A Bayesian DSGE Model of
Stock Market Bubbles and Business Cycles[*]

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Abstract

We present an estimated DSGE model of stock market bubbles and business cycles using Bayesian methods. Bubbles emerge through a positive feedback loop mechanism supported by self-fulfilling beliefs. We identify a sentiment shock which drives the movements of bubbles and is transmitted to the real economy through endogenous credit constraints. This shock explains most of the stock market fluctuations and sizable fractions of the variations in real quantities. It generates the comovement between stock prices and the real economy and is the dominant force behind the internet bubbles and the Great Recession.

Keywords: Stock Market Bubbles, Bayesian Estimation, DSGE, Credit Constraints, Business Cycles, Sentiment Shock

JEL codes: E22, E32, E44

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1. Introduction

The U.S. stock market is highly volatile (Shiller (1981)). The volatility of stock prices relative to output from 1975:Q1 to 2010:Q4 is 6.36.[1] The U.S. stock market comoves with macroeconomic quantities. The boom phase is often associated with strong output, consumption, investment, and hours worked, while the bust phase is often associated with economic downturns. Stock prices, consumption, investment, and hours worked are procyclical, i.e., they exhibit a positive contemporaneous correlation with output. In particular, the correlation between stock prices and output from 1975:Q1 to 2010:Q4 is 0.42.

These observations raise several questions. What are the key forces driving the boom-bust episodes? Are they driven by economic fundamentals, or are they bubbles? What explains the comovement between the stock market and the macroeconomic quantities? These questions are challenging for macroeconomists. Standard macroeconomic models treat the stock market as a sideshow. One can derive the stock price that supports the Pareto optimal allocation in a competitive equilibrium and the stock price is equal to the capital stock multiplied by Tobin's Q (Hayashi (1982)). We call this value the *fundamental value.* Much attention has been devoted to the equity premium puzzle (Hansen and Singleton (1983) and Mehra and Prescott (1985)). However, the preceding questions have remained underexplored.

Since the capital stock is a slow-moving state variable, a large movement of Tobin's Q is needed to explain the stock market volatility. Instead of trying to identify such a mechanism, we purse the idea that the stock market value contains a bubble component in addition to the fundamental value. Our goal is to provide an estimated dynamic stochastic general equilibrium (DSGE) model of stock market bubbles using Bayesian methods to address the aforementioned questions.[2] To the best of our knowledge, our paper provides the first structural analysis of bubbles using the Bayesian DSGE framework. Our model-based, full-information econometric methodology has several advantages over the single-equation or the vector autoregression (VAR) approach used in the early literature to identify bubbles. First, because neither bubbles nor fundamentals are observable, the literature fails to differentiate between misspecified fundamentals and bubbles (see Gurkaynak (2008) for a recent survey). By contrast, we treat bubbles as a latent variable in a DSGE model. The state space representation of the DSGE model allows us to conduct Bayesian inference of the latent variables by using observable data. We can answer the question of whether bubbles are important by comparing the marginal data densities of a DSGE model with bubbles and an alternative DSGE model without bubbles. Second, our DSGE model is theoretically coherent in the sense that decision rules of economic agents are derived from assumptions about preferences and technologies and some fundamental principles such as intertemporal optimization, rational expectations, and competitive equilibrium. Our model helps us better understand the mechanism behind the macroeconomic impact of bubbles. Third, because our model is structural, we can do counterfactual analysis to

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[1]The quarterly data are in logs and HP-filtered. See Section 3.1 for a description of the data.

[2]See An and Schorfheide (2007), Del Negro and Schorfheide (2011), and Herbst and Schorfheide (2015) for an introduction to Bayesian analysis of DSGE models.

examine the role of bubbles in generating fluctuations in macroeconomic quantities.

We set up a real business cycle (RBC) model with three standard elements: habit formation, investment adjustment costs, and variable capacity utilization. The novel element of our model is the assumption that firms are subject to idiosyncratic investment efficiency shocks and face endogenous credit constraints as in Miao and Wang (2011, 2012, 2014, 2015), Miao, Wang, and Xu (2012a), and Miao, Wang, and Zhou (2015). Under this assumption, a stock market bubble can emerge through a positive feedback loop mechanism supported by self-fulfilling beliefs. The intuition is as follows. Suppose that households have optimistic beliefs about the stock market value of a firm. The firm uses its assets as collateral to borrow from the lender. If both the lender and the firm believe that firm assets have high value, then the firm can borrow more and make more investment. This makes firm value indeed high, supporting people's initial optimistic beliefs. Bubbles can burst if people believe they can. By no arbitrage, if a bubble in an asset bursts, a new one in the same asset cannot emerge. To facilitate recurrent bubbles in the model, we introduce exogenous entry and exit. New entrants bring new bubbles to the economy, making the total bubble in the economy stationary.

