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RESEARCH ARTICLE

The perils of credit booms

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Abstract We present a dynamic general equilibrium model of production economies with adverse selection in the financial market to study the interaction between funding liquidity and market liquidity and its impact on business cycles. Entrepreneurs can take on short-term collateralized debt and trade long-term assets to finance investment. Funding liquidity can erode market liquidity. High funding liquidity discourages firms from selling their good long-term assets since these good assets have to subsidize lemons when there is information asymmetry. This can cause a liquidity dry-up in the market for long-term assets and even a market breakdown, resulting in a financial crisis. Multiple equilibria can coexist. Credit booms combined with changes in beliefs can cause equilibrium regime shifts, leading to an economic crisis or expansion.

Keywords Adverse selection · Liquidity · Collateral · Bubbles · Credit boom · Financial crises · Multiple equilibria

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1 Introduction

The goal of this paper is to study the interaction between funding liquidity and market liquidity and its impact on business cycles. Following Brunnermeier and Pedersen (2009), we define funding liquidity as the ease with which entrepreneurs can borrow, and market liquidity as the ease with which they can obtain funds through trading of assets. Our key idea is that funding liquidity can erode market liquidity in the presence of information asymmetries and adverse selection in the financial markets (Akerlof 1970). Allowing for more short-term borrowing backed by collateralized real assets raises funding liquidity, which alleviates resource misallocation by providing more efficient firms with more liquidity for investment and production. High funding liquidity, however, reduces the need for liquidity from long-term assets and is costly when adverse selection exists in the market for long-term assets. High funding liquidity discourages firms from selling their good long-term assets since good assets have to subsidize lemons. This can cause a liquidity dry-up in the market for long-term assets and even a market breakdown, resulting in a financial crisis.

To formalize our idea, we build an infinite-horizon general equilibrium model of production economies in which there is a continuum of entrepreneurs subject to idiosyncratic investment efficiency shocks. Entrepreneurs are borrowing constrained and can use their physical capital as collateral to borrow (Kiyotaki and Moore 1997). They can also trade two types of long-term assets to finance real investment. One type is a lemon, which is intrinsically useless and does not deliver any payoffs. The other is a good asset and can deliver positive payoffs. Sellers know the quality of the assets but buyers do not.

In the benchmark model under symmetric information, equilibrium is unique and lemons are not traded. Although funding liquidity competes away some market liquidity, the total liquidity still rises so that a credit boom always leads to an economic expansion. By contrast, when there is asymmetric information about asset quality, three types of equilibrium can arise. In a *pooling equilibrium*, both good assets and lemons are traded at a positive pooling price. In a *bubbly lemon equilibrium*, lemons are traded at a positive price and drive the good assets out of the market. In a *frozen equilibrium*, the market for long-term assets breaks down. These equilibria can be ranked in a decreasing order of steady-state capital stock.

We show that in some region of the parameter space all three types of equilibrium can coexist depending on people's self-fulfilling beliefs. In this region neither the payoff (fundamentals) of the good assets nor the proportion of lemons can be too high or too low. If the fundamentals of the good assets are too weak, holders of the good assets will prefer to sell them to finance real investment when a sufficiently high investment efficiency shock arrives, instead of holding them to obtain low payoffs. Thus a bubbly lemon equilibrium cannot exist. But if the fundamentals of the good assets are too strong, holders of these assets will prefer to hold on to them, and these assets will never get traded. Thus a pooling equilibrium cannot exist. On the other hand, if the proportion of lemons is too low, then the pooling price of the good assets will be high enough for entrepreneurs with high investment efficiency to sell these assets to finance their real investment. Thus a bubbly lemon equilibrium cannot exist. By contrast, if the proportion of lemons is too high, then the adverse selection problem will be so severe that sellers are unwilling to sell their good assets at a low pooling price. Thus a pooling equilibrium cannot exist. In this case intrinsically useless lemons drive the good assets out of the market.

The mechanism for the coexistence of the three types of equilibrium is as follows. When all agents optimistically believe that the asset price is high, entrepreneurs with sufficiently high investment efficiency shocks will want to sell their good assets to finance investment. This raises the proportion of good assets in the market and hence raises the asset price, supporting the initial optimistic belief about the asset price. A pooling equilibrium can arise. On the other hand, when all agents pessimistically believe that the asset price is sufficiently low, all entrepreneurs will not sell their good assets, but sell lemons only. In this case the market will consist of lemons only. Entrepreneurs with low investment efficiency shocks are willing to buy lemons at a low positive price because they expect to sell lemons at a high positive price in the future to finance investment when they are hit by a sufficiently high investment efficiency shock. Then a bubbly lemon equilibrium can arise. In the extreme case, when all agents believe that the assets have no value, no assets will be traded and the financial market will break down, leading to a frozen equilibrium.

A credit boom through increased collateralized borrowing can cause a regime to shift from one type of equilibrium to another. For example, when the economy is initially at a pooling equilibrium, a sufficiently large permanent credit boom can cause a large competing effect so that no good assets will be traded due to adverse selection. The economy will enter a bubbly lemon equilibrium in which the intrinsically useless lemon is traded as a bubble asset at a positive price. Since market liquidity has dried up, a financial crisis will arise.

Even a temporary credit boom can cause a regime shift through changes in confidence or beliefs. In standard models with a unique steady state, a temporary change in parameter values will cause the economy to return to the original steady state eventually. By contrast, given that three types of equilibrium can coexist in our model, a change in confidence or beliefs can cause the economy to switch from the original equilibrium to another type of equilibrium as discussed previously. In a numerical example, we show that when the economy is initially at a good pooling steady state and when agents pessimistically believe the economy will soon switch to the bubbly lemon equilibrium in response to a temporary credit boom, market liquidity will drop discontinuously and the economy will enter a recession eventually.

Not all credit booms end in a financial crisis. When there is no regime shift, a modest credit boom can boost total liquidity and cause an economic expansion. If there is a regime shift, the expansion can be large. For example, when the economy is initially at a bubbly lemon equilibrium, a permanent modest credit boom can cause the economy to switch to a pooling equilibrium. The asset price and output will rise to permanently higher levels eventually.

Our model can help explain the phenomenon that some credit booms lead to expansions and others end in recessions. The idea that financial crises are due to credit booms gone wrong dates back to Minsky (1977) and Kindleberger (1978) and is supported by some empirical studies on emerging and advanced economies (McKinnon and Pill 1997; Kaminsky and Reinhart 1999; Reinhart and Rogoff 2009; Gourinchas and Obstfeld 2012; Schularick and Taylor 2012). However, using a sample of 61 emerging and industrial countries over the 1960–2010 period, Mendoza and Terrones (2012) find that the odds are about 1–4 that once a country enters a credit boom it will end in a currency or a banking crisis, and a little less than 1–4 that it will end in a sudden stop. Gorton and Ordoñez (2015) find that there are 87 credit booms in their sample of 34 countries over 1960–2010, of which 33 ended in financial crises. Our model suggests that the interaction between funding liquidity and market liquidity under adverse selection is useful to understand the preceding evidence.

Our paper is closely related to the one by Gorton and Ordoñez (2015) who also study the question of why some credit booms result in financial crises while others do not. Unlike us, they build an overlapping-generations model with adverse selection in which borrowers and lenders have asymmetric information about the collateral quality. Firms finance investment opportunities with short-term collateralized debt. If agents do not produce information about the collateral quality, a credit boom develops, accommodating firms with lower quality projects and increasing the incentives of lenders to acquire information about the collateral, eventually triggering a crisis. When the average quality of investment opportunities also grows, the credit boom may not end in a crisis because the gradual adoption of low-quality projects is not strong enough for lenders to acquire information about the collateral.

Our idea that funding liquidity can erode market liquidity is related to Malherbe (2014). He builds a three-date adverse selection model of liquidity in which cash holding by some agents imposes a negative externality on others because it reduces future market liquidity. The intuition for why holding cash worsens adverse selection is best understood from a buyer's point of view: the more cash a seller is expected to have on hand, the less likely it is that he is trading to raise cash, and the more likely it is that he is trying to pass on a lemon. The impact of funding liquidity in our paper is like that of cash holding in his paper, but our model is very different from his. In particular, Malherbe (2014) assumes risk aversion for the agents to make optimal portfolio choice decisions, while we do not need risk aversion. Moreover, his model admits two types of equilibria, while ours admits three types.

Brunnermeier and Pedersen (2009) argue that funding liquidity and market liquidity are mutually reinforcing in an endowment economy when margin requirements are endogenously determined by the value-at-risk control. Unlike our paper, they do not consider real investment and short-term debt backed by collateralized real assets. As in their paper, we show that liquidity can be fragile because market liquidity can drop discontinuously due to equilibrium regime shifts.

More broadly, our paper is related to the recent literature that uses adverse selection models to explain financial crises and business cycles.[1] Kurlat (2013) provides a dynamic model with adverse selection in which firms are allowed to accumulate capital only and cannot trade other types of assets. A fraction of capital can become useless lemons. Sellers know the quality of capital, but buyers do not. In his model there can

[1] Our paper is also related to the large literature that studies business cycles with credit market frictions (Kiyotaki and Moore 1997; Carlstrom and Fuerst 1997; Bernanke et al. 1999).

be only two types of equilibrium: either capital is traded at a positive price or there is no trade at all. One key difference between Kurlat (2013) and our paper is that the former shuts down the channel of funding liquidity and focuses on the effect of adverse selection on market liquidity, while our paper incorporates trades in short-term and long-term assets and studies the interaction between funding and market liquidity under information asymmetry. Bigio (2015) studies an economy where asymmetric information about the quality of capital endogenously determines liquidity. He presents a theory where liquidity-driven recessions follow from surges in the dispersion of collateral quality.

Our paper is also related to Guerrieri and Shimer (2014) who study dynamic adverse selection in asset markets.[2] Their model has a unique equilibrium in which better quality assets trade at higher prices but with a lower price-dividend ratio in less liquid markets. They also study how asset purchase and subsidy programs may raise prices and liquidity and reverse the flight to quality. But their model does not have real investment and production and does not study the impact of credit booms on the real economy, which is the focus of our paper.[3]

As intrinsically useless lemons can have a positive price in our model, our paper is related to the literature on rational bubbles. Since the seminal study by Santos and Woodford (1997), it has been widely believed that it is hard to generate rational bubbles in competitive models with infinitely lived agents. Recently, there has been a growing literature that introduces borrowing constraints to study bubbles in infinite-horizon models with production.[4] This literature does not resolve the coexistence puzzle, i.e., why bubbles like fiat money can coexist with interest-bearing assets. In our model the intrinsically useless lemons can coexist with good assets with positive payoffs in a pooling equilibrium due to adverse selection. This is related to some recent papers in the search and monetary economics literature [see, e.g., Williamson and Wright (1994), Lester et al. (2012), Li et al. (2012), and the survey by Lagos et al. (2017)].

2 The model

Consider a discrete-time infinite-horizon model based on Kiyotaki and Moore (2008). The economy is populated by a continuum of identical workers with a unit measure and a continuum of ex ante identical entrepreneurs with a unit measure. Each entrepreneur runs a firm that is subject to idiosyncratic shocks to its investment efficiency, so entrepreneurs are ex post heterogeneous. There is no aggregate uncertainty about fundamentals. Assume that a law of large numbers holds so that aggregate variables are deterministic.

[2] Guerrieri et al. (2010) combine search frictions and adverse selection in a static model.

[3] Other related papers include Eisfeldt (2004), Tomura (2012), Gorton and Ordoñez (2014), Benhabib et al. (2014), Li and Whited (2014), and House and Masatlioglu (2015), among others.

[4] See, e.g., Zhao (2015), Miao et al. (2016), and Ikeda and Phan (2016). Also see Miao (2014) for a survey.

2.1 Setup

Each worker supplies one unit of labor inelastically. For simplicity, we assume that workers have no access to financial markets, and thus they simply consume their wage income in each period. Entrepreneurs are risk neutral and indexed by $j \in [0, 1]$. Entrepreneur j derives utility from a consumption stream $\{C_{jt}\}$ according to

$$\sum_{t=0}^{\infty} \beta^t C_{jt}, \ C_{jt} \geq 0, \tag{1}$$

where $\beta \in (0, 1)$ represents the common subjective discount factor. He owns a constant-returns-to-scale technology to produce output according to

$$y_{jt} = Ak_{jt}^{\alpha} n_{jt}^{1-\alpha}, \ \ \alpha \in (0, 1), \tag{2}$$

where A, k_{jt}, and n_{jt} represent productivity, capital input, and labor input, respectively.

Solving the static labor choice problem

$$R_{kt} k_{jt} \equiv \max_{n_{jt} \geq 0} \ Ak_{jt}^{\alpha} n_{jt}^{1-\alpha} - W_t n_{jt}$$

gives labor demand

$$n_{jt} = \left[\frac{(1-\alpha) A}{W_t} \right]^{\frac{1}{\alpha}} k_{jt}, \tag{3}$$

and the capital return

$$R_{kt} = \alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{W_t} \right]^{\frac{1-\alpha}{\alpha}}, \tag{4}$$

where W_t is the wage rate.

Entrepreneur j can make investment i_{jt} to raise his capital stock so that the law of motion for his capital is given by

$$k_{jt+1} = (1-\delta) k_{jt} + i_{jt} \varepsilon_{jt}, \tag{5}$$

where $\delta \in (0, 1)$ represents the depreciation rate and ε_{jt} represents an investment efficiency shock that is independent across firms and over time. Let the cumulative distribution function of ε_{jt} be F on $[\varepsilon_{\min}, \varepsilon_{\max}] \subset [0, \infty)$. Assume that there is no insurance market against the idiosyncratic investment shock ε_{jt} and that investment is irreversible at the firm level so that $i_{jt} \geq 0$.

Entrepreneurs cannot trade physical capital due to its illiquidity, but can trade two types of financial assets. First, they can borrow or save by trading a one-period risk-free bond with zero net supply. Let R_{ft} denote the market interest rate. Second, they can trade long-term assets, which can be of high or low quality. The high quality asset, called the good asset, delivers a positive payoff c in every period. One may interpret this asset as a console bond with coupon payment c or land with rents c. The low-quality asset, called lemon, does not deliver any payoff. It may represent a toxic asset or useless land. The proportion of lemons in the economy is π. Assume that sellers know the quality of their own assets, but buyers cannot distinguish between the lemons and the good assets. Moreover, no one who owned an asset previously remembers it. This assumption is similar to that in Guerrieri and Shimer (2014) and simplifies the analysis. Due to this information asymmetry, assets are sold at the same price P_t.

Entrepreneur j's budget constraint is given by

$$C_{jt} + i_{jt} + \frac{b_{jt+1}}{R_{ft}} = R_{kt} k_{jt} + P_t \left(s_{jt}^g + s_{jt}^l - x_{jt} \right) + c h_{jt}^g + b_{jt}, \tag{6}$$

where $s_{jt}^g \geq 0$, $s_{jt}^l \geq 0$, $h_{jt}^g \geq 0$, $x_{jt} \geq 0$, and b_{jt} represent the sale of the good asset, the sale of the lemon, the holdings of the good asset, the total purchase of the two assets, and the bond holdings, respectively. When $b_{jt} < (\geq) \, 0$, it is interpreted as borrowing (saving). Assume that entrepreneurs are borrowing constrained. There are many different ways to introduce borrowing constraints in the literature. We adopt the following:

$$\frac{b_{jt+1}}{R_{ft}} \geq -\mu_t k_{jt}, \tag{7}$$

where $\mu_t \in [0, 1]$. The interpretation is that entrepreneur j can use a fraction of his physical capital as collateral to borrow from other firms (Kiyotaki and Moore 1997). We allow μ_t to be time varying to capture the credit market condition. We interpret an increase in μ_t as an exogenous credit boom by relaxing credit constraints.

Because buyers do not observe the quality of the assets, their purchased assets may contain both lemons and good assets. Let Θ_t denote the fraction of good assets in the market. Then the laws of motion for the holdings of the good asset and the lemon are given by

$$h_{jt+1}^g = h_{jt}^g - s_{jt}^g + \Theta_t x_{jt}, \tag{8}$$

$$h_{jt+1}^l = h_{jt}^l - s_{jt}^l + (1 - \Theta_t) \, x_{jt}. \tag{9}$$

Entrepreneur j's problem is to choose a nonnegative sequence of $\left\{ i_{jt}, s_{jt}^g, s_{jt}^l, x_{jt}, C_{jt} \right\}$ to maximize his utility in (1) subject to (5), (6), (7), (8), (9) and the short-sales constraints

$$0 \leq s_{jt}^g \leq h_{jt}^g, \; 0 \leq s_{jt}^l \leq h_{jt}^l. \tag{10}$$

2.2 Equilibrium definition

Let $K_t = \int k_{jt} \mathrm{d}i$, $I_t = \int i_{jt} \mathrm{d}j$, $C_t = \int C_{jt} \mathrm{d}j$, and $Y_t = \int y_{jt} \mathrm{d}j$. A competitive equilibrium under asymmetric information consists of sequences of aggregate quantities $\{C_t, K_t, I_t, Y_t\}$, individual quantities $\left\{ C_{jt}, i_{jt}, s_{jt}^g, s_{jt}^l, x_{jt}, b_{jt} \right\}$, $j \in [0, 1]$, prices $\left\{ W_t, P_t, R_{kt}, R_{ft} \right\}$, and the market proportion of good assets $\{\Theta_t\}$ such that:

(i) The sequences $\left\{C_{jt},\ i_{jt}, s^g_{jt}, s^l_{jt}, x_{jt}, b_{jt}\right\}$ solve each entrepreneur j's optimization problem taking $\left\{W_t, P_t, R_{kt}, R_{ft}\right\}$ and $\{\Theta_t\}$ as given.

