Convergence, Financial Development, and Policy Analysis*

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Abstract

We study the relationship among inflation, economic growth, and financial development in a Schumpeterian overlapping-generations model with credit constraints. In the baseline case money is super-neutral. When the financial development exceeds some critical level, the economy catches up and then converges to the growth rate of the world technology frontier. Otherwise, the economy converges to a poverty trap with a growth rate lower than the frontier and with inflation decreasing with the level of financial development. We then study efficient allocation and identify the sources of inefficiency in a market equilibrium. We show that a particular combination of monetary and fiscal policies can make a market equilibrium attain the efficient allocation.

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1 Introduction

The industrial revolution marked a dramatic turning point in the economic progress of nations. During the nineteenth century, a number of technological leaders in the Western Europe and North America leapt ahead of the rest of the world, while others lagged behind and became colonies or semi-colonies of the Western powers. After the WWII, most developing countries obtained political independence and started their industrialization and modernization process. One might expect that, with the spread of technology and the advantage of backwardness (Gerschenkron (1962)), the world should have witnessed convergence in income and living standard. Instead, the post WWII was a period of continued and accelerated divergence (Pritchett (1997)). According to Maddison (2008), the per capita GDP in the U.S., the most advanced countries in the 20th century, grew at an average annual growth rate of 2.1% in the period between 1950 and 2008. While some OECD and East Asian economies were able to narrow the per capita GDP gap with an annual growth rate higher than that of the U.S. in the catch up process, most other countries in Latin America, Asia, and Africa failed to achieve so.[1]

Why some countries fail to converge in growth rates despite the possibility of technology transfer has been a puzzle. There are several explanations in the literature.[2] In this paper we focus on the explanation of Aghion, Howitt, and Mayer-Foulkes (henceforth AHM) (2005) and Acemoglu, Aghion, and Zilibotti (2006) based on a Schumpeterian overlapping-generations (OLG) model of economic growth with credit constraints.[3]

We contribute to this literature by analyzing the relationship among growth, inflation, and financial development. Figure 1 presents the cross-sectional evidence on the sample of 71 countries over the period 1960-1995.[4] Panels A and B show that the average inflation rate is negatively related to the average per capita GDP growth rate and positively related to the average money growth rate. Panel C shows that the average inflation rate is negatively related to the average level of financial development and this relationship vanishes at a high level of financial development (about 50%). Panel D displays the countries that fail to converge to the world frontier growth rate, identified by AHM (2005). These countries have a low average level of financial development and their inflation is negatively related to the average level of financial development.

Motivated by the evidence above, we introduce a monetary authority and a government to a closed-economy version of the AHM model. We modify this model in several ways. First, we introduce money by assuming money enters utility (Sidrauski (1967)). This money-in-the-utility

[1] In the period of 1950-2008, the average per capita GDP growth rates for the whole Latin America, Asia, and Africa were respectively 1.8%, 1.6%, and 1.2% (Maddison (2008)).

[2] See Banerjee and Duflo (2005) for a survey.

[3] AHM (2005) provide empirical evidence to support the importance of the credit constraints for convergence or divergence.

[4] Appendix B presents data description.

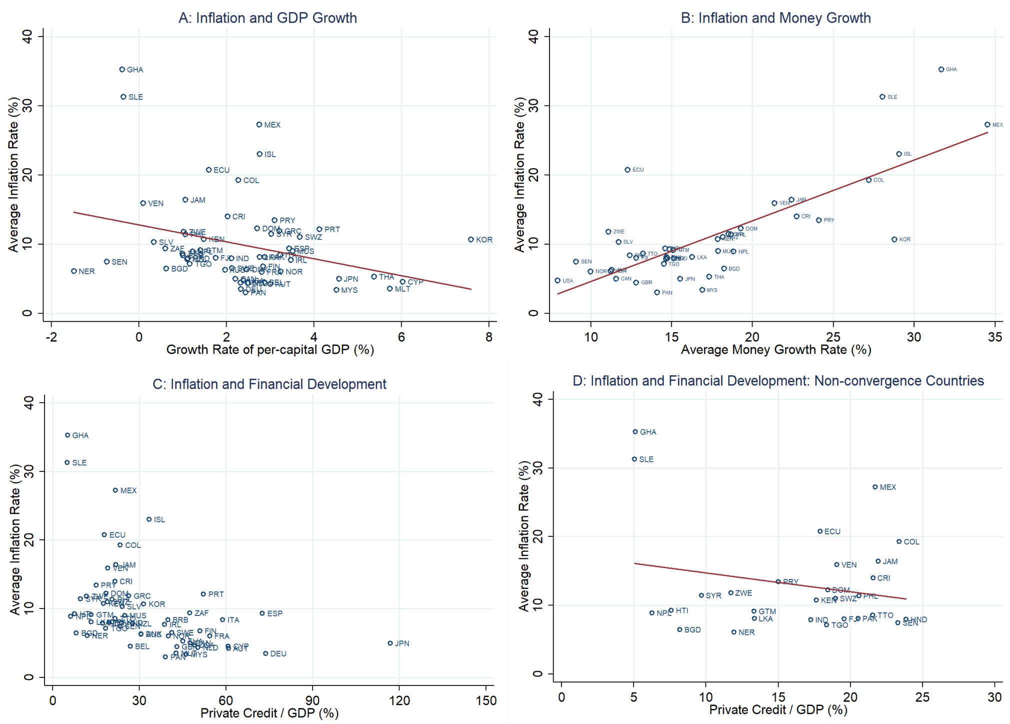

Figure 1: The average inflation rate, the average per capita GDP growth rate, the average money growth rate, and the average level of financial development, 1960-1995.

approach can be microfounded in several ways once one takes into account the role of medium of exchange (McCallum (1983)). Although money can be valued in the OLG model as a store of value (Samuelson (1958)), the equilibrium net nominal interest rate is zero and hence one cannot analyze monetary policy in terms of interest rate rules. Our modeling of money avoids this issue.[5] Second, we introduce intra-generational heterogeneity so that there are savers and borrowers (entrepreneurs) in each period. We can then endogenize the nominal interest rate in a credit market and study how credit market imperfections affect interest rates. Third, we assume savers are risk averse so that we can derive their consumption and portfolio choices. In each period a young saver must choose optimal consumption, money holdings, and saving in terms of nominal bonds.

We show that the market equilibrium in our model can be summarized by a system of four nonlinear difference equations for four sequences of variables: the nominal interest rate, the inflation rate, the normalized R&D investment, and the proximity to the technological frontier. For this equilibrium system, monetary policy is modeled by a money supply rule. If one uses an interest rate rule as in the dynamic new Keynesian literature (Woodford (2003)), then money supply is endogenous and the nominal interest rate is replaced by the money growth rate in the equilibrium system. Due to the complexity of our model, we cannot reduce this system to a scalar one for

[5] Another approach is to introduce a cash-in-advance constraint.

the proximity variable alone as in the AHM model. However we are still able to provide a full characterization of the steady state along a balanced growth path, which is consistent with the evidence presented in Figure 1.

It turns out that how money supply is introduced to the economy is critical for how money affects the equilibrium allocation and long-run growth. We first show that, if money increments are transferred to the old agents in an amount proportional to their pre-transfer money holdings, then money is super-neutral in the sense that monetary policy does not affect long-run growth and the equilibrium allocation along a balanced growth path.[6] This result dates back to Lucas's (1972) model, in which there is no endogenous growth. The intuition is that the demand for money and saving depends on the ratio of the nominal interest rate and the money growth rate and hence the real interest rate in the long run. Thus only real variables are determined in the steady state.

We show that there are three dynamic patterns as in the AHM model with the difference that our model incorporates inflation:

1. When the credit market is perfect so that the credit constraint does not bind, the economy converges to the world frontier growth rate and there is no marginal effect of financial development.

2. When the credit constraint binds, but is not tight enough, the economy converges to the world frontier growth rate with a level effect of financial development.

3. When the credit constraint is sufficiently tight, there is divergence in growth rates with a growth effect of financial development. In this case the economy enters an equilibrium with poverty trap.

Using numerical examples, we find that the steady states for all these three cases are saddle points. For any given initial value of the proximity to the frontier, there exists a unique saddle path such that the economy will transition to the steady state. For the first two cases, the transition paths display the feature of the advantage of backwardness (Gerschenkron (1962)). Moreover, the inflation rate rises during the transition. But for the third case, the economy exhibits the feature of the disadvantage of backwardness and falls into the poverty trap with low economic growth, low innovation, and high inflation. The inflation rate declines during the transition. Moreover, the long-run rate of productivity growth increases with financial development and the long-run inflation rate decreases with financial development.

Next we study efficient allocation. Suppose that there is a social planner who maximizes the sum of discounted utilities of all agents in the present and future generations. We derive the efficient allocation and long-run growth rate. By comparing with the efficient allocation, we find

[6] Money growth has a short-run effect on the transition path.

there are four sources of inefficiency in a market equilibrium. First, there is monopoly inefficiency in the production of intermediate goods. The resulting price distortion generates an inefficiently low level of final net output when taken the innovation rate as given. Second, the private return to innovation ignores the dynamic externality or spillover effect of technology. Third, the credit market imperfection prevents innovators to obtain necessary funds for R&D. Finally, the OLG framework itself may cause dynamic inefficiency.

Can a combination of monetary and fiscal policies correct the preceding inefficiencies and make the market equilibrium attain the efficient allocation? We show that when money increments are transferred to the entrepreneur, money is not super-neutral and there is a particular nominal interest rate such that the market equilibrium can achieve innovation efficiency, but it cannot achieve output and consumption efficiency. The intuition is that money growth is like an inflation tax and there is a wealth effect when the tax is not proportionally distributed to the agents according to their pre-transfer money holdings. Money affects the real economy through the redistribution channel. We then introduce fiscal policies to attain the efficient allocation. We find different policies are needed in different development stages. When the economy faces severe credit market imperfections, the government should try to loosen credit constraints by ensuring better contract enforcements or better monitoring of borrowers. For example, the government can make direct lending to entrepreneurs financed by lump-sum taxes on savers. When the government has better monitoring technologies than private agents, the credit constraints can be overcome. The economy can then avoid the equilibrium with poverty traps.

Our paper is related to several strands of literature. First, it is related to the literature on poverty traps and convergence or divergence in economies with credit market imperfections (e.g., Banerjee and Newman (1993), Galor and Zeira (1993), Howitt (2000), Mookherjee and Ray (2001), Azariadis and Stachurski (2005), Howitt and Mayer-Foulkes (2005), Aghion, Howitt, and Mayer-Foulkes (2005) and Acemoglu, Aghion, and Zilibotti (2006)). As pointed out by Azariadis and Stachurski (2005) in their survey, this literature typically studies models of self-reinforcing mechanisms that cause poverty to persist. In these models there is no technical progress and therefore no positive long-run growth. As discussed earlier, our paper is most closely related to Aghion, Howitt, and Mayer-Foulkes (2005) and Acemoglu, Aghion, and Zilibotti (2006), which incorporate long-run growth. Unlike these two papers, we introduce money, endogenize interest rates, and provide a policy analysis. Howitt and Mayer-Foulkes (2005) also derive three convergence patterns analogous to those in our paper, but the disadvantage of backwardness that prevents convergence in that paper arises from low levels of human capital rather than from credit-market imperfections.

Second, our paper is related to the literature that analyzes the effects of financial constraints or financial intermediation on long-run growth. Early contributions include Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), and King and Levine (1993). None of these papers studies technology transfer and the associated policy issues which are our focus.

