Asset Bubbles, Collateral, and Policy Analysis

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Abstract

This paper provides a theory of credit-driven asset bubbles in an infinite-horizon production economy. Entrepreneurs face idiosyncratic investment distortions and credit constraints. An intrinsically useless asset such as land serves as collateral for borrowing. A land bubble can form because land commands a liquidity premium. The land bubble can provide liquidity and relax credit constraints, but can also generate inefficient overinvestment. Its net effect is to reduce welfare. Property taxes, Tobin's taxes, macroprudential policy, and credit policy can prevent the formation of a land bubble.

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1. Introduction

Many countries have experienced asset bubbles. As evidence, Figure 1 presents the real housing price indexes, the price-income ratios, and the price-rental ratios for the United States, Japan, Spain, and Greece. This figure reveals that the three series comove for each country, indicating that fluctuations in housing prices may not be driven entirely by fundamentals (i.e., incomes or rents). The collapse of housing bubbles is often accompanied by a financial crisis. It is widely believed that the credit crisis resulting from the bursting of the housing bubble is the primary cause of the 2007–2009 recession in the United States. The collapse of the Japanese housing bubble contributed to the so-called "Lost Decade." The collapse of housing bubbles in European countries may be partly to blame for the European sovereign debt crisis.

What causes an asset bubble? What is its welfare effect? If an asset bubble reduces welfare, what policies can prevent a bubble from forming? The goal of this paper is to present a theoretical study to address these questions by providing a model of credit-driven asset bubbles in an infinite-horizon production economy. To be concrete, we focus on bubbles on an intrinsically useless asset such as land bubbles.[1] The model economy is populated by a continuum of identical households. Each household is an extended family consisting of a continuum of entrepreneurs and a continuum of workers. Each entrepreneur runs a firm and workers work for the firms. There is no aggregate uncertainty about fundamentals.

There are three key assumptions in our model. First, entrepreneurs face borrowing constraints because of financial market imperfections. In particular, they have limited commitment and contract enforcement is imperfect. They must pledge land as collateral and borrow against at most a fraction of the land value. That is, they must make down payments in order to purchase land. This kind of borrowing constraint is often called a leverage constraint or a margin constraint. It is related to the idea put forth by Kiyotaki and Moore (1997) and Brunnermeier and Pederson (2009), among others.

Second, entrepreneurs face idiosyncratic distortions on the investment good price. For example, governments may offer different tax credits or subsidies to different firms financed by lump sum

[1] Davis and Heathcote (2007) document that fluctuations of housing prices are largely driven by those of land prices. Thus the emergence and crash of land bubbles can help explain the large fluctuations of housing prices.

taxes on households. As Restuccia and Rogerson (2008) and Klenow and Hsieh (2009) argue, policy distortions can generate resource misallocations and are widespread in many developed and developing countries. In this paper we consider idiosyncratic investment subsidies, e.g., investment tax credit (ITC), which are an important policy tool to stimulate investment.[2]

Third, land trading is illiquid. Following Kiyotaki and Moore (2008), we assume that entrepreneurs face a resaleability constraint, which means that they can resell at most a fraction of their existing land. In addition, they cannot short sell land.

Land plays two important roles in the model. First, it is an asset that allows resources to be transferred intertemporally and generate capital gains or losses. Second, it is used as collateral to facilitate borrowing. In general, land may be productive and useful for producing agriculture products. In this paper we abstract away from this role of land and focus on its first two roles instead. In particular, we assume that land is intrinsically useless so that its fundamental value is zero. We will show that land can have a positive value in equilibrium, which represents a bubble.

In standard models with infinitely-lived agents, bubbles can typically be ruled out by transversality conditions. Why can a land bubble exist in our model? The reason is that in our model entrepreneurs face borrowing constraints and land can provide liquidity. Hence land commands a liquidity premium. Consider the special case where entrepreneurs cannot borrow. Since they face idiosyncratic ITC, those with high ITC are willing to invest more. Resources should be reallocated from entrepreneurs with low ITC to those with high ITC. In the absence of a credit market, land as an asset plays the role of transferring resources among entrepreneurs and also over time. As a result, land is valuable just like money. In the presence of a credit market, land also serves as collateral for borrowing and a high land value can relax the credit constraint. Hence land generates a collateral yield. The two benefits provided by land constitute the liquidity premium.

Since liquidity depends at least partly on beliefs, so does the existence of a land bubble. If no one believes that land is valuable, then no one will trade it or use it as collateral. Then land

[2] As Hassett and Hubbard (2002) point out, since 1962, the mean duration of a typical state in the United States in which an ITC is in effect has been about three and a half years, and the mean duration of the no-ITC state has been about the same length. Goolsbee (1998) documents evidence that the ITC varies across time and across assets and firms. In October 2003, China's government provided investment tax credits to six industries of the manufacturing sector in Northeastern provinces and later the tax reform was expanded to more industries in more provinces (Chen, He, and Zhang (2013)).

is indeed valueless in equilibrium. Thus our model features two types of equilibria: the bubbly equilibrium and the bubbleless equilibrium.[3] Which type is more efficient? Having discussed the good side of a land bubble in terms of providing liquidity and relaxing credit constraints, we now turn to its bad side. Our model features idiosyncratic tax/subsidy distortions. The existence of a land bubble allows entrepreneurs with high ITC to make more investment. This creates inefficient overinvestment and resources misallocation, which reduces welfare. The overall welfare effect of a land bubble is ambiguous. We prove that a land bubble can reduce welfare in some special cases and provide numerical examples for more general cases.

Given that land bubbles can reduce welfare, what policies can prevent the formation of a bubble? In the standard models of rational bubbles (e.g., Tirole (1985)), the return on the bubble is equal to the capital gains only since the bubble does not deliver any payoffs. In a deterministic model, this implies that the interest rate is equal to the growth rate of the bubble. By contrast, in our model the return on the bubble is equal to capital gains plus the liquidity premium. This asset pricing equation has important policy implications. In particular, we focus on fiscal and macroprudential policies that can reduce the liquidity premium and hence the benefit of having a land bubble. If the benefit is sufficiently small, the bubbly steady state cannot exist and the economy reaches the unique bubbleless equilibrium. We study four types of policies: (i) limit the loan-to-value (LTV) ratio to a sufficiently low level or raise the down payment to a sufficiently high level; (ii) raise property taxes to a sufficiently high level and transfer the tax revenue to households; (iii) raise property transaction taxes (or Tobin's taxes) to a sufficiently high level and transfer the tax revenue to households; and (iv) the government purchases private bonds financed by lump sum taxes. These policies have been implemented in some countries, though empirical studies are needed to see whether they are effective in eliminating asset bubbles.