(ii) The sequences $\left\{n_{jt}, R_{kt}\right\}$ satisfy (3) and (22).

(iii) All markets clear,

$$\int x_{jt} \mathrm{d}j = \int \left(s^g_{jt} + s^l_{jt}\right) \mathrm{d}j, \tag{11}$$

$$\int h^l_{jt} \mathrm{d}j = \pi,\ \int h^g_{jt} \mathrm{d}j = 1 - \pi,\ \int b_{jt} \mathrm{d}j = 0, \tag{12}$$

$$\int n_{jt} \mathrm{d}j = 1,\ W_t + C_t + I_t = Y_t + (1-\pi)\, c. \tag{13}$$

(iv) The law of motion for aggregate capital satisfies

$$K_{t+1} = (1-\delta)\, K_t + \int \varepsilon_{jt} i_{jt} \mathrm{d}j. \tag{14}$$

(v) The market proportion of good assets is consistent with individual entrepreneurs' selling decisions,

$$\Theta_t = \frac{\int s^g_{jt} \mathrm{d}j}{\int s^l_{jt} \mathrm{d}j + \int s^g_{jt} \mathrm{d}j}. \tag{15}$$

3 Symmetric information benchmark

Before deriving solutions to our model with information asymmetry, we first consider a benchmark with symmetric information. Suppose that both the buyers and sellers know the quality of the long-term assets so that there are separate prices P^g_t and P^l_t associated with the good asset and the lemon, respectively. Moreover, buyers can purchase the lemon or the good asset separately. In this case entrepreneur j's decision problem is to choose $\left\{C_{jt}, i_{jt}, s^g_{jt}, s^l_{jt}, x^g_{jt}, x^l_{jt}, b_{jt}\right\}$ to maximize (1) subject to (5), (10), and

$$\begin{aligned}
C_{jt} + i_{jt} + \frac{b_{jt+1}}{R_{ft}} &= R_{kt} k_{jt} + P^g_t \left(s^g_{jt} - x^g_{jt}\right) + P^l_t \left(s^l_{jt} - x^l_{jt}\right) + c h^g_{jt} + b_{jt}, \\
h^g_{jt+1} &= h^g_{jt} - s^g_{jt} + x^g_{jt}, \\
h^l_{jt+1} &= h^l_{jt} - s^l_{jt} + x^l_{jt}, \\
k_{jt+1} &= (1-\delta)\, k_{jt} + \varepsilon_{jt} i_{jt}, \\
\frac{b_{jt+1}}{R_{ft}} &\geq -\mu_t k_{jt}, \\
0 \leq s^g_{jt} &\leq h^g_{jt},\ 0 \leq s^l_{jt} \leq h^l_{jt},\ C_{jt}, k_{jt}, i_{jt} \geq 0,
\end{aligned}$$

where x^g_{jt} and x^l_{jt} represent the purchase of the good asset and the lemon asset, respectively.

A competitive equilibrium under symmetric information consists of sequences of aggregate quantities $\{C_t, K_t, I_t, Y_t\}$, individual quantities $\left\{C_{jt}, i_{jt}, s^g_{jt}, s^l_{jt}, x^g_{jt}, x^l_{jt}, b_{jt}\right\}$, $j \in [0, 1]$, and prices $\left\{W_t, R_{kt}, R_{ft}, P^g_t, P^l_t\right\}$ such that:

(i) The sequences $\left\{C_{jt}, i_{jt}, s^g_{jt}, s^l_{jt}, x^g_{jt}, x^l_{jt}, b_{jt}\right\}$ solve each entrepreneur j's optimization problem taking $\left\{W_t, R_{kt}, P^g_t, P^l_t\right\}$ as given.

(ii) The sequences $\left\{n_{jt}, R_{kt}\right\}$ satisfy (3) and (22).

(iii) All markets clear so that equations

$$\int x^g_{jt}\mathrm{d}j = \int s^g_{jt}\mathrm{d}j, \int x^l_{jt}\mathrm{d}j = \int s^l_{jt}\mathrm{d}j,$$

(12), and (13) hold.

(iv) The law of motion for aggregate capital satisfies (14).

The following proposition characterizes the equilibrium system under symmetric information.

Proposition 1 *In a competitive equilibrium with symmetric information, let*

$$\varepsilon^*_t = \frac{1}{Q_t} \in (\varepsilon_{\min}, \varepsilon_{\max}).$$

Then:

1. *Firms with $\varepsilon_{jt} \geq \varepsilon^*_t$ make real investment, sell all of their good assets and lemons, and exhaust their borrowing limit.*
2. *Firms with $\varepsilon_{jt} < \varepsilon^*_t$ do not invest. They are willing to buy any amount of good assets and lemons and are indifferent between borrowing and saving.*
3. *$\left(Q_t, P^g_t, P^l_t, R_{kt}, R_{ft}, K_t, I_t\right)$ satisfy*

$$Q_t = \beta\left\{(1-\delta) Q_{t+1} + R_{kt+1} + (R_{kt+1} + \mu_{t+1}) \int_{\varepsilon^*_{t+1}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*_{t+1}} - 1\right) \mathrm{d}F(\varepsilon)\right\}, \tag{16}$$

$$P^g_t = \frac{P^g_{t+1} + c}{R_{ft}}, \tag{17}$$

$$P^l_t = \frac{P^l_{t+1}}{R_{ft}}, \tag{18}$$

$$\frac{1}{R_{ft}} = \beta\left[1 + \int_{\varepsilon^*_{t+1}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*_{t+1}} - 1\right) \mathrm{d}F(\varepsilon)\right], \tag{19}$$

$$K_{t+1} = (1-\delta) K_t + \left[R_{kt}K_t + \pi P^l_t + (1-\pi)\left(P^g_t + c\right) + \mu_t K_t\right] \int_{\varepsilon^*_t}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F(\varepsilon), \tag{20}$$

$$I_t = \left[R_{kt}K_t + (1-\pi)\left(P^g_t + c\right) + \pi P^l_t + \mu_t K_t\right]\left[1 - F\left(\varepsilon^*_t\right)\right], \tag{21}$$

$$R_{kt} = \alpha A K_t^{\alpha-1}, \tag{22}$$

and the usual transversality conditions.

Here Q_t represents Tobin's marginal Q or the shadow price of capital. Equation (16) is the asset pricing equation for capital. Each firm j makes real investment if and only if its investment efficiency shock ε_{jt} exceeds an investment threshold $\varepsilon_t^* = 1/Q_t$. That is, the firm's marginal Q exceeds the investment cost $1/\varepsilon_{jt}$ in terms of consumption units. Equations (17) and (18) are the asset pricing equations for the good asset and the lemon, respectively. The lemon represents a pure bubble asset because it does not deliver any fundamental payoffs. If agents believe it will not have value in the future, $P_{t+1}^l = 0$, then it has no value today $P_t^l = 0$. Equation (19) is the asset pricing equation for the bond. In our deterministic model the discount rates for the good asset and the lemon are the same and equal to the interest rate R_{ft}. The interest rate is determined by the condition that the marginal entrepreneur is indifferent between consuming today and investing tomorrow. This condition also holds for the model with information asymmetry studied in Sect. 4.

The integral term in (16) and (19) represents the liquidity premium because capital and bonds can help the firm relax its borrowing constraints by raising its net worth. We focus on the interpretation of (19). Purchasing a unit of bonds costs $1/R_{ft}$ at time t. At time $t+1$, when the investment efficiency shock $\varepsilon_{jt+1} \geq \varepsilon_{t+1}^* = 1/Q_{t+1}$, firm j receives one unit of the payoff from the bond and then uses this payoff to finance real investment, which generates profits $\varepsilon_{jt+1} Q_{t+1} - 1$. The average profits are given by the integral term, which also represents the option value of investment in the next period. Equation (19) shows that in equilibrium the marginal cost must be equal to the marginal benefit. Equations (20) and (21) give the law of motion for capital and aggregate investment. They reflect the fact that firms can use internal funds, short-term debt, and long-term assets to finance real investment.

If there were no good asset in the model, then a lemon bubble could emerge and the bubble and bonds could coexist (Miao et al. 2015). In this case the lemon and the bond are perfect substitutes and the net interest rate on bonds must be zero ($R_f = 1$) in the steady state. However, in the presence of an asset with positive payoffs, a lemon bubble cannot exist. To see this we study the steady state and use a variable without a subscript to denote its steady-state value. We maintain the following assumption throughout the paper.

Assumption 1 Let

$$\beta \left[1 + \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon_{\min}} - 1 \right) \mathrm{d}F(\varepsilon) \right] > 1. \tag{23}$$

This assumption states that the marginal benefit from one unit of liquidity when an entrepreneur invests for all efficiency levels is larger than one. It is equivalent to $\beta E(\varepsilon) > \varepsilon_{\min}$, which is a weak restriction. In particular, it is satisfied when β is sufficiently close to 1. The following lemma will be repeatedly used.

Lemma 1 *Under Assumption 1, there exists a unique solution, denoted by* ε_b^*, *to* $\varepsilon^* \in (\varepsilon_{\min}, \varepsilon_{\max})$ *in the equation*

$$\beta\left(1+\int_{\varepsilon^*}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*}-1\right)\mathrm{d}F(\varepsilon)\right)=1. \tag{24}$$

This lemma states that there is an interior investment cutoff ε_b^* such that the marginal benefit from one unit of liquidity is exactly equal to one in the steady state. At this cutoff the steady-state interest rate is equal to one. The following proposition characterizes the steady-state equilibrium of the economy under symmetric information.

Proposition 2 *Let Assumption 1 hold. When* μ *is sufficiently small, there exists a unique steady-state equilibrium in which* $P^l = 0$,

$$R_f = R_f\left(\varepsilon^*\right) \equiv \frac{1}{\beta\left[1+\int_{\varepsilon^*}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*}-1\right)\mathrm{d}F(\varepsilon)\right]} > 1, \tag{25}$$

$$P^g = \frac{c}{R_f(\varepsilon^*)-1}, \tag{26}$$

$$K = K\left(\varepsilon^*\right) \equiv \left\{\frac{\left(\frac{1}{\beta}-1+\delta\right)\frac{1}{\varepsilon^*}-\left[\int_{\varepsilon^*}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*}-1\right)\mathrm{d}F(\varepsilon)\right]\mu}{\alpha A\left[1+\int_{\varepsilon^*}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*}-1\right)\mathrm{d}F(\varepsilon)\right]}\right\}^{\frac{1}{\alpha-1}}, \tag{27}$$

where $\varepsilon^* \in \left(\varepsilon_b^*, \varepsilon_{\max}\right)$ *is the unique solution to the equation*

$$D\left(\varepsilon^*\right) \equiv \frac{\delta K(\varepsilon^*)}{\int_{\varepsilon^*}^{\varepsilon_{\max}}\varepsilon \mathrm{d}F(\varepsilon)} - \alpha A K\left(\varepsilon^*\right)^{\alpha} - \mu K\left(\varepsilon^*\right) - (1-\pi)c = \frac{(1-\pi)c}{R_f(\varepsilon^*)-1}. \tag{28}$$

Moreover, $\frac{\partial \varepsilon^*}{\partial \mu} > 0$, $\frac{\partial K}{\partial \mu} > 0$, $\frac{\partial Y}{\partial \mu} > 0$, $\frac{\partial R_f}{\partial \mu} > 0$, *and* $\frac{\partial P^g}{\partial \mu} < 0$.

Equation (26) shows that the price of the good asset is equal to the discounted present value of dividends and the discount rate is the interest rate. Since $\varepsilon^* > \varepsilon_b^*$, the interest rate $R_f(\varepsilon^*) > 1$. Equation (27) gives the steady-state capital stock and is derived from Eq. (16) using $\varepsilon^* = 1/Q$. Since we will show later that (16) also holds under asymmetric information, the steady-state capital stock has the same functional form $K(\cdot)$. The expression on the left-hand side of (28) represents the aggregate demand $D(\varepsilon^*)$ for outside liquidity from the market for long-term assets and the expression on the right-hand side represents the aggregate supply $S(\varepsilon^*)$ of such outside liquidity. The demand comes from the investment spending net of internal profits, short-term debt backed by collateralized capital, and dividends from the good asset. The supply comes from the sale of the good asset. The existence of an equilibrium ε^* can be easily proved using Eq. (28) by the intermediate value theorem. For uniqueness we impose a sufficient condition that μ is sufficiently small so that the demand for and the supply of outside liquidity are monotonic in ε^*. For all our numerical examples studied later, we choose values of μ to ensure uniqueness.

Under symmetric information the good asset drives the bad. If the two types of assets coexisted in the steady state in the sense that $P^l > 0$ and $P^g > 0$, Eqs. (17) and (18) would imply that

$$\frac{P^g + c}{P^g} = R_f,\ 1 = R_f.$$

These two equations cannot hold at the same time whenever $c > 0$. This means that the lemon must have no value in the steady state, $P^l = 0$. Anticipating zero price in the long run, the market would not value the lemon at any time; that is, $P_t^l = 0$ for all t by Eq. (18) [see Miao et al. (2015) for a formal proof]. This result illustrates the coexistence puzzle in the literature on rational bubbles and in monetary theory.

We can measure market liquidity in two ways. First, Brunnermeier and Pedersen (2009) measure market liquidity as the difference between the market price and the fundamental value. Although the fundamental value has various different meanings in the literature, they define it as the asset value in an economy without frictions. According to their definition, market liquidity is given by $P_t^g - \frac{\beta c}{1-\beta}$. Since the fundamental value is constant, we can simply use the market price as a proxy for market liquidity. Second, we can use trading volume to measure market liquidity. Since only the good asset is traded when $\varepsilon_{jt} > \varepsilon_t^*$, trading volume is given by $(1-\pi)\left[1 - F\left(\varepsilon_t^*\right)\right]$. We can show that these two measures are positively correlated.

Turning to a comparative statics analysis in the steady state, we consider the impact of an increase in μ, which can be interpreted as a permanent credit boom. Proposition 2 shows that a permanent credit boom raises the interest rate and drives down market liquidity. Even though funding liquidity erodes market liquidity, total liquidity will rise. This improves investment efficiency and alleviates resource misallocation by raising ε^*, leading to increased output and investment.

We close this section by analyzing the transition dynamics of an unexpected permanent credit boom using a numerical example.[5] We do not intend to match data quantitatively and set parameter values as follows: $\beta = 0.97$, $\alpha = 0.38$, $A = 1$, $\delta = 0.15$, $\pi = 0.25$, and $c = 0.06$. We also set F as the uniform distribution over $[0, 1]$. As shown in Fig. 1, a credit boom through an increase in μ_t improves investment efficiency by raising ε_t^*, which lowers the liquidity premium, and therefore the market liquidity (P_t^g) decreases. Meanwhile, since people have rational expectations about the unique steady state in which $P^l = 0$, there is no belief supporting a positive sequence for P_t^l in the transition dynamics. Moreover, the credit expansion drives up the total liquidity, which boosts investment, output and consumption in the long run. Consumption drops initially because investment jumps on impact, but total output does not change as it is determined by predetermined capital only.

[5] We use Dynare to compute all numerical examples in the paper based on the nonlinear shooting algorithm for solving deterministic dynamic models described in Adjemian et al. (2011).