Third, our paper is related to the literature on the relation between money and growth. Recent papers include Gomme (1991), Marquis and Reffett (1994), Chu and Cozzi (2014), Jones and Manuelli (1995), Miao and Xie (2013), and Chu et al. (2017), among others. These papers typically introduce money via cash-in-advance constraints in infinite-horizon models, which do not feature poverty traps. By contrast, we follow the money-in-the-utility function approach of McCallum (1983) and Abel (1987) in the OLG framework. Our focus is on how monetary and fiscal policies can attain efficient allocation and avoid poverty traps.

2 The Model

We consider a monetary overlapping generations model of a closed economy based on Aghion, Howitt, and Mayer-Foulkes (2005) and Acemoglu, Aghion, and Zilibotti (2006). Time is discrete and runs forever. Time is denoted by $t = 1, 2, \ldots$. Each generation has a unit measure of identical entrepreneurs and a unit measure of identical savers. Each agent lives for two periods. Only entrepreneurs can conduct innovation, but they face borrowing constraints. Savers lend funds to entrepreneurs, but they cannot innovate. As a benchmark, we follow Lucas (1972) and assume that the government (or central bank) directly transfers money to all agents and the monetary transfer is proportional to each agent's pre-transfer money holdings.

2.1 Production

All agents work for the producers who combine labor and a continuum of specialized intermediate goods to produce a general good according to the production function,

$$Z_t = L_t^{1-\alpha} \int_0^1 A_t(i)^{1-\alpha} x_t(i)^\alpha \, di, \tag{1}$$

where L_t is labor demand, $x_t(i)$ is the input of the latest version of intermediate good i, and $A_t(i)$ is the productivity parameter associated with it. The general good is used for consumption, as an input to R&D and also as an input to the production of intermediate goods. The general good is produced under perfect competition. Suppose that the aggregate labor supply is normalized to one and the real price of the general good is also normalized to one. Then the equilibrium real price of each intermediate good equals its marginal product:

$$p_t(i) = \alpha \left(\frac{x_t(i)}{A_t(i)} \right)^{\alpha-1}. \tag{2}$$

For each intermediate good i there is one entrepreneur born each period t who is capable of producing an innovation for the next period. If he succeeds in innovating, then he will be the ith incumbent in period $t+1$. Let $\mu_t(i)$ be the probability that he succeeds. Then the technology evolves according to

$$A_{t+1}(i) = \begin{cases} \bar{A}_{t+1} & \text{with probability } \mu_t(i) \\ A_t(i) & \text{with probability } 1-\mu_t(i) \end{cases},$$

where $\bar{A}_{t+1}$ is the world frontier technology, which grows at the exogenously given constant rate $g > 0$. That a successful innovator gets to implement $\bar{A}_{t+1}$ is a manifestation of technology transfer in the sense that domestic R&D makes use of ideas developed elsewhere in the world. If an innovation fails, the intermediate good sector i uses the technology in the previous period.

In each intermediate good sector where an innovation has just occurred, the incumbent can produce one unit of the intermediate good using one unit of the general good as the only input. In each intermediate sector there are an unlimited number of people capable of producing copies of the latest generation of that intermediate good at a unit cost of $\chi > 1$. The fact that $\chi > 1$ implies that the fringe is less productive than the incumbent producer. The parameter χ captures technological factors as well as government regulation affecting entry. A higher χ corresponds to a less competitive market. So in sectors where an innovation has just occurred, the incumbent will be the sole producer, at a price equal to the unit cost of the competitive fringe, whereas in noninnovating sectors where the most recent incumbent is dead, production will take place under perfect competition with a price equal to the unit cost of each producer. In either event the price will be χ, and according to the demand function (2) the quantity demanded will be

$$x_t(i) = \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_t(i). \tag{3}$$

It follows that an unsuccessful innovator will earn zero profits next period, whereas the real profit of a successful incumbent will be

$$\Psi_t(i) = p_t(i)\, x_t(i) - x_t(i) = (\chi - 1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \bar{A}_t \equiv \psi \bar{A}_t,$$

where ψ represents the normalized profit:

$$\psi = (\chi - 1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}.$$

2.2 Entrepreneurs

An entrepreneur born in period $t \geq 1$ is endowed with $\lambda \in (0,1)$ units of labor when young and supplies labor inelasically to the general good producers. He derives utility from consumption c_{t+1}^e when old according to

$$\beta \log\left(E_t c_{t+1}^e\right),$$

where $\beta \in (0, 1)$ is the subjective discount factor. This utility function is an increasing transformation of a risk-neutral utility function. We will see the role of the log transformation in Section 4.

An innovation costs N_t units of general good in period t. The young entrepreneur receives labor income λw_t, which may not be sufficient to cover the innovation cost N_t. Suppose that the entrepreneur borrows B_t dollars at the nominal interest rate R_{ft} between periods t and $t+1$ from the savers so that

$$N_t = \frac{B_t}{P_t} + \lambda w_t, \tag{4}$$

where P_t denotes the price level and w_t is the real wage rate.

We follow Aghion, Banerjee, and Piketty (1999) and Aghion, Howitt, and Mayer-Foulkes (2005) to model financial market imperfections. Suppose that the entrepreneur can hide a successful innovation at a real cost κN_t so that he can avoid repaying debt. The parameter $\kappa \in (0, 1)$ reflects the degree of financial development. A higher value of κ means that it is more costly for the entrepreneur to misbehave. It measures the degree of creditor protection. To implement the contract without default, the entrepreneur faces an incentive constraint

$$\beta \left(\mu_t \psi \bar{A}_{t+1} - R_{ft} \frac{B_t}{P_{t+1}} \right) \geq \beta \mu_t \psi \bar{A}_{t+1} - \kappa N_t,$$

where the expression on the left-hand side of the inequality is the discounted expected consumption if the entrepreneur behaves and the expression on the right-hand side is the discounted expected consumption if he is dishonest. Simplifying yields the borrowing constraint

$$\frac{B_t}{P_t} \leq \frac{\kappa N_t}{\beta R_{ft} / \Pi_{t+1}}, \tag{5}$$

where $\Pi_{t+1} = P_{t+1}/P_t$ denotes the inflation rate. By (4) this constraint is equivalent to

$$N_t \leq \frac{\beta R_{ft} / \Pi_{t+1}}{\beta R_{ft} / \Pi_{t+1} - \kappa} \lambda w_t \tag{6}$$

for $\beta R_{ft} / \Pi_{t+1} > \kappa$. Thus R&D investment is limited by a multiple of the entrepreneur's net worth λw_t. This multiple is called the credit multiplier by AHM and increases with κ, but decreases with the real interest rate.

Suppose that

$$N_t = \Phi(\mu_t) \bar{A}_{t+1},$$

where the function Φ is twice continuously differentiable and satisfies $\Phi(0) = 0$ and $\Phi' > 0$ and $\Phi'' > 0$. The factor $\bar{A}_{t+1}$ reflects the "fishing-out" effect: the further ahead the frontier moves, the more difficult it is to innovate. This effect is important to have a balanced growth path. We can also rewrite the preceding equation as

$$\mu_t = F \left(N_t / \bar{A}_{t+1} \right), \tag{7}$$

where $F = \Phi^{-1}$ satisfies $F(0) = 0$, $F' > 0$, and $F'' < 0$.

The entrepreneur's expected consumption is given by

$$E_t c_{t+1}^e = \mu_t \psi \bar{A}_{t+1} - \frac{R_{ft} B_t}{P_{t+1}} = F\left(\frac{N_t}{\bar{A}_{t+1}}\right) \psi \bar{A}_{t+1} - \frac{R_{ft} P_t}{P_{t+1}} (N_t - \lambda w_t).$$

The entrepreneur's objective is to solve the following problem

$$\max_{N_t} F\left(N_t / \bar{A}_{t+1}\right) \psi \bar{A}_{t+1} - \frac{R_{ft} P_t}{P_{t+1}} (N_t - \lambda w_t)$$

subject to (6). When the credit constraint (6) does not bind, the first-order condition is given by

$$F'\left(\frac{N_t}{\bar{A}_{t+1}}\right) \psi = \frac{R_{ft}}{\Pi_{t+1}}, \tag{8}$$

where $\Pi_{t+1} = P_{t+1}/P_t$ denotes the inflation rate. This condition says that the expected marginal return to R&D is equal to the real interest rate.

The initial old entrepreneur at time t does not have labor income and hence does not conduct innovation. We assume that he simply consumes his money endowment M_0^e and the government proportional transfer $M_0^e z_1$.

2.3 Savers

A saver born at time $t \geq 1$ is endowed with $1 - \lambda$ units of labor when young and supplies labor inelastically to the general good producers. He has the utility function

$$\log(c_t^y) + \beta \log(c_{t+1}^o) + \gamma \log(M_t / P_t), \ \gamma > 0,$$

where β is the discount factor, c_t^y (c_{t+1}^o) denotes consumption at time t ($t + 1$) when the saver is young (old), M_t denotes money holdings chosen in period t. He faces the following budget constraints

$$c_t^y + \frac{S_t}{P_t} + \frac{M_t}{P_t} = (1 - \lambda) w_t,$$

$$c_{t+1}^o = \frac{S_t R_{ft}}{P_{t+1}} + \frac{M_t (1 + z_{t+1})}{P_{t+1}},$$

where S_t denotes saving and $z_{t+1} \geq 0$ denotes the proportional rate of the monetary transfer from the government. Note that the above utility specification does not have a satiation level of real balances as in Friedman (1969).

The first-order conditions give

$$\frac{1}{c_t^y} = \beta \frac{1}{c_{t+1}^o} \frac{P_t}{P_{t+1}} R_{ft},$$

and

$$\frac{1}{c_t^y} = \frac{\gamma}{M_t / P_t} + \frac{\beta}{c_{t+1}^o} \frac{P_t (1 + z_{t+1})}{P_{t+1}}.$$

Using these conditions and the budget constraints, we can derive that

$$c_t^y = \frac{(1-\lambda)w_t}{1+\beta+\gamma}, \tag{9}$$

$$\frac{M_t}{P_t} = \frac{\gamma(1-\lambda)w_t}{1+\beta+\gamma}\frac{1}{1-(1+z_{t+1})/R_{ft}}, \tag{10}$$

$$\frac{S_t}{P_t} = \frac{(1-\lambda)w_t}{1+\beta+\gamma}\left[\beta - \frac{\gamma}{R_{ft}/(1+z_{t+1})-1}\right]. \tag{11}$$

Thus, consumption, the money demand, and the saving demand are all proportional to the saver's real wealth $(1-\lambda)w_t$. Moreover the money demand decreases with $R_{ft}/(1+z_{t+1})$ and the saving demand increases with $R_{ft}/(1+z_{t+1})$. This property is important for the long-run super-neutrality of money because $R_{ft}/(1+z_{t+1})$ is proportional to the real interest rate in the steady state, which is independent of the inflation rate.

We assume that

$$R_{ft} > (1+z_{t+1})\left(1+\frac{\gamma}{\beta}\right). \tag{12}$$

This assumption ensures that the money demand $M_t/P_t > 0$ and the saving demand $S_t/P_t > 0$.