We show that the interest rate in the bubbleless steady state is lower than that in the bubbly steady state. The reason is that the land bubble crowds out the bond demand, thereby reducing the bond price and raising the interest rate. This implies that all four policies will reduce the interest rate in the long run after the bubble is eliminated. This seems to contradict the conventional

[3] There may exist a third type of equilibria with stochastic bubbles (see, e.g., Weil (1987) and Miao and Wang (2013)). Stochastic bubbles can increase consumption volatility and reduce welfare. We will not study this type of equilibria in the paper.

wisdom that a low interest rate may cause a land bubble because a low interest rate encourages excessive mortgage borrowing.[4] But this can be reconciled by noting that the conventional wisdom ignores the general equilibrium effect of the land bubble.

Related literature. Our paper is related to a growing literature on rational bubbles.[5] Most models of rational bubbles adopt the overlapping generations framework (Tirole (1985) and Weil (1987)). Introducing rational bubbles into an infinite-horizon model is generally nontrivial due to the transversality conditions (Santos and Woodford (1997)). Kocherlakota (1992) shows that infinite-horizon models of endowment economies with trading frictions or borrowing constraints can generate bubbles. Recently, there has been a growing interest in introducing rational bubbles into production economies with borrowing constraints. Examples include Caballero and Krishnamurthy (2006), Farhi and Tirole (2012), and Martin and Venture (2012) in the overlapping generations framework and Kocherlakota (2009), Wang and Wen (2012), Miao and Wang (2012, 2013, 2014, 2015), Miao, Wang, and Xu (2013a), Miao, Wang, and Xu (2013b), and Hirano and Yanagawa (2013) in the infinite-horizon growth framework. In particular, Miao and Wang (2013) study how a variety of endogenous credit constraints derived from optimal contracts with limited commitment can generate stock price bubbles. They show that stock price bubbles can relax credit constraints and generate dividend/collateral yields, which also represent the liquidity premium. Unlike their study, this paper focuses on leverage constraints and pure bubbles attached to intrinsically useless assets.

Fiat money is a pure bubble. Kiyotaki and Moore (2008) provide a model in which money is valued due to its liquidity. Our idea is similar to theirs. But land is different from money because land is illiquid and serves as collateral and also because land is not produced by the government.

Our paper is more closely related to the literature on housing or land bubbles.[6] Kocherlakota (2009) provides a model of land bubbles based on Kiyotaki and Moore (2008). In his model firms face idiosyncratic productivity shocks and collateral constraints. Land is intrinsically useless, but

[4] Glaeser, Gottlieb, and Gyourko (2010) document evidence that low interest rates cannot explain the housing bubble between 2001 and the end of 2005 in the US. Adam, Kuang and Marcet (2011) present a model that relates the low real interest rates at the start of the millennium to the observed housing booms in the G7 economies.

[5] See Brunnermierer and Oehmke (2013) and Miao (2014) for surveys.

[6] See DSGE models of housing without bubbles, e.g., Iacoviello (2005), Kiyotaki, Michaelides, and Nikolov (2011), Liu, Wang, and Zha (2013), and Liu, Miao, and Zha (2013), among others.

serves as collateral as in Kiyotaki and Moore (1997). Thus land bubbles improve welfare. He, Wright, and Zhu (2013) build a model of housing bubbles in a monetary economics framework. Their model does not incorporate real investment and the credit constraint applies to households instead of firms. As in our paper, the existence of a housing bubble is due to the liquidity premium. Unlike our model with two steady states, their model delivers a unique steady state. Housing in their model can also provide direct utility. Arce and Lopez-Salido (2011) study housing bubbles in an overlapping generations framework with credit constraints. The interest rate is equal to the growth of the bubble in their model. In contrast to our result, they show that the interest rate in the bubbly steady state is lower than that in the bubbleless steady state. They also incorporate utility from housing and show that the housing price in the bubbly equilibrium is less than the discounted value of the utility flow (or dividends).

Bubbles must provide some benefits to economic agents, or else, they could not exist in the first place. However, policymakers and researchers are more concerned about the welfare costs of bubbles. Potential costs include volatility and fire sales after the collapse of bubbles (Caballero and Krishnamurthy (2006) and Miao and Wang (2015)) and misallocation of resources in the presence of market distortions such as externality (Grossman and Yanagawa (1993) and Miao and Wang (2014)). In this paper we focus on the cost generated by resource misallocation in the presence of idiosyncratic tax policy distortions. Most papers in the literature discuss the role of monetary policy in preventing bubbles. In an overlapping generations model, Galí (2014) studies how monetary policy can affect the fluctuations of bubbles. But monetary policy cannot eliminate bubbles.[7] Because the asset pricing equation for the bubble includes the liquidity premium in our model, we argue that other policy tools can be used to lower this premium and eliminate bubbles.

Property tax policy and LTV policy are often discussed by the policymakers and the general public. For example, the Chinese government has implemented these policies to curb the growth of housing prices and to prevent housing bubbles.[8] Our analysis provides a theoretical foundation for

[7] See Bernanke and Gertler (1999) for a related study. While Galí (2014) argues that the leaning against the wind policy may not be justified theoretically, Adam and Woodford (2013) provide a model with housing, where that policy can be Ramsey optimal.

[8] For example, a new nationwide real estate sales tax was introduced in late 2009. Families purchasing a second home were required to make at least a 40% downpayment in 2010, and legislation for a property tax was passed in November 2013.

these policies. The asset purchase policy proposed in our paper is related to those in Kocherlakota (2009), Hirano, Inaba, and Yanagawa (current issue), and Miao and Wang (2013). Kocherlakota (2009) discusses credit policy to restore the bubbly equilibrium. Miao and Wang (2013) provide a credit policy to achieve the first-best allocation. Hirano, Inaba, and Yanagawa (current issue) study bailout policy and welfare implications for workers who are taxpayers.

The key difference between our paper and some of the aforementioned papers is that our model adopts the infinite-horizon growth framework, which is amenable to quantitative studies (see, e.g., Miao, Wang, and Xu (2013b) and Miao, Wang, and Zha (2014)). In addition, the borrowing constraint in our model differs from those often used in the literature on housing prices. Many papers adopt the Kiyotaki and Moore (1997) collateral constraint, which ensures that the debt repayment does not exceed the collateral value so that the borrower will never default. The borrowing constraint in this paper is a type of margin constraint, consistent with the institutional feature in the mortgage market. We show that given the margin constraint, the Kiyotaki-Moore collateral constraint is always satisfied so that default never occurs in our model. The margin constraint is also adopted in Arce and Lopez-Salido (2011) and some references therein.

2. The Baseline Model

To preserve the tractability of the representative agent framework and also allow for firm heterogeneity, we consider an economy populated by a continuum of identical households of unit mass. Each household is an extended family consisting of a continuum of ex ante identical entrepreneurs of unit mass and a continuum of identical workers also of unit mass. Each entrepreneur runs a firm. There is a government that subsidizes entrepreneurial investment and the subsidy is financed by lump-sum taxes on households. Following Restuccia and Rogerson (2008), we assume that this policy distortion is idiosyncratic and take it as a given institutional feature throughout the analysis. For simplicity, we do not consider other forms of government spending.