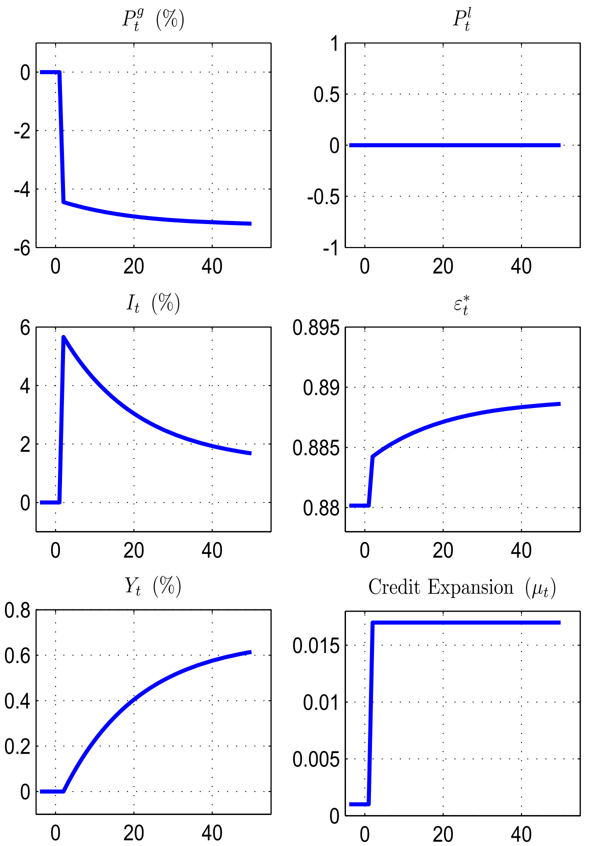

Fig. 1 Transition dynamics in response to a permanent credit shock under symmetric information when μ_t rises from 0.001 to 0.0165 from period 1 onward. The vertical axes for variables other than ε_t^* and μ_t describe percentage changes

4 Asymmetric information

When there is information asymmetry, three types of equilibrium (pooling equilibrium, bubbly lemon equilibrium, and frozen equilibrium) can arise. We will first study an entrepreneur's decision problem and then study these equilibria.

4.1 Decision problem

Suppose that lemons and good assets are traded at the pooling price $P_t > 0$. Entrepreneurs take sequences of prices $\left\{W_t, P_t, R_{ft}\right\}$ and the market proportion of good assets $\{\Theta_t\}$ as given. The following proposition characterizes their decision problems.

Proposition 3 *Under asymmetric information, in a competitive equilibrium with $P_t > 0$ for all t, let*

$$\varepsilon_t^* = \frac{1}{Q_t} \in (\varepsilon_{\min}, \varepsilon_{\max}), \ \varepsilon_t^{**} = \min\left\{\frac{p_t^g \varepsilon_t^*}{P_t}, \varepsilon_{\max}\right\} > \varepsilon_t^*,$$

where $\left\{Q_t, p_t^g, p_t^l, P_t, R_{ft}\right\}$ satisfy Eqs. (16), (19), and

$$p_t^g = \frac{c}{R_{ft}} + \beta p_{t+1}^g \left[1 + \int_{\varepsilon_{t+1}^{**}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon_{t+1}^{**}} - 1\right) \mathrm{d}F(\varepsilon)\right], \tag{29}$$

$$p_t^l = \frac{P_{t+1}}{R_{ft}}, \tag{30}$$

$$P_t = \Theta_t p_t^g + (1 - \Theta_t) p_t^l. \tag{31}$$

1. *If $\varepsilon_{jt} \geq \varepsilon_t^*$, firm j exhausts its borrowing limit to make investment, sells all its lemons ($s_{jt}^l = h_{jt}^l$), and does not buy any asset. It sells all its good assets ($s_{jt}^g = h_{jt}^g$) if $\varepsilon_{jt} \geq \varepsilon_t^{**}$, but does not sell any good assets ($s_{jt}^g = 0$) if $\varepsilon_t^* \leq \varepsilon_{jt} < \varepsilon_t^{**}$.*
2. *If $\varepsilon_{jt} < \varepsilon_t^*$, firm j does not invest, does not sell any good assets ($s_{jt}^g = 0$), sells all its lemons, is willing to purchase any amount of assets, and is indifferent between saving and borrowing.*
3. *The optimal investment rule is given by*

$$i_{jt} = \begin{cases} R_{kt} k_{jt} + P_t \left(s_{jt}^g + h_{jt}^l\right) + c h_{jt}^g + \mu_t k_{jt} + b_{jt} & \text{if } \varepsilon_{jt} \geq \varepsilon_t^* \\ 0 & \text{otherwise} \end{cases}.$$

Unlike in the symmetric information case, p_t^g and p_t^l are shadow prices of the good asset and the lemon asset, respectively, which represent the holding value of the assets. They must satisfy equilibrium restrictions (29) and (30). Both assets are traded at the common market price P_t, which is a weighted average of p_t^l and p_t^g.

The cutoff value $\varepsilon_t^* = 1/Q_t$ is the investment threshold as in the symmetric information case. Unlike in the symmetric information case, information asymmetry induces firms to sell all of their lemons for any level of efficiency shocks ε_{jt}. The reason is that the market price P_t is at least as high as the shadow price p_t^l of the lemon. Equation (30) shows that p_t^l is equal to the future selling price P_{t+1} discounted by the interest rate R_{ft}.

Because P_t is also not higher than the shadow price p_t^g of the good asset, firms will not sell the good asset unless there are other benefits from selling in addition to the price. The extra benefits come from profits generated by funded additional real investment. The total benefits are given by $Q_t \varepsilon_t P_t = P_t \varepsilon_t / \varepsilon_t^*$. When these benefits exceed the shadow price p_t^g, the firm will sell the good asset. This gives the second cutoff ε_t^{**} given in the proposition. The right-hand side of (29) reflects dividends c and the total benefit from selling the good asset in the next period. Note that it is possible that $p_t^g \varepsilon_t^* / P_t \geq \varepsilon_{\max}$ or $\varepsilon_t^{**} = \varepsilon_{\max}$. In this case no firm will sell any good asset so that no good asset will be traded in the market. We will analyze this case in the next subsection.

Next we analyze an entrepreneur's decision problem when the market for long-term assets breaks down. In this case firms can use internal funds, short-term debt, and payoffs from the good asset to finance real investment. Since long-term financial assets are not traded in a frozen equilibrium, $h_{jt}^{g} = h_{j0}^{g}$ for all t.

Proposition 4 *In a competitive equilibrium in which the market for long-term assets breaks down, let*

$$\varepsilon_t^* = \frac{1}{Q_t} \in (\varepsilon_{\min}, \varepsilon_{\max}),$$

where Q_t satisfies (16)*. Then the optimal investment rule is given by*

$$i_{jt} = \begin{cases} R_{kt}k_{jt} + ch_{jt}^{g} + \mu_t k_{jt} + b_{jt} & \text{if } \varepsilon_{jt} \geq \varepsilon_t^* \\ 0 & \text{otherwise} \end{cases}.$$

4.2 Bubbly lemon equilibrium

Now we impose the market-clearing conditions and derive the equilibrium system when only lemons are traded in the market. This happens when $\varepsilon_t^{**} = \varepsilon_{\max}$ and hence $\Theta_t = 0$ and $P_t = p_t^{l}$.

Proposition 5 *The dynamical system for a bubbly lemon equilibrium is given by eight Eqs.* (16), (19), (22)*, $\varepsilon_t^* = 1/Q_t$, and*

$$P_t = \frac{P_{t+1}}{R_{ft}}, \tag{32}$$

$$p_t^{g} = \beta p_{t+1}^{g} + \frac{c}{R_{ft}}, \tag{33}$$

$$K_{t+1} = (1-\delta) K_t + [R_{kt}K_t + \pi P_t + (1-\pi) c + \mu_t K_t] \int_{\varepsilon_t^*}^{\varepsilon^{\max}} \varepsilon \mathrm{d}F(\varepsilon), \tag{34}$$

$$I_t = [R_{kt}K_t + \pi P_t + (1-\pi) c + \mu_t K_t] \left[1 - F\left(\varepsilon_t^*\right)\right], \tag{35}$$

for eight variables $\left\{Q_t, p_t^{g}, P_t, R_{kt}, R_{ft}, K_t, I_t, \varepsilon_t^*\right\}$ satisfying the restrictions

$$0 < P_t \leq \frac{\varepsilon_t^*}{\varepsilon_{\max}} p_t^{g}, \quad \varepsilon_{\min} < \varepsilon_t^* < \varepsilon_{\max}. \tag{36}$$

Once we know the eight equilibrium variables $\left\{Q_t, p_t^{g}, P_t, R_{kt}, R_{ft}, K_t, I_t, \varepsilon_t^*\right\}$, we can derive other equilibrium variables easily. Here p_t^{g} denotes the shadow price of the good asset and P_t is the market price of the lemon. When $P_t Q_t \varepsilon_{\max} = P_t \varepsilon_{\max}/\varepsilon_t^* \leq p_t^{g}$, it is not profitable even for the most efficient firm to sell the good asset to finance real investment. Thus the good asset is not traded in the market. Why can the intrinsically useless lemon asset be traded at a positive price? The reason is that firms with high investment efficiency want to sell this asset at a positive price to finance investment. Firms with low investment efficiency want to buy this asset because they believe that they can sell lemons at a positive price to finance future investment if a high investment efficiency shock arrives in the future.

To derive the existence of such an equilibrium, we first analyze the steady state. Define

$$c_H \equiv \frac{\delta K\left(\varepsilon_b^*\right)}{\int_{\varepsilon_b^*}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F\left(\varepsilon\right)} - \alpha A\left[K\left(\varepsilon_b^*\right)\right]^{\alpha} - \mu K\left(\varepsilon_b^*\right), \; c_L \equiv \frac{c_H\left(1-\beta\right)\varepsilon_{\max}}{\varepsilon_b^*}, \tag{37}$$

$$\underline{c}^B\left(\pi\right) \equiv \frac{c_H c_L}{\pi c_H + (1-\pi) c_L}, \quad \bar{c}^B\left(\pi\right) \equiv \frac{c_H}{1-\pi}, \tag{38}$$

where ε_b^* is defined in Lemma 1 and $K(\cdot)$ is defined in (27). Note that $\underline{c}^B\left(\pi\right)$ and $\bar{c}^B\left(\pi\right)$ also depend on other parameters in the model, especially μ.

Proposition 6 *Let Assumption 1 hold. If*

$$0 < \underline{c}^B\left(\pi\right) \leq c \leq \bar{c}^B\left(\pi\right), \tag{39}$$

then there exists a unique steady-state equilibrium with bubbly lemons in which the investment threshold is $\varepsilon_b^* \in (\varepsilon_{\min}, \varepsilon_{\max})$, the aggregate capital stock is $K\left(\varepsilon_b^*\right)$, the interest rate $R_f = 1$, the shadow price of the good asset is given by

$$p^g = \frac{c}{1-\beta}, \tag{40}$$

and the market price of the lemon P satisfies

$$\frac{\delta K\left(\varepsilon_b^*\right)}{\int_{\varepsilon_b^*}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F\left(\varepsilon\right)} - \alpha A K\left(\varepsilon_b^*\right)^{\alpha} - \mu K\left(\varepsilon_b^*\right) - (1-\pi)\, c = \pi P. \tag{41}$$

The intuition for condition (39) is as follows. If $0 < c < \underline{c}^B\left(\pi\right)$, then the fundamentals of the good asset are too weak so that its shadow price (or holding value) is too low. Thus it is more profitable for firms with sufficiently high investment shocks to sell the good asset to finance real investment. This means that the good asset will be traded in the market and the bubbly lemon equilibrium cannot exist. On the other hand, if $c > \bar{c}^B\left(\pi\right)$, then firms can use the payoffs c from the good asset to finance real investment, and there is no room for the emergence of a lemon bubble to finance real investment.

Since the lemon asset does not have any payoff, the interest rate in the bubbly lemon steady state must be exactly equal to one by (32). In this case the investment cutoff is equal to ε_b^* derived in Lemma 1. Equation (40) states that the shadow price of the good asset is equal to the present value of dividends discounted by the subjective discount factor β. Equation (41) states that the demand for outside liquidity from the market for long-term assets is equal to the liquidity provided by the lemon asset only because the good asset is not traded.

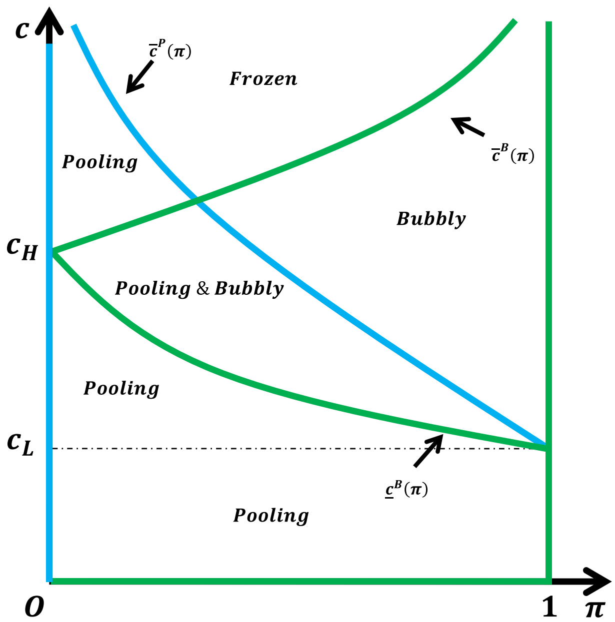

Fig. 2 Three types of equilibrium

Consider the impact of the parameter π, holding c as well as other parameters constant. If the proportion π of lemons is too low, then the price of the financial assets will be high enough for firms with high investment efficiency to sell their good assets to finance their real investment. Thus a bubbly lemon equilibrium cannot exist and a pooling equilibrium may arise.

Figure 2 illustrates the region of the parameters for the existence of a bubbly lemon equilibrium. We can easily show that $\bar{c}^B(\pi)$ is an increasing function of π on [0, 1] and $\bar{c}^B(0) = c_H$ and $\lim_{\pi \to 1} \bar{c}^B(\pi) = \infty$. But $\underline{c}^B(\pi)$ is a decreasing function of π on [0, 1] and $\underline{c}^B(0) = c_H$ and $\lim_{\pi \to 1} \underline{c}^B(\pi) = c_L$. In addition, $\bar{c}^B(\pi) > \underline{c}^B(\pi)$ for $\pi \in (0, 1]$. A unique bubbly lemon equilibrium exists for parameter values of (π, c) in the region between the lines $c = \bar{c}^B(\pi)$ and $c = \underline{c}^B(\pi)$.

4.3 Pooling equilibrium

The following proposition characterizes a pooling equilibrium.

Proposition 7 *The dynamical system for a pooling equilibrium is given by 11 Eqs.* (16), (19), (22), (29), (30), (31),

$$\varepsilon_t^* = \frac{1}{Q_t}, \quad \varepsilon_t^{**} = \frac{p_t^g}{P_t}\varepsilon_t^*,$$
$$\Theta_t = \frac{(1-\pi)\left[1 - F\left(\varepsilon_t^{**}\right)\right]}{\pi + (1-\pi)\left[1 - F\left(\varepsilon_t^{**}\right)\right]}, \tag{42}$$

$$K_{t+1} = (1-\delta) K_t + (1-\pi) P_t \int_{\varepsilon_t^{**}}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F(\varepsilon)$$
$$+ [R_{kt} K_t + \pi P_t + (1-\pi) c + \mu_t K_t] \int_{\varepsilon_t^{*}}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F(\varepsilon), \quad (43)$$
$$I_t = [R_{kt} K_t + \pi P_t + (1-\pi) c + \mu_t K_t] \left[1 - F\left(\varepsilon_t^*\right)\right]$$
$$+ (1-\pi) P_t \left[1 - F\left(\varepsilon_t^{**}\right)\right], \quad (44)$$

for 11 variables $\left\{Q_t, p_t^g, p_t^l, P_t, R_{kt}, R_{ft}, \Theta_t, K_t, I_t, \varepsilon_t^*, \varepsilon_t^{**}\right\}$ *satisfying the restrictions*

$$\varepsilon_{\min} < \varepsilon_t^* < \varepsilon_t^{**} < \varepsilon_{\max}. \quad (45)$$

In a pooling equilibrium both the good asset and the lemon are traded at the pooling price P_t to finance real investment. There is an interior threshold ε_t^{**} for selling the good asset. Thus the proportion of good assets in the market is given by (42). Equation (44) reveals that aggregate investment is financed by internal funds, the lemon, and the good asset.