We introduce a sentiment shock that drives the fluctuations in the bubble and hence the stock price. This shock reflects households' beliefs about the relative size of the old bubble to the new bubble. This shock is transmitted to the real economy through credit constraints. Its movements affect the tightness of the credit constraints and hence a firm's borrowing capacity. This in turn affects a firm's investment decisions and hence output.[3] Specifically, in response to a positive sentiment shock, the bubble and the stock price rise. This relaxes firms' credit constraints and raises their investments. Importantly, the rise in the bubble has a capital reallocation effect, making resources move to more productive firms. Tobin's Q falls as the capital stock rises, causing the capacity utilization rate and labor demand to rise. The increased hours and capacity utilization together raise output. Consumption also rises due to the wealth effect.

We also incorporate five other shocks often studied in the literature: permanent and transitory labor-augmenting technology (or TFP) shocks, the permanent investment-specific technology (IST) shock, the labor supply shock, and the financial shock (a shock to the external financing constraint). We estimate our model using Bayesian methods to fit six U.S. time series data of consumption, investment, hours, the relative price of investment goods, stock prices, and the Chicago Fed's National Financial Conditions Index (NFCI). Our full-information, model-based empirical strategy for identifying the sentiment shock exploits the impulse response property that the model generated observable variables react differently to different types of shocks. We then use our estimated model to address the questions raised earlier. We also use our model to shed light on two major bubble and crash episodes: (i) the internet bubble during the late 1990s and its subsequent crash, and (ii)

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[3]Chirinko and Schaller (2001), Goyal and Yamada (2004), and Gilchrist, Himmelberg, and Huberman (2005) find empirical evidence that investment responds to the stock market value beyond the fundamentals. See Gan (2007) and Chaney, Sraer, and Thesmar (2012) for empirical evidence on the relation between collateral constraints and investment.

the recent stock market bubble caused by the housing bubble and the subsequent Great Recession.

Our baseline estimation results show that the sentiment shock explains most of the fluctuations in the stock price at the business cycle frequency. It also explains a sizable fraction of the variations in investment, consumption, and output. Consistent with the RBC literature, the two TFP shocks together explain most of the variations in these quantities. Historical decomposition of shocks shows that the sentiment shock explains almost all of the stock market booms and busts. It also accounts for a sizable share of the fall in consumption and investment during the Great Recession and a large share of the rise in consumption and investment during the internet bubble. For both boom-bust episodes, the labor supply shock, instead of the sentiment shock, is the major driving force behind the movements in labor hours.

The sentiment shock and the financial shock work through a similar channel in that both shocks are transmitted to the real economy through the credit constraints. One difference is that, unlike the sentiment shock, the financial shock cannot generate the comovement among stock prices, investment, and consumption as well as the excessive volatility in the stock market by impulse response analyses. Another difference is that the sentiment shock directly affects stock prices. Without using the stock price data in the estimation, the financial shock is important, while the sentiment shock is not. However, when the stock price data is included in the estimation, the sentiment shock displaces the financial shock, making the impact of the latter much smaller.

We emphasize that the sentiment shock is not simply a residual used to explain the stock market volatility. When we shut down this shock and introduce measurement errors into the measurement equation for the stock price data, we find that the measurement errors explain most of the variation in the stock prices. But they cannot explain the comovement between the stock market and the real economy.

It is challenging for standard DSGE models to explain this comovement and the stock market booms and busts. One often needs a large investment adjustment cost parameter to make Tobin's Q highly volatile. One also has to introduce other sources of shocks to drive the comovement between Tobin's Q and real quantities because many shocks studied in the literature fail to generate either the right comovement or the right relative volatility in the data. For example, the TFP shock cannot generate large volatility of the stock price, while the IST shock generates counterfactual comovements of Tobin's Q (hence stock prices) and the relative price of investment goods if both series are used as observable data. The financial shock typically makes investment and consumption move in opposite directions and causes stock prices to move countercyclically.