The initial old saver is endowed with money holdings M_0^s and derives utility according $\log(c_1^o)$, where

$$c_1^o = \frac{M_0^s(1+z_1)}{P_1}.$$

2.4 Competitive Equilibrium

Define the aggregate technology as

$$A_t = \int A_t(i)\,di.$$

In equilibrium the probability of innovation will be the same in each sector: $\mu_t(i) = \mu_t$ for all i. Thus average productivity evolves according to

$$A_{t+1} = \mu_t \bar{A}_{t+1} + (1-\mu_t)A_t. \tag{13}$$

Define the normalized productivity as $a_t = A_t/\bar{A}_t$. Normalized productivity is an inverse measure of the country's distance to the technological frontier, or its technology gap. It describes the proximity to the technological frontier and satisfies the dynamics

$$a_{t+1} = \mu_t + \frac{1-\mu_t}{1+g}a_t. \tag{14}$$

Equation (13) implies that

$$\frac{A_{t+1}-A_t}{A_t} = \mu_t\left(\frac{1+g}{a_t}-1\right).$$

Thus there is an advantage of backwardness (Gerschenkron (1962)) in the sense that the further the country is behind the frontier, the faster the country grows. On the other hand, the country's growth rate also depends on innovation μ_t. More innovation allows more firms to adopt the frontier technology and hence enhancing growth. Thus the net effect depends on both a_t and μ_t. Here μ_t or R&D investment is like the role of human capital that determines a country's "absorptive capacity" (Nelson and Phelps (1966)).

In equilibrium $L_t = 1$. We then use (1), (2), and $p_t(i) = \chi$ to derive aggregate output of the general good

$$Z_t = \zeta A_t, \text{ where } \zeta \equiv \left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}}.$$

The wage rate is given by

$$w_t = (1-\alpha) Z_t = (1-\alpha)\zeta A_t. \tag{15}$$

The equilibrium interest rate R_{ft} and the price level P_t are determined by the market-clearing conditions for credit and money: $B_t = S_t$ and $M_t = (1+z_t) M_{t-1}$ for $t \geq 1$, where z_t is the money growth rate controlled by the central bank and $M_0 = M_0^s + M_0^e$ is given.

By (4), (11), and the market-clearing condition $B_t = S_t$, we have

$$N_t - \lambda w_t = \frac{(1-\lambda) w_t}{1+\beta+\gamma}\left[\beta - \frac{\gamma(1+z_{t+1})}{R_{ft} - (1+z_{t+1})}\right]. \tag{16}$$

Value added in the general sector is wage income, whereas value added in the intermediate sectors is profit income. Total GDP is the sum of value added in all sectors:

$$Y_t = w_t + \mu_{t-1}\psi\bar{A}_t = (1-\alpha)\zeta A_t + \mu_{t-1}\psi\bar{A}_t. \tag{17}$$

3 Equilibrium Balanced Growth Paths

In this section we solve for competitive equilibrium and derive equilibrium balanced growth path.

3.1 Perfect Credit Markets

Suppose that the credit constraint (6) does not bind so that the credit market is perfect. It follows from (8) that the optimal innovation is determined by the condition

$$F'(n_t)\psi = \frac{R_{ft}}{\Pi_{t+1}}, \tag{18}$$

where we define $n_t = N_t/\bar{A}_{t+1}$. We can rewrite (14) as

$$a_{t+1} = F(n_t) + \frac{1-F(n_t)}{1+g} a_t. \tag{19}$$

Conjecture that the economy will grow at the rate of the world technology frontier along a balanced growth path so that $A_{t+1} = (1+g) A_t$. Using (10) to compute the ratio M_{t+1}/M_t and then imposing the money market-clearing condition $M_{t+1} = M_t(1+z_{t+1})$, we obtain

$$(1+z_{t+1})\frac{P_t}{P_{t+1}} = \frac{M_{t+1}/P_{t+1}}{M_t/P_t} = \frac{\frac{w_{t+1}}{1-(1+z_{t+2})/R_{ft+1}}}{\frac{w_t}{1-(1+z_{t+1})/R_{ft}}}.$$

Using (15), $a_t = A_t/\bar{A}_t$, and $A_{t+1} = (1+g)\,A_t$, we can simplify the preceding equation as

$$\Pi_{t+1} = (1+z_{t+1})\,\frac{1-(1+z_{t+2})\,/R_{ft+1}}{1-(1+z_{t+1})\,/R_{ft}}\frac{a_t}{a_{t+1}\,(1+g)}. \tag{20}$$

Thus the inflation rate is determined by money demand and money supply, which in turn are determined by the nominal interest rate, the growth rate of domestic productivity, and the growth rate of money supply. Using $n_t = N_t/\bar{A}_{t+1}$, (15), and (16), we derive that

$$n_t = \frac{(1-\alpha)\,\zeta a_t}{1+g}\left[\lambda + \frac{(1-\lambda)}{1+\beta+\gamma}\left(\beta - \frac{\gamma(1+z_{t+1})}{R_{ft}-(1+z_{t+1})}\right)\right]. \tag{21}$$

Now the competitive equilibrium under perfect credit markets can be summarized by a system of four difference equations (18), (19), (20), and (21) for four sequences $\{R_{ft}\}, \{a_t\}, \{\Pi_{t+1}\}$, and $\{n_t\}$ such that (12) and (6) are satisfied.

We first study the balanced growth path in the steady state. We use a variable without time subscript to denote its steady state value. It follows from (20) that the steady-state inflation is given by

$$\Pi = \frac{1+z}{1+g}, \tag{22}$$

which increases with the money growth rate, but decreases with the productivity growth rate. We then obtain a system of three equations

$$\begin{aligned} n &= \frac{(1-\alpha)\,\zeta a}{1+g}\left\{\lambda + \frac{(1-\lambda)}{1+\beta+\gamma}\left[\beta - \frac{\gamma}{R_f/\,(1+z)-1}\right]\right\}, && (23)\\ R_f &= \frac{1+z}{1+g}F'(n)\psi, && (24)\\ a &= \frac{(1+g)\,F(n)}{g+F\,(n)}, && (25) \end{aligned}$$

to determine three variables n, R_f, and a.

From equations (23), (24), and (25), we can show that n is determined by the equation

$$n = \frac{(1-\alpha)\,\zeta F(n)}{g+F(n)}\left\{\lambda + \frac{(1-\lambda)}{1+\beta+\gamma}\left[\beta - \frac{\gamma}{F'(n)\psi/\,(1+g)-1}\right]\right\}.$$

Equivalently, μ is determined by the equation

$$\Phi'\,(\mu) = \frac{\psi}{R_f/\Pi}, \tag{26}$$

where

$$\frac{R_f}{\Pi} = (1+g)\left\{1 + \frac{\gamma}{\beta - \left[\frac{\Phi(\mu)(g+\mu)}{(1-\alpha)\zeta\mu} - \lambda\right]\frac{1+\beta+\gamma}{1-\lambda}}\right\}.$$

We can check that the real interest rate R_f/Π is decreasing in μ and $\Phi'\,(\mu)$ is increasing in μ. Assuming that

$$\Phi'\,(0) < \frac{\psi}{1+g}\left\{1 + \frac{\gamma}{\beta - \left[\frac{\Phi'(0)g}{(1-\alpha)\zeta} - \lambda\right]\frac{1+\beta+\gamma}{1-\lambda}}\right\}^{-1}, \tag{27}$$

and

$$\Phi'(1) > \frac{\psi}{1+g}\left\{1 + \frac{\gamma}{\beta - \left[\frac{\Phi(1)(g+1)}{(1-\alpha)\zeta} - \lambda\right]\frac{1+\beta+\gamma}{1-\lambda}}\right\}^{-1}, \tag{28}$$

we use the intermediate value theorem to deduce that there is a unique solution $\mu^* \in (0,1)$ to (26).[7] The associated R&D investment is given by $n^* = \Phi(\mu^*)$ and hence R_f^* and a^* are determined by (24) and (25). We also assume that the condition

$$\frac{R_f^*}{1+z} > 1 + \frac{\gamma}{\beta} \tag{29}$$

is satisfied so that (12) holds along the balanced growth path. We will verify later that this condition is indeed satisfied.

Using (23) and (15), we can rewrite the credit constraint (6) along a balanced growth path as

$$n\left(\frac{\beta R_f}{\Pi} - \kappa\right) \leq \frac{\beta R_f}{\Pi}\frac{\lambda(1-\alpha)\zeta a}{1+g}.$$

The critical value of κ such that the credit constraint just binds in equilibrium is given by

$$\kappa^* = \frac{\beta R_f^*}{\Pi}\left[1 - \frac{\lambda(1-\alpha)\zeta a^*}{n^*(1+g)}\right]. \tag{30}$$

When $\kappa > \kappa^*$, the credit constraint does not bind. It follows from (26) that money supply does not affect the equilibrium innovation rate μ^*. An increase in the money growth rate raises the nominal interest rate one for one and hence does not affect savings. Thus the supply of funds for innovation does not depend on monetary policy. We summarize the result below.

Proposition 1 *Suppose that the monetary transfer is given to the old generation only in a quantity proportional to the pre-transfer money holdings of each. Let conditions (27) and (28) hold. If $\kappa \geq \kappa^*$, where κ^* satisfies (30), then there is a unique steady state with $\mu^* \in (0,1)$ such that the credit constraint does not bind and the productivity grows at the rate g. In this steady state money is super-neutral in the sense that the steady-state real quantities are independent of money growth rate z. They are also independent of κ.*

Since $\mu^* \in (0,1)$, the economy can never reach the world technology frontier in that $a^* \in (0,1)$. For the economy to reach the frontier, we must have condition (28) hold with equality so that $\mu^* = a^* = 1$. This case can happen when innovation profits are sufficiently large, i.e., ψ is sufficiently large.

We use a numerical example to illustrate the transition dynamics. As in AHM (2006), we set $\Phi(\mu) = \phi\mu + \frac{\delta}{2}\mu^2$ and $F(n) = \frac{1}{\delta}\left(\sqrt{2n\delta + \phi^2} - \phi\right)$. We choose parameter values as $\alpha = 0.8$, $\chi = 1.15$, $\phi = 0.0134$, $\delta = 0.2604$, $\lambda = 0.01$, $g = 0.04$, $\beta = 0.96$, and $\gamma = 0.017$. We assume that

[7] We do not consider the knife-edge case of boundary solutions.

money supply grows at a constant rate $z = 0.06$. Our simple two-period lived OLG model cannot be calibrated to confront with data. We use our numerical example to illustrate the working of our model. We find that the critical value $\kappa^* = 0.678$. We choose an arbitrary $\kappa > \kappa^*$. Then the steady state values are given by $R_f^* = 1.08$, $\Pi^* = 1.0192$, $a^* = 0.5$, $\mu^* = 0.037$, $n^* = 0.0007$. Moreover, the GDP Y_t normalized by $\bar{A}_t$ is equal to 0.024. The steady state is a saddle point. Only a_t is a predetermined variable. Figure 2 illustrates the transition dynamics for the case of perfect credit markets when the economy starts at $a_1 = 0.3$. We find that μ_t, a_t, and Π_t gradually increases to their steady state values, but R_{ft} decreases to its steady state value. Given that we take the the money growth rate fixed, the inflation rate moves inversely with the growth rate of productivity. The transition path illustrates the advantage of backwardness. When the economy initially falls behind the world frontier, both its technology and innovation grow faster. Thus its GDP also grows faster. They eventually catch up with the growth rate of the world frontier.