Time is discrete and denoted by $t = 0, 1, ...$ There is no aggregate uncertainty about fundamentals. Assume that a law of large numbers holds so that aggregate variables are deterministic.

2.1. Entrepreneurs

An entrepreneur is indexed by $j \in [0,1]$. Each entrepreneur j runs a firm using a constant-returns-to-scale technology to produce output according to $Y_{jt} = K_{jt}^{\alpha} N_{jt}^{1-\alpha}, \quad \alpha \in (0,1)$, where K_{jt} and N_{jt} represent capital and labor inputs, respectively. Entrepreneurs can borrow and lend among themselves by trading one-period riskless bonds. They can also trade land. Normalize land supply to one. For simplicity, assume that land is intrinsically useless in that it does not deliver any payoff or direct utility.[9] Land can be used by entrepreneurs as collateral for borrowing. Assume that each entrepreneur is initially endowed with zero bond and one unit of land, i.e., $B_{j0} = 0$ and $H_{j0} = 1$ for all j. Assume that land does not depreciate.

Solving the static labor choice problem,

$$R_{kt} K_{jt} \equiv \max_{N_{jt}} \; K_{jt}^{\alpha} N_{jt}^{1-\alpha} - W_t N_{jt},$$

gives

$$N_{jt} = \left(\frac{1-\alpha}{W_t} \right)^{\frac{1}{\alpha}} K_{jt}, \quad R_{kt} = \alpha \left(\frac{1-\alpha}{W_t} \right)^{\frac{1-\alpha}{\alpha}}, \tag{1}$$

where W_t denotes the wage rate. We will show later that R_{kt} is equal to the rental rate of capital.

Entrepreneur j's dividends are given by

$$D_{jt} = R_{kt} K_{jt} - \tau_{jt} I_{jt} - P_t (H_{jt+1} - H_{jt}) + \frac{B_{jt+1}}{R_{ft}} - B_{jt}, \tag{2}$$

where I_{jt}, P_t, H_{jt}, and R_{ft} denote the investment level, the land price, land holdings, and the (gross) interest rate, respectively. In addition, B_{jt} represents the debt level if it is positive; and savings, otherwise. Note that τ_{jt} represents distortions on the investment good price and is an important variable in the model. When $\tau_{it} > 1$, it represents capital market distortions, e.g., transactions costs. When $\tau_{it} \in (0,1)$, we interpret $1 - \tau_{jt} > 0$ as a subsidy to investment, e.g., ITC, that must be financed by taxes on households. For simplicity, suppose that τ_{jt} is independently and identically distributed across firms and over time, and is drawn from a fixed distribution with the density function f on $[\tau_{\min}, \tau_{\max}]$. This modeling captures the fact that distortions like tax subsidies vary

[9] See Tirole (1985), Kocherlakota (2009), Miao and Wang (2013) for the discussions of how to introduce rents into the asset with bubbles. One way is to introduce economic growth and assume that rents grow at a rate lower than economic growth. See Miao, Wang and Zha (2014) for another way.

across time and across assets and firms. Goolsbee (1998) documents extensive empirical evidence on this fact and points out that the ITC varies by asset for many years in his sample. Motor vehicles and aircraft, for example, normally have lower rates of credit. Song and Wu (2015) estimate that idiosyncratic distortions on the investment good price can generate capital misallocation and cause aggregate revenue losses of 20 percent for Chinese firms.

Entrepreneur j's capital accumulation equation is given by

$$K_{jt+1} = (1-\delta)K_{jt} + I_{jt}, \; K_{j0} \text{ given,} \tag{3}$$

where $\delta \in (0,1)$ denotes the depreciation rate. One could shut down τ_{jt} and introduce an idiosyncratic investment efficiency shock ε_{jt} to (3) so that I_{jt} units of investment would generate $\varepsilon_{jt} I_{jt}$ units of capital as in Miao, Wang, and Xu (2013a). In this case asset bubbles could still emerge and improve investment efficiency. But no taxes would be levied on households and hence asset bubbles would be welfare improving.

Entrepreneurs face several constraints due to real and financial frictions. First, there is empirical evidence that equity financing is more costly than debt financing. For simplicity, we assume that equity financing is so costly that entrepreneurs cannot raise new equity. We thus impose the constraint,

$$D_{jt} \geq 0. \tag{4}$$

Second, due to imperfect contract enforcement, there is a down payment restriction or margin requirement on land purchases:

$$\frac{B_{jt+1}}{R_{ft}} \leq \theta P_t H_{jt+1}, \tag{5}$$

where $\theta \in (0,1)$ represents the LTV ratio and $1-\theta$ represents the down payment or margin requirement. Land is used as collateral. To ensure that entrepreneur j will not default in the next period, we require that

$$B_{jt+1} \leq P_{t+1} H_{jt+1}. \tag{6}$$

This constraint ensures that debt repayments do not exceed the collateral value in the next period. Kiyotaki and Moore (1997) introduce this constraint, but ignore the margin constraint (5). The financial friction modeled in (4)-(6) is the most important part of the model and is crucial for the existence of a bubble. Firms in our model do not have sufficient liquidity to finance real investment. Land as an asset provides liquidity and alleviates the asset shortage problem.

Third, land trading is illiquid. Following Kiyotaki and Moore (2008), we impose the following resaleability constraint

$$H_{jt+1} \geq \omega H_{jt}, \tag{7}$$

where $\omega > 0$ represents the restriction on liquidity of land trading. This constraint means that entrepreneurs can sell at most a fraction $1 - \omega$ of their existing land. We also rule out short sales of land so that $H_{jt} \geq 0$.

The last constraint is that investment is irreversible at the firm level, i.e.,

$$I_{jt} \geq 0. \tag{8}$$

As will become clear later, this assumption is useful for deriving optimal investment given constant-returns-to-scale technology.

Now we describe entrepreneur j's decision problem by dynamic programming. Entrepreneur j's value function is denoted by $V_t(\tau_{jt}, K_{jt}, H_{jt}, B_{jt})$, where we suppress aggregate state variables as arguments in the value function. The dynamic programming problem is given by

$$V_t(\tau_{jt}, K_{jt}, H_{jt}, B_{jt}) = \max \; D_{jt} + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(\tau_{jt+1}, K_{jt+1}, H_{jt+1}, B_{jt+1}), \tag{9}$$

subject to (2), (3), (4), (5), (6), (7), and (8). Here E_t represents the conditional expectation operator with respect to the idiosyncratic shock and Λ_t is the representative household's marginal utility.