We now analyze the steady state of a pooling equilibrium. We will first prove the existence of the two steady-state thresholds ε^* and ε^{**}. By (29), the steady-state shadow price of the good asset is given by

$$p^g\left(\varepsilon^*, \varepsilon^{**}\right) \equiv \frac{\beta\left(1 + \int_{\varepsilon^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*} - 1\right) \mathrm{d}F(\varepsilon)\right)}{1 - \beta\left(1 + \int_{\varepsilon^{**}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^{**}} - 1\right) \mathrm{d}F(\varepsilon)\right)} c. \quad (46)$$

By the definition of ε^* and ε^{**}, the pooling price is given by

$$P\left(\varepsilon^*, \varepsilon^{**}\right) = \frac{\varepsilon^*}{\varepsilon^{**}} p^g\left(\varepsilon^*, \varepsilon^{**}\right). \quad (47)$$

Equation (30) in the steady state gives

$$p^l\left(\varepsilon^*, \varepsilon^{**}\right) = \beta\left[1 + \int_{\varepsilon^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*} - 1\right) \mathrm{d}F(\varepsilon)\right] P\left(\varepsilon^*, \varepsilon^{**}\right). \quad (48)$$

Equation (31) in the steady state implies that

$$P\left(\varepsilon^*, \varepsilon^{**}\right) = \Theta\left(\varepsilon^{**}\right) p^g\left(\varepsilon^*, \varepsilon^{**}\right) + (1 - \Theta\left(\varepsilon^{**}\right)) p^l\left(\varepsilon^*, \varepsilon^{**}\right), \quad (49)$$

where it follows from (42) that

$$\Theta\left(\varepsilon^{**}\right) = \frac{(1-\pi)\left[1 - F\left(\varepsilon^{**}\right)\right]}{\pi + (1-\pi)\left[1 - F\left(\varepsilon^{**}\right)\right]}.$$

Equations (47), (48), and (49) imply that

$$1 = \Theta\left(\varepsilon^{**}\right) \frac{\varepsilon^{**}}{\varepsilon^*} + (1 - \Theta\left(\varepsilon^{**}\right))\beta\left[1 + \int_{\varepsilon^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*} - 1\right) \mathrm{d}F(\varepsilon)\right]. \quad (50)$$

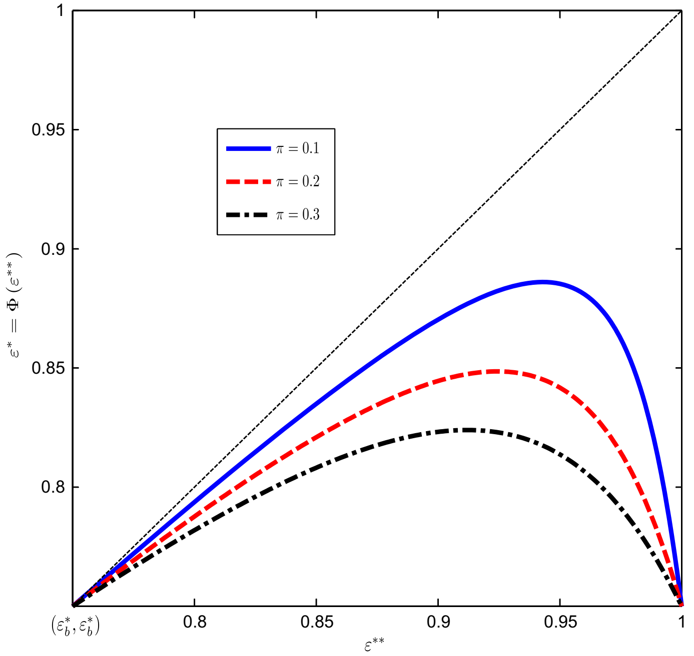

Fig. 3 A numerical illustration of $\varepsilon^* = \Phi\left(\varepsilon^{**}\right)$. We set $\beta = 0.97$ and $F(\varepsilon) = \varepsilon$ for $\varepsilon \in [0, 1]$

Lemma 2 *Let Assumption 1 hold. For any* $\varepsilon^{**} \in \left(\varepsilon_b^*, \varepsilon_{\max}\right)$*, there exists a unique solution, denoted by* $\varepsilon^* = \Phi(\varepsilon^{**})$ *, to* $\varepsilon^* \in \left(\varepsilon_b^*, \varepsilon^{**}\right)$ *in Eq.* (50).

Figure 3 illustrates the function Φ. It is not a monotonic function on $\left(\varepsilon_b^*, \varepsilon_{\max}\right)$ and satisfies the property that

$$\lim_{\varepsilon^{**} \downarrow \varepsilon_b^*} \Phi\left(\varepsilon^{**}\right) = \varepsilon_b^* = \lim_{\varepsilon^{**} \uparrow \varepsilon_{\max}} \Phi\left(\varepsilon^{**}\right).$$

Now we prove the existence of ε^{**} using a single equation. To derive this equation, we first rewrite equation (43) in the steady state as

$$D\left(\varepsilon^*\right) = S\left(\varepsilon^*, \varepsilon^{**}\right) \equiv \frac{\int_{\varepsilon^{**}}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F(\varepsilon)}{\int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F(\varepsilon)} (1-\pi) P\left(\varepsilon^*, \varepsilon^{**}\right) + \pi P\left(\varepsilon^*, \varepsilon^{**}\right), \quad (51)$$

where $D(\varepsilon^*)$ represents the demand for outside liquidity defined in Sect. 3 and $S(\varepsilon^*, \varepsilon^{**})$ represents the supply of outside liquidity. The supply comes from the sale of the good asset and the lemon. The lemon is always sold at the price $P(\varepsilon^*, \varepsilon^{**})$, but the good asset is sold only when $\varepsilon \geq \varepsilon^{**}$.

Substituting $\varepsilon^* = \Phi(\varepsilon^{**})$ into (51) and (47) yields an equation for ε^{**},

$$D\left(\Phi\left(\varepsilon^{**}\right)\right) = S\left(\Phi\left(\varepsilon^{**}\right), \varepsilon^{**}\right).$$

We can also rewrite this equation as

$$\Gamma\left(\varepsilon^{**};\pi\right)=c, \tag{52}$$

where

$$\Gamma\left(\varepsilon^{**};\pi\right) = \frac{\frac{\delta K(\Phi(\varepsilon^{**}))}{\int_{\Phi(\varepsilon^{**})}^{\varepsilon_{\max}}\varepsilon \mathrm{d}F(\varepsilon)} - \alpha A\left[K\left(\Phi\left(\varepsilon^{**},\pi\right)\right)\right]^{\alpha} - \mu K\left(\Phi\left(\varepsilon^{**}\right)\right)}{(1-\pi)+\frac{\Phi(\varepsilon^{**})}{\varepsilon^{**}}\left[\pi+(1-\pi)\frac{\int_{\varepsilon^{**}}^{\varepsilon_{\max}}\varepsilon \mathrm{d}F(\varepsilon)}{\int_{\Phi(\varepsilon^{**})}^{\varepsilon_{\max}}\varepsilon \mathrm{d}F(\varepsilon)}\right]\left[\frac{\beta\left(1+\int_{\Phi(\varepsilon^{**})}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\Phi(\varepsilon^{**},\pi)}-1\right)\mathrm{d}F(\varepsilon)\right)}{1-\beta\left(1+\int_{\varepsilon^{**}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^{**}}-1\right)\mathrm{d}F(\varepsilon)\right)}\right]}. \tag{53}$$

Proposition 8 *Let Assumption 1 hold and $c_H > 0$ where c_H is given in (37). For a sufficiently small μ and any $\pi \in (0,1)$, there exists a solution, denoted by ε_p^{**}, to $\varepsilon^{**} \in \left(\varepsilon_b^*, \varepsilon_{\max}\right)$ in Eq. (52) if and only if $0 < c < \overline{c}^P(\pi)$, where*

$$\overline{c}^P(\pi) = \max_{\varepsilon^{**}\in\left[\varepsilon_b^*,\varepsilon_{\max}\right]} \Gamma\left(\varepsilon^{**};\pi\right). \tag{54}$$

In this case a pooling steady-state equilibrium exists and the steady-state capital stock is given by $K\left(\varepsilon_p^*\right)$, where $\varepsilon_p^* = \Phi\left(\varepsilon_p^{**}\right)$.

The intuition behind this proposition is as follows. If c is close to zero, then the good asset is similar to the lemon and buyers cannot distinguish between these two types of assets. Thus both types of assets can be traded at a pooling price in equilibrium. But if c exceeds $\overline{c}^P(\pi)$, then the fundamentals of the good asset are too strong. Holders of the good asset will not want to sell it and the good asset will not be traded in the market. Thus a pooling equilibrium cannot exist and a bubbly lemon equilibrium may arise.

Figure 4 illustrates the function Γ and the determination of the equilibrium threshold ε^{**}. We can show that

$$\lim_{\varepsilon^{**}\downarrow\varepsilon_b^*} \Gamma\left(\varepsilon^{**};\pi\right)=0, \quad \lim_{\varepsilon^{**}\uparrow\varepsilon_{\max}} \Gamma\left(\varepsilon^{**};\pi\right)=\underline{c}^B(\pi). \tag{55}$$

The function $\Gamma(\varepsilon^{**};\pi)$ may not be monotonic in ε^{**}. There may be multiple solutions for ε^{**}, and hence there may exist multiple pooling equilibria, each of which corresponds to a solution for ε^{**}.

Figure 2 illustrates the existence condition in the parameter space of (π, c). When $\pi \to 1$, we must have $\Theta \to 0$, $\varepsilon^* \to \varepsilon_b^*$, and $\varepsilon^{**} \to \varepsilon_{\max}$. Thus $\Gamma(\varepsilon^{**};\pi) \to c_H$ so that $\overline{c}^P(1) = c_H$. On the other hand, when $\pi \to 0$, we must have $\Theta \to 1$ so that $\varepsilon^* = \varepsilon^{**}$. Then $\Gamma(\varepsilon^{**};\pi)$ reaches the maximum of infinity when $\varepsilon^{**} = \varepsilon_b^*$. It follows from (54) that $\overline{c}^P(0) = \infty$. A pooling equilibrium exists for parameter values of (π, c) in the region below the line $c = \overline{c}^P(\pi)$. The function $\overline{c}^P(\pi)$ is downward

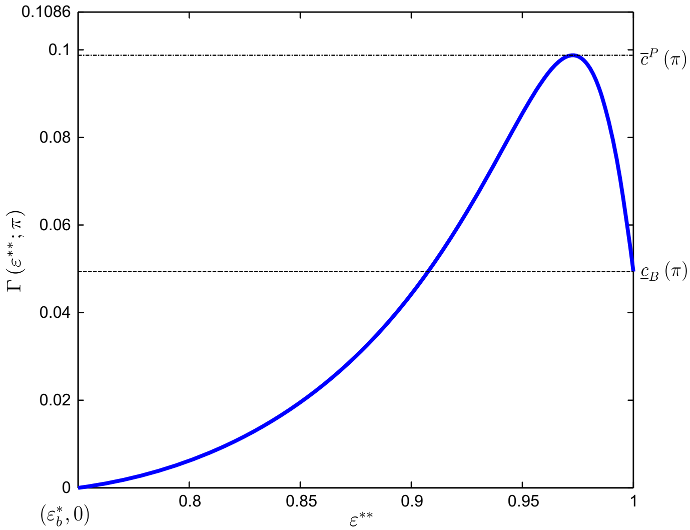

Fig. 4 A numerical illustration of $\Gamma\left(\varepsilon^{**};\pi\right)$. We set $\beta = 0.97$, $\alpha = 0.38$, $A = 1$, $\delta = 0.15$, $\pi = 0.25$, and $F(\varepsilon) = \varepsilon$ on $[0, 1]$

sloping. Holding c as well as other parameters constant, if the proportion π of lemons is too high, then the adverse selection problem will be so severe that trading the good asset as a way to subsidize the lemon would be highly discouraged. Thus a pooling equilibrium cannot exist.

4.4 Frozen equilibrium

We finally analyze the frozen equilibrium in which agents expect the asset price to be zero. Then no sellers will want to sell their assets at a zero price and no assets will be traded. The market for long-term assets will completely break down. The following proposition characterizes the equilibrium system.

Proposition 9 *The dynamical system for a frozen equilibrium is given by five Eqs.* (16)*,* (22)*,* $\varepsilon_t^* = 1/Q_t$*, and*

$$K_{t+1} = (1-\delta)K_t + [R_{kt}K_t + (1-\pi)c + \mu_t K_t] \int_{\varepsilon_t^*}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F(\varepsilon), \tag{56}$$

$$I_t = [R_{kt}K_t + (1-\pi)c + \mu_t K_t]\left[1 - F\left(\varepsilon_t^*\right)\right], \tag{57}$$

for five variables $\left\{Q_t, R_{kt}, K_t, I_t, \varepsilon_t^*\right\}$ *satisfying the restriction* $\varepsilon_t^* \in (\varepsilon_{\min}, \varepsilon_{\max})$.

The following proposition characterizes the steady state.

Proposition 10 *There exists a unique steady state for the frozen equilibrium in which the steady-state capital stock is equal to* $K\left(\varepsilon_a^*\right)$ *defined in* (27) *where* $\varepsilon_a^* \in (\varepsilon_{\min}, \varepsilon_{\max})$ *is the unique solution to* $\varepsilon^* \in (\varepsilon_{\min}, \varepsilon_{\max})$ *in the equation* $D(\varepsilon^*) = 0$*, i.e.,*

$$\frac{\delta K\left(\varepsilon^{*}\right)}{\int_{\varepsilon^{*}}^{\varepsilon_{\max}} \varepsilon \mathrm{d} F(\varepsilon)}-\alpha A K\left(\varepsilon^{*}\right)^{\alpha}-\mu K\left(\varepsilon^{*}\right)-(1-\pi) c=0. \tag{58}$$

Equation (58) shows that the supply of outside liquidity from the market for long-term assets is zero.

5 Steady-state properties

We now combine the previous analyses and present the parameter space of (π, c) for the existence of the three types of equilibrium using Fig. 2. First, we note that a frozen equilibrium always exists on the whole parameter space. Next, by (54) and (55), we have $\underline{c}^{B}(\pi) \leq \overline{c}^{P}(\pi)$. Thus the curve $c=\overline{c}^{P}(\pi)$ is always above the curve $c=\underline{c}^{B}(\pi)$. We highlight two important regions. We can see that a bubbly lemon equilibrium and a pooling equilibrium coexist when (π, c) lies in the region $\left\{(\pi, c) \mid \underline{c}^{B}(\pi) \leq c \leq \min \left(\overline{c}^{B}(\pi), \overline{c}^{P}(\pi)\right)\right\}$. But in the region $\left\{(\pi, c) \mid \overline{c}^{P}(\pi) \leq c \leq \overline{c}^{B}(\pi)\right\}$, the bad asset drives out the good one in the sense that a bubbly lemon equilibrium exists but a pooling equilibrium does not.

Proposition 11 *Let Assumption 1 hold. Suppose that the parameter values are such that the three types of equilibrium under asymmetric information coexist. Then*

$$K\left(\varepsilon_{a}^{*}\right)<K\left(\varepsilon_{b}^{*}\right)<K\left(\varepsilon_{p}^{*}\right),$$

where ε_{a}^{*}, ε_{b}^{*}, and ε_{p}^{*} denote the investment thresholds in the frozen equilibrium, bubbly lemon equilibrium, and pooling equilibrium, respectively.

The intuition behind Proposition 11 is the following. As characterized previously, the demand side for outside liquidity from the market for long-term assets is the same for all types of equilibria. What differs is the supply side. The liquidity supplied in a pooling equilibrium is larger than that supplied in a bubbly lemon equilibrium, which in turn is larger than that supplied in a frozen equilibrium. Thus the steady-state capital stock is the largest in a pooling equilibrium and the smallest in a frozen equilibrium.

We have so far characterized the steady states and their existence conditions. We now use some numerical examples to illustrate the impact of a permanent credit boom on the steady states. We illustrate the effect of μ on asset prices and output in Fig. 5. For the parameter values given in Sect. 3, all three types of steady-state equilibria coexist for $\mu \in [0, 0.016]$. There are two steady-state pooling equilibria. We will focus on the 'good' pooling steady state with a higher asset price and larger output because this steady state is stable (a saddle point) and the other is unstable. When μ rises, funding liquidity rises and imposes a negative externality on the market for the long-term asset. The asset price and market liquidity decline. The total liquidity (sum of market liquidity and funding liquidity) may not be monotonic with μ and hence real investment and output are not monotonic either. There may exist an optimal level of μ that strikes a balance between funding and market liquidity. As illustrated on the right panel of Fig. 5, the effect of μ on asset prices and output is indeed non-monotone and output is maximized at $\mu = 0.009$.