Our finding that the usual macroeconomic risks such as the TFP and IST shocks do not explain much of the variations in the stock market is consistent with that in Li, Li, and Yu (2013). Without incorporating the stock price data, Li, Li, and Yu (2013) estimate the DSGE model of Christiano, Trabandt and Walentin (2010) using Bayesian methods and extract the TFP, IST, and monetary policy shocks from this model. They find that these shocks predict the future stock returns with the adjusted R-squared ranging from 0.02 to 0.04 for one-quarter horizon.

Recently, two types of shocks have drawn wide attention: the news shock and the risk (or uncertainty) shock. The news shock cannot generate the comovement in a standard RBC model (Barro and King (1984) and Wang (2012)). To generate the comovements, Beaudry and Portier (2004) incorporate multisectoral adjustment costs, Christiano et al. (2008) introduce nominal rigidities and inflation-targeting monetary policy, and Jaimovich and Rebelo (2009) consider preferences that exhibit a weak short-run wealth effect on the labor supply. These three papers study calibrated DSGE models and do not examine the empirical importance of the news shock.[4] Fujiwara, Hirose, and Shintani (2011) and Schmitt-Grohe and Uribe (2012) study this issue using the Bayesian DSGE approach. Most Bayesian DSGE models do not incorporate stock prices as observable data for estimation. As Schmitt-Grohe and Uribe (2012) point out, "as is well known, the neoclassical model does not provide a fully adequate explanation of asset price movements."[5]

By incorporating the stock price data, Christiano, Motto, Rostagno (2010, 2014) argue that the risk shock, related to that in Bloom (2009), displaces the marginal efficiency of investment shock and is the most important shock driving business cycles.[6] They also introduce a news shock to the risk shock, instead of TFP. Their models are based on Bernanke, Gertler and Gilchrist (1999) and identify the credit constrained entrepreneurs' net worth as the stock market value in the data. By contrast, we use the aggregate market value of the firms in the model as the stock price index in the data, which is more consistent with the conventional measurement.

As in Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Jermann and Quadrini (2012), and Liu, Wang and Zha (LWZ for short) (2013), financial frictions play an important role in our model. Unlike these studies, our model features firm heterogeneity. Some firms are financially constrained, while others are not. In the aggregate, firms can be self-financing. This feature is consistent with the empirical evidence documented by Chari, Christiano, and Kehoe (2008) and Ohanian (2010). Unlike the representative firm setup, our model features a capital reallocation channel for the financial frictions to impact the real economy.

Our paper is closely related to the literature on rational bubbles (Tirole (1982), Weil (1987), and Santos and Woodford (1997)). The recent Great Recession has generated renewed interest in this literature. Recent important contributions include Kocherlakota (2009), Farhi and Tirole (2012), Hirano and Yanagawa (2013), Martin and Ventura (2011, 2012), Wang and Wen (2012a), Miao and Wang (2011, 2012, 2014, 2015), Miao, Wang and Xu (2012a), and Miao, Wang and Zhou (2015). Most papers in this literature are theoretical, while Wang and Wen (2012a) provide some calibration exercises. Except for Miao and Wang (2011, 2012, 2014, 2015), and Miao, Wang and Xu (2012a), all other papers study bubbles in intrinsically useless assets or assets with exogenously given payoffs. See Miao (2014) for a survey of the recent literature.

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[4]Beaudry and Portier (2006) study the empirical implications of the news shock using the VAR approach.

[5]In Section 6.8 of their paper, Schmitt-Grohe and Uribe (2012) discuss briefly how the share of unconditional variance explained by anticipated shocks will change when stock prices are included as observable data. But they do not include stock prices in their baseline estimation.

[6]It is difficult for shocks to the TFP shock's variance (uncertainty shocks) to generate comovements among investment, consumption, hours, and stock prices in standard DSGE models (see, e.g., Basu and Bundick (2011)).

Our paper is also related to the studies by Farmer (2012a,b), who argues that multiple equilibria supported by self-fulfilling beliefs can help understand the recent Great Recession. Farmer provides a search model and replaces the Nash bargaining equation for the wage determination with an equation to determine the expected stock future price. In particular, he assumes that the expected future stock price relative to the price level or the real wage is determined by an exogenously given variable representing beliefs. The evolution of this variable is determined by a belief function. Unlike Farmer's approach, we model beliefs as a sentiment shock to the relative size of the old bubble to the new bubble. We then derive a no-arbitrage equation for the bubble in equilibrium. No extra equation is imposed exogenously.