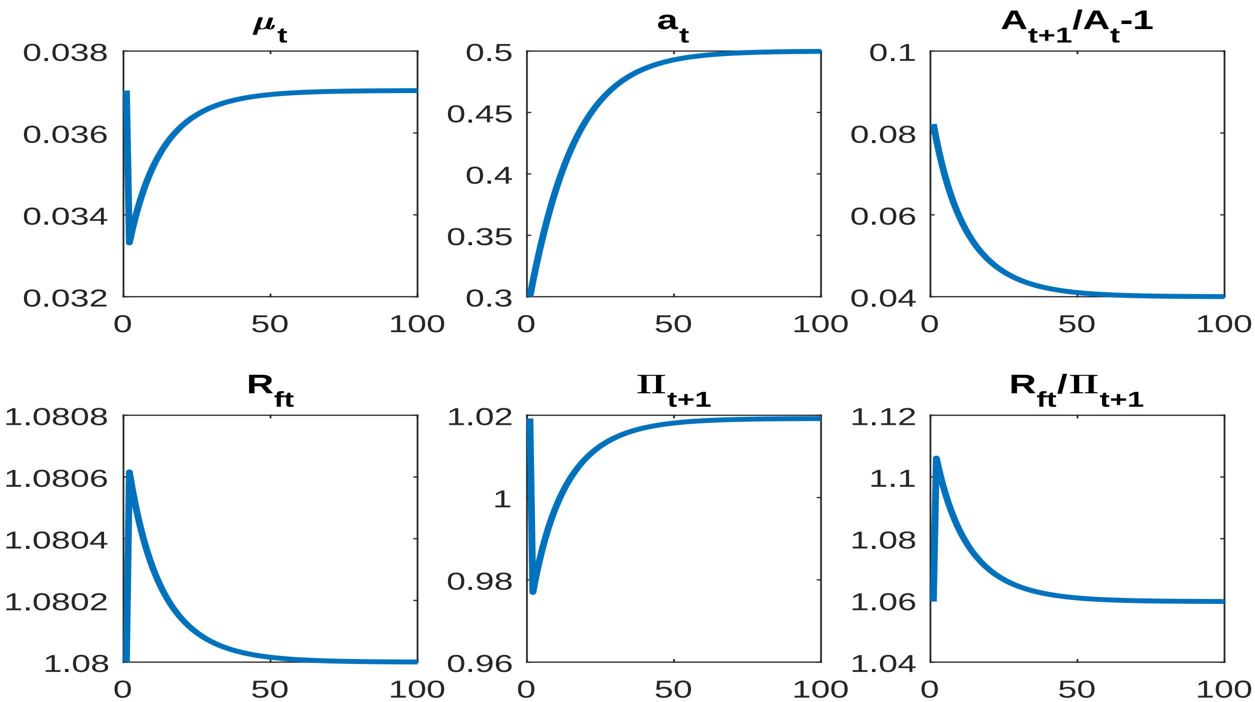

Figure 2: Transition dynamics for the case of perfect credit markets.

The steady-state proximity to frontier a^* depends on the preference and technology parameters. A crucial parameter is the marginal cost of innovation ϕ given the quadratic specification of Φ. By (26), a higher ϕ raises the marginal cost and reduces the marginal benefit by reducing the real interest rate, thereby reducing the innovation rate μ^*. This causes the economy's absorptive capacity to be smaller so that a^* is smaller.

3.2 Binding Credit Constraints

Suppose that the credit constraint (6) binds. Using (16) and (6) we obtain

$$w_t \left\{ \lambda + \frac{(1-\lambda)}{1+\beta+\gamma} \left[\beta - \frac{\gamma(1+z_{t+1})}{R_{ft} - (1+z_{t+1})} \right] \right\} = \frac{\beta R_{ft}/\Pi_{t+1}}{\beta R_{ft}/\Pi_{t+1} - \kappa} \lambda w_t. \tag{31}$$

We also require that

$$F'(n_t)\,\psi > \frac{R_{ft}}{\Pi_{t+1}}. \tag{32}$$

Now the equilibrium system consists of equations (19), (20), (21), and (31) for four sequences $\{R_{ft}\}, \{a_t\}, \{\Pi_{t+1}\}$, and $\{n_t\}$ such that (12) and (32) hold.

In the steady state the equilibrium system becomes (22), (23), (25), and the following equation

$$\underbrace{\lambda + \frac{1-\lambda}{1+\beta+\gamma} \left[\beta - \frac{\gamma}{R_f/(1+z) - 1} \right]}_{\text{supply}} = \underbrace{\frac{\lambda \beta R_f/\Pi}{\beta R_f/\Pi - \kappa}}_{\text{limit}}. \tag{33}$$

The expression on the left-hand side of equation (33) is increasing in R_f and the expression on the right-hand side is decreasing in R_f. Thus there is a unique solution for R_f to the equation above such that

$$\frac{R_f}{1+z} > \max\left\{ 1 + \frac{\gamma}{\beta}, \frac{\kappa}{\beta(1+g)} \right\}. \tag{34}$$

Let R_f^{**} denote the solution. Using equations (23), (25) and (33), we derive that

$$n = \frac{\lambda(1-\alpha)\zeta F(n)}{g + F(n)} \frac{\beta R_f^{**}/\Pi}{\beta R_f^{**}/\Pi - \kappa}.$$

We can equivalently rewrite this equation in terms of μ as

$$\Phi(\mu)(g+\mu) = \frac{(1-\alpha)\zeta\lambda\beta R_f^{**}/\Pi}{\beta R_f^{**}/\Pi - \kappa}\mu. \tag{35}$$

Assume that

$$\Phi'(0)\, g < \frac{(1-\alpha)\zeta\lambda\beta R_f^{**}/\Pi}{\beta R_f^{**}/\Pi - \kappa} < \Phi(1)(g+1). \tag{36}$$

Then it follows from the intermediate value theorem that there is a unique solution, denoted by $\mu^{**} \in (0,1)$, to the equation (35). The corresponding R&D investment level is denoted by $n^{**} = \Phi(\mu^{**})$.

Define the critical values κ^{**} and $\bar{\kappa}$ for κ such that

$$\Phi'(0)\, g = \frac{(1-\alpha)\zeta\lambda\beta R_f^{**}/\Pi}{\beta R_f^{**}/\Pi - \kappa^{**}},$$

$$\frac{(1-\alpha)\zeta\lambda\beta R_f^{**}/\Pi}{\beta R_f^{**}/\Pi - \bar{\kappa}} = \Phi(1)(g+1).$$

where R_f^{**} is the solution to equation (33) and is a function of κ. We can verify that the expression

$$\frac{R_f^{**}/\Pi}{R_f^{**}/\Pi - \kappa}$$

increases with κ along the supply curve in Figure 3. Thus the values κ^{**} and $\bar{\kappa}$ are unique.

We summarize the result below.

Proposition 2 *Suppose that the monetary transfer is given to the old generation only in a quantity proportional to the pre-transfer money holdings of each. If* $\kappa^{**} < \kappa < \min\{\kappa^*, \bar{\kappa}\}$, *then there is a unique steady state such that the credit constraint binds and the productivity grows at the rate* g. *In this steady state we have* $\mu^{**} \in (0,1)$, $0 < n^{**} < n^*$, *and* $R_f^{**} < R_f^*$. *Moreover, money is super-neutral, and* n^{**}, μ^{**}, *and* R_f^{**} *increase with* κ.

We use Figure 3 to illustrate the determination of the steady-state nominal interest rate. The curve labeled "Supply" describes the supply of funds normalized by the wage rate, which is given by the expression on the left-hand side of equation (33). This curve increases with the nominal interest rate R_f. The curve labeled "Demand" describes the demand for funds normalized by the wage rate, which is given by

$$\frac{N_t}{w_t} = \frac{n}{\frac{(1-\alpha)\zeta F(n)}{g+F(n)}}. \quad \text{(Demand)}$$

It increases with n. Using (24) to substitute for n, we can show that the above expression decreases with R_f. The curves labeled "Limit $\kappa > \kappa^*$" and "Limit $\kappa < \kappa^*$" describe the borrowing limits normalized by the wage rate for $\kappa > \kappa^*$ and $\kappa < \kappa^*$, respectively, which are given by the expression on the right-hand side of equation (33). This expression decreases with R_f.

When $\kappa > \kappa^*$, the equilibrium nominal interest rate R_f^* is determined by the intersection of the demand curve and the supply curve. In this case the credit constraint does not bind and a change in κ does not affect equilibrium. When $\kappa < \kappa^*$, the credit constraint binds so that the equilibrium nominal interest rate R_f^{**} is determined by the intersection of the supply curve and the borrowing limit curve. From the figure we can see that $R_f^{**} < R_f^*$ and an increase in κ raises R_f^{**}. However the change in κ does not have a growth effect.

For a numerical illustration, we choose the same parameter values as in Section 3.1 except that we set $\kappa = 0.5$. Then the credit constraint binds. We find the steady state values $R_f^{**} = 1.0796$, $\Pi^{**} = 1.0192$, $a^{**} = 0.327$, $\mu^{**} = 0.018$, and $n^{**} = 0.0003$. The normalized GDP is equal to 0.016. Compared to the case of perfect credit markets, credit market imperfections enlarges the distance to the frontier even though grow rates are the same in that $a^{**} < a^*$, $\mu^{**} < \mu^*$, $n^{**} < n^*$, and normalized GDP are all smaller. We find that the steady state is also a saddle point. Figure 4 illustrates the transition dynamics, which also display the advantage of backwardness.

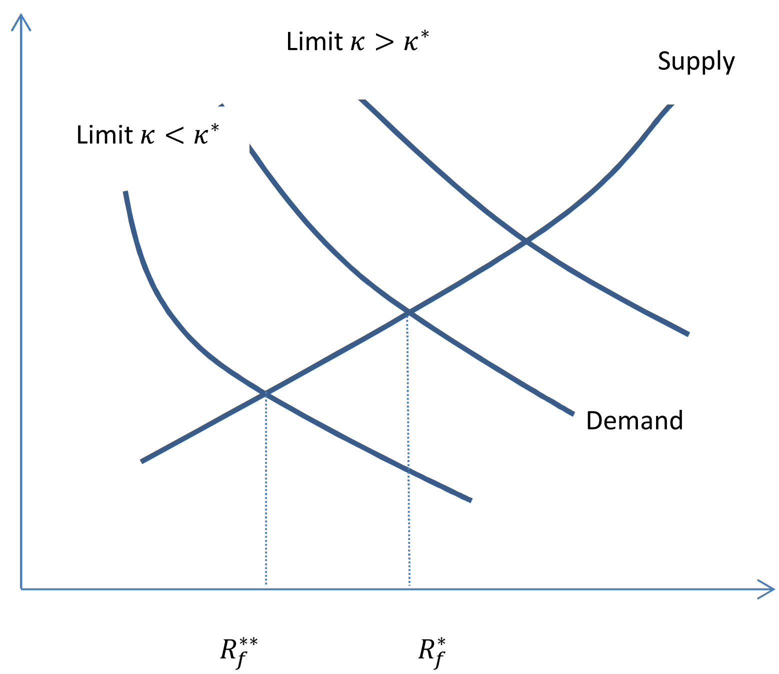

Figure 3: Determination of the steady-state equilibrium nominal interest rates. The curves labeled "Supply" and "Demand" describe the supply of and demand for funds normalized by the wage rate, respectively. The curves labeled "Limit $\kappa > \kappa^*$" and "Limit $\kappa > \kappa^*$" describe the borrowing limits normalized by the wage rate for $\kappa > \kappa^*$ and $\kappa < \kappa^*$, respectively.