2.2. Households

Assume that labor supply is inelastic and normalized to one. Entrepreneurs and workers hand over their dividends and wages to their family. The family pool their income and distribute it equally among family members. A representative household chooses family consumption C_t to maximize its life-time expected utility, $\sum_{t=0}^{\infty} \beta^t \ln(C_t)$, subject to $C_t = W_t N_t + D_t - \Gamma_t$, where

$N_t = 1$, $D_t = \int D_{jt} dj$ denotes the total dividends from all firms, and Γ_t denotes lump-sum taxes satisfying $\Gamma_t = \int (1 - \tau_{jt}) I_{jt} dj$.

Given the utility function above, we can derive marginal utility $\Lambda_t = 1/C_t$. For simplicity, we have assumed that households do not borrow or save as in Kocherlakota (2009) and Kiyotaki and Moore (2008). We can relax this assumption and suppose that households can save, but cannot borrow against their future incomes. Then households will optimally choose not to save because, as we will show later, the equilibrium interest rate is too low, i.e., $R_{ft} < \Lambda_t / (\beta \Lambda_{t+1})$. Consequently, none of our results will change.

2.3. Competitive equilibrium

A competitive equilibrium consists of sequences of individual quantities $\{I_{jt}, N_{jt}, K_{jt+1}, Y_{jt}, H_{jt+1}\}$ and aggregate quantities $\{C_t, I_t, N_t, Y_t\}$ and prices $\{W_t, R_{kt}, R_{ft}, P_t\}$ such that (i) households optimize; (ii) workers and entrepreneurs optimize; and (iii) the markets for labor, land, bonds, and consumption goods all clear, i.e., $C_t + I_t = Y_t$,

$$N_t = \int_0^1 N_{jt} dj = 1, \ \int_0^1 H_{jt} dj = 1, \ \int_0^1 B_{jt} dj = 0,$$

where $I_t = \int I_{jt} dj$ and $Y_t = \int Y_{jt} dj$.

3. Model Solution

We first solve entrepreneurs' decision problem and then characterize the equilibrium system. Finally, we analyze the steady state and local dynamics of the system.

3.1. Entrepreneurs' Decision Problem

We conjecture entrepreneur j's value function takes the form:

$$V_t(\tau_{jt}, K_{jt}, H_{jt}, B_{jt}) = v_t(\tau_{jt}) K_{jt} + p_t(\tau_{jt}) H_{jt} - \varphi_t(\tau_{jt}) B_{jt},$$

where $v_t(\tau_{jt})$, $p_t(\tau_{jt})$, and $\varphi_t(\tau_{jt})$ are to be determined and satisfy the following restrictions:

$$P_t = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \int p_{t+1}(\tau) f(\tau)\, d\tau, \quad \frac{1}{R_{ft}} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \int \varphi_{t+1}(\tau) f(\tau)\, d\tau. \tag{10}$$

These restrictions are asset pricing equations derived from optimality and equilibrium (see the online appendix, which also contains proofs of all results in the paper). They imply that the land price P_t and the bond price $1/R_{ft}$ reflect the marginal valuation of land and bonds. Define $Q_t \equiv \beta \frac{\Lambda_{t+1}}{\Lambda_t} \int v_{t+1}(\tau) f(\tau)\, d\tau$ as Tobin's marginal Q or the marginal value of one additional unit of installed capital. We substitute the conjecture above into the Bellman equation and derive the following result:

Proposition 1 *(i) For* $\tau_{jt} \leq Q_t$,

$$\begin{aligned} I_{jt} &= \frac{1}{\tau_{jt}} \left[R_{kt} K_{jt} + (1 - \omega + \theta \omega) P_t H_{jt} - B_{jt} \right], \\ \frac{B_{jt+1}}{R_{ft}} &= \theta P_t H_{jt+1}, \ \ H_{jt+1} = \omega H_{jt}. \end{aligned}$$

For $\tau_{jt} > Q_t$, $I_{jt} = 0$, *and entrepreneur* j *is indifferent among any choices of* H_{jt+1} *and* B_{jt+1} *satisfying (5), (7) and*

$$0 \leq R_{kt} K_{jt} + \frac{B_{jt+1}}{R_{ft}} - B_{jt} - P_t (H_{jt+1} - H_{jt}).$$

(ii) The land price, Tobin's Q, and the interest rate satisfy

$$P_t = \beta \frac{\Lambda_{t+1}}{\Lambda_t} P_{t+1} \left\{ 1 + \underbrace{(1 - \omega + \omega \theta) \int_{\tau \leq Q_{t+1}} \frac{Q_{t+1} - \tau}{\tau} f(\tau) d\tau}_{\textit{liquidity premium}} \right\}, \tag{11}$$

$$Q_t = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left[(1 - \delta) Q_{t+1} + R_{kt+1} + \underbrace{R_{kt+1} \int_{\tau \leq Q_{t+1}} \frac{Q_{t+1} - \tau}{\tau} f(\tau) d\tau}_{\textit{liquidity premium}} \right], \tag{12}$$

$$\frac{1}{R_{ft}} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left[1 + \underbrace{\int_{\tau \leq Q_{t+1}} \frac{Q_{t+1} - \tau}{\tau} f(\tau) d\tau}_{\textit{liquidity premium}} \right], \tag{13}$$

and the transversality conditions hold

$$\lim_{i\to\infty}\beta^i\frac{\Lambda_{t+i}}{\Lambda_t}Q_{t+i}K_{jt+i+1}=\lim_{i\to\infty}\beta^i\frac{\Lambda_{t+i}}{\Lambda_t}\frac{B_{jt+i+1}}{R_{ft+i}}=\lim_{i\to\infty}\beta^i\frac{\Lambda_{t+i}}{\Lambda_t}P_{t+i}H_{jt+i+1}=0. \tag{14}$$

We first discuss the intuition behind the optimal investment policy given in part (i) of the proposition. Due to idiosyncratic policy distortions, one dollar of investment costs τ_{jt} dollars. Its benefit is given by Tobin's marginal Q. Thus, when $\tau_{jt}\leq Q_t$, investing is profitable and optimal investment reaches the upper limit. In addition, entrepreneur j borrows as much as possible to finance investment so that the credit constraint (5) binds. Because $\theta\in(0,1)$, he also wants to sell land as much as possible to finance investment so that (7) binds. When $\tau_{jt}>Q_t$, investing is not profitable so that $I_{jt}=0$. Due to constant-returns-to-scale technology, the entrepreneur is indifferent among any choices of B_{jt+1} and H_{jt+1} in the set of the feasibility constraints.