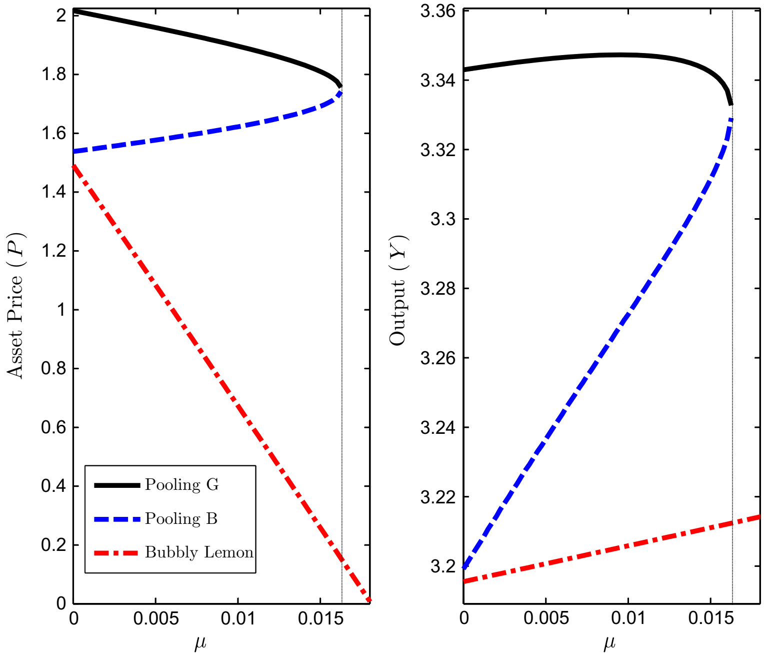

Fig. 5 The impact of μ on the steady-state asset price and output. We set $\beta = 0.97$, $\alpha = 0.38$, $A = 1$, $\delta = 0.15$, $\pi = 0.25$, $c = 0.06$, and $F(\varepsilon) = \varepsilon$ on $[0, 1]$

When $\mu \in [0.016, 0.017]$, funding liquidity is so large that entrepreneurs have no incentive to trade good assets because good assets must subsidize lemons due to adverse selection. Entrepreneurs are willing to trade lemons as a bubble because the bubble can raise their net worth and help them finance investment. In this case only the bubbly lemon steady state exists. Asset prices and output are discontinuous at $\mu = 0.016$. A small change of μ near $\mu = 0.016$ can cause an equilibrium regime shift and hence liquidity can be fragile.

When $\mu > 0.017$, entrepreneurs not only have no incentive to sell their good assets, but also have no interest in trading lemons because they have sufficient funding liquidity to finance investment and there is no need to trade lemons as a bubble asset. In this case neither the pooling steady state nor the bubbly lemon steady state can be supported and the market for long-term assets breaks down.

6 Transition dynamics

In this section we study transition dynamics when the economy moves from one steady state to another. This transition can be caused by either shifts in beliefs or shocks to fundamentals. Although financial markets are typically disrupted in recessions with trading volume and asset prices plummeting, seldom do we observe a complete market collapse. Therefore we focus on pooling and bubbly lemon equilibria. Meanwhile, since the fluctuations of financial markets are turbulent, and even seemingly discontinuous, there may exist regime switch from one type of equilibrium to another. Thus our numerical examples proceed with the coexistence parameter space defined in the previous section. We do not intend to match data, but use numerical examples to illustrate the workings of the model.

6.1 Belief-driven regime shifts

We first consider the case where a change in beliefs can cause a regime shift without any shock to fundamentals. We set the parameter values as in Sect. 3 and also $\mu = 0.001$. There are two pooling steady states as shown in Fig. 5. Suppose that the economy is initially at the good pooling steady state at $t = 1$.

Suppose that agents pessimistically believe that the economy will suddenly shift to a bubbly lemon equilibrium at $t = 1$. The change in beliefs is also unexpected. Figure 6 describes the transition dynamics from the pooling steady state to the bubbly lemon steady state. The figure shows that the asset price drops discontinuously. Entrepreneurs do not trade good assets but rather trade lemons only. Thus market liquidity declines. Real investment also drops discontinuously initially and gradually rises to a lower bubbly lemon steady-state level. Output also drops gradually to a lower bubbly lemon

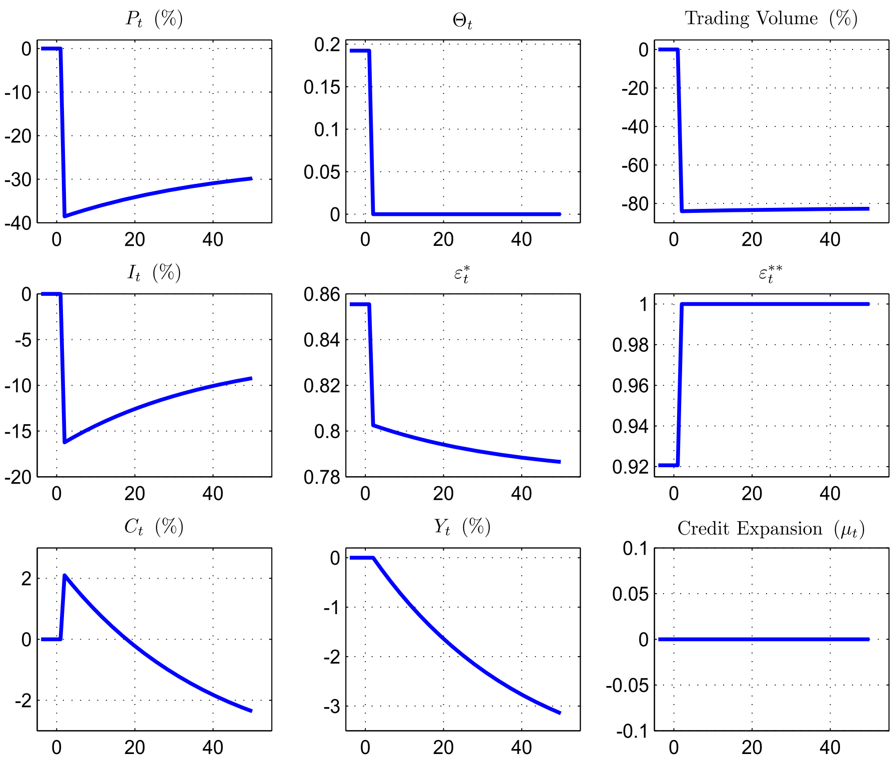

Fig. 6 Belief-driven regime shift from the good pooling steady state to the bubbly lemon steady state. We set $\beta = 0.97$, $\alpha = 0.38$, $A = 1$, $\delta = 0.15$, $\pi = 0.25$, $c = 0.06$, and $F(\varepsilon) = \varepsilon$ on [0, 1]

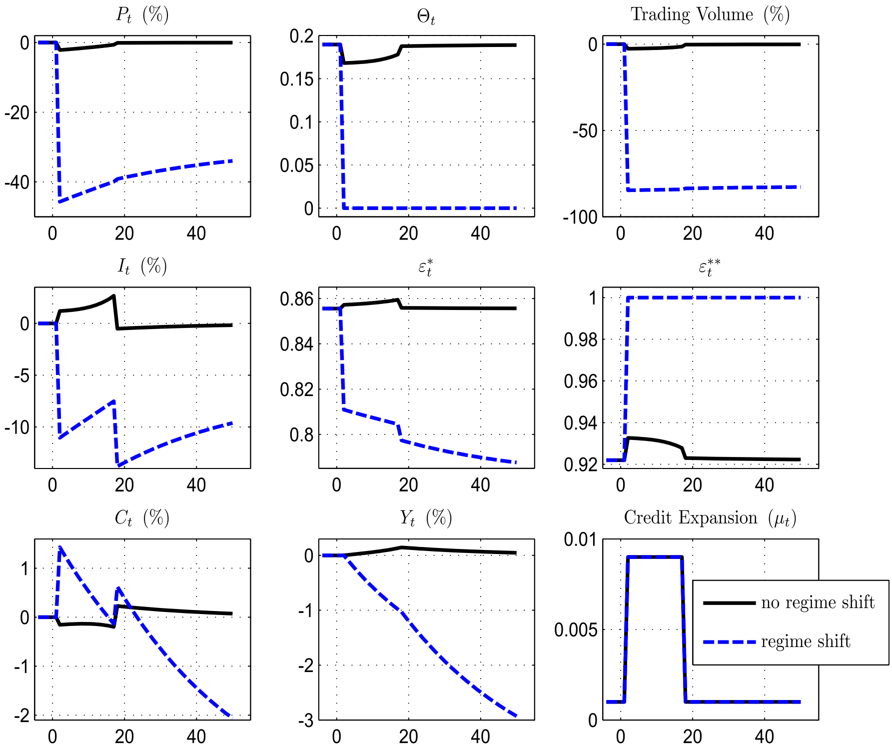

Fig. 7 The impact of a temporary credit boom. The *solid lines* describe the transition dynamics from the good pooling steady state to the same steady state. The *dashed lines* describe the transition dynamics from the good pooling steady state to the bubbly lemon steady state. We set $\beta = 0.97$, $\alpha = 0.38$, $A = 1$, $\delta = 0.15$, $\pi = 0.25$, $c = 0.06$, and $F(\varepsilon) = \varepsilon$ on [0, 1]

steady-state level. But consumption counterfactually rises on impact by the resource constraint. This is because labor is exogenously fixed and capital is predetermined. One way to fix this problem is to introduce variable labor and capacity utilization so that output can fall on impact.

This example shows that a change in beliefs without any fundamental shock can cause a liquidity dry-up and a recession. On the other hand, an optimistic belief shift can cause a boom without any fundamental shock. Suppose that the economy is initially at the bubbly lemon steady state. But people optimistically believe that the economy will switch to a pooling equilibrium immediately. When an entrepreneur optimistically believes that other entrepreneurs will trade their good assets, he is willing to do the same. Thus market liquidity increases and the asset price rises. As more entrepreneurs are willing to sell their good assets, the average asset quality in the market improves, which drives up the pooling price, and in turn justifies the initial optimistic belief. The increased market liquidity then boosts investment, accelerates capital accumulation, and eventually raises both output and consumption. The transitional dynamics look like the paths in Fig. 6 flipped at the horizontal axis.

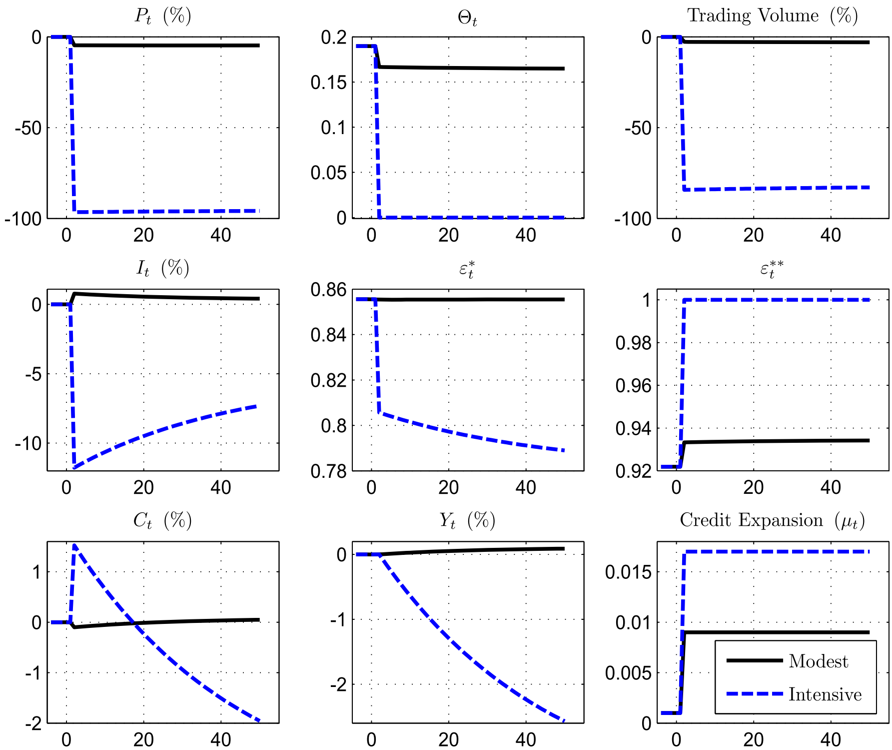

Fig. 8 The impact of a permanent credit boom. The *solid lines* describe the transition dynamics from the good pooling steady to another good pooling steady state. The *dashed lines* describe the transition dynamics from the good pooling steady state to the bubbly lemon steady state. We set $\beta = 0.97$, $\alpha = 0.38$, $A = 1$, $\delta = 0.15$, $\pi = 0.25$, $c = 0.06$, and $F(\varepsilon) = \varepsilon$ on [0, 1]

6.2 Good or bad credit booms

Now we study the impact of a change in fundamentals through a change in funding liquidity. In particular, we consider the impact of a credit boom when μ_t rises. Agents have perfect foresight about the path of μ_t. We will show that a credit boom can cause either a boom in the real economy or a financial crisis depending on agents' beliefs.

First, we consider the impact of an unexpected temporary credit boom when μ_t rises from 0.001 to 0.009 initially and lasts for 16 periods and then returns to the original level forever. Suppose that the economy is initially at the good pooling steady state and agents fully anticipate the change of μ_t. Figure 7 shows the transition dynamics. The solid lines describe the case where there is no regime shift and the economy will return to the original pooling steady state eventually.

The dashed lines describe the case where agents pessimistically believe that the economy will unexpectedly shift to the bubbly lemon equilibrium. In this case the asset price drops discontinuously and gradually returns to a lower level in the bubbly lemon steady state. Real investment also drops discontinuously, rebounds slightly, and drops again with μ_t. It then gradually rises to its lower steady-state level. Output also eventually decreases to a lower steady-state level, resulting in a crisis.

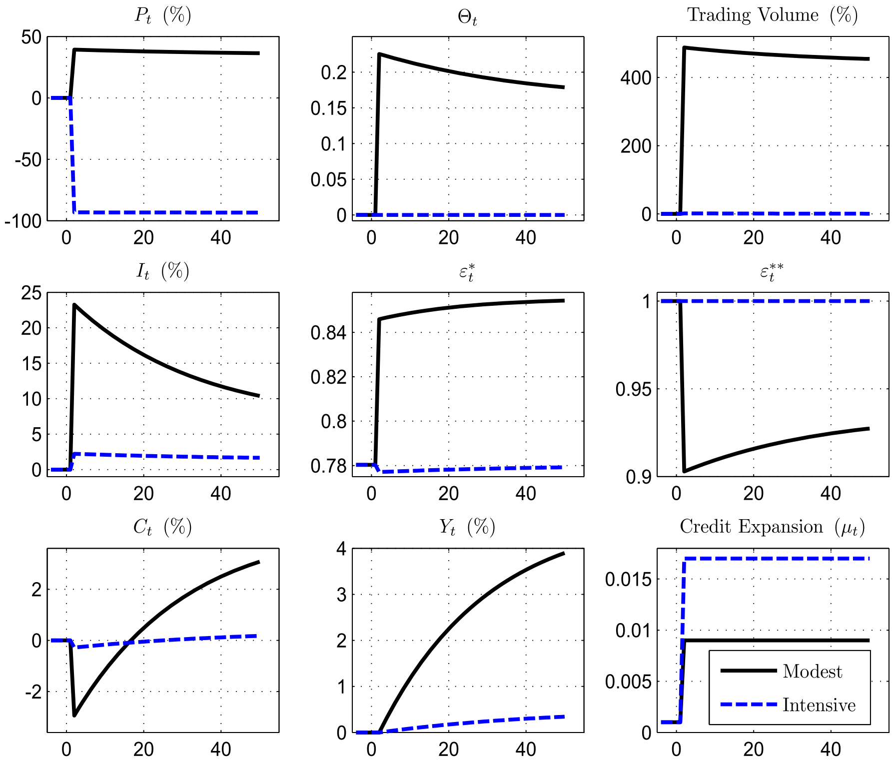

Fig. 9 The impact of a permanent credit boom. The *solid lines* describe the transition dynamics from the bubbly lemon steady state to the good pooling steady state. The *dashed lines* describe the transition dynamics from the bubbly lemon steady state from another bubbly lemon steady state. We set $\beta = 0.97$, $\alpha = 0.38$, $A = 1$, $\delta = 0.15$, $\pi = 0.25$, $c = 0.06$, and $F(\varepsilon) = \varepsilon$ on $[0, 1]$

Second, we consider the impact of an unexpected permanent credit boom. Suppose that the economy is initially at the pooling steady state. Figure 8 illustrates that a modest credit boom can lead to a boom in the real economy while a large one can lead to an economic recession. The solid lines in the figure show the transition dynamics from one good pooling steady state to another pooling steady state along a pooling equilibrium path when μ_t rises immediately from 0.001 to 0.009 and stays there forever. Even though funding liquidity reduces market liquidity, the total liquidity rises so that investment and output also rise slightly.

When μ_t rises immediately from 0.001 to 0.0165, the credit boom is so large that the pooling steady state cannot be supported, as shown in Fig. 5. Then the economy transits to a bubbly lemon steady state. The asset price and investment drop discontinuously on impact. During the transition path, asset prices, investment, and output fall, leading to a recession.