The remainder of the paper proceeds as follows. Section 2 presents the baseline model. Section 3 estimates model parameters using Bayesian methods. Section 4 analyzes the estimated model's economic implications. Section 5 extends the model by incorporating the consumer sentiment index data. Section 6 concludes. Technical details are relegated to an online supplemental material.

2.   The Baseline Model

We consider an infinite-horizon economy that consists of households, firms, capital goods producers, and financial intermediaries. Households supply labor to firms, deposit funds in competitive financial intermediaries, and trade firm shares in a stock market. Firms produce final goods that are used for consumption and investment. Capital goods producers produce investment goods subject to adjustment costs. Firms purchase investment goods from capital goods producers subject to credit constraints. Firms finance investment using internal funds, new equity issuance, and external borrowing. Firms and households can save in competitive financial intermediaries (or banks), which make one-period loans to borrowers. As a starting point, we assume that there is no friction in financial intermediaries so that we treat them as a veil. In addition, we do not consider money or monetary policy and study a real model of business cycles.

2.1.   Households

There is a continuum of identical households of measure unity. Each household derives utility from consumption and leisure according to the following expected utility function:

$$E \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - hC_{t-1}) - \psi_t N_t \right], \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, $h \in (0,1)$ is the habit persistence parameter, C_t denotes consumption, N_t denotes labor, and ψ_t represents a labor supply shock. This shock accounts for the labor wedge and may proxy for a variety of labor market frictions that could be important in the real world. Assume that $\ln \psi_t$ follows an AR(1) process. The specification of linear disutility of labor reflects indivisible labor in the RBC literature and helps generate large

fluctuations in hours worked relative to productivity.

The representative household's budget constraint is given by

$$C_t + P_t^s s_{t+1} + \frac{d_{t+1}}{R_{dt}} = W_t N_t + \Pi_t + (D_t + P_t^s)s_t + d_t, \ s_0 = 1, \ d_0 = 0, \tag{2}$$

where s_t, P_t^s, d_t, R_{dt}, W_t, Π_t, and D_t denote share holdings, the aggregate stock price of all final goods firms, deposits in the financial intermediaries, the deposit rate, the wage rate, the profit from capital goods producers, and the aggregate dividend, respectively. The household is subject to a borrowing constraint, $d_{t+1} \geq 0$. Without a borrowing constraint, a bubble cannot exist (e.g., Kocherlakota (1992, 2009)). In equilibrium, $s_t = 1$. The household's first-order conditions are given by

$$\Lambda_t W_t = \psi_t, \tag{3}$$

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta E_t \frac{h}{C_{t+1} - hC_t}, \tag{4}$$

$$\frac{1}{R_{dt}} \geq \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}, \ \text{with equality when } d_{t+1} > 0, \tag{5}$$

where Λ_t represents the marginal utility of consumption.

2.2. Firms

There is a continuum of final goods firms of measure unity. Suppose that households believe that each firm's stock may contain a bubble. They also believe that the bubble may burst with some probability. By rational expectations, a bubble cannot reemerge in the same firm after bursting. Otherwise there would be an arbitrage opportunity. This means that none of the firms would contain any bubble once all bubbles have burst if no new firms enter the economy. As a result, we follow Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), and Gertler and Kiyotaki (2010), and assume exogenous entry and exit, for simplicity. A firm may die with an exogenously given probability δ_e each period. After death, its value is zero and a new firm enters the economy without costs so that the total measure of firms is fixed at unity in each period. A new firm entering at date t starts with an initial capital stock K_{0t} and then operates in the same way as an incumbent firm. The new firm may bring a new bubble into the economy.[7]

An incumbent firm $j \in [0,1]$ combines capital K_t^j and labor N_t^j to produce final goods Y_t^j using the following production function:[8]

$$Y_t^j = (u_t^j K_t^j)^\alpha \left(A_t N_t^j \right)^{1-\alpha}, \tag{6}$$

where $\alpha \in (0,1)$, u_t^j denotes the capacity utilization rate, and A_t denotes the labor-augmenting

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[7]See Martin and Ventura (2012) for a related overlapping generations model with recurrent bubbles.

[8]A firm can be identified by its age. Hence, we may use the notation $K_{t,\tau}$ to denote firm j's capital stock K_t^j if its age is τ. Because we want to emphasize the special role of bubbles, we only use such a notation for the bubble.

technology shock. Given the Cobb-Douglas production function, we may also refer to A_t as a total factor productivity (TFP) shock. For a new firm entering at date t, we set $K_t^j = K_{0t}$. Assume that A_t is composed of a permanent component A_t^p and a transitory (mean-reverting) component A_t^m such that $A_t = A_t^p A_t^m$, where $\ln \lambda_{at} \equiv \ln \left(A_t^p / A_{t-1}^p \right)$ and $\ln A_t^m$ follow independent AR(1) processes.