Next we consider part (ii) of the proposition, which gives the asset pricing equations for the land price, Tobin's Q and the interest rate. The left-hand side of equation (11) represents the cost of buying one unit of land. The right-hand side of this equation represents the benefit of holding this unit of land. It consists of two components. The first component is the usual resale value. The second component is a special feature of our model. It represents the role of liquidity and collateral played by land as an asset. Specifically, to finance investment, the entrepreneur can sell $(1-\omega)$ units of land and borrow against the value of $\omega\theta$ units of land. The entrepreneur makes investment if and only if $\tau\leq Q_{t+1}$. The expected return from one dollar of the investment is given by $\int_{\tau\leq Q_{t+1}}\frac{Q_{t+1}-\tau}{\tau}f(\tau)d\tau$. Thus the total expected return from the investment is given by the second component on the right-hand side of (11). We call this component the "liquidity premium" in the land price.

It is straightforward to show that the Lagrange multiplier associated with the dividend constraint (4) is equal to $(Q_t-\tau_{jt})/\tau_{jt}$ if $Q_t>\tau_{jt}$; and 0, otherwise. This Lagrange multiplier is also equal to that associated with the borrowing constraint (5). Thus the liquidity premium essentially reflects the shadow value of relaxing external financing constraints by an additional dollar. Land has liquidity value because it can relax these constraints.

Alternatively we may interpret (11) when $P_t>0$ as a standard Euler equation, $1=\beta\frac{\Lambda_{t+1}}{\Lambda_t}R^H_{t+1}$, where R^H_{t+1} denotes the return on land. This return consists of two components: capital gains

P_{t+1}/P_t and the liquidity premium in returns defined as the liquidity premium in the land price multiplied by P_{t+1}/P_t. Note that land is intrinsically useless and does not deliver any rent. The liquidity premium is generated from the belief about the future value of land $P_{t+1} > 0$.

In the traditional literature on bubbles (e.g., Tirole (1985)), there is no liquidity premium so that the return on land is equal to the capital gains or the growth rate of the land price. The transversality condition (14) for infinitely-lived agents then rules out the existence of a bubble.[10] Because of the liquidity premium, the transversality condition cannot rule out bubbles in our model.

Equation (12) is the asset pricing equation for Tobin's Q. The dividend generated from capital consists of rents R_{kt+1} and a liquidity premium for capital. Due to the credit constraint, a unit of capital generates R_{kt+1} units of internal funds (or liquidity) which can be used to finance investment. The investment generates expected return given by the last component in (12).

Equation (13) shows that the bond price also carries a liquidity premium due to credit constraints. The liquidity premium causes the equilibrium interest rate to be lower than the implicit interest rate $\Lambda_t / (\beta \Lambda_{t+1})$ in an economy without any frictions. This result proves our previous claim in Section 2.2.

Note that liquidity premium has three different expressions in (11), (12), and (13). They reflect different degrees of liquidity provided by land, capital, and bonds. Two special cases merit discussions. First, when $\omega = 0$, land trading is liquid. Land as an asset is a perfect substitute for bonds and they earn the same liquidity premium. Second, when $\theta = 1$, entrepreneurs can borrow against the full value of the non-resaleable land. Even though land trading may be illiquid, the non-resaleable land is effectively traded through bond trading. Thus land trading is effectively liquid. In this case land and bonds are also perfect substitutes and earn the same liquidity premium.

3.2. Equilibrium System

We now aggregate individual decision rules and impose market-clearing conditions. Define aggregate capital as $K_t \equiv \int K_{jt} dj$. We can characterize the equilibrium system as follows:

Proposition 2 *The equilibrium system is given by the following nine equations: (11), (12), (13),*

[10] The transversality conditions are necessary for infinite-horizon optimization problems with discounting and finite value functions (see, e.g., Ekeland and Scheinkman (1986)).

and

$$I_t = [R_{kt}K_t + (1-\omega+\theta\omega)\, P_t] \int_{\tau \le Q_t} \frac{1}{\tau} f(\tau) d\tau, \tag{15}$$

$$C_t + I_t = Y_t = K_t^{\alpha}, \tag{16}$$

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{17}$$

$$W_t = (1-\alpha)\, K_t^{\alpha},\ R_{kt} = \alpha K_t^{\alpha-1}, \tag{18}$$

for nine variables $\{C_t, I_t, Y_t, K_{t+1}, W_t, R_{kt}, R_{ft}, Q_t, P_t\}$. *The usual transversality conditions hold.*

Equation (15) shows that only firms with tax distortions $\tau \le Q_t$ contribute to aggregate investment. Other firms do not invest. Aggregate investment is financed by internal funds $R_{kt}K_t$, land sales $(1-\omega)\, P_t$, and external borrowing $\theta\omega P_t$. Equations (16)-(18) are standard as in the literature.

We have already explained the three asset pricing equations (11), (12), (13). We observe that $P_t = 0$ for all t always satisfies equation (11). we call such an equilibrium a bubbleless equilibrium. Later we will show that there can exist an equilibrium in which $P_t > 0$ for all t. We call such an equilibrium a bubbly equilibrium. It is straightforward to verify that, in a bubbly equilibrium, the Kiyotaki-Moore type collateral constraint (6) is always satisfied and hence our omission of this constraint in Section 3.1 is without loss of generality.

We now describe how the two types of equilibria work in the model. In a bubbleless equilibrium, land has no value and will not be traded. The credit market is essentially shut down because no collateral is available. For highly subsidized entrepreneurs with $\tau_{jt} \le Q_t$, investment is profitable. These entrepreneurs use internal funds to finance investment. For entrepreneurs with $\tau_{jt} > Q_t$, investment is not profitable and hence they do not invest.

In a bubbly equilibrium, entrepreneurs with $\tau_{jt} \le Q_t$ borrow and sell land to finance investment as much as possible until both the borrowing and resaleability constraints bind. Entrepreneurs with $\tau_{jt} > Q_t$ do not invest. They are indifferent between saving and borrowing, and between buying and selling land. To clear the bond and land markets, their aggregate behavior is to save and lend to highly subsidized entrepreneurs and also buy land from them. In the special case of $\omega = 0$, land trading is liquid. Highly subsidized entrepreneurs with $\tau_{jt} \le Q_t$ sell all their land to finance investment. They will not borrow because they have no land collateral. To clear the land market, entrepreneurs with $\tau_{jt} > Q_t$ must purchase land. Borrowing and saving take place within these firms.

3.3. Bubbleless Steady State

We use a subscript f to denote a variable in an equilibrium without bubble. We also remove the time subscript for any variable in the steady state. Using the steady-state version of equations (15) and (17), we can show that

$$R_{kf} = \frac{\delta}{\int_{\tau \leq Q_f} \frac{1}{\tau} f(\tau) d\tau}. \tag{19}$$

Substituting R_{kf} into the steady-state version of equation (12) yields an equation for Q_f.

