varepsilon_t^a, \varepsilon_t^\mu, \varepsilon_t^m\}$ are exogenously given.

1. Monetary policy,

$$R_t^n = \rho_r R_{t-1}^n + (1-\rho_r)\left(R^n + \phi_\pi \log\frac{\Pi_t}{\Pi} + \phi_y \log\frac{Y_t}{Y}\right) + v_t^m, \tag{B.1}$$

   where

$$v_t^m = \rho_m v_{t-1}^m + \varepsilon_t^m.$$

2. Supply of long-term bonds,

$$B_t = B. \tag{B.2}$$

3. Macroprudential policy tax on long-term bonds,

$$\tau_t^l = \tau^l,$$

   or

$$\tau_t^l = \phi_l(R_t^n - R^n). \tag{B.3}$$

4. Government budget constraint,

$$Q_{t-1}^l B_{t-1} R_t^l = S_t^g + Q_t^l B_t. \tag{B.4}$$

5. Real return on long-term government bonds,

$$R_t^l = \frac{1 + \rho Q_t^l}{Q_{t-1}^l} \Pi_t^{-1}. \tag{B.5}$$

6. Labor supply,

$$u_{C_t} W_t = \chi L_t^{\varphi}, \tag{B.6}$$

where

$$u_{C_t} = \frac{1}{C_t - hC_{t-1}}.$$

7. Household's first-order condition for deposit,

$$\mathbb{E}_t \Lambda_{t,t+1} R_{t+1} = 1, \tag{B.7}$$

where

$$\Lambda_{t,t+1} \equiv \beta \frac{u_{C_{t+1}}}{u_{C_t}}.$$

8. Household's first-order condition for long-term bonds,

$$1 + \kappa \left(\frac{B_{ht}}{B_t} - \eta_B \right) = \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l. \tag{B.8}$$

9. Household's first-order condition on firm equity,

$$1 + \kappa \left(\frac{S_{ht}}{S_t} - \eta_S \right) = \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^k. \tag{B.9}$$

10. Real interest rate,

$$R_t = \frac{R_{t-1}^n}{\Pi_t}. \tag{B.10}$$

11. Real return on equity,

$$R_t^k = \frac{Z_t + (1-\delta) Q_t^k}{Q_{t-1}^k}. \tag{B.11}$$

12. Law of motion of bank net worth,

$$N_t = \sigma \Bigg[\left(R_t^k - R_t\right) Q_{t-1}^k S_{b,t-1} - R_t \frac{\psi}{2} \left(\frac{Q_{t-1}^k S_{b,t-1}}{N_{t-1}} \right)^2 N_{t-1} + \left(R_t^l - R_t(1+\tau_{t-1}^l)\right) Q_{t-1}^l B_{b,t-1} + R_t N_{t-1} \Bigg] + \omega N_{t-1}. \tag{B.12}$$

13. Bank's no-arbitrage condition,

$$\mathbb{E}_t \Omega_{t+1} \left(R_{t+1}^k - R_{t+1} \left(1 + \psi \frac{Q_t^k S_{bt}}{N_t} \right) \right) = \mathbb{E}_t \Omega_{t+1} \left(R_{t+1}^l - R_{t+1}(1+\tau_t^l) \right), \tag{B.13}$$

where

$$\Omega_{t,t+1} = \Lambda_{t,t+1} \left[(1-\sigma) + \sigma \phi_{t+1} \right].$$

14. Marginal value of bank net worth,

$$\phi_t = \frac{\theta \mathbb{E}_t \Omega_{t+1} R_{t+1} \left(1 + \frac{\psi}{2} \left(\frac{Q_t^k S_{bt}}{N_t} \right)^2 \right)}{\theta - \mathbb{E}_t \Omega_{t+1} (R_{t+1}^l - R_{t+1}(1+\tau_t^l))}. \tag{B.14}$$

15. Borrowing constraint for banks,

$$\theta Q_t^k S_{bt} + \theta Q_t^l B_{bt} \leq \phi_t N_t, \tag{B.15}$$

with equality holds if $\mathbb{E}_t \Omega_{t,t+1} \left(R_{t+1}^l - R_{t+1}(1+\tau_t^l) \right) > 0$.