Assume that the capital depreciation rate between period t and period $t+1$ is given by $\delta_t^j = \delta(u_t^j)$, where δ is a twice continuously differentiable convex function that maps a positive number into $[0,1]$. We do not need to parameterize the function δ since we use the log-linearization solution method. We only need it to be such that the steady-state capacity utilization rate is normalized to 1. The capital stock evolves according to

$$K_{t+1}^j = (1 - \delta_t^j)K_t^j + \varepsilon_t^j I_t^j, \tag{7}$$

where I_t^j denotes investment and ε_t^j measures the efficiency of the investment. Assume that investment is irreversible at the firm level so that $I_t^j \geq 0$. Assume that ε_t^j is IID across firms and over time and is drawn from the fixed cumulative distribution Φ over $[\varepsilon_{\min}, \varepsilon_{\max}] \subset (0, \infty)$ with mean 1 and the probability density function ϕ. This shock induces firm heterogeneity in the model. For tractability, assume that the capacity utilization decision is made before the observation of investment efficiency shock ε_t^j. Consequently, the optimal capacity utilization does not depend on the idiosyncratic shock ε_t^j.

Given the wage rate w_t and the capacity utilization rate u_t^j, the firm chooses labor demand N_t^j to solve the following problem:

$$R_t u_t^j K_t^j = \max_{N_t^j} \ (u_t^j K_t^j)^\alpha (A_t N_t^j)^{1-\alpha} - W_t N_t^j, \tag{8}$$

where

$$R_t \equiv \alpha \left[\frac{(1 - \alpha) A_t}{W_t} \right]^{\frac{1-\alpha}{\alpha}}. \tag{9}$$

In each period t, firm j can make investment I_t^j by purchasing investment goods from capital producers at the price P_t. Its flow-of-funds constraint is given by

$$D_t^j + L_t^j + P_t I_t^j = u_t^j R_t K_t^j + \frac{L_{t+1}^j}{R_{ft}}, \tag{10}$$

where $L_{t+1}^j > 0(< 0)$ represents borrowing (savings), R_{ft} represents the interest rate, and $D_t^j > 0(< 0)$ represents dividends (new equity issuance). Assume that external financial markets are imperfect so that firms are subject to the following constraint on new equity issuance:

$$D_t^j \geq -\eta_t K_t^j, \tag{11}$$

where η_t is an exogenous stochastic shock to equity issuance. In addition, external borrowing is

subject to a credit constraint:

$$E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(K_{t+1}^j, L_{t+1}^j) \geq E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(K_{t+1}^j, 0) - E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(\xi_t K_t^j, 0), \quad (12)$$

where $\bar{V}_{t,\tau}(k_t, l_t) \equiv \int V_{t,\tau}(k_t, l_t, \varepsilon) \, d\Phi(\varepsilon)$ represents the ex ante value after integrating out ε and $V_{t,\tau}(k, l, \varepsilon)$ represents the cum-dividends stock market value of the firm with assets k, debt l, and idiosyncratic investment efficiency shock ε at time t with age τ. Here, ξ_t represents a collateral shock that reflects the friction in the credit market as in Jermann and Quadrini (2012) and LWZ (2013). Note that τ represents the age of firm j. We will show below that equity value depends on the age because it contains a bubble component which is age dependent.

Following Miao and Wang (2011), we can interpret (12) as an incentive constraint in a contracting problem between the firm and the lender when the firm has limited commitment.[9] In any period t, firm j chooses to borrow L_{t+1}^j / R_{ft}. It may default on debt L_{t+1}^j at the beginning of period $t + 1$ before the realization of the idiosyncratic investment efficiency shock and conditional on its surviving in period $t + 1$. If it does not default, it obtains continuation value $\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(K_{t+1}^j, L_{t+1}^j)$. If it defaults, debt is renegotiated and the repayment is relieved. Firm value is $\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(K_{t+1}^j, 0)$. The lender can seize the collateralized asset $\xi_t K_t^j$ and keep the firm running with these assets by reorganizing the firm.[10] Thus the threat value to the lender is $\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(\xi_t K_t^j, 0)$. Following Jermann and Quadrini (2012), assume that the firm has a full bargaining power. Then the expression on the right-hand side of (12) is the value of the firm if it chooses to default. Thus constraint (12) ensures firm j has no incentive to default in equilibrium.[11]