Proposition 3 *The equation*

$$1 - \beta(1-\delta) = \beta\delta \frac{\int \max\left(\frac{1}{\tau}, \frac{1}{Q_f}\right) f(\tau) d\tau}{\int_{\tau \leq Q_f} \frac{1}{\tau} f(\tau) d\tau} \tag{20}$$

has a unique solution for $Q_f \in (\tau_{\min}, \tau_{\max})$. If R_{kf} in (19) satisfies

$$R_{kf} > \alpha\delta, \tag{21}$$

then there is a unique bubbleless steady state and Q_f is equal to Tobin's Q in the bubbleless steady state.

Given Q_f, we can derive the steady-state rental rate of capital R_{kf} from equation (19). We then use (18) to determine the steady-state capital stock K_f. The steady-state investment, output, and consumption are given by $I_f = \delta K_f$, $Y_f = K_f^{\alpha}$, and $C_f = Y_f - I_f$, respectively. Condition (21) ensures that $C_f > 0$. A sufficient condition for it in terms of primitives is given by $\alpha \int \frac{1}{\tau} f(\tau) d\tau < 1$, because $Q_f \in (\tau_{\min}, \tau_{\max})$.

3.4. Bubbly Steady State

In this subsection we study the bubbly steady state in which $P_t = P > 0$ for all t. We remove the time subscript and use a subscript b to indicate a bubbly steady state. The following proposition provides a characterization.

Proposition 4 *Suppose that*

$$\frac{\beta^{-1}-1}{1-\omega(1-\theta)} < \tau_{\max}\int \frac{1}{\tau} f(\tau)d\tau - 1, \tag{22}$$

and that R_{kb} satisfies

$$R_{kb} = \frac{1-\beta(1-\delta)}{\beta\int \max\left(\frac{1}{\tau},\frac{1}{Q_b}\right) f(\tau)d\tau} > \alpha\delta. \tag{23}$$

Then the bubbly and bubbleless steady states coexist if and only if

$$1 < \beta\left[1+(1-\omega+\theta\omega)\int_{\tau\leq Q_f}\frac{Q_f-\tau}{\tau}f(\tau)d\tau\right], \tag{24}$$

where Q_f is determined by (20).

Condition (23) ensures that $C_b > 0$. If this condition is satisfied, then (21) also holds since we can show that $R_{kb} < R_{kf}$. A sufficient condition for (23) in terms of primitives when $\tau_{\min} > 0$ is given by $\tau_{\min}[1-\beta(1-\delta)] > \alpha\beta\delta$. If $\tau_{\min}$ were too low, then the ITC would be too large, causing firms to make too much investment so that steady-state consumption would be negative. Condition (24) ensures that $P > 0$. To interpret this condition, we recall the discussion following Proposition 1. The right-hand side of (24) represents the steady-state benefit of purchasing one unit of land when Tobin's Q is equal to the bubbleless steady state value Q_f. When this benefit is larger than the unit cost, a land bubble can exist. Condition (24) will be crucial for our policy analysis in Section 5.

The following proposition compares the two steady states.

Proposition 5 *If the bubbleless and bubbly steady states coexist, then $Q_f > Q_b$, $R_{kb} < R_{kf}$, $R_{fb} > R_{ff}$, $K_b > K_f$, $I_b > I_f$, and $Y_b > Y_f$.*

This proposition shows that the existence of a land bubble in the steady state allows entrepreneurs to finance more investment and accumulate more capital stock. This causes the rental rate of capital and Tobin's marginal Q to be lower and output to be higher in the bubbly steady state than in the bubbleless steady state. However, it is not necessarily true that consumption is higher in the bubbly steady state than in the bubbleless steady state. The intuition is that a land bubble may cause entrepreneurs to overinvest, causing fewer resources to be allocated to consumption. Thus a land bubble may reduce welfare. We will study this issue in Section 4.

Note that when $\omega = 0$, land trading is liquid. Equations (11) and (13) imply that $R_{fb} = 1$ in the bubbly steady state and $R_{ff} < 1$ in the bubbleless steady state. But when $\omega > 0$, we must have $R_{fb} < 1$ by (11) and (13). The intuition is that when $\omega = 0$, land and bonds are perfect substitutes. Since land is intrinsically useless in the model, the net interest rate of bonds must be zero. But when $\omega > 0$, land is an illiquid asset. For land and bonds to coexist in a bubbly equilibrium, the net interest rate of bonds must be negative. To generate a positive steady-state net interest rate, we can introduce economic growth by assuming that aggregate productivity grows at a constant rate. See Miao and Wang (2013) for a related analysis.

Proposition 5 shows that the interest rate in the bubbleless steady state is lower than that in the bubbly steady state. The reason is that the land bubble crowds out the bond demand, thereby reducing the bond price and raising the interest rate. This result has an important policy implication as we will show in Section 5.

3.5. Local Dynamics

We now study local dynamics around the bubbly and bubbleless steady states. Because of the complexity of the model, we are unable to provide a full characterization. The following proposition characterizes the bubbleless steady state for general distribution functions.

Proposition 6 *When both the bubbly and bubbleless steady states exist, then the local equilibrium around the bubbleless steady state is indeterminate of degree one. When only the bubbleless steady state exists, then it is a saddle point and there is a unique bubbleless equilibrium converging to it.*

To prove this proposition, we use Proposition 2 to simplify the equilibrium system to a system of four nonlinear difference equations for four unknown variables C_t, K_t, Q_t, and P_t. Only K_t is a predetermined variable. The other three variables are nonpredetermined. We then linearize the equilibrium system around the bubbleless steady state to obtain a linear system $MX_{t+1} = X_t$, where $X_t = (\hat{C}_t, \hat{K}_t, \hat{Q}_t, P_t)'$ and a hatted variable denotes log deviation from the steady state. P_t is the deviation from 0. We check properties of the eigenvalues of the coefficient matrix M. We can show that when both the bubbly and bubbleless steady states exist, there are two eigenvalues outside the unit circle and two eigenvalues inside the unit circle. This means that the local equilibrium around the bubbleless steady state is indeterminate of degree one. In particular, given K_0 and for any initial value $P_0 > 0$ in the neighborhood of the bubbleless steady state, there is a unique equilibrium path (C_t, K_t, Q_t, P_t) converging to the bubbleless steady state. That is, the land bubble eventually bursts but its initial value P_0 is indeterminate. However, when only the bubbleless steady state exists, the matrix M has three eigenvalues inside the unit circle and one eigenvalue outside the unit circle. This means that the bubbleless steady state is determinate and there is a unique equilibrium converging to this steady state. Since $P_t = 0$ always satisfies equation (11), this equilibrium must be bubbleless.

We now turn to the bubbly steady state. We are able to derive the following theoretical result for a general distribution in the special case of $\omega(1-\theta) = 0$ and $\delta = 1$.