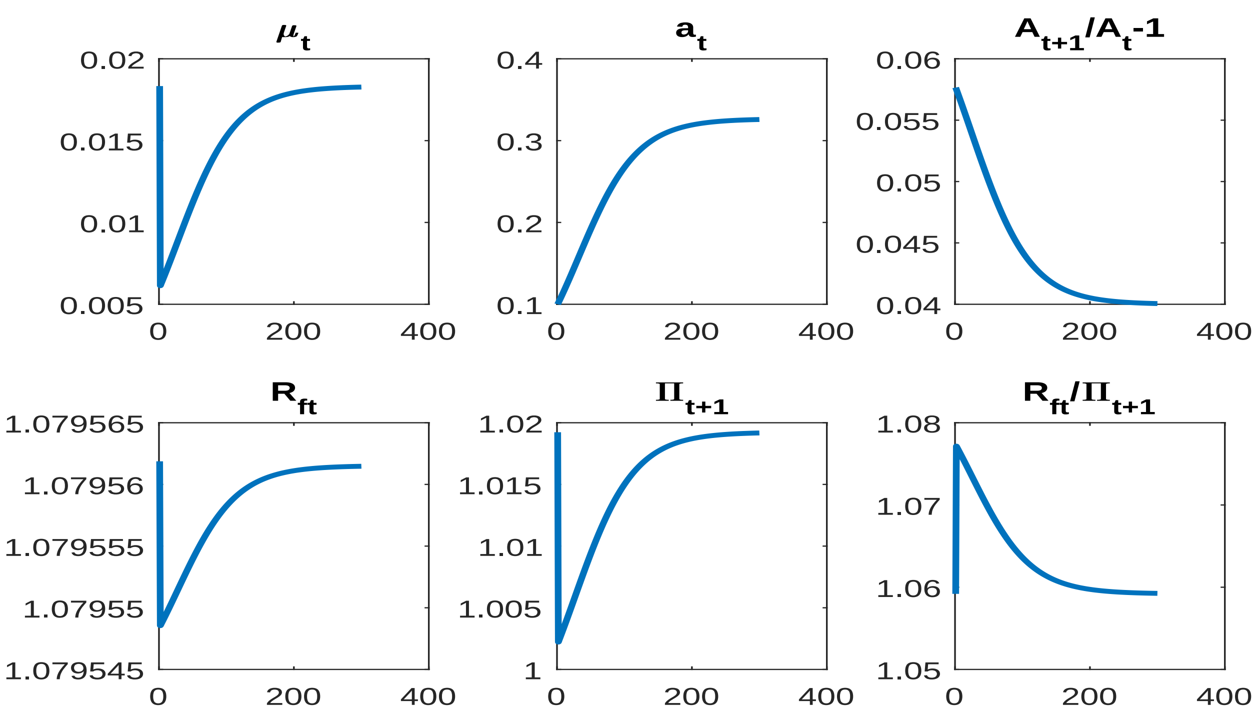

Figure 4: Transition dynamics for the case of binding credit constraints.

3.3 Poverty Trap

Since $\Phi(0) = 0$ by assumption, we deduce that there is another solution to (35) denoted by $\hat{\mu} = 0$. In this case the corresponding $\hat{a} = \hat{n} = 0$. The economy enters a poverty trap. When κ is sufficiently small, condition (36) is violated. Then the only steady state equilibrium is the poverty trap. In the poverty trap the economy still grows but at a rate lower than g. In Proposition 3 below we show that

$$\lim_{t\to\infty} \frac{A_{t+1}}{A_t} = (1+g) \lim_{t\to\infty} \frac{a_{t+1}}{a_t} = F'(0)\, \frac{(1-\alpha)\,\zeta\lambda\beta R_f/\Pi}{\beta R_f/\Pi - \kappa} + 1,$$

where R_f is the steady-state nominal interest rate. By (20) the steady-state inflation rate satisfies

$$\Pi = \frac{1+z}{F'(0)\, \frac{(1-\alpha)\zeta\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa} + 1}, \tag{37}$$

instead of equation (22). The poverty-trap steady state is characterized by a system of two equations (33) and (37) for two variables R_f and Π. In the appendix we prove that there is a unique solution, denoted by $\hat{R}_f$ and $\hat{\Pi}$.

Proposition 3 *Suppose that the monetary transfer is given to the old generation only in a quantity proportional to the pre-transfer money holdings of each. If* $0 < \kappa < \kappa^{**}$*, then there is a unique steady-state equilibrium, which enters the poverty trap with* $\hat{\mu} = \hat{a} = \hat{n} = 0$*. Money is super-neutral. The steady-state productivity growth rate is given by*

$$\lim_{t\to\infty} \frac{A_{t+1}}{A_t} = F'(0)\, \frac{(1-\alpha)\,\zeta\lambda\beta \hat{R}_f/\hat{\Pi}}{\beta \hat{R}_f/\hat{\Pi} - \kappa} + 1,$$

which is between 0 and $1+g$ *and increases with* $\kappa \in (0, \kappa^{**})$*. The steady-state inflation rate* $\hat{\Pi} > (1+z)/(1+g)$ *and decreases with* $\kappa \in (0, \kappa^{**})$*.*

To illustrate this proposition numerically, we use the same parameter values as in Section 3.1 except that we set $\kappa = 0.1$. We then find the poverty trap equilibrium with the steady-state values $\hat{R}_f = 1.0792$ and $\hat{\Pi} = 1.0205$. In the steady-state, the normalized GDP is equal to 0 and the technology growth rate is 1.0387. The steady-state inflation rate is higher than the two cases studied in Sections 3.1 and 3.2. We also find the poverty-trap steady state is a saddle point. Figure 5 illustrates the transition dynamics when the economy starts at $a_1 = 0.5$. It shows that the economy falls further behind the world frontier. Both a_t and μ_t decrease to zero. The inflation rate Π_t and the nominal interest rate R_{ft} also decrease to their steady-state values, but the real interest rate R_{ft}/Π_{t+1} increases to its steady-state value. The growth rate of productivity increases to a level lower than the world frontier. The economy falls into a poverty trap with low economic growth and high inflation. Thus there is a disadvantage of backwardness when the level financial development is extremely low.

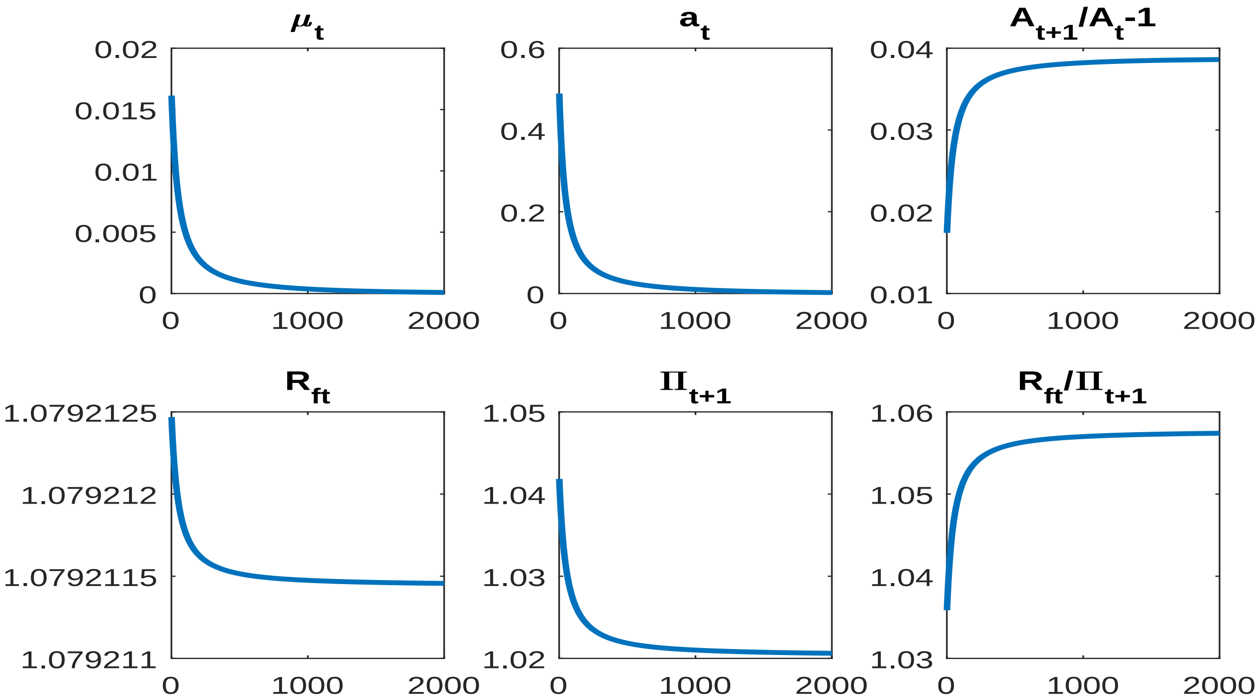

Figure 5: Transition dynamics for the case of poverty trap.

4 Efficient Allocation

In this section we study efficient allocation. Following Abel (1987), suppose that a social planner maximizes the sum of discounted utility of all agents in the economy

$$\omega u\left(c_1^e\right) + u\left(c_1^o\right) + \sum_{t=1}^{\infty} \beta^{t-1}\left[u\left(c_t^y\right) + \beta u\left(c_{t+1}^o\right) + \omega\beta u\left(c_{t+1}^e\right)\right] \tag{38}$$

$$= \sum_{t=1}^{\infty} \beta^{t-1}\left[u\left(c_t^y\right) + u\left(c_t^o\right) + \omega u\left(c_t^e\right)\right],$$

where the planner assigns the utility weight ω to the entrepreneur and discounts utilities of future generations by β. Here we set $u(c) = \log(c)$. As in the dynamic new Keynesian framework, we consider a cashless limit and ignore money in the utility (Woodford (2003) and Gali (2008)). The resource constraint is given by

$$c_t^y + c_t^o + c_t^e + N_t = L_t^{1-\alpha} \int_0^1 A_t(i)^{1-\alpha} x_t(i)^{\alpha}\, di - \int \chi_t(i) x_t(i)\, di, \tag{39}$$

where $\chi_t(i) = 1$, when an innovation occurs in sector i, and $\chi_t(i) = \chi$, otherwise.

Maximizing the expressions on the right-hand side of equation (39) yields the efficient labor input $L_t = 1$ and the efficient intermediate goods input

$$x_t(i) = \begin{cases} \alpha^{\frac{1}{1-\alpha}} A_t(i) & \text{if an innovation occurs} \\ \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_t(i) & \text{otherwise} \end{cases}. \tag{40}$$

We can then compute the GDP (net output):

$$
\begin{aligned}
Y_t^e &= \int_0^1 A_t(i)^{1-\alpha} x_t(i)^\alpha \, di - \int \chi_t(i) x_t(i) \, di \\
&= \left(\frac{1}{\alpha} - 1\right) \left[\alpha^{\frac{1}{1-\alpha}} \mu_{t-1} \bar{A}_t + (1 - \mu_{t-1}) \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi A_{t-1} \right].
\end{aligned}
\tag{41}
$$

The resource constraint (39) becomes

$$
c_t^y + c_t^o + c_t^e + N_t = Y_t^e. \tag{42}
$$

where $\mu_0 = 0$ and A_0 is exogenously given.