What happens if the economy is initially at the bubbly lemon steady state? Consider the impact of an unexpected permanent modest credit boom when μ_t rises immediately from 0.001 to 0.009 and stays there forever. Suppose that the agents are optimistic and the economy switches immediately from a bubbly lemon steady-state equilibrium to

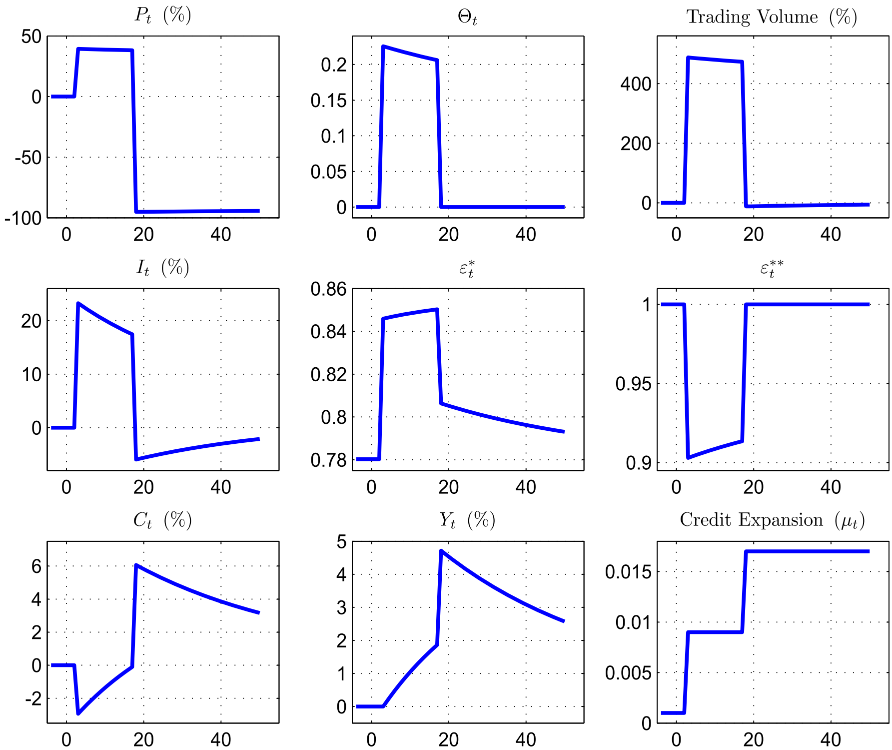

Fig. 10 Transition dynamics starting from the bubbly lemmon steady state in response to a gradual credit boom. We set $\beta = 0.97$, $\alpha = 0.38$, $A = 1$, $\delta = 0.15$, $\pi = 0.25$, $c = 0.06$, and $F(\varepsilon) = \varepsilon$ on $[0, 1]$

a pooling equilibrium. The solid lines in Fig. 9 show that asset prices, investment, and output all rise, resulting in an economic expansion.

By contrast, if the credit boom is too large (e.g., μ_t increases from 0.001 to 0.0165), then a pooling steady state can no longer be supported. Therefore a large credit boom worsens market liquidity by discouraging entrepreneurs from selling their good assets in the market, which in turn has a negative effect on investment, output, and consumption. Note that, when $\mu = 0.0165$, the bubbly lemon equilibrium and the frozen equilibrium coexist. We only consider the former equilibrium because trading volume is still positive in this case, albeit reduced significantly. This is more realistic than the frozen equilibrium.

Combining the insights from the previous numerical examples in Figs. 6 and 9 yields the richer dynamics in Fig. 10 that are closer to the empirical evidence. Suppose that the economy is initially at the bubbly lemon steady state with $\mu = 0.001$. There is a modest unexpected credit boom in that μ_t increases from 0.001 to 0.009 from $t = 1$ to $t = 16$ and then there is another unexpected large credit boom where μ_t increases from 0.009 to 0.0165 from $t = 17$ on. As shown in Sect. 5, a pooling steady-state equilibrium can be sustained at a modest level of μ_t but a large level. Suppose that people are optimistic and hence the economy immediately switches to a pooling equilibrium regime that lasts until $t = 16$. But since a pooling steady state cannot be supported at $\mu = 0.0165$, people's optimistic beliefs cannot be sustained either and the economy reverts to the bubbly lemon regime. This causes asset prices and output to fall. In summary, Fig. 10 shows that a credit boom is initially associated with a boom in asset prices, market liquidity, investment and output, but they all fall later on and a crisis follows.

7 Conclusion

We have provided a macroeconomic model with adverse selection to study the interaction between market liquidity and funding liquidity in a production economy. Our key idea is that funding liquidity can erode market liquidity. High funding liquidity discourages firms from selling their good long-term assets since these good assets have to subsidize lemons when there is information asymmetry between sellers and buyers. This can cause a liquidity dry-up in the market for long-term assets and even a market breakdown, resulting in a financial crisis. We show that three types of equilibrium can coexist. Credit booms combined with changes in beliefs can cause equilibrium regime shifts, leading to an economic crisis or expansion.

One limitation of our model is that it is stylized and cannot be confronted with data. Moreover, we have not studied policy questions because of space limitation. An important implication of our model is that economic booms and busts are not only driven by fundamentals but also self-fulfilling beliefs. How to design policies to eliminate equilibrium multiplicity is important for economic stability. We leave this topic for future research.

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Appendix

A Proofs

Proof of Proposition 1 We first consider an entrepreneur's decision problem. For ease of notation, we suppress the subscript j. Let $V_t\left(k_t, \varepsilon_t, h_t^g, h_t^l, b_t\right)$ denote the value function, where we have suppressed the aggregate state variables. Then V_t satisfies the following Bellman equation

$$V_t\left(k_t, \varepsilon_t, h_t^g, h_t^l, b_t\right) = \max C_t + \beta E_t V_{t+1}\left(k_{t+1}, \varepsilon_{t+1}, h_{t+1}^g, h_{t+1}^l, b_{t+1}\right), \qquad \text{(A.1)}$$

subject to the constraints described in Sect. 3, where the conditional expectation is taken with respect to ε_{t+1}. Conjecture that the value function V_t takes the following form:

$$V_t\left(k_t,\varepsilon_t,h_t^g,h_t^l,b_t\right)=q_t(\varepsilon_t)k_t+\phi_t^g(\varepsilon_t)h_t^g+\phi_t^l(\varepsilon_t)h_t^l+\phi_t^b\left(\varepsilon_t\right)b_t, \tag{A.2}$$

where $q_t(\varepsilon_t)$, $\phi_t^g(\varepsilon_t)$, $\phi_t^l(\varepsilon_t)$, and $\phi_t^b(\varepsilon_t)$ are to be determined.

Then V_{t+1} is also linear and we can write

$$\beta E_t\left[V_{t+1}\left(k_{t+1},\varepsilon_{t+1},h_{t+1}^g,h_{t+1}^l,b_{t+1}\right)\right]=Q_tk_{t+1}+p_t^gh_{t+1}^g+p_t^lh_{t+1}^l+p_t^bb_{t+1},$$

where we define

$$Q_t=\beta E\left[q_{t+1}(\varepsilon_{t+1})\right],\ p_t^g=\beta E\left[\phi_{t+1}^g(\varepsilon_{t+1})\right], \tag{A.3}$$

$$p_t^l=\beta E\left[\phi_{t+1}^l(\varepsilon_{t+1})\right],\ p_t^b=\beta E\left[\phi_{t+1}^b\left(\varepsilon_{t+1}\right)\right]. \tag{A.4}$$

We use the flow-of-funds constraint and other constraints in Sect. 3 to derive

$$\begin{aligned}
&C_{jt}+\beta E_tV_{t+1}\left(k_{t+1},\varepsilon_{t+1},h_{t+1}^g,h_{t+1}^l,b_{t+1}\right)\\
&\quad=R_{kt}k_t+P_t^g\left(s_t^g-x_t^g\right)+P_t^l\left(s_t^l-x_t^l\right)+ch_t^g+b_t\\
&\quad\quad-i_t-\frac{b_{t+1}}{R_{ft}}+Q_tk_{t+1}+p_t^gh_{t+1}^g+p_t^lh_{t+1}^l+p_t^bb_{t+1}\\
&\quad=R_{kt}k_t-i_t+P_t^g\left(s_t^g-x_t^g\right)+P_t^l\left(s_t^l-x_t^l\right)+ch_t^g-\frac{b_{t+1}}{R_{ft}}+b_t\\
&\quad\quad+Q_t\left[(1-\delta)k_t+i_t\varepsilon_t\right]+p_t^g\left(h_t^g-s_t^g+x_t^g\right)+p_t^l\left(h_t^l-s_t^l+x_t^l\right)+p_t^bb_{t+1}\\
&\quad=\left[R_{kt}+(1-\delta)Q_t\right]k_t+\left(p_t^g+c\right)h_t^g+p_t^lh_t^l+b_t+\left(p_t^b-\frac{1}{R_{ft}}\right)b_{t+1}\\
&\quad\quad+\left(Q_t\varepsilon_t-1\right)i_t+\left(p_t^g-P_t^g\right)x_t^g+\left(p_t^l-P_t^l\right)x_t^l+\left(P_t^g-p_t^g\right)s_t^g+\left(P_t^l-p_t^l\right)s_t^l.
\end{aligned}$$

If $p_t^g>P_t^g$, then all entrepreneurs would purchase as much good assets as possible. If $p_t^g<P_t^g$, then no entrepreneurs would purchase any good asset. In both cases a competitive equilibrium could not exist. Thus we must have $p_t^g=P_t^g$ and $p_t^l=P_t^l$. If $p_t^b>1/R_{ft}$, then all entrepreneurs would prefer to buy bonds and an equilibrium could not exist. If $p_t^b<1/R_{ft}$, then all entrepreneurs would borrow until the borrowing constraint binds. In this case all entrepreneurs would also want to purchase as much financial assets as possible in order to take leverage. But this would not constitute an equilibrium. Thus $1/R_{ft}=p_t^b$.

We can simplify the last equality to derive

$$
\begin{aligned}
& C_t + \beta E_t V_{t+1}\left(k_{t+1}, \varepsilon_{t+1}, h_{t+1}^g, h_{t+1}^l, b_{t+1}\right) \\
& \quad = \left[R_{kt} + (1-\delta) Q_t\right] k_t + \left(P_t^g + c\right) h_t^g + P_t^l h_t^l + b_t + \left(Q_t \varepsilon_t - 1\right) i_t.
\end{aligned}
$$

Let $\varepsilon_t^* = 1/Q_t$. Since $i_t \geq 0$, it is optimal to make as much investment as possible if and only if $\varepsilon_t \geq \varepsilon_t^*$.

By the flow-of-funds constraint and the borrowing constraint,

$$
\begin{aligned}
i_t & = R_{kt} k_t + P_t^g \left(s_t^g - x_t^g\right) + P_t^l \left(s_t^l - x_t^l\right) + c h_t^g + b_t - C_t - \frac{b_{t+1}}{R_{ft}} \\
& \leq R_{kt} k_t + P_t^g s_t^g + P_t^l s_t^l - \left(P_t^g x_t^g + P_t^l x_t^l\right) + \mu_t k_t + c h_t^g + b_t .
\end{aligned}
$$

Since a firm with $\varepsilon_t > \varepsilon_t^*$ wants to invest using as many resources as possible, it will not purchase any asset and will sell all its assets; that is

$$
x_t^g = x_t^l = 0,\ s_t^g = h_t^g,\ s_t^l = h_t^l .
$$

Moreover, it will borrow as much as possible up to the borrowing limit. A firm with $\varepsilon_t < \varepsilon_t^*$ will not invest. Since $p_t^b = 1/R_{ft}$, $p_t^l = P_t^l$, and $p_t^g = P_t^g$, the firm is indifferent between saving and borrowing and is indifferent between buying and selling assets. We then obtain the optimal investment rule

$$
i_t = \begin{cases} R_{kt} k_t + \left(P_t^g + c\right) h_t^g + P_t^l h_t^l + \mu_t k_t + b_t & \text{if } \varepsilon_t > \varepsilon_t^* \\ 0 & \text{otherwise} \end{cases} .
$$

Thus we can derive aggregate investment and the law of motion for capital in Eqs. (20) and (21), where we have used the market-clearing condition for bonds, i.e., $\int b_{jt} \mathrm{d} j = 0$.

Substituting the decision rules back into (A.1) and using the conjectured value function, we can derive

$$
\begin{aligned}
& q_t(\varepsilon_t) k_t + \phi_t^g(\varepsilon_t) h_t^g + \phi_t^l(\varepsilon_t) h_t^l + \phi_t^b(\varepsilon_t) b_t \\
& \quad = \left[(1-\delta) Q_t + R_{kt}\right] k_t + \left(P_t^g + c\right) h_t^g + P_t^l h_t^l + b_t \\
& \qquad + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*} - 1, 0\right)\left[R_{kt} k_t + \left(P_t^g + c\right) h_t^g + P_t^l h_t^l + b_t + \mu_t k_t\right] \\
& \quad = \left\{(1-\delta) Q_t + R_{kt}\left[1 + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*} - 1, 0\right)\right] + \mu_t \max\left(\frac{\varepsilon_t}{\varepsilon_t^*} - 1, 0\right)\right\} k_t \\
& \qquad + \left(P_t^g + c\right)\left[1 + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*} - 1, 0\right)\right] h_t^g \\
& \qquad + P_t^l \left[1 + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*} - 1, 0\right)\right] h_t^l + \left[1 + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*} - 1, 0\right)\right] b_t .
\end{aligned}
$$

Matching coefficients yields

$$
\begin{aligned}
q_t(\varepsilon_t) &= (1-\delta)\,Q_t + R_{kt}\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right) + \mu_t \max\left(\frac{\varepsilon_t}{\varepsilon_t^*}-1,0\right),\\
\phi_t^g(\varepsilon_t) &= \left(P_t^g + c\right)\left[1+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*}-1,0\right)\right],\\
\phi_t^l(\varepsilon_t) &= P_t^l\left[1+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*}-1,0\right)\right],\\
\phi_t^b(\varepsilon_t) &= 1+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*}-1,0\right).
\end{aligned}
$$

Using the preceding definition of Q_t, p_t^g, p_t^l and p_t^b and noting that $p_t^g = P_t^g$, $p_t^l = P_t^l$, $p_t^b = 1/R_{ft}$, we can derive their asset pricing equations given in Proposition 1.

Since firms with $\varepsilon_t \le \varepsilon_t^*$ are indifferent between buying and selling assets, we allow them to purchase assets so that asset markets can clear

$$
\begin{aligned}
\int_{\varepsilon_t \le \varepsilon_t^*} x_t^g(\varepsilon_t)\,\mathrm{d}F(\varepsilon) &= \left[1-F\left(\varepsilon_t^*\right)\right](1-\pi),\\
\int_{\varepsilon_t \le \varepsilon_t^*} x_t^l(\varepsilon_t)\,\mathrm{d}F(\varepsilon) &= \left[1-F\left(\varepsilon_t^*\right)\right]\pi.
\end{aligned}
$$

Without loss of generality, we can set individual purchasing choice as

$$
x_t^g = \begin{cases} \frac{[1-F(\varepsilon_t^*)](1-\pi)}{F(\varepsilon_t^*)} & \text{if } \varepsilon_t < \varepsilon_t^* \\ 0 & \text{otherwise} \end{cases},\quad x_t^l = \begin{cases} \frac{[1-F(\varepsilon_t^*)]\pi}{F(\varepsilon_t^*)} & \text{if } \varepsilon_t < \varepsilon_t^* \\ 0 & \text{otherwise} \end{cases}.
$$

Moreover firms with $\varepsilon_t < \varepsilon_t^*$ are indifferent between saving and borrowing. □

Proof of Lemma 1 It is straightforward to check that $\beta\left[1+\int_{\varepsilon^*}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*}-1\right)\mathrm{d}F(\varepsilon)\right]$ decreases with ε^*. Since

$$
\beta\left[1+\int_{\varepsilon_{\max}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon_{\max}}-1\right)\mathrm{d}F(\varepsilon)\right] = \beta < 1,\ \beta\left[1+\int_{\varepsilon_{\min}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon_{\min}}-1\right)\mathrm{d}F(\varepsilon)\right] > 1,
$$

where the second inequality comes from Assumption 1, it follows from the intermediate value theorem that there exists a unique solution, denoted by $\varepsilon_b^* \in (\varepsilon_{\min}, \varepsilon_{\max})$, to the equation $\beta\left[1+\int_{\varepsilon^*}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*}-1\right)\mathrm{d}F(\varepsilon)\right] = 1.$ □

Proof of Proposition 2: Equation (17) in the steady state gives (26). For $P^g > 0$, we need

$$
\frac{1}{R_f} = \beta\left[1+\int_{\varepsilon^*}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*}-1\right)\mathrm{d}F(\varepsilon)\right] < 1.
$$

If follows from Eq. (18) that $P^l = 0$. By Lemma 1, the condition above is equivalent to $\varepsilon^* > \varepsilon_b^*$. Using $Q = 1/\varepsilon^*$ and Eq. (16), we can derive the steady-state capital stock in Eq. (27). Using Eq. (20) in the steady state yields

$$\delta K = \left[\alpha A K^{\alpha} + \mu K + (1-\pi)\left(P^{g} + c\right)\right] \int_{\varepsilon^*}^{\varepsilon^{\max}} \varepsilon \mathrm{d}F(\varepsilon).$$

Substituting (26) for P^g and (27) for $K = K(\varepsilon^*)$ into the equation above gives an equation for ε^*, (28). We need the following lemma to complete the proof.