16. Production function,

$$\Delta_t Y_t = A_t (K_t)^\alpha L_t^{1-\alpha}, \tag{B.16}$$

where

$$\log A_t = \rho_a \log A_{t-1} + \varepsilon_t^a.$$

17. Labor demand,

$$W_t = P_{wt}(1-\alpha) \frac{Y_t}{L_t} \Delta_t. \tag{B.17}$$

18. Profits of capital,

$$Z_t = P_{wt} \alpha \frac{Y_t}{K_t} \Delta_t. \tag{B.18}$$

19. Capital price,

$$1 = Q_t^k \left[1 - \Omega_k \left(\frac{I_t}{K_t} - \delta\right)\right].$$

20. Optimal pricing of retailers,

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\Gamma_t^a}{\Gamma_t^b}. \tag{B.19}$$

21. Numerator of the pricing rule,

$$\Gamma_t^a = P_{wt} Y_t + \mathbb{E}_t \gamma \Lambda_{t,t+1} \Pi^{-\varepsilon} \Pi_{t+1}^{\varepsilon} \Gamma_{t+1}^a. \tag{B.20}$$

22. Denominator of the pricing rule,

$$\Gamma_t^b = (1 - \tau_t^c) Y_t + \mathbb{E}_t \gamma \Lambda_{t,t+1} \Pi^{1-\varepsilon} \Pi_{t+1}^{\varepsilon - 1} \Gamma_{t+1}^b, \tag{B.21}$$

where

$$\tau_t^c = 1 - \exp\left[-\frac{\mu_t}{\kappa_\pi} + \log\left(\frac{\varepsilon}{\varepsilon - 1}\right)\right],$$

$\kappa_\pi = (1 - \beta\gamma)(1 - \gamma)/\gamma$, and

$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu + \rho_{ma} \varepsilon_{t-1}^\mu.$$

23. Inflation and the pricing rule,

$$1 = \left[\gamma \left(\frac{\Pi}{\Pi_t}\right)^{1-\varepsilon} + (1 - \gamma) p_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{B.22}$$

24. Price dispersion,

$$\Delta_t = (1 - \gamma) p_t^{*-\varepsilon} + \gamma \left(\frac{\Pi}{\Pi_t}\right)^{-\varepsilon} \Delta_{t-1}. \tag{B.23}$$

25. Resource constraint,

$$Y_t = C_t + I_t + G. \tag{B.24}$$

26. Law of motion of capital,

$$K_{t+1} = (1 - \delta) K_t + I_t - \frac{\Omega_k}{2} \left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{B.25}$$

27. Issuance of equity claims,

$$K_t = S_{t-1}. \tag{B.26}$$

28. Market clearing for firm equity,

$$S_t = S_{ht} + S_{bt}. \tag{B.27}$$

29. Market clearing for long-term bonds,

$$B_t = B_{ht} + B_{bt}. \tag{B.28}$$

C Algorithm to Compute Bank-Run Equilibrium

This section describes the numerical method to compute the transition dynamics of the economy subject to bank runs. We study the following experiment. Suppose that the economy is at the no-run equilibrium steady state at $t = 0$. A one-time positive cost-push shock innovation $\varepsilon_1^{\mu} = 0.01$ hits the economy at $t = 1$, followed by two monetary policy innovations $\varepsilon_1^m = \varepsilon_2^m = 0.0025$. We assume that there is no technology shock through out the experiment. Thus μ_t and v_t^m follow deterministic processes that converge to the steady state in period $T + 1$ where T is a large enough number.

Suppose that a bank run occurs only in one period if possible. We suppose that it occurs in period J and check $x_J < 1$ is satisfied. After the cost-push shock and interest rate hikes, the economy moves along the transition path until a bank run occurs in period J. After the bank run, the economy starts a new transition path from period $J + 1$ until it converges back to the no-run equilibrium steady state in period $T + 1$.