Now the planner's problem is to maximize (38) subject to (7), (13), and (42). Let $\beta^t \Lambda_t$ and $\beta^t \Lambda_t q_t$ be the Lagrange multipliers associated with (42) and (13), respectively. The variable q_t represents the shadow value of the technology A_{t+1}. The first-order conditions are given by

$$
\begin{aligned}
\omega u'(c_t^e) &= u'(c_t^o) = u'(c_t^y) = \Lambda_t, \\
\Lambda_t &= \beta \Lambda_{t+1} (\frac{1}{\alpha} - 1) \left[\alpha^{\frac{1}{1-\alpha}} \bar{A}_{t+1} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi A_t \right] \frac{F'\left(N_t / \bar{A}_{t+1}\right)}{\bar{A}_{t+1}} \\
&\quad + \Lambda_t q_t [\bar{A}_{t+1} - A_t] \frac{F'\left(N_t / \bar{A}_{t+1}\right)}{\bar{A}_{t+1}}, \\
\Lambda_t q_t &= \beta (1 - \mu_{t+1}) \Lambda_{t+1} q_{t+1} + \beta^2 \Lambda_{t+2} (1 - \mu_{t+1}) (\frac{1}{\alpha} - 1) \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi.
\end{aligned}
$$

We can immediately derive that

$$
c_t^e = \omega c_t^y = \omega c_t^o. \tag{43}
$$

The efficiency condition for N_t says that the marginal social cost of R&D Λ_t must be equal to the marginal social benefit, which reflects the fact that an increase in N_t raises the current innovation probability μ_t, which in turn raises the domestic productivity. Rising productivity has two benefits. First, it raises profits. Second, it causes the domestic productivity to be closer to the world frontier.

The efficiency condition for A_{t+1} is similar to an asset pricing equation for capital. It says that the shadow price of technology is equal to the discounted present value of future dividends, which are equal to the profits contributed by domestic technology alone when no innovation occurs. We emphasize that innovation has a positive long-lasting externality effect because it raises all future productivities.

Since $\bar{A}_{t+1}/\bar{A}_t = 1 + g$, we conjecture that, on the balanced growth path, $a_t = A_t/\bar{A}_t$, μ_t, $N_t/\bar{A}_{t+1} = n_t$, and q_t are constant over time, but c_t^y, c_t^o, and c_t^e all grow at the rate g. We then have

$$
q = \frac{\beta}{1+g}(1-\mu) q + \left(\frac{\beta}{1+g}\right)^2 (1-\mu)(\frac{1}{\alpha} - 1) \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi,
$$

and

$$\frac{1}{F'(n)} = \beta \frac{1}{1+g}(\frac{1}{\alpha}-1)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi \frac{a}{1+g}\right] + q\left[1-\frac{a}{1+g}\right].$$

Equation (13) implies

$$a = \frac{F(n)(1+g)}{g+F(n)}. \tag{44}$$

Using the above three equations we can derive

$$\begin{aligned}\frac{1}{F'(n)} &= \frac{\beta}{1+g}(\frac{1}{\alpha}-1)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi\right] \\ &\quad + \frac{\beta}{1+g-\beta(1-F(n))}(\frac{1}{\alpha}-1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi \frac{g}{g+F(n)}.\end{aligned}$$

This equation is equivalent to

$$\begin{aligned}\Phi'(\mu) &= \frac{\beta}{1+g}(\frac{1}{\alpha}-1)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi\right] \\ &\quad + \frac{\beta}{1+g-\beta(1-\mu)}(\frac{1}{\alpha}-1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \frac{\chi g}{g+\mu}.\end{aligned} \tag{45}$$

We can easily check that the expression on the left-hand side of the equation is an increasing function of μ and the expression on the right-hand side is a decreasing function of μ. Given the following assumption, there is a unique solution, denoted by $\mu_{FB} \in (0,1)$, to the above equation.

Assumption 1 *The parameter values satisfy*

$$\Phi'(0) < \frac{\beta}{1+g}(\frac{1}{\alpha}-1)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi\right] + \frac{\beta}{1+g-\beta}(\frac{1}{\alpha}-1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi,$$

and

$$\Phi'(1) > \beta\frac{1}{1+g}(\frac{1}{\alpha}-1)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi\right] + \frac{\beta}{1+g}(\frac{1}{\alpha}-1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \frac{\chi g}{g+1}.$$

We then obtain the efficient innovation rate μ_{FB}, the efficient normalized R&D investment $n_{FB} = \Phi(\mu_{FB})$, and the efficient normalized productivity a_{FB} using (44). Moreover, the implied real interest rate is given by

$$R^r_{FB} = \frac{u'(c^y_t)}{\beta u'(c^o_{t+1})} = \frac{1+g}{\beta}. \tag{46}$$

We summarize the preceding analysis below.

Proposition 4 *Under Assumption 1, there is a unique efficient allocation with $\mu_{FB} \in (0,1)$, $a_{FB} \in (0,1)$, and $n_{FB} > 0$ along the balanced growth path with the productivity growth rate being g.*[8] *Moreover μ_{FB} is independent of ω.*

[8] In the knife-edge case where the second inequality in Assumption 1 holds as an equality, the efficient innovation rate $\mu_{FB} = 1$. In this case $a_{FB} = 1$ and the economy reaches the world frontier technology level.

By the analysis in the previous section, we can immediately see that competitive equilibrium allocation is generally not efficient. There are four sources of inefficiency in the market economy studied in Sections 2 and 3. First, entrepreneurs face credit constraints, which distorts innovation investments. Second, innovators are monopolists. Third, private innovation does not take into account of the externality effect on future productivity. When choosing innovation investment, entrepreneurs only maximize expected monopoly profits in the next period. As we have shown above, efficient innovation not only causes profits in the next period to rise, but also causes future productivity to rise, which raises future profits. Fourth, there is intertemporal inefficiency in the sense that the equilibrium real interest rate and the implied efficient rate may be different.

In general the competitive innovation may be either higher or lower than the efficient innovation depending on the parameter values. To see this fact we consider the case with the perfect credit market. Comparing the equilibrium condition (24) with (45), we can see clearly how the market equilibrium generates inefficiency.[9] First, the equilibrium real interest rate R_f^*/Π may not be equal to the efficient rate R_{FB}^r. Second, the private return to innovation (the normalized monopoly profit) ψ may not be equal to the one-period social return described by the expression (excluding $\beta/(1+g)$) on the first line of equation (45). Third, the positive externality effect captured by the expression on the second line does not appear in the above equilibrium condition.

In fact we can show that the private return to innovation ψ is smaller than the one-period social return and hence smaller than the total social return.[10] But the market real interest rate may be either higher or lower than the efficient rate R_{FB}^r. When γ is sufficiently large, savers have a sufficiently large preference for money so that his saving is sufficiently low. In this case the market real interest can be higher than the efficient rate and hence the market equilibrium innovation is lower than the efficient level.

Using the parameter values in Section 3.1, we computer the efficient steady-state values: $a_{FB} = 0.858$, $\mu_{FB} = 0.1886$, and $n_{FB} = 0.0072$. The normalized GDP is 0.047. The implied real interest rate is 1.0833. Even though the implied real interest rate is higher than the market real interest rate, it turns out that the market equilibrium gives low levels of innovation and GDP.

[9] Notice that $F'(n) = 1/\Phi'(\mu)$.

[10] We need to prove that

$$\psi = (\chi - 1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} < (\frac{1}{\alpha} - 1)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi\right].$$

This is equivalent to

$$\frac{\alpha}{1-\alpha} < \frac{\chi\left(\chi^{\frac{\alpha}{1-\alpha}} - 1\right)}{\chi - 1}.$$

The expression on the right-hand side is equal to $\alpha/(1-\alpha)$ when $\chi = 1$ and increases with $\chi > 1$.

5 Monetary and Fiscal Policies

In this section we study monetary and fiscal polices under which a competitive equilibrium can achieve the efficient allocation along the balanced growth path derived in the previous section. For simplicity we set $\omega = 1$. Our analysis can be similarly applied to other values of ω. In Sections 2 and 3 we have shown that monetary policy is super-neutral if money is transferred to savers in an amount proportional to their pre-transfer holdings and entrepreneurs do not hold any money. In this section we relax this assumption. Since entrepreneurs face credit constraints, we naturally assume that the government transfers money increments to young entrepreneurs in a lump-sum manner.

5.1 Perfect Credit Market

We first study the case where the credit market is perfect so that the credit constraint is slack. We consider the following policy tools such that the government's budget balances in each period t:

- The central bank sets a constant nominal interest rate R_f and transfers the money increments τ_{et} to the old entrepreneur.
- The government subsidizes the production of the final good by imposing a tax credit $1 - \tau_t(i)$ on the intermediate input.
- The government subsidizes the old entrepreneur's expected profits from innovation at the rate τ_N.
- The government levies a lump-sum tax $T_N \bar{A}_t$ on the old entrepreneur's income.
- The government levies a lump-sum tax $T_w \bar{A}_t$ on the wage income.

When the central bank sets a nominal interest rate, the money growth rate z will be endogenous. Equivalently, we can assume that the money growth rate is an exogenous policy instrument so that the nominal interest rate is endogenous as in Sections 2 and 3. Our focus on the interest rate policy is consistent with the practice in many countries and also with the dynamic new Keynesian literature.

We will show that, by a suitable choice of the above policy instruments, a competitive equilibrium can achieve the efficient allocation along the balanced growth path. We first show that monetary policy alone can achieve the efficient innovation, but cannot achieve production and consumption efficiency.

Assumption 2 *The parameter values are such that*

$$n_{FB} > \frac{\lambda(1-\alpha)\zeta}{1+g} \frac{(1+g)F(n_{FB})}{g+F(n_{FB})}$$

and

$$\frac{\psi F'\left(n_{FB}\right)}{1+g} > 1 + \frac{\gamma}{\beta}.$$

The previous assumption ensures that the efficient innovation cannot be self-financed by the entrepreneur's wage income alone and both monetary transfers and external credit are needed.

Proposition 5 *Suppose that money increments are transferred to young entrepreneurs. Under Assumption 2, there is a nominal interest rate $\bar{R}_f > 1 + \gamma/\beta$ such that the market equilibrium under a perfect credit market achieves the efficient innovation n_{FB} along the balanced growth path with the productivity growth rate being g.*

The intuition for this result can be seen from the equilibrium optimality condition for innovation, (24). The central bank can choose a particular nominal interest rate (or money growth rate) such that the private marginal benefit from efficient innovation is equal to the real interest rate.

Next we show that for any given μ_{t-1} the efficient GDP is higher than the equilibrium GDP because of the monopoly distortion. To show this result, we first observe that the efficient GDP Y_t^e is given by (41). By (17), the equilibrium GDP in the market economy is given by

$$\begin{aligned} & Y_t = w_t + \mu_{t-1}\psi \bar{A}_t \\ = \quad & (1-\alpha)\left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}}\left[\mu_{t-1}\bar{A}_t + (1-\mu_{t-1})A_{t-1}\right] + (\chi - 1)\,\mu_{t-1}\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\bar{A}_t, \end{aligned}$$

where we have substituted equations (13) and (15) and the expressions for ζ and ψ. We can easily verify that $Y_t^e > Y_t$.

To achieve the efficient GDP, the government can subsidize the final good firm's input expenditure. Let $\tau_{xt}(i)$ be the subsidy to input i in period t. Then the final good producer's problem is given by

$$\max L_t^{1-\alpha}\int_0^1 A_t\left(i\right)^{1-\alpha} x_t\left(i\right)^{\alpha} di - \int_0^1 \tau_{xt}(i)p_t(i)x_t(i)di - w_t L_t.$$

This leads to

$$x_t(i) = \left(\frac{\tau_{xt}(i)p_t\left(i\right)}{\alpha}\right)^{\frac{1}{\alpha-1}} A_t\left(i\right). \tag{47}$$

Since $p_t\left(i\right) = \chi$, it follows from (40) and (47) that setting

$$\tau_{xt}(i) = \begin{cases} \frac{1}{\chi} & \text{if an innovation occurs} \\ 1 & \text{otherwise} \end{cases}$$

achieves the efficient intermediate input level and final GDP Y_t^e.