Lemma 3 *For a sufficiently small μ, $K(\varepsilon^*)$ increases with ε^* on $(\varepsilon_{\min}, \varepsilon_{\max})$.*

Proof Let

$$h\left(\varepsilon^*\right) = \frac{1/\beta - 1 + \delta - \mu \int_{\varepsilon^*}^{\varepsilon_{\max}} (\varepsilon - \varepsilon^*) \, \mathrm{d}F(\varepsilon)}{\varepsilon^* + \int_{\varepsilon^*}^{\varepsilon_{\max}} (\varepsilon - \varepsilon^*) \, \mathrm{d}F(\varepsilon)}.$$

We can compute that

$$h'\left(\varepsilon^*\right) = \frac{\mu\left[(1 - F(\varepsilon^*))\,\varepsilon^* + \int_{\varepsilon^*}^{\varepsilon_{\max}} (\varepsilon - \varepsilon^*) \, \mathrm{d}F(\varepsilon)\right] - F(\varepsilon^*)\,(1/\beta - 1 + \delta)}{\left[\varepsilon^* + \int_{\varepsilon^*}^{\varepsilon_{\max}} (\varepsilon - \varepsilon^*) \, \mathrm{d}F(\varepsilon)\right]^2}.$$

For a sufficiently small $\mu \in (\varepsilon_{\min}, \varepsilon_{\max})$, $h'(\varepsilon^*) < 0$. Thus by (27),

$$K\left(\varepsilon^*\right) = \left[\frac{h(\varepsilon^*)}{\alpha A}\right]^{\frac{1}{\alpha-1}}$$

increases with ε^*. □

Simple algebra shows that the expression

$$\frac{\delta}{\int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F(\varepsilon)} - \alpha A K\left(\varepsilon^*\right)^{\alpha-1} - \mu = \frac{\delta}{\int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F(\varepsilon)}$$
$$- \frac{\left(\frac{1}{\beta} - 1 + \delta\right) \frac{1}{\varepsilon^*} - \mu \int_{\varepsilon^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*} - 1\right) \mathrm{d}F(\varepsilon)}{1 + \int_{\varepsilon^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*} - 1\right) \mathrm{d}F(\varepsilon)} - \mu$$

increases with ε^* on $\left(\varepsilon_b^*, \varepsilon_{\max}\right)$. Let $D(\varepsilon^*)$ denote the expression on the left-hand side of (28). Then since $D(\varepsilon^*)$ is the product of the preceding expression and $K(\varepsilon^*)$, it increases with ε^*.

We can check that

$$S\left(\varepsilon^*\right) \equiv \frac{(1-\pi)\,c}{R_f(\varepsilon^*) - 1}$$

decreases with ε^* on $\left(\varepsilon_b^*, \varepsilon_{\max}\right)$. As ε^* decreases to ε_b^*, $S(\varepsilon^*)$ approaches infinity since $R_f\left(\varepsilon_b^*\right) = 1$ by Lemma 1, but $D\left(\varepsilon_b^*\right)$ is finite. As ε^* increases to $\varepsilon_{\max}$, $D(\varepsilon^*)$ approaches infinity, but the limit of $S(\varepsilon^*)$ is finite. By the intermediate value theorem, there is a unique solution to $\varepsilon^* \in \left(\varepsilon_b^*, \varepsilon_{\max}\right)$ in Eq. (28).

Differentiating the expressions on the two sides of Eq. (28) yields

$$\frac{\partial D\left(\varepsilon^{*}\right)}{\partial \varepsilon^{*}} \frac{\partial \varepsilon^{*}}{\partial \mu}-K\left(\varepsilon^{*}\right)=\frac{\partial S\left(\varepsilon^{*}\right)}{\partial \varepsilon^{*}} \frac{\partial \varepsilon^{*}}{\partial \mu}.$$

We then have

$$\frac{\partial \varepsilon^{*}}{\partial \mu}\left[\frac{\partial D\left(\varepsilon^{*}\right)}{\partial \varepsilon^{*}}-\frac{\partial S\left(\varepsilon^{*}\right)}{\partial \varepsilon^{*}}\right]=K\left(\varepsilon^{*}\right).$$

Since $\frac{\partial D(\varepsilon^*)}{\partial \varepsilon^*} > 0$ and $\frac{\partial S(\varepsilon^*)}{\partial \varepsilon^*} < 0$ for small μ, we have $\frac{\partial \varepsilon^*}{\partial \mu} > 0$. Since $K(\varepsilon^*)$ and $R_f(\varepsilon^*)$ increase with ε^*, Y increases with μ and P^g decreases with μ. □

Proof of Proposition 3: We can write down an entrepreneur's decision problem by dynamic programming as in (A.1) subject to the constraints given in Sect. 2. We suppress the subscript j throughout the proof. Conjecture that the value function takes the form as in (A.2). Then we have

$$\beta E\left[V_{t+1}(k_{t+1}, \varepsilon_{t+1}, h_{t+1}^{g}, h_{t+1}^{l}, b_{t+1})\right]=Q_{t} k_{t+1}+p_{t}^{g} h_{t+1}^{g}+p_{t}^{l} h_{t+1}^{l}+p_{t}^{b} b_{t+1},$$

where Q_t, p_t^g, P_t^l, and p_t^b are defined as in (A.3) and (A.4).

Using the flow-of-funds constraint and the preceding equation, we can derive

$$\begin{aligned}
& C_{t}+\beta E_{t} V_{t+1}\left(k_{t+1}, \varepsilon_{t+1}, h_{t+1}^{g}, h_{t+1}^{l}, b_{t+1}\right) \\
& =R_{kt} k_{t}-i_{t}+P_{t}\left(s_{t}^{g}+s_{t}^{l}-x_{t}\right)+c h_{t}^{g}-\frac{b_{t+1}}{R_{ft}}+b_{t} \\
& \quad +Q_{t} k_{t+1}+p_{t}^{g} h_{t+1}^{g}+p_{t}^{l} h_{t+1}^{l}+p_{t}^{b} b_{t+1} \\
& =R_{kt} k_{t}-i_{t}+P_{t}\left(s_{t}^{g}+s_{t}^{l}-x_{t}\right)+c h_{t}^{g}-\frac{b_{t+1}}{R_{ft}}+b_{t} \\
& \quad +Q_{t}\left[(1-\delta) k_{t}+i_{t} \varepsilon_{t}\right]+p_{t}^{g}\left(\Theta_{t} x_{t}+h_{t}^{g}-s_{t}^{g}\right) \\
& \quad +p_{t}^{l}\left[\left(1-\Theta_{t}\right) x_{t}+h_{t}^{l}-s_{t}^{l}\right]+p_{t}^{b} b_{t+1} \\
& =\left[(1-\delta) Q_{t}+R_{kt}\right] k_{t}+\left(p_{t}^{g}+c\right) h_{t}^{g}+p_{t}^{l} h_{t}^{l} \\
& \quad +\left(Q_{t} \varepsilon_{t}-1\right) i_{t}+\left[\Theta_{t} p_{t}^{g}+\left(1-\Theta_{t}\right) p_{t}^{l}-P_{t}\right] x_{t} \\
& \quad +\left(P_{t}-p_{t}^{g}\right) s_{t}^{g}+\left(P_{t}-p_{t}^{l}\right) s_{t}^{l}+b_{t}+\left(p_{t}^{b}-\frac{1}{R_{ft}}\right) b_{t+1}.
\end{aligned}$$

By a similar argument to the proof of Proposition 1, for the entrepreneur's optimal decisions to be compatible with a competitive equilibrium, we must have

$$P_{t}=\Theta_{t} p_{t}^{g}+\left(1-\Theta_{t}\right) p_{t}^{l}, \quad p_{t}^{b}=\frac{1}{R_{ft}}.$$

Thus we have

$$\begin{aligned}
&\max_{i_t, s_t^l, s_t^g, x_t, C_t, b_{t+1}} C_t + \beta E_t V_{t+1}\left(k_{t+1}, \varepsilon_{t+1}, h_{t+1}^g, h_{t+1}^l, b_{t+1}\right) \\
&= \max_{i_t, s_t^l, s_t^g} [(1-\delta) Q_t + R_{kt}] k_t + \left(p_t^g + c\right) h_t^g + p_t^l h_t^l + b_t \\
&\quad + (Q_t \varepsilon_t - 1) i_t + \left(P_t - p_t^g\right) s_t^g + \left(P_t - p_t^l\right) s_t^l.
\end{aligned}$$

Since $i_t \geq 0$, it is optimal for the firm to make real investment if and only if $\varepsilon_t \geq 1/Q_t = \varepsilon_t^*$. When making the investment, the firm will invest as much as possible. By the flow-of-funds constraint (6) and the borrowing constraint (7), we have

$$\begin{aligned}
i_t &= R_{kt} k_t + P_t\left(s_t^g + s_t^l\right) - P_t x_t + c h_t^g + b_t - C_t - \frac{b_{t+1}}{R_{ft}} \\
&\leq R_{kt} k_t + P_t\left(s_t^g + s_t^l\right) + c h_t^g + b_t + \mu_t k_t - P_t x_t.
\end{aligned}$$

To leave the maximum resource for investing, the firm will not purchase any asset; that is, $x_t = 0$. The borrowing constraint must also bind when $\varepsilon_t > 1/Q_t = \varepsilon_t^*$. Thus we obtain the investment rule

$$i_t = \begin{cases} R_{kt} k_t + P_t\left(s_t^g + s_t^l\right) + c h_t^g + b_t + \mu_t k_t & \text{for } \varepsilon_t \geq \varepsilon_t^* \\ 0 & \varepsilon_t < \varepsilon_t^* \end{cases}.$$

Substituting this investment rule into the right-hand side of the Bellman equation in (A.1), we can derive that for $\varepsilon_t > \varepsilon_t^*$,

$$\begin{aligned}
&\max_{i_t, s_t^l, s_t^g, x_t, C_t, b_{t+1}} C_t + \beta E_t V_{t+1}\left(k_{t+1}, \varepsilon_{t+1}, h_{t+1}^g, h_{t+1}^l, b_{t+1}\right) \\
&= \max_{s_t^l, s_t^g} [(1-\delta) Q_t + R_{kt}] k_t + \left(p_t^g + c\right) h_t^g + p_t^l h_t^l + b_t \\
&\quad + (Q_t \varepsilon_t - 1)\left[R_{kt} k_t + P_t\left(s_t^g + s_t^l\right) + c h_t^g + b_t + \mu_t k_t\right] \\
&\quad + \left(P_t - p_t^g\right) s_t^g + \left(P_t - p_t^l\right) s_t^l \\
&= \max_{s_t^l, s_t^g} [(1-\delta) Q_t + Q_t \varepsilon_t R_{kt} + \mu_t (Q_t \varepsilon_t - 1)] k_t + \left(p_t^g + Q_t \varepsilon_t c\right) h_t^g + p_t^l h_t^l \\
&\quad + Q_t \varepsilon_t b_t + \left(Q_t \varepsilon_t P_t - p_t^g\right) s_t^g + \left(Q_t \varepsilon_t P_t - p_t^l\right) s_t^l \\
&= [(1-\delta) Q_t + Q_t \varepsilon_t R_{kt} + \mu_t (Q_t \varepsilon_t - 1)] k_t + Q_t P_t \varepsilon_t h_t^l + Q_t \varepsilon_t b_t \\
&\quad + \left[p_t^g + Q_t \varepsilon_t c + \max\left(Q_t P_t \varepsilon_t - p_t^g, 0\right)\right] h_t^g,
\end{aligned}$$

where in the last equality we have used the fact that $s_t^l = h_t^l$ since $Q_t \varepsilon_t P_t \geq P_t \geq p_t^l$ and that $s_t^g = h_t^g$ if $Q_t P_t \varepsilon_t \geq p_t^g$ and $s_t^g = 0$, otherwise.

If $\varepsilon_t \leq \varepsilon_t^*$, then $i_t = 0$ and we have

$$
\begin{aligned}
&\max_{i_t, s_t^l, s_t^g, x_t, C_t, b_{t+1}} C_t + \beta E_t V_{t+1}\left(k_{t+1}, \varepsilon_{t+1}, h_{t+1}^g, h_{t+1}^l, b_{t+1}\right) \\
&= \max_{s_t^g, s_t^l} \left[(1-\delta) Q_t + R_{kt}\right] k_t + \left(p_t^g + c\right) h_t^g + p_t^l h_t^l + b_t \\
&\quad + \left(P_t - p_t^g\right) s_t^g + \left(P_t - p_t^l\right) s_t^l \\
&= \left[(1-\delta) Q_t + R_{kt}\right] k_t + \left(p_t^g + c\right) h_t^g + P_t h_t^l + b_t,
\end{aligned}
$$

where the second equality follows from the fact that $s_t^g = 0$ since $P_t \leq p_t^g$ and that $s_t^l = h_t^l$ since $P_t \geq p_t^l$.

We now combine the preceding two cases for all $\varepsilon_t \in [\varepsilon_{\min}, \varepsilon_{\max}]$. If

$$
P_t \geq \frac{p_t^g}{Q_t \varepsilon_{\max}} = \frac{\varepsilon_t^*}{\varepsilon_{\max}} p_t^g,
$$

then

$$
\begin{aligned}
&\max_{i_t, s_t^l, s_t^g, x_t, C_t, b_{t+1}} C_t + \beta E_t V_{t+1}\left(k_{t+1}, \varepsilon_{t+1}, h_{t+1}^g, h_{t+1}^l, b_{t+1}\right) \\
&= \left[(1-\delta) Q_t + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*}, 1\right) R_{kt} + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*} - 1, 0\right) \mu_t\right] k_t \\
&\quad + \left[p_t^g + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*}, 1\right) c + Q_t P_t \max\left(\varepsilon_t - \frac{p_t^g}{p_t Q_t}, 0\right)\right] h_t^g \\
&\quad + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*}, 1\right) P_t h_t^l + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*}, 1\right) b_t \\
&= \left[(1-\delta) Q_t + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*}, 1\right) R_{kt} + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*} - 1, 0\right) \mu_t\right] k_t \\
&\quad + \left[\max\left(\frac{\varepsilon_t}{p_t^g / (P_t Q_t)}, 1\right) p_t^g + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*}, 1\right) c\right] h_t^g \\
&\quad + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*}, 1\right) P_t h_t^l + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*}, 1\right) b_t.
\end{aligned}
$$

If

$$
P_t < \frac{p_t^g}{Q_t \varepsilon_{\max}} = \frac{\varepsilon_t^*}{\varepsilon_{\max}} p_t^g,
$$

then

$$
\begin{aligned}
&\max_{i_t, s_t^l, s_t^g, x_t, C_t, b_{t+1}} C_t + \beta E_t V_{t+1}\left(k_{t+1}, \varepsilon_{t+1}, h_{t+1}^g, h_{t+1}^l, b_{t+1}\right) \\
&= \left[(1-\delta) Q_t + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*}, 1\right) R_{kt} + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*} - 1, 0\right) \mu_t\right] k_t
\end{aligned}
$$

$$
\begin{aligned}
&+\left[p_t^g+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)c\right]h_t^g+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)P_t h_t^l+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)b_t\\
&=\left[(1-\delta)\,Q_t+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)R_{kt}+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*}-1,0\right)\mu_t\right]k_t\\
&+\left[\max\left(\frac{\varepsilon_t}{\varepsilon^{\max}},1\right)p_t^g+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)c\right]h_t^g+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)P_t h_t^l\\
&+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)b_t.
\end{aligned}
$$

Let $\varepsilon_t^{**}\equiv\min\left(\frac{p_t^g}{P_tQ_t},\varepsilon^{\max}\right)$. Then for any $\varepsilon_t\in\left(\varepsilon^{\min},\varepsilon^{\max}\right)$, we can write

$$
\begin{aligned}
&\max_{i_t,s_t^l,s_t^g,x_t,C_t,b_{t+1}} C_t+\beta E_t V_{t+1}\left(k_{t+1},\varepsilon_{t+1},h_{t+1}^g,h_{t+1}^l,b_{t+1}\right)\\
&=\left[(1-\delta)\,Q_t+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)R_{kt}+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*}-1,0\right)\mu_t\right]k_t\\
&+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)P_t h_t^l\\
&+\left[\max\left(\frac{\varepsilon_t}{\varepsilon_t^{**}},1\right)p_t^g+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)c\right]h_t^g+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)b_t.
\end{aligned}
$$

Substituting the preceding equation into the Bellman equation and using (A.2), we match coefficients to derive that for any $\varepsilon_t\in(\varepsilon_{\min},\varepsilon_{\max})$,

$$
\begin{aligned}
q_t(\varepsilon_t)&=(1-\delta)\,Q_t+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)R_{kt}+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*}-1,0\right)\mu_t,\\
\phi_t^g(\varepsilon_t)&=\max\left(\frac{\varepsilon_t}{\varepsilon_t^{**}},1\right)p_t^g+\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)c,\\
\phi_t^l(\varepsilon_t)&=\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right)P_t,\\
\phi_t^b(\varepsilon_t)&=\max\left(\frac{\varepsilon_t}{\varepsilon_t^*},1\right).
\end{aligned}
$$