We compute the equilibrium path in the following steps:

1. Compute the transition path of the economy after the cost-push and monetary shocks when no bank run ever happens, denoted as $\{X_t\}_{t=1}^T$ where X_t stands for the vector of all endogenous variables.

2. Compute the transition path of the economy recovering from a bank run. Suppose that a bank run occurs in period J, $1 \le J \le T$. New banks restart in period $J + 1$. We denote the transition path after a period-J bank run as $\{{}_J X_t^*\}_{t=J+1}^T$.

3. Using the equilibrium conditions in period J, we solve for the endogenous variables X_J^*.

4. Finally, the transition path when a bank run takes place in period J is the combination of

three pieces:

$$\left\{ \{X_t\}_{t=1}^{J-1}, \quad X_J^*, \quad \{_J X_t^*\}_{t=J+1}^{T} \right\}.$$

We use the usual nonlinear deterministic simulation in Dynare to compute the transition path without a bank run $\{X_t\}_{t=1}^{T}$ using the equilibrium conditions in Appendix B. We next describe the method for computing the transition path after the bank run $\{_J X_t^*\}_{t=J+1}^{T}$ and the variables during the bank run X_J^*.

C. 1 Transition Path after Bank Run

The transition path after the bank run $\{_J X_t^*\}_{t=J+1}^{T}$ solves the deterministic version of the equilibrium conditions in Appendix B with one modification: the bank starts with net worth $N_{J+1} = N^{new}$. We need to determine the proper boundary conditions for $t = J$.

In addition to exogenous shocks, the model has 10 endogenous predetermined state variables. We need to determine their values at time J, $\{Q_J^l, Q_J^k, S_{b,J}, B_{b,J}, N_J, B_J, S_J, C_J, R_J^n, \Delta_J\}$. First, we have $S_{b,J} = B_{b,J} = N_J = 0$ since banks do not operate during the period of the bank run. We also have $B_J = B$ by our assumed fiscal policy. We are left with the vector of six values, $\mathcal{S} = \{Q_J^k, Q_J^l, S_J, C_J, R_J^n, \Delta_J\}$, to be determined. We use the following iterative procedure to compute $\mathcal{S}$ together with other variables in X_J^*.

Step 1. Take the vector of the steady-state values $\mathcal{S}^0 \equiv \{Q^k, Q^l, S, C, R^n, \Delta\}$, as an initial guess for $\mathcal{S}$.

Step 2. Given the values for the endogenous state variables in the i-th iteration $\mathcal{S}^i$, $i \geq 0$, we compute the transition path after the bank run: $\{_J X_t^{*i}\}_{t=J+1}^{J+T}$. Using the equilibrium conditions at $t = J$, we can solve for the vector of endogenous variables X_J^{*i}. In the next subsection we describe the detailed procedure. We then update the values for the state variables in the $(i+1)$-th iteration $\mathcal{S}^{i+1}$ using the solution in X_J^{*i}.

Step 3. We repeat the above two steps until convergence according to the criterion $||\mathcal{S}^i - \mathcal{S}^{i+1}|| < 10^{-6}$. After convergence, we obtain X_J^* from the last iteration.

C. 2 Computing the Updated Values

In this subsection we describe the procedure to compute the updated value of the state vector $\mathcal{S}^+ = \{Q_J^{k+}, Q_J^{l+}, S_J^+, C_J^+, R_J^{n+}, \Delta_J^+\}$ given the value from the previous iteration $\mathcal{S} = \{Q_J^k, Q_J^l, S_J, C_J, R_J^n, \Delta_J\}$. For simplicity, we suppress the number of iteration superscript i. As described above, we can solve for the transition path after the bank run in period J given state $\mathcal{S}$, $\{_J X_t^*\}_{t=J+1}^{J+T}$. We can also compute the transition path before the bank run $\{X_t\}_{t=1}^{J-1}$. We now use the following equilibrium conditions at $t = J$ to compute $\mathcal{S}^+$ as well as other variables in X_J^*.

1. Compute C_J^+ using
$$C_J^+ = \frac{\Lambda_{J,J+1}C_{J+1} + \beta h C_{J-1}}{\beta + \Lambda_{J,J+1}h},$$
where $\Lambda_{J,J+1} = 1/R_{J+1}$.