In this case a successful innovator produces intermediate good $x_t\left(i\right) = \alpha^{\frac{1}{1-\alpha}}\bar{A}_t$ and earns monopoly profits

$$p_t\left(i\right)x_t\left(i\right) - x_t\left(i\right) = \chi\alpha^{\frac{1}{1-\alpha}}\bar{A}_t - \alpha^{\frac{1}{1-\alpha}}\bar{A}_t = \psi^*\bar{A}_t,$$

where $\psi^* \equiv \alpha^{\frac{1}{1-\alpha}}(\chi-1) > \psi$. Since the final good firm earns zero profit, the real wage under the government policies is given by

$$w_t = (1-\alpha)\left[\alpha^{\frac{\alpha}{1-\alpha}}\mu_{t-1}\bar{A}_t + (1-\mu_{t-1})\left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}} A_{t-1}\right]. \tag{48}$$

The total subsidy is given by

$$\int_0^1 (1-\tau_t(i))\, p_t(i)\, x_t(i)\, di = \alpha^{\frac{1}{1-\alpha}}\mu_{t-1}\bar{A}_t(\chi-1).$$

Let the after-tax wage rate be $w_{Dt} = w_t - T_w\bar{A}_t$. We then derive the saver's decision rules as

$$c_t^y = \frac{(1-\lambda)w_{Dt}}{1+\beta+\gamma}, \tag{49}$$

$$c_{t+1}^o = \frac{\beta}{1+\beta+\gamma}\frac{R_{ft}}{\Pi_{t+1}}(1-\lambda)w_{Dt}, \tag{50}$$

$$\frac{M_t}{P_t} = \frac{\gamma}{1+\beta+\gamma}\frac{R_{ft}}{R_{ft}-1}(1-\lambda)w_{Dt}, \tag{51}$$

$$\frac{S_t}{P_t} = \frac{1}{1+\beta+\gamma}\frac{\beta R_{ft}-\beta-\gamma}{R_{ft}-1}(1-\lambda)w_{Dt}, \tag{52}$$

where we assume that

$$R_{ft} > 1 + \frac{\gamma}{\beta},$$

so that savings and money demand are positive. Unlike in the model of Section 2, here the money demand decreases with the nominal interest rate R_{ft} and the saving demand increases with R_{ft}. This property allows money to be not super-neutral.

The entrepreneur's budget constraint (4) when young becomes

$$N_t = \frac{B_t}{P_t} + \lambda w_{Dt} + \tau_{et}, \tag{53}$$

where

$$\frac{M_t - M_{t-1}}{P_t} = \tau_{et}. \tag{54}$$

The entrepreneur's problem is to maximize his expected consumption when old:

$$\max\,(1+\tau_N)F\left(N_t/\bar{A}_{t+1}\right)\psi^*\bar{A}_{t+1} - T_N\bar{A}_{t+1} - R_{ft}\frac{P_t}{P_{t+1}}[N_t - \lambda w_{Dt} - \tau_t^e].$$

Suppose that credit constraint is slack. The first-order condition implies that

$$(1+\tau_N)F'(n_t)\psi^* = R_{ft}\frac{P_t}{P_{t+1}}. \tag{55}$$

By the market-clearing condition for loans, (53), (54), (51), and (52), we derive that

$$N_t = \lambda w_{Dt} + \frac{z_t}{1+z_t}\frac{(1-\lambda)\gamma}{1+\beta+\gamma}\frac{R_{ft}}{R_{ft}-1}w_{Dt} + \frac{1-\lambda}{1+\beta+\gamma}\frac{\beta R_{ft}-\beta-\gamma}{R_{ft}-1}w_{Dt}. \tag{56}$$

The three terms on the right-hand side of this equation give three sources of funds for the R&D investment: internal funds (wage), government monetary transfers, and external debt.

In the steady state equations (25) and (22) still hold and (55) becomes

$$(1+\tau_N)F'(n)\psi^* = \frac{R_f}{\Pi}. \tag{57}$$

Using (15) and (25), we rewrite (56) as

$$n = \eta\left[\lambda + \frac{z}{1+z}\frac{(1-\lambda)}{1+\beta+\gamma}\frac{\gamma R_f}{R_f-1} + \frac{1-\lambda}{1+\beta+\gamma}\frac{\beta R_f - \beta - \gamma}{R_f-1}\right], \tag{58}$$

where it follows from (48) that

$$\eta \equiv \frac{w_{Dt}}{\bar{A}_{t+1}} = (1-\alpha)\left[\frac{\alpha^{\frac{\alpha}{1-\alpha}}\mu}{1+g} + (1-\mu)\left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}}\frac{a}{(1+g)^2}\right] - \frac{T_w}{1+g} \tag{59}$$

is constant along a balanced growth path.

By (53), and (54), the credit constraint (5) becomes

$$N_t \leq \frac{\beta R_{ft}/\Pi_{t+1}}{\beta R_{ft}/\Pi_{t+1} - \kappa}\left[\lambda w_{Dt} + \frac{z_t}{1+z_t}\frac{\gamma(1-\lambda)}{1+\beta+\gamma}\frac{R_{ft}}{R_{ft}-1}w_{Dt}\right]. \tag{60}$$

In the steady state this constraint becomes

$$n \leq \frac{\eta\beta R_f/\Pi}{\beta R_f/\Pi - \kappa}\left[\lambda + \frac{z}{1+z}\frac{1-\lambda}{1+\beta+\gamma}\frac{\gamma R_f}{R_f-1}\right]. \tag{61}$$

For simplicity we do not consider government spending and government debt. The following government budget constraint must be satisfied:

$$\tau_N F\left(N_{t-1}/\bar{A}_t\right)\psi^*\bar{A}_t + \alpha^{\frac{1}{1-\alpha}}\mu_{t-1}\bar{A}_t(\chi-1) + \tau_{et} = T_N\bar{A}_t + T_w\bar{A}_t + \frac{M_t - M_{t-1}}{P_t}.$$

By (54), this constraint along a balanced growth path becomes

$$\tau_N F(n)\psi^* + \alpha^{\frac{1}{1-\alpha}}\mu(\chi-1) = T_N + T_w. \tag{62}$$

The steady-state competitive equilibrium under fiscal and monetary policy instruments $\{R_f, \tau_x(i), T_w, \tau_N, T_N\}$ consists of equations (22), (25), (57), and (58) for four variables n, a, z, and Π such that (61) and (62) hold.[11]

To implement the efficient allocation along a balanced growth path discussed in Section 4, we first use (43) and $\omega = 1$ to show

$$c_t^y = \frac{1}{3}\left(Y_t^e - N_t\right).$$

[11] During the transition path, we may use the interest rate rule

$$R_{ft} = R_f\left(\frac{\Pi_t}{\Pi}\right)^{\theta}.$$

Thus we use (49) to set the labor income tax as

$$T_w \bar{A}_t = w_t - \frac{1+\beta+\gamma}{3(1-\lambda)} \left(Y_t^e - N_t \right).$$

Along a balanced growth path, we have

$$\begin{aligned} T_w &= \left[1 - \frac{1+\beta+\gamma}{3(1-\lambda)}\right](1-\alpha)\left[\alpha^{\frac{\alpha}{1-\alpha}}\mu + (1-\mu)\left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}} \frac{a}{(1+g)}\right] \\ &\quad + \frac{1+\beta+\gamma}{3(1-\lambda)}(1+g)\, n. \end{aligned} \tag{63}$$

We set T_N such that the government budget constraint (62) is satisfied.

It remains to choose τ_N and R_f such that the competitive equilibrium implies efficient production and innovation such that $a = a_{FB}$, $n = n_{FB}$, and $\mu = F(n_{FB})$. We maintain the following assumption such that the efficient R&D investment cannot be self-financed by the entrepreneur's wage income.

Assumption 3 *Parameter values are such that*

$$n_{FB} > \lambda \eta_{FB},$$

where η_{FB} *is defined in (59) and where* T_w *is defined in (63) with* $\mu = \mu_{FB}$ *and* $a = a_{FB}$.

We summarize the result below.

Proposition 6 *Suppose that money increments are transferred to young entrepreneurs. Suppose that Assumption 3 holds and* $0 < \gamma < 1 - \beta$. *Then, for* $\kappa \geq \kappa^*$, *the steady-state efficient allocation can be implemented by the competitive equilibrium along a balanced growth path under the monetary and fiscal policies* $R_f^* > 1 + \gamma/\beta$, τ_N^*, T_N^*, *and* T_w^* *such that*

$$n_{FB} = \eta_{FB} \left[\lambda + \frac{1-\lambda}{1+\beta+\gamma} \frac{\beta(\gamma+\beta)\left(R_f^* - 1\right) - \gamma}{\beta\left(R_f^* - 1\right)}\right], \tag{64}$$

$$\tau_N^* = \frac{(1+g)/\beta}{F'(n_{FB})\psi^*} - 1, \tag{65}$$

and

$$T_N^* = \tau_N^* F(n_{FB})\psi^* + \alpha^{\frac{1}{1-\alpha}} F(n_{FB})(\chi - 1) - T_w^*.$$

Here T_w^* *is given by (63) and* η_{FB} *is given by (59), where* $n = n_{FB}$, $\mu = F(n_{FB})$, *and* $a = a_{FB}$. *Moreover,* κ^* *satisfies*

$$n_{FB} = \frac{(1+g)\eta_{FB}}{1+g-\kappa^*} \left[\lambda + \frac{\beta R_f^* - 1}{\beta R_f^*} \frac{1-\lambda}{1+\beta+\gamma} \frac{\gamma(1+g)}{1+g-\beta}\right]. \tag{66}$$

The intuition behind the proposition is that we set the nominal interest rate such that the real interest rate is given by (46). At this rate, we ensure intertemporal efficiency so that $c_t^y = c_t^o$. The implied money growth rate z^* and inflation rate in the steady state satisfy

$$z^* = \beta R_f^* - 1, \Pi^* = \frac{1+z^*}{1+g}.$$

Moreover, the tax rate τ_N^* ensures that it is optimal for the entrepreneur to choose the efficiency level of innovation. Finally, the taxes or transfers T_N^* and T_w^* ensure that $c_t^e = c_t^y = c_t^o$ and the government budget balances. Notice that the signs of τ_N^*, T_N^* and T_w^* are ambiguous. Thus they may be either interpreted as taxes or subsidies.

The restriction on γ is a sufficient condition to ensure that the government monetary transfer combined with wages are not sufficient for entrepreneurs to finance the efficient level of the R&D investment. Thus external debt is needed through the credit market. If γ is too large, then the savers' money demand is large enough so that the government can transfer a sufficient amount of money to the entrepreneurs and the credit market is not needed. In this paper we will not consider this uninteresting case.

5.2 Binding Credit Constraints

When the credit constraint binds so that the credit market is imperfect, the government should improve credit markets to raise κ by imposing better creditor protection and better contract enforcement. If we take κ as given, we can introduce another policy instrument to overcome the credit constraint. Once the credit constraint is slack, we then use the policy tools studied in the previous subsection to achieve the efficient allocation.

Consider the case of $\kappa < \kappa^*$, where κ^* is defined in (66). Suppose that the government can make direct lending D_t at the nominal interest rate R_{ft} to the young entrepreneur. Suppose that the government has better monitoring abilities than private agents so that the entrepreneur cannot hide or divert the government funds. Then the credit constraint (60) only applies to $N_t - D_t$. The government finances the loans by levying lump-sum taxes on the young saver and then makes transfer $D_t R_{ft}$ to the old saver. In this case the saver's consumption and portfolio choices are not affected.