2.3. Decision Problem

We describe firm j's decision problem by dynamic programming:

$$
\begin{aligned}
V_{t,\tau}\left(K_t^j, L_t^j, \varepsilon_t^j\right) &= \max_{I_t^j, u_t^j, L_{t+1}^j} R_t u_t^j K_t^j - P_t I_t^j - L_t^j + \frac{L_{t+1}^j}{R_{ft}} \\
&\quad + (1 - \delta_e) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1,\tau+1}\left(K_{t+1}^j, L_{t+1}^j, \varepsilon_{t+1}^j\right),
\end{aligned}
$$

subject to (7), (12), and

$$0 \leq P_t I_t^j \leq u_t^j R_t K_t^j + \eta_t K_t^j - L_t^j + \frac{L_{t+1}^j}{R_{ft}}, \quad (13)$$

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[9]Miao and Wang (2011) show that other types of credit constraints such as self-enforcing debt constraints can also generate bubbles.

[10]Using $\xi_t K_{t+1}^j$ as collateral does not change our key insight, but makes the analysis slightly more complicated (see Miao and Wang (2011)).

[11]Miao and Wang (2011) discuss other forms of credit constraints under which a bubble can exist. The key idea is that a bubble helps relax credit constraints.

where we have used equations (10) and (11). We conjecture and verify that the value function takes the following form in the proof of Proposition 1 (see the online supplemental material Appendix A):

$$V_{t,\tau}(K_t^j, L_t^j, \varepsilon_t^j) = v_t(\varepsilon_t^j)K_t^j + b_{t,\tau}(\varepsilon_t^j) - v_{Lt}(\varepsilon_t^j)L_t^j, \tag{14}$$

where $v_t(\varepsilon_t^j)$, $b_{t,\tau}(\varepsilon_t^j) \geq 0$, and $v_{Lt}(\varepsilon_t^j)$ depend only on idiosyncratic shock ε_t^j and aggregate state variables. The form in (14) is intuitive following Hayashi (1982). Since we assume competitive markets with constant-returns-to-scale technology, it is natural that firm value takes a linear functional form. However, in the presence of credit constraints (12), firm value may contain a speculative component, $b_{t,\tau}(\varepsilon_t^j)$. Either $b_{t,\tau}(\varepsilon_t^j) = 0$ or $b_{t,\tau}(\varepsilon_t^j) > 0$ can be an equilibrium solution depending on agents' beliefs (note that the preceding dynamic programming problem does not give a contraction mapping). As in Miao and Wang (2011), we may interpret this component as a bubble.

Define the date-t ex-dividend stock price of the firm of age τ as

$$P_{t,\tau}^{s,j} = (1 - \delta_e)E_t \frac{\beta\Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(K_{t+1}^j, L_{t+1}^j).$$

Given the above conjectured form in (14), we have

$$P_{t,\tau}^{s,j} = Q_t K_{t+1}^j + B_{t,\tau} - \frac{1}{R_{ft}} L_{t+1}^j, \tag{15}$$

where we define

$$Q_t = (1 - \delta_e)E_t \frac{\beta\Lambda_{t+1}}{\Lambda_t} v_{t+1}(\varepsilon_{t+1}^j), \quad B_{t,\tau} = (1 - \delta_e)E_t \frac{\beta\Lambda_{t+1}}{\Lambda_t} b_{t+1,\tau+1}\left(\varepsilon_{t+1}^j\right). \tag{16}$$

Note that Q_t and $B_{t,\tau}$ do not depend on idiosyncratic shocks because they are integrated out. We interpret Q_t and $B_{t,\tau}$ as the (shadow) price of installed capital (Tobin's marginal Q) and the average bubble of the firm, respectively. Note that marginal Q and the investment goods price P_t are different in our model due to financial frictions and idiosyncratic investment efficiency shocks. In addition, marginal Q is not equal to average Q in our model because of the existence of a bubble. Given (14), (15), and (16), the credit constraint (12) becomes

$$\frac{1}{R_{ft}} L_{t+1}^j \leq Q_t \xi_t K_t^j + B_{t,\tau}. \tag{17}$$

We then have the following proposition:

Proposition 1 *(i) The optimal investment level I_t^j of firm j with a bubble satisfies*

$$P_t I_t^j = \begin{cases} u_t R_t K_t^j + \eta_t K_t^j + Q_t \xi_t K_t^j + B_{t,\tau} - L_t^j & \text{if } \varepsilon_t^j \geq \frac{P_t}{Q_t} \\ 0 & \text{otherwise} \end{cases}. \tag{18}$$

(ii) Each firm chooses the same capacity utilization rate u_t satisfying

$$R_t(1 + G_t) = Q_t \delta'(u_t), \tag{19}$$

where

$$G_t = \int_{\varepsilon \geq P_t/Q_t} (Q_t/P_t \varepsilon - 1) d\Phi(\varepsilon). \tag{20}$$

(iii) The bubble, the price of installed capital, and the lending rate satisfy

$$B_{t,\tau} = \beta(1 - \delta_e) E_t \frac{\Lambda_{t+1}}{\Lambda_t} B_{t+1,\tau+1} (1 + G_{t+1}), \tag{21}$$

$$Q_t = \beta(1 - \delta_e) E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[u_{t+1} R_{t+1} + Q_{t+1}(1 - \delta_{t+1}) + (u_{t+1} R_{t+1} + \xi_{t+1} Q_{t+1} + \eta_{t+1}) G_{t+1} \right], \tag{22}$$

$$\frac{1}{R_{ft}} = \beta(1 - \delta_e) E_t \frac{\Lambda_{t+1}}{\Lambda_t} (1 + G_{t+1}), \tag{23}$$

where $\delta_t = \delta(u_t)$.

The intuition behind the investment rule given in equation (18) is the following. The cost of one unit of investment is the purchasing price P_t. The associated benefit is the marginal Q multiplied by the investment efficiency ε_t^j. If the benefit exceeds the cost $Q_t \varepsilon_t^j \geq P_t$, the firm will invest at full capacity. Otherwise, the firm makes zero investment. This investment rule implies that firm-level investment is lumpy, which is similar to the case with fixed adjustment costs. Equation (18) shows that the investment rate increases with cash flows R_t, marginal Q, Q_t, and the bubble, $B_{t,\tau}$.

Equation (17) shows that the existence of a bubble $B_{t,\tau}$ relaxes the credit constraint, and hence allows the firm to make more investment. Thus the bubble term $B_{t,\tau}$ enters the investment rule in (18). In addition, the existence of a bubble in the aggregate economy affects the equilibrium Q_t and P_t and hence the investment threshold $\varepsilon_t^* \equiv P_t/Q_t$. This also implies that the bubble has an extensive margin effect by affecting the number of investing firms. We call this effect of the bubble the capital reallocation effect.

The bubble must satisfy the no-arbitrage condition given in (21). Having a bubble at time t costs $B_{t,\tau}$ dollars. The benefit consists of two components: (i) The bubble has the value $B_{t+1,\tau+1}$ at $t+1$. (ii) The bubble can help the firm generate dividends $B_{t+1,\tau+1} G_{t+1}$. The intuition is that a dollar of the bubble increases the borrowing capacity by one dollar as revealed by (17). This allows the firm to make more investment, generating additional dividends $(\varepsilon Q_t/P_t - 1)$ for the efficiency shock $\varepsilon \geq P_t/Q_t$. The expected investment benefit is given by (20). Thus $B_{t+1,\tau+1} (1 + G_{t+1})$ represents the sum of "dividends" and the reselling value of the bubble. Using the stochastic discount factor $\beta \Lambda_{t+1}/\Lambda_t$ and considering the possibility of firm death, equation (21) says that the cost of having the bubble is equal to the expected benefit.

Note that the bubble $B_{t,\tau}$ is non-predetermined. Clearly, $B_{t,\tau} = B_{t+1,\tau+1} = 0$ is a solution to (21). If no one believes in a bubble, then a bubble cannot exist. We shall show below an equilibrium

10

with bubble $B_{t,\tau} > 0$ exists. Both types of equilibria are self-fulfilling. Note that the transversality condition cannot rule out a bubble because of the additional benefit G_{t+1} generated by the bubble.

The right-hand side of equation (19) gives the tradeoff between the cost and the benefit of a unit increase in the capacity utilization rate for a unit of capital. A high utilization rate makes capital depreciate faster. But it can generate additional profits and also additional investment benefits.