Proposition 7 *Let $\omega(1-\theta) = 0$ and $\delta = 1$. Suppose that the bubbly steady state exists. Then there is a locally unique bubbly equilibrium converging to the bubbly steady state.*

The idea of the proof is similar to that for Proposition 6. For general distributions and parameter values, we are unable to derive theoretical results. However, we have verified numerically that the results in Propositions 6 and 7 hold for a wide range of parameter values and for many different types of distributions for the idiosyncratic shock. Note that Tirole (1985) and Miao and Wang (2013) prove similar results in other models of bubbles.

4. Welfare Analysis

In this section we study the welfare implications of the bubbleless and bubbly equilibria. Both equilibria are inefficient due to idiosyncratic policy distortions and credit constraints. We will take these distortions as a given institutional feature and compare welfare between the bubbleless and bubbly equilibria.

4.1. Welfare Comparison

Let $U_f(K_0)$ and $U_b(K_0)$ denote the household life-time utility level in the bubbleless equilibrium and in the bubbly equilibrium, respectively, given the economy starts at the aggregate capital stock K_0. Then the bubbleless and bubbly steady-state life-time utility levels are given by $U_f(K_f)$ and $U_b(K_b)$, respectively.

We first provide a theoretical result for a special case.

Proposition 8 *Let $\delta = 1$ and $\omega(1-\theta) = 0$. Suppose that both bubbly and bubbleless steady states exist. If $\int_{\tau \le Q_f} \frac{1}{\tau} f(\tau)\, d\tau > 1$, then $U_f(K_f) > U_b(K_b)$ and $U_f(K_0) > U_b(K_0)$.*

Here we sketch the key idea of the proof. We show that the saving rate $s_t \equiv I_t/Y_t = s$ must be at the respective constant steady state value for all t in both the bubbly and bubbleless equilibria. In addition, Tobin's Q must be at the constant steady state value during the transition. Importantly, the bubble-to-output ratio is also constant over time in the bubbly equilibrium. However, the capital stock, investment, consumption, and output change over time. In particular, the law of motion for capital satisfies $K_{t+1} = I_t = s_t Y_t = s K_t^{\alpha}$ and consumption is given by $C_t = (1-s) K_t^{\alpha}$. We can then write the life-time utility level as

$$\sum_{t=0}^{\infty} \beta^t \ln(C_t) = \frac{\ln(1-s)}{1-\beta} + \frac{\alpha}{1-\alpha\beta}\left[\frac{\beta}{1-\beta}\ln(s) + \ln(K_0)\right]. \tag{25}$$

To compare welfare, we only need to compare the equilibrium saving rate and initial capital stock. It turns out that the saving rate in the bubbly equilibrium is too high, generating too much investment. This causes welfare to be lower.

4.2. Examples

In the online appendix we provide an explicitly solved example to illustrate Propositions 3-8. Let $\delta = 1$, $\omega(1-\theta) = 0$, and $f(\tau) = \eta\tau^{\eta-1}$, $\eta > 1$, for $0 \le \tau \le 1$. We can check the conditions in Proposition 4. For (22) to hold, we need $\eta < 1/(1-\beta)$. For (23) to hold, we need $(1/\beta - 1)^{\frac{1}{\eta}} (\eta - 1)^{\frac{1}{\eta}} > \alpha$. To ensure $P/Y_b > 0$, i.e. for (24) to hold, we need $\eta < \beta/(1-\beta)$. If the preceding conditions are satisfied, then the bubbly and bubbleless steady states coexist. It can be verified numerically that these two conditions hold for a wide range of parameter values.

We now provide some numerical examples for general values of ω and θ. Suppose that one period corresponds to a quarter. We set $\alpha = 0.3$, $\beta = 0.99$, $\delta = 0.025$, $\omega = 0.2$, and $\theta = 0.75$. We set $\eta = 5.7$ so that the bubbleless steady-state capital to output ratio is equal to 10 as in the US data. We find the following numerical results: in the bubbleless steady state, $K_f = 28.67$, $Y_f = 2.737$, $I_f = 0.7166$, $C_f = 2.020$, $s_f = 0.2619$, $Q_f = 0.9324$, $R_{kf} = 0.02864$, $R_{ff} = 0.8839$, and $U_f(K_f) = 70.31$; in the bubbly steady state, $K_b = 46.11$, $Y_b = 3.156$, $I_b = 1.153$, $C_b = 2.003$, $s_b = 0.3653$, $Q_b = 0.5912$, $R_{kb} = 0.02053$, $R_{fb} = 0.9995$, $P = 10.84$, and $U_b(K_b) = 69.48$. Clearly the saving rate in the bubbly steady state is 39% higher than that in the bubbleless steady state. But the life-time utility level in the bubbly steady state is about 1.2% lower than that in the bubbleless steady state. We can measure the welfare cost as a proportional compensation for consumption in the bubbly equilibrium such that the household is indifferent between the bubbly and bubbleless equilibria. We find that the steady-state welfare cost is 0.83% of consumption.

The welfare cost is even larger during the transition period. Figure 2 plots the paths of life-time utility levels in the bubbly and bubbleless equilibria for two initial values of the capital stock, $K_0 = 1.05K_b$ and $K_0 = 0.95K_f$. The initial utility gap is large and then gradually shrinks over time. In the long run, the difference in utility is still significant. When measured in terms of consumption compensation, the initial welfare cost is equal to 7.45% and 7.48% for $K_0 = 1.05K_b$ and $K_0 = 0.95K_f$, respectively.

5. Policy Analysis

In the previous section we have shown that land bubbles generate excessive investment and reduce welfare. In this section we will study the policies that can eliminate the bubbly steady state and allow the economy to achieve the unique bubbleless equilibrium. We will introduce one policy at a time in the baseline model presented in Section 2. We emphasize that both the bubbly and bubbleless equilibria are inefficient because of the presence of idiosyncratic tax distortions and credit market imperfections. We take these distortions as a given institutional feature. To achieve the first-best allocation, one has to remove the idiosyncratic policy distortions and credit market imperfections. Study of such policies is beyond the scope of this paper.

5.1. Loan-to-Value Ratio

Recently some countries, such as Hungary, Norway, Sweden and the UK, have adopted maximum loan-to-value (LTV) ratios for mortgages as a macroprudential instrument to regulate the housing market. The intuition is that the LTV ratio can control the credit limit and hence stabilize the credit market. In our model lowering the LTV ratio θ reduces the credit limit and hence reduces the collateral yield generated by the land bubble when land trading is illiquid $\omega > 0$. This can reduce the benefit of holding land. When θ is sufficiently small, the benefit is sufficiently small so that the expression on the right-hand side of (24) is smaller than 1, causing the existence condition for a bubbly equilibrium to be violated. In this case a bubbly equilibrium cannot exist. This result is consistent with the general view that one important cause of the housing or land bubble is excessive credit. If the policymaker can adequately control credit, a land bubble cannot exist.