To implement the efficient allocation along the balanced growth path, we suppose $D_t = D\bar{A}_{t+1}$ and set D at a value higher than the expression below:

$$\left[\frac{(1+g)\,\eta_{FB}}{1+g-\kappa^*} - \frac{(1+g)\,\eta_{FB}}{1+g-\kappa}\right]\left[\lambda + \frac{\beta R_f^* - 1}{\beta R_f^*}\frac{1-\lambda}{1+\beta+\gamma}\frac{\gamma\,(1+g)}{1+g-\beta}\right].$$

Then the constraint (61) is slack at the equilibrium allocation and interest rate R_f^* described in Proposition 5. Now we can apply the analysis in Section 5.1 and Proposition 5 to achieve the efficient allocation.

6 Conclusion

An important feature of developing countries is that credit markets are imperfect due to reasons such as weak contract enforcement, weak creditor protection, and agency issues. We follow AHM (2005) and incorporate this feature into a Schumpeterian overlapping-generations model of economic growth to explain convergence and divergence. Our contribution is to introduce money and study how monetary and fiscal policies can achieve efficient allocation in a market equilibrium. We find that how money increments are transferred to agents is important for their long-run impact on economic growth. When money increments are transferred to agents in an amount proportional to their pre-transfer holdings, money is super-neutral. For a sufficiently low level of financial development, the economy can enter a poverty trap with low economic growth and high inflation. When money increments are transferred to young entrepreneurs, to whom money is most needed, it is not super-neutral. Monetary policy affects the real economy through a redistribution channel. The government should first improve credit market conditions so that entrepreneurs are not credit constrained. Then there is a combination of monetary and fiscal policies such that the economy can avoid the poverty trap and achieve efficient allocation. In this case the economy will grow at a faster rate for some period of time and then gradually converge to the same rate as the world frontier.

One limitation of our model is that we have assumed that the world frontier technology grows at an exogenously given constant rate. In the future research, it is desirable to relax this assumption and treat the technological innovation as endogenously determined in both advanced countries and developing countries. In the new setup, a developing country with well-functioning financial market, appropriate fiscal and monetary policies, and the advantage of backwardness in technological innovation, may achieve absolute convergence and become an advanced country.

Appendix

A Proofs

Proof of Proposition 1: See the analysis in the main text. It remains to prove that condition (27) is satisfied. We leave it to the proof of Proposition 2 next. Note that even though the solution that $\mu = n = a = 0$ and

$$R_f = \frac{1+z}{1+g} F'(0)\,\psi$$

satisfies equations (23), (24), and (25), we can check that this solution violates condition (27) and hence is not an equilibrium. Q.E.D.

Proof of Proposition 2: In the main text, we have shown that there is a unique solution to (33) such that condition (34) is satisfied. From Figure 1, we can see that the unconstrained equilibrium interest rate is higher than the constraint one. Thus (34) and hence (27) must hold for the unconstrained equilibrium. If $\kappa^{**} < \kappa < \min\{\kappa^*, \bar{\kappa}\}$, then the unconstraint equilibrium derived in Proposition 1 violates the credit constraint and condition (36) is satisfied. For (32) to hold, we need

$$\psi F'(n^{**}) > \frac{R_f^{**}}{\Pi}.$$

Since (24) holds at n^* and R_f^* and since $n^{**} < n^*$ and $R_f^{**} < R_f^*$, the above condition follows from the concavity of F. The rest of the proof follows from the analysis in the main text using Figure 1. In particular, an increase in κ raises the nominal interest rate R_f^{**} and hence raises n^{**} by (23). It also raises the corresponding a^{**}. But there is no growth effect because the economy still grows at the rate g. Q.E.D.

Proof of Proposition 3: If $0 < \kappa < \kappa^{**}$, then condition (36) is violated and the poverty trap equilibrium is the only steady state. The following algebra shows that the productivity growth will converge to a rate between 0 and $1+g$ for $\kappa < \kappa^{**}$:

$$\begin{aligned}
\lim_{t\to\infty} \frac{A_{t+1}}{A_t} &= (1+g) \lim_{t\to\infty} \frac{a_{t+1}}{a_t} = (1+g) \lim_{t\to\infty} \left(\frac{\mu_t}{a_t} + \frac{1-\mu_t}{1+g} \right) \\
&= (1+g) \lim_{t\to\infty} \frac{F(n_t)}{a_t} + 1 = (1+g) \lim_{a\to 0} \frac{F(n)}{a} + 1 \\
&= (1+g) F'(0) \frac{\partial n}{\partial a}\Big|_{a\to 0} = F'(0) \frac{(1-\alpha)\,\zeta\lambda\beta R_f/\Pi}{\beta R_f/\Pi - \kappa} + 1 \\
&< F'(0)\,\Phi'(0)\, g + 1 = g + 1,
\end{aligned}$$

where we have used equations (23) and (33) and $\mu_t \to 0$ to derive the second last equality. The last inequality holds because condition (36) is violated. It follows from (37) that $\hat{\Pi} > (1+z)/(1+g)$.

We modify Figure 2 to show the existence of the unique solution for $\hat{\Pi}$ and $\hat{R}_f$. Now the horizontal axis shows the real interest rate R_f/Π instead of the nominal interest rate R_f. The borrowing-limit curve still describes the expression on the right-hand side of equation (33) as a decreasing function of R_f/Π. The supply curve describes the expression on the left-hand side of (33), which is written as

$$\begin{aligned} & \lambda + \frac{1-\lambda}{1+\beta+\gamma}\left[\beta - \frac{\gamma}{R_f/(1+z)-1}\right] \\ = \; & \lambda + \frac{1-\lambda}{1+\beta+\gamma}\left[\beta - \frac{\gamma}{\frac{R_f}{\Pi}\frac{\Pi}{1+z}-1}\right]. \end{aligned}$$

By (37), Π is an increasing function of R_f/Π. Thus the above expression increases with R_f/Π. There is a unique intersection point between the borrowing-limit and supply curves, which determines the equilibrium real interest rate. Then $\hat{\Pi}$ and $\hat{R}_f$ are determined. Q.E.D.

Proof of Proposition 4: See the main text. Q.E.D.

Proof of Proposition 5: Suppose that there is no fiscal policy. When money increments are transferred to entrepreneurs, the saver's consumption and portfolio choices are given by equations (49)-(52) with w_{Dt} replaced by w_t, where w_t is given by (15). The competitive equilibrium for a given interest rate sequence $\{R_{ft}\}$ under perfect credit markets can be summarized by a system of four difference equations (18), (19), (20), and

$$n_t = \frac{(1-\alpha)\zeta a_t}{1+g}\left[\lambda + \frac{z}{1+z}\frac{(1-\lambda)}{1+\beta+\gamma}\frac{\gamma R_{ft}}{R_{ft}-1} + \frac{(1-\lambda)}{1+\beta+\gamma}\frac{(1-\lambda)}{1+\beta+\gamma}\frac{\beta R_{ft}-\beta-\gamma}{R_{ft}-1}\right]. \tag{A.1}$$

for four sequences $\{z_t\}, \{a_t\}, \{\Pi_{t+1}\}$, and $\{n_t\}$ such that (6) and $R_{ft} > 1+\gamma/\beta$ are satisfied. In the steady state the system becomes three equations (24), (25), and

$$n = \frac{(1-\alpha)\zeta a}{1+g}\left[\lambda + \frac{z}{1+z}\frac{(1-\lambda)}{1+\beta+\gamma}\frac{\gamma R_f}{R_f-1} + \frac{(1-\lambda)}{1+\beta+\gamma}\frac{\beta R_f-\beta-\gamma}{R_f-1}\right], \tag{A.2}$$

for three unknowns n, z, and a. Let $n = n_{FB}$. Using this system we derive one equation for one unknown R_f :

$$n_{FB} = \frac{(1-\alpha)\zeta a_{FB}}{1+g}\left[\lambda + \frac{(1-\lambda)}{1+\beta+\gamma}\frac{(\gamma+\beta)(R_f-1) - \gamma\psi F'(n_{FB})/(1+g)}{R_f-1}\right],$$

where

$$a_{FB} = \frac{(1+g)F(n_{FB})}{g+F(n_{FB})}.$$

Given Assumption 2, we show that there is a unique solution, denoted by $\bar{R}_f > 1+\gamma/\beta$, to the above equation. Given $\bar{R}_f$, we can also easily show that $n = n_{FB}$ is the only equilibrium solution. Q.E.D.

Proof of Proposition 6: Given the monetary and fiscal policies described in Proposition 5, a system of four equations (25), (22), (57), and (58) determine the four equilibrium variables n, a, μ, and z. This system is

$$a = \frac{F(n)(1+g)}{g+F(n)},$$

$$\Pi = \frac{1+z}{1+g},$$

$$\frac{F'(n)}{F'(n_{FB})} = \frac{R_f^*}{\Pi}\frac{\beta}{1+g},$$

$$n = \eta\left[\lambda + \frac{z}{1+z}\frac{(1-\lambda)}{1+\beta+\gamma}\frac{\gamma R_f^*}{R_f^*-1} + \frac{1-\lambda}{1+\beta+\gamma}\frac{\beta R_f^*-\beta-\gamma}{R_f^*-1}\right].$$

We can simplify this system to one equation for n :

$$\frac{n}{\eta} = \lambda + \left[1 - \frac{F'(n)}{\beta R_f^* F'(n_{FB})}\right]\frac{(1-\lambda)}{1+\beta+\gamma}\frac{\gamma R_f^*}{R_f^*-1} + \frac{1-\lambda}{1+\beta+\gamma}\frac{\beta R_f^*-\beta-\gamma}{R_f^*-1}, \qquad \text{(A.3)}$$

where

$$\eta = \frac{1+\beta+\gamma}{3(1-\lambda)}\left[(1-\alpha)\left[\frac{\alpha^{\frac{\alpha}{1-\alpha}}F(n)}{1+g} + (1-F(n))\frac{\left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}}F(n)}{(1+g)(g+F(n))}\right] - (1+g)\,n\right].$$

We can easily check that $n = n_{FB}$ is a solution to equation (A.3). We next show that this is only solution. Since $F(n)$ is concave and $F(0) = 0$, we can easily show that $F(n)/n$ decreases with n. Thus η/n decreases with n or n/η increases with n. We also know that the expressions on the right-hand side of (A.3) increase with n. Two monotonic curves can only have one intersection point if there is any. Thus there is a unique solution $n = n_{FB}$ to equation (A.3).

We can then verify that the solution to the above system is given by

$$a = a_{FB}, n = n_{FB}, z = \beta R_f^* - 1, \Pi = \frac{\beta R_f^*}{1+g}.$$

Since the above system has a unique solution, the preceding solution is the only steady-state equilibrium. We can also verify that at this equilibrium the credit constraint does not bind. Q.E.D.

B Data Description

For Figure 1 we follow Levine, Loayza, and Beck (2000) and AHM (2005) and consider cross-sectional data on 71 countries over the period 1960–1995. As in their papers, we use private credit, defined as the value of credits by financial intermediaries to the private sector, divided by GDP, as our preferred measure of financial development. We construct this measure using the updated 2017 version of the Financial Development and Structure Database. We have also used other measures of financial development and the pattern in Figure 1 does not change. We construct the average per capita GDP growth rates using the Penn World Table and construct the average inflation rates and the average (broad) money growth rates using the World Bank WDI database. We delete outliers with average inflation rates higher than 40%, but the pattern in Figure 1 still holds for the full sample. The outliers are Argentina, Bolivia, Brazil, Chile, Israel, Peru, and Uruguay. The non-convergence countries used in Panel D of Figure 1 are identified according to Table II of AHM (2005).

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