Substituting these equations into the previous definitions of Q_t, p_t^g, p_t^l, and p_t^b, we obtain their asset pricing equations as in the proposition. □

Proof of Proposition 4: In a frozen equilibrium, $P_t=0$ for all t. No firms want to sell any good assets since the holding value $p_t^g>0$. In a frozen equilibrium, the market for long-term assets breaks down. We conjecture that the value function V_t takes the following form:

$$
V_t\left(k_t,\varepsilon_t,h_t^g,h_t^l,b_t\right)=q_t(\varepsilon_t)k_t+\phi_t^g(\varepsilon_t)h_t^g+\phi_t^l(\varepsilon_t)h_t^l+\phi_t^b(\varepsilon_t)b_t.
$$

Then we can write

$$\beta E\left[V_{t+1}\left(k_{t+1},\varepsilon_{t+1},h^g_{t+1},h^l_{t+1},b_{t+1}\right)\right]=Q_t k_{t+1}+p^g_t h^g_{t+1}+p^l_t h^l_{t+1}+p^b_t b_{t+1},$$

where we define $Q_t,\ p^g_t,\ p^l_t$, and p^b_t as before. The Bellman equation is given by

$$V_t\left(k_t,\varepsilon_t,h^g_t,h^l_t,b_t\right)=\max_{i_t}\ C_t+\beta E\left[V_{t+1}\left(k_{t+1},\varepsilon_{t+1},h^g_{t+1},h^l_{t+1},b_{t+1}\right)\right] \quad \text{(A.5)}$$

subject to (5), $b_{t+1}/R_{ft}\geq -\mu_t k_t$, and

$$i_t=R_{kt}k_t+ch^g_t+b_t-\frac{b_{t+1}}{R_{ft}}-C_t.$$

Using the flow-of-funds constraint, we can compute the objective function in (A.5) as

$$\begin{aligned}&C_t+\beta E\left[V_{t+1}\left(k_{t+1},\varepsilon_{t+1},h^g_{t+1},h^l_{t+1},b_{t+1}\right)\right]\\&\quad=R_{kt}k_t-i_t+ch^g_t+Q_t\left[(1-\delta)\,k_t+i_t\varepsilon_t\right]+p^g_t h^g_{t+1}+p^l_t h^l_{t+1}\\&\quad=\left[R_{kt}+(1-\delta)\,Q_t\right]k_t+(Q_t\varepsilon_t-1)\,i_t+\left(p^g_t+c\right)h^g_t+p^l_t h^l_t,\end{aligned}$$

where we have used the fact that $h^g_{t+1}=h^g_t=h^g_0$ and $h^l_t=h^l_t=h^l_0$ for all t. We then obtain the investment rule in the proposition. Substituting this investment rule back into (A.5) and matching coefficients, we obtain

$$\begin{aligned}q_t(\varepsilon_t)&=(1-\delta)\,Q_t+R_{kt}\left[1+\max\left(\frac{\varepsilon_t}{\varepsilon^*_t}-1,0\right)\right]+\max\left(\frac{\varepsilon_t}{\varepsilon^*_t}-1,0\right)\mu_t,\\ \phi^g_t(\varepsilon_t)&=p^g_t+c\left[1+\max\left(\frac{\varepsilon_t}{\varepsilon^*_t}-1,0\right)\right],\\ \phi^l_t(\varepsilon_t)&=p^l_t,\\ \phi^b_t\left(\varepsilon_t\right)&=1+\max\left(\frac{\varepsilon_t}{\varepsilon^*_t}-1,0\right).\end{aligned}$$

Using the definitions of Q_t, p^g_t, p^l_t, and p^b_t, we can derive (16), (17), and $p^l_t=\beta E_t\left(p^l_{t+1}\right)$. By the transversality condition, we deduce that $p^l_t=0$ for all t. □

Proof of Proposition 5: By (36), $Q_t P_t\varepsilon_t\leq Q_t P_t\varepsilon_{\max}<p^g$. No firms want to sell the good assets so that $\varepsilon^{**}_t=\varepsilon_{\max}$ and $\Theta_t=0$. Thus $P_t=p^l_t$ by (31). We then use (29) to derive (33) and use (30) to derive (32).

We use Proposition 3 to derive Eq. (44) for aggregate investment. We then obtain the law of motion for aggregate capital in Eq. (34). Using (3), (4), and the labor market-clearing condition $N_t=1$, we can derive that

$$W_t=(1-\alpha)\,A\left(\frac{K_t}{N_t}\right)^{\alpha}=(1-\alpha)\,AK^{\alpha}_t,\ R_{kt}=\alpha A\left(\frac{N_t}{K_t}\right)^{1-\alpha}=\alpha AK^{\alpha-1}.$$

In addition,

$$Y_t = \int y_{jt} \mathrm{d}j = \int A k_{jt}^{\alpha} n_{jt}^{1-\alpha} \mathrm{d}j = AK^{\alpha} N^{1-\alpha} = AK^{\alpha}.$$

By the decision rule in Proposition 3 and the market-clearing condition for financial assets,

$$\int x_t \mathrm{d}F(\varepsilon) = \int \left(s_t^g + s_t^l\right) \mathrm{d}F(\varepsilon),$$

we can derive

$$\int_{\varepsilon_t \le \varepsilon_t^*} x_t \mathrm{d}F(\varepsilon) = \pi + (1-\pi)\left[1 - F\left(\varepsilon_t^{**}\right)\right].$$

Since x_t is indeterminate at the individual firm level, we can set

$$x_{jt} = \begin{cases} \frac{\pi + (1-\pi)\left[1 - F\left(\varepsilon_t^{**}\right)\right]}{F\left(\varepsilon_t^*\right)} & \text{if } \varepsilon_{jt} < \varepsilon_t^{**} \\ 0 & \text{otherwise} \end{cases},$$

for all j. □

Proof of Proposition 6: By Lemma 1, there exists a unique cutoff value $\varepsilon_b^* \in (\varepsilon_{\min}, \varepsilon_{\max})$ to Eq. (24). In the bubbly lemon steady state, $P > 0$ and hence Eq. (32) is equivalent to Eq. (24). This implies that ε_b^* is the investment threshold in the bubbly lemon steady state. By (24) and (33), we can derive p^g as in (40). Using Eqs. (16) and (22), we can show that the steady-state capital stock is equal to $K\left(\varepsilon_b^*\right)$ where $K(\cdot)$ is given in (27). Using Eqs. (22) and (34), we can solve for P as in (41). We need to verify that the condition

$$0 < P < \frac{\varepsilon_b^*}{\varepsilon_{\max}} p^g$$

holds in the steady state. But this is equivalent to (39). □

Proof of Proposition 7: In a pooling equilibrium the restriction in (45) must hold. Firms with $\varepsilon_{jt} \ge \varepsilon_t^{**}$ sell their good assets. By Proposition 3 and the market-clearing conditions for assets, we can compute Θ_t as in the proposition. Using the decision rule for investment in Proposition 3 and aggregating individual decision rules, we obtain (43) and (44). □

Proof of Lemma 2: Under Assumption 1, Lemma 1 establishes the existence of a unique solution ε_b^* to Eq. (24). Since $\beta\left(1 + \int_{\varepsilon^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*} - 1\right) \mathrm{d}F(\varepsilon)\right)$ decreases with ε^*, it follows that

$$\beta\left(1 + \int_{\varepsilon^{**}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^{**}} - 1\right) \mathrm{d}F(\varepsilon)\right) < \beta\left(1 + \int_{\varepsilon^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*} - 1\right) \mathrm{d}F(\varepsilon)\right) = 1$$

for $\varepsilon^{**} > \varepsilon_b^*$. Thus we deuce that

$$\lim_{\varepsilon^*\uparrow\varepsilon^{**}} \Theta\left(\varepsilon^{**}\right)\left(\frac{\varepsilon^{**}}{\varepsilon^*}\right) + (1-\Theta\left(\varepsilon^{**}\right))\beta\left(1+\int_{\varepsilon^*}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*}-1\right)\mathrm{d}F\left(\varepsilon\right)\right) < 1$$
$$\lim_{\varepsilon^*\downarrow\varepsilon_b^*} \Theta\left(\varepsilon^{**}\right)\left(\frac{\varepsilon^{**}}{\varepsilon^*}\right) + (1-\Theta\left(\varepsilon^{**}\right))\beta\left(1+\int_{\varepsilon^*}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*}-1\right)\mathrm{d}F\left(\varepsilon\right)\right) > 1.$$

Since the expression on the right-hand side of Eq. (50) decreases continuously with ε^*, it follows from the intermediate value theorem that there exists a unique solution to ε^* in $\left(\varepsilon_b^*, \varepsilon^{**}\right)$ in Eq. (50). □

Proof of Proposition 8: Following the strategy used in the context, we know pooling equilibrium can be supported if and only if

$$0 < c < \overline{c}^P\left(\pi\right),$$

where $\overline{c}^P\left(\pi\right) = \max_{\varepsilon^{**}\in\left[\varepsilon_b^*,\varepsilon_{\max}\right]} \Gamma\left(\varepsilon^{**},\pi\right)$, and

$$\Gamma\left(\varepsilon^{**},\pi\right) \equiv \frac{\frac{\delta K\left(\Phi\left(\varepsilon^{**}\right)\right)}{\int_{\Phi\left(\varepsilon^{**}\right)}^{\varepsilon_{\max}}\varepsilon\mathrm{d}F(\varepsilon)} - \alpha AK\left(\Phi\left(\varepsilon^{**}\right)\right)^{\alpha} - \mu K\left(\Phi\left(\varepsilon^{**}\right)\right)}{(1-\pi)+\frac{\Phi\left(\varepsilon^{**}\right)}{\varepsilon^{**}}\left[\pi+(1-\pi)\frac{\int_{\varepsilon^{**}}^{\varepsilon_{\max}}\varepsilon\mathrm{d}F(\varepsilon)}{\int_{\Phi\left(\varepsilon^{**}\right)}^{\varepsilon_{\max}}\varepsilon\mathrm{d}F(\varepsilon)}\right]\left[\frac{\beta\left(1+\int_{\Phi\left(\varepsilon^{**}\right)}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\Phi\left(\varepsilon^{**}\right)}-1\right)\mathrm{d}F(\varepsilon)\right)}{1-\beta\left(1+\int_{\varepsilon^{**}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^{**}}-1\right)\mathrm{d}F(\varepsilon)\right)}\right]}$$

As in the proof of Proposition 2 and Lemma 3, for a sufficiently small μ, the expression

$$\frac{\delta K\left(\varepsilon^*\right)}{\int_{\varepsilon^*}^{\varepsilon_{\max}}\varepsilon\mathrm{d}F\left(\varepsilon\right)} - \alpha A\left[K\left(\varepsilon^*\right)\right]^{\alpha} - \mu K\left(\varepsilon^*\right)$$

increases with ε^*. Thus the numerator of the expression for Γ given above satisfies

$$\frac{\delta K\left(\varepsilon^*\right)}{\int_{\varepsilon^*}^{\varepsilon_{\max}}\varepsilon\mathrm{d}F\left(\varepsilon\right)} - \alpha AK\left(\varepsilon^*\right)^{\alpha} - \mu K\left(\varepsilon^*\right)$$
$$\geq \frac{\delta K\left(\varepsilon_b^*\right)}{\int_{\varepsilon_b^*}^{\varepsilon_{\max}}\varepsilon\mathrm{d}F\left(\varepsilon\right)} - \alpha AK\left(\varepsilon_b^*\right)^{\alpha} - \mu K\left(\varepsilon_b^*\right) = c_H > 0$$

for any $\varepsilon^* \geq \varepsilon_b^*$. In addition, it follows from Lemma 1 that

$$1-\beta\left(1+\int_{\varepsilon^{**}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^{**}}-1\right)\mathrm{d}F\left(\varepsilon\right)\right) > 0$$

for $\varepsilon^{**} > \varepsilon_b^*$ so that the denominator of the expression for Γ given above is also positive. We deduce that

$$\Gamma\left(\pi,\varepsilon^{**}\right) \geq 0$$

for all $\varepsilon^{**} \in \left(\varepsilon_b^*, \varepsilon_{\max}\right)$. Since

$$\lim_{\varepsilon^{**} \downarrow \varepsilon_b^*} \beta \left(1 + \int_{\varepsilon^{**}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^{**}} - 1\right) \mathrm{d}F\left(\varepsilon\right)\right) = 1$$

and other limits are finite, we have

$$\lim_{\varepsilon^{**} \downarrow \varepsilon_b^*} \Gamma\left(\pi, \varepsilon^{**}\right) = 0.$$

By the intermediate value theorem, there exists a solution to ε^{**} in $\left(\varepsilon_b^*, \varepsilon_{\max}\right)$ in Eq. (52).

We can verify that

$$\Gamma\left(\varepsilon_{\max}\right) = \frac{c_H c_L}{\pi c_H + (1-\pi) c_L} = \underline{c}^B\left(\pi\right),$$

where the first equality uses the fact that $\Phi\left(\varepsilon_{\max}\right) = \varepsilon_b^*$ and the second uses the definition of $\underline{c}^B\left(\pi\right)$ by Eq. (38). Therefore we know that $\bar{c}^p\left(\pi\right) > \underline{c}^B\left(\pi\right)$.

The steady-state capital stock $K\left(\varepsilon_p^*\right)$ is derived from Eq. (16) using $\varepsilon_p^* = 1/Q$. □

Proof of Proposition 9: We apply Proposition 4. Aggregation leads to the equations for aggregate capital and investment in the proposition. □

Proof of Proposition 10: By Eq. (16), we can derive the steady-state capital stock $K\left(\varepsilon^*\right)$ defined in (27). We need to solve for ε^*. By (22) and (56), we can derive Eq. (58). As in the proof of Proposition 8, we know that the right-hand side of (58) strictly increases with ε^*. In addition, we can show that

$$\lim_{\varepsilon^{**} \uparrow \varepsilon_{\max}} \frac{\delta K\left(\varepsilon^*\right)}{\int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F\left(\varepsilon\right)} - \alpha A K\left(\varepsilon^*\right)^{\alpha} - \mu K\left(\varepsilon^*\right) = +\infty$$

and

$$\lim_{\varepsilon^* \downarrow \varepsilon_{\min}} \frac{\delta K\left(\varepsilon^*\right)}{\int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F\left(\varepsilon\right)} - \alpha A K\left(\varepsilon^*\right)^{\alpha} - \mu K\left(\varepsilon^*\right)$$
$$= K\left(\varepsilon_{\min}\right)\left[\frac{\delta}{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F\left(\varepsilon\right)} - \frac{1/\beta - 1 + \delta}{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F\left(\varepsilon\right)}\right] = K\left(\varepsilon_{\min}\right) \frac{1 - 1/\beta}{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F\left(\varepsilon\right)} < 0.$$

Therefore there exists a unique solution $\varepsilon^* \in \left(\varepsilon_{\min}, \varepsilon_{\max}\right)$ to Eq. (58) for any $c > 0$.

Proof of Proposition 11: By Lemma 3, for a sufficiently small μ, $K\left(\varepsilon^*\right)$ increases with ε^*. To prove $K\left(\varepsilon_p^*\right) > K\left(\varepsilon_b^*\right) > K\left(\varepsilon_a^*\right)$, we only need to show that $\varepsilon_p^* > \varepsilon_b^* > \varepsilon_a^*$ when μ is small enough. By Lemma 2 and Proposition 8, $\varepsilon_p^* > \varepsilon_b^*$.

By definition,

$$D\left(\varepsilon^*\right) \equiv \frac{\delta K\left(\varepsilon^*\right)}{\int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon \mathrm{d}F\left(\varepsilon\right)} - \left[\alpha A K\left(\varepsilon^*\right)^{\alpha} - \mu K\left(\varepsilon^*\right) + \left(1-\pi\right)c\right].$$

By (37) and (38), ε_b^* satisfies the equation

$$D\left(\varepsilon_b^*\right) = \left(1-\pi\right)\left(\bar{c}^B\left(\pi\right) - c\right). \tag{A.6}$$

By Proposition 10, ε_a^* satisfies

$$D\left(\varepsilon_a^*\right) = 0. \tag{A.7}$$

As shown in Proposition 6, a bubbly lemon steady-state equilibrium can be supported if $c < \bar{c}^B\left(\pi\right)$. Therefore Eqs. (A.6) and (A.7) jointly imply

$$D\left(\varepsilon_b^*\right) > D\left(\varepsilon_a^*\right). \tag{A.8}$$

As in the proof of Proposition 2, $D\left(\varepsilon^*\right)$ strictly increases with ε^* for a sufficiently small μ. Then Eq. (A.8) implies that $\varepsilon_b^* > \varepsilon_a^*$. □

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