Note that this result depends on the assumption that $\omega > 0$. When land trading is liquid (i.e., $\omega = 0$), entrepreneurs can sell all of their land holdings to finance investment and be left with no collateral for borrowing. In this case controlling the LTV ratio is an ineffective way to eliminate a bubble.

5.2. Property Tax

Next we consider the impact of the property tax. Suppose that the government taxes the property and transfers the tax revenue to households in a lump-sum manner. Then the entrepreneur's flow-of-funds constraint becomes

$$D_{jt} = R_{kt}K_{jt} - \tau_{jt}I_{jt} - P_t(H_{jt+1} - H_{jt}) + \frac{B_{jt+1}}{R_{ft}} - B_{jt} - \tau_H P_t H_{jt}, \tag{26}$$

where τ_H represents the tax rate on the property. As in Proposition 4, we can show that the bubbly and bubbleless steady states coexist if and only if

$$1 < \beta \left[1 - \tau_H + (1 - \omega + \theta\omega - \tau_H) \int_{\tau \leq Q_f} \frac{Q_f - \tau}{\tau} f(\tau) d\tau \right],$$

where Q_f is Tobin's Q in the bubbleless equilibrium. Since Q_f is independent of τ_H, this condition will be violated when τ_H is sufficiently large. In this case a bubbly equilibrium cannot exist. The intuition is that when the property tax rate is sufficiently large, the benefit of holding land will be less than the cost of purchasing land so that land will not be traded at a positive price.

5.3. Property Transaction Tax

It is often argued that the Tobin tax on financial transactions can stabilize the financial market. We now consider the impact of the Tobin tax or the property transaction tax in the land market. Suppose that the transaction of land is taxed at the rate $\phi \in (0,1)$ and that the tax revenue is rebated to households in a lump-sum manner. Then entrepreneur j's flow-of-funds constraint becomes

$$D_{jt} = R_{kt}K_{jt} - \tau_{jt}I_{jt} - P_t(H_{jt+1} - H_{jt}) + \frac{B_{jt+1}}{R_{ft}} - B_{jt} - \phi P_t|H_{jt+1} - H_{jt}|. \tag{27}$$

His decision problem is to solve (9) subject to (3), (4), (5), (6), (7), (8), and (27). In the online appendix we derive the equilibrium system and a necessary and sufficient condition for the existence of a bubbly steady state. We show that when ϕ is sufficiently large, this condition is violated and a land bubble cannot exist. The intuition is that when the tax rate on land transactions is sufficiently large, the benefit of trading land will be less than the cost of purchasing land so that land will not be traded at a positive price.

5.4. Asset Purchases

The government can affect interest rates by intervening in the private bond market. When the government participates in trading in the private bond market, the bond market-clearing condition becomes $\int_0^1 B_{jt}dj = B_{gt}$, where $B_{gt} > (<)0$ represents government purchases (sales) of bonds. The government budget constraint is given by

$$\int (1-\tau_{jt})I_{jt}dj + \frac{B_{gt+1}}{R_{ft}} = B_{gt} + \Gamma_t,$$

where Γ_t represents lump-sum taxes net of government spending.

By Proposition 1 and the bond market clearing condition, we can derive aggregate investment

$$I_t = [R_{kt}K_t + (1-\omega+\omega\theta)P_t - B_{gt}] \int_{\tau \le Q_t} \frac{1}{\tau} f(\tau) d\tau.$$

Other equilibrium conditions described in Proposition 2 remain unchanged.

The government purchases of bonds do not affect the borrowing constraint (5) directly, but affect the entrepreneurs' debt liabilities.[11] When setting $B_{gt} = (1 - \omega + \omega\theta)P_t$, then the debt liabilities to the government will offset the liquidity benefit provided by land. In this case investment I_t is effectively financed by internal funds only. Hence, a land bubble does not provide any liquidity to entrepreneurs and hence it cannot exist. The economy will reach the bubbleless equilibrium. Once the bubbleless equilibrium is reached, the government does not need to purchase any private bonds since $B_{gt} = 0$.

The interest rate in the bubbleless steady state is lower than that in the bubbly steady state by Proposition 5. The intuition is that the government purchase of private bonds raises the demand for private bonds and hence the bond price. This means that the asset purchase policy will not only eliminate land bubbles, but also lower the interest rate in the long run. This result contradicts the usual view that the central bank should increase interest rates in response to a growing bubble. Galí (2014) also makes this point in an overlapping generations model of bubbles.

6. Conclusion

In this paper we have presented a theory of credit-driven land bubbles in an infinite-horizon production economy when firms face idiosyncratic distortions on the investment good price. We assume that land is intrinsically useless, but can serve as collateral for borrowing. A land bubble can form because it commands a liquidity premium. The land bubble can provide liquidity and relax credit constraint, but it can also generate inefficient overinvestment. Property taxes, Tobin's taxes, macroprudential policy, and asset purchase policy can prevent the formation of land bubbles. These policies can be beneficial when land bubbles reduce social welfare. Our model also applies to other intrinsically useless bubbly assets similar to land. Although our model shows that asset bubbles can reduce welfare, some other models suggest that asset bubbles can improve welfare depending on the modeling of investment and the role of assets. It would be interesting to document empirical evidence on the welfare implications of asset bubbles.

[11] In the online appendix we study the impact of foreign purchases of bonds and show that they raise the domestic land prices and consumption, but reduce interest rates.

To compare the four types of policies analyzed in the paper, we note that asset purchase policy could be costly when asset purchases are financed by distortionary taxes. If taxes are on workers only, workers may be worse off. Property taxes and Tobin's taxes can reduce the welfare of the land owners and traders (or entrepreneurs). The LTV policy can reduce leverage and hence investment. Thus, although these policies can eliminate bubbles in the long run, they are costly in the short run. Our model is too stylized to study all the pros and cons of the four types of policies. We leave this question for future research.

For future research, it would be interesting to introduce rents and study the disconnect between housing prices and rents. Miao, Wang, and Zha (2014) have provided such a study. For simplicity, we have ignored aggregate uncertainty and the volatility generated by asset bubbles. Excessive volatility is also a potential cost of asset bubbles. In addition, asset bubbles may contribute to business cycles. Introducing bubbles into the dynamic stochastic general equilibrium framework and studying their quantitative implications should be an exciting research topic. Miao, Wang, and Xu (2013a) and Miao, Wang, and Zha (2014) have initiated such research.

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Figure 1: Real housing price indexes, price-income ratios, and price-rental ratios. See the online appendix for the data description.

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Figure 2: Transition paths of life-time utility levels. Parameter values are given by $\alpha = 0.3$, $\beta = 0.99$, $\delta = 0.025$, $\omega = 0.2$, $\theta = 0.75$, and $\eta = 5.7$.