2. Compute Q_J^{k+} and Q_J^{l+} using
$$Q_J^{k+} = \Lambda_{J,J+1}(Z_{J+1} + (1-\delta)Q_{J+1}^k)\left[1 + \kappa\left(\frac{S_{hJ}}{S_J} - \eta_S\right)\right]^{-1},$$
$$Q_J^{l+} = \Lambda_{J,J+1}(1 + \rho Q_{J+1}^l)\Pi_{J+1}^{-1}\left[1 + \kappa\left(\frac{B_{hJ}}{B_J} - \eta_B\right)\right]^{-1},$$
where $S_{hJ} = S_J, K_J = S_{J-1}$ and $B_{hJ} = B_J = B$.

3. Compute S_J^+ using
$$S_J^+ = (1-\delta)K_J + I_J - \frac{\Omega_k}{2}\left(\frac{I_J}{K_J} - \delta\right)^2 K_J.$$
where
$$I_J = \left[\frac{1}{\Omega_k}\left(1 - \frac{1}{Q_J^{k+}}\right) + \delta\right]K_J.$$

4. Compute the following variables recursively
$$\begin{aligned}
Y_J &= C_J^+ + I_J + G, \\
L_J &= \left(\frac{\Delta_J Y_J}{A_J K_J^\alpha}\right)^{\frac{1}{1-\alpha}}, \\
W_J &= \chi L_J^\varphi (C_J^+ - hC_{J-1}), \\
P_{wJ} &= \frac{W_J L_J}{(1-\alpha)Y_J\Delta_J}, \\
\Gamma_J^a &= P_{wJ}Y_J + \gamma\Lambda_{J,J+1}\Pi^{-\varepsilon}\Pi_{J+1}^{\varepsilon}\Gamma_{J+1}^a, \\
\Gamma_J^b &= (1-\tau_J^c)Y_J + \gamma\Lambda_{J,J+1}\Pi^{1-\varepsilon}\Pi_{J+1}^{\varepsilon-1}\Gamma_{J+1}^b, \\
p_J^* &= \frac{\varepsilon}{\varepsilon - 1}\frac{\Gamma_J^a}{\Gamma_J^b}, \\
\Pi_J &= \left[\frac{1 - (1-\gamma)p^{*1-\varepsilon}}{\gamma}\right]^{\frac{1}{\varepsilon-1}}\Pi.
\end{aligned}$$

5. Compute R_J^{n+} using

$$R_J^{n+} = \rho_r R_{J-1}^n + (1-\rho_r)\left(R^n + \phi_\pi \log\frac{\Pi_J}{\Pi} + \phi_y \log\frac{Y_J}{Y}\right) + v_J^m.$$

6. Compute Δ_J^+ using

$$\Delta_J^+ = (1-\gamma)p_J^{*-\varepsilon} + \gamma\left(\frac{\Pi}{\Pi_J}\right)^{-\varepsilon}\Delta_{J-1}.$$

7. We can determine the remaining variables in X_J^* :

$$R_J = R_{J-1}^n/\Pi_J,\ Z_J = P_{wJ}\alpha Y_J \Delta_J^+/K_J,$$

$$R_J^k = \frac{Z_J + (1-\delta)Q_J^{k+}}{Q_{J-1}^k},$$

$$R_J^l = \frac{1+\rho Q_J^{l+}}{Q_{J-1}^l}\frac{1}{\Pi_J},$$

$$S_J^g = Q_{J-1}^l B_{J-1} R_J^l - Q_J^{l+} B_J,$$

where $B_{J-1} = B_J = B$.

8. We have $N_J = S_{bJ} = B_{bJ} = 0$, and ϕ_J and τ_J^l are invalid as all banks fail in period J.

In sum, we have computed the updated state vector $\mathcal{S}^+ = \{Q_J^{k+}, Q_J^{l+}, S_J^+, C_J^+, R_J^{n+}, \Delta_J^+\}$ together with the updated vector of endogenous variables X_J^*.