Equation (22) is an asset pricing equation of marginal Q. The dividends from capital consist of the rental rate $u_{t+1}R_{t+1}$ in efficiency units and the investment benefit $(u_{t+1}R_{t+1}+\xi_{t+1}Q_{t+1})G_{t+1}$ of an additional unit increase in capital. The reselling value of undepreciated capital is $Q_{t+1}(1 - \delta_{t+1})$.

Equation (23) is an asset pricing equation for the interest rate. For firms that decide not to invest and save (buying the bonds issued by other firms). For every one dollar saved today, the firm will earn R_{ft} in the next period. The firm may receive a favorable investment shock in the next period and invest R_{ft} to generate additional dividends $(\varepsilon Q_{t+1}/P_{t+1} - 1)$ in the next period. Hence the total return on saving will be $R_{ft}(1 + G_{t+1})$.

2.4. Sentiment Shock

To model households' beliefs about the movements of the bubble, we introduce a sentiment shock. Suppose that households believe that the new firm in period t may contain a bubble of size $B_{t,0} = b_t^* > 0$ with probability ω. Then the total new bubble is given by $\omega \delta_e b_t^*$.

Suppose that households believe that the relative size of the bubbles at date $t + \tau$ for any two firms born at date t and $t + 1$ is given by θ_t, i.e.,

$$\frac{B_{t+\tau,\tau}}{B_{t+\tau,\tau-1}} = \theta_t, \quad t \geq 0, \ \tau \geq 1, \tag{24}$$

where θ_t follows an exogenously given process:

$$\ln \theta_t = (1 - \rho_\theta)\,\bar{\theta} + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta,t}, \tag{25}$$

where $\bar{\theta}$ is the mean, $\rho_\theta \in (-1,1)$ is the persistence parameter, and $\varepsilon_{\theta,t}$ is an IID normal random variable with mean zero and variance σ_θ^2. We interpret this process as a *sentiment shock*, which reflects household beliefs about the fluctuations in bubbles.[12] These beliefs may change randomly over time. It follows from (24) that

$$B_{t,0} = b_t^*, \ \ B_{t,1} = \theta_{t-1}b_t^*, \ \ B_{t,2} = \theta_{t-1}\theta_{t-2}b_t^*, ..., \ t \geq 0. \tag{26}$$

This equation implies that the sizes of new bubbles and old bubbles are linked by the sentiment shock. The sentiment shock changes the relative sizes. Note that the growth rate $B_{t+1,\tau+1}/B_{t,\tau}$ of

---

[12]In a different formulation available upon request, we may interpret θ_t as the probability that the bubble survives in the next period. This formulation is isomorphic to the present model. In particular, m_t in equation (32) can be interpreted as the mass of firms having bubbles. Equation (34) is the asset pricing equation for the bubble B_t^a/m_t. The advantage of the present setup is that we allow θ_t to be greater than 1.

the bubble in the same firm born at any given date $t - \tau$ must satisfy the equilibrium restriction derived in equation (21).

2.5. Capital Producers

Capital goods producers create new investment goods using input of final output subject to adjustment costs. They sell new investment goods to firms with investing opportunities at the price P_t. The objective function of a capital producer is to choose $\{I_t\}$ to solve:

$$\max_{\{I_t\}} \ E \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} \left\{ P_t I_t - \left[1 + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 \right] \frac{I_t}{Z_t} \right\},$$

where $\bar{\lambda}_I$ is the steady-state growth rate of aggregate investment, $\Omega > 0$ is the adjustment cost parameter, and Z_t represents an IST shock as in Greenwood, Hercowitz and Krusell (1997). The growth rate $\bar{\lambda}_I$ will be determined in Section 3. Following Justiniano, Primiceri, and Tambalotti (2011), we assume that $Z_t = Z_{t-1}\lambda_{zt}$, where $\ln \lambda_{zt}$ follows an AR(1) process. The optimal level of investment goods satisfies the first-order condition:

$$
\begin{aligned}
Z_t P_t \ = \ & 1 + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 + \Omega \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right) \frac{I_t}{I_{t-1}} \\
& - \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \Omega \left(\frac{I_{t+1}}{I_t} - \bar{\lambda}_I \right) \frac{Z_t}{Z_{t+1}} \left(\frac{I_{t+1}}{I_t} \right)^2 .
\end{aligned}
\tag{27}
$$

2.6. Aggregation and Equilibrium

Let $K_t = \int K_t^j dj$ denote the aggregate capital stock of all firms in the end of period $t - 1$ before the realization of the death shock. Let X_t denote the aggregate capital stock after the realization of the death shock, but before new investment and depreciation take place. Then

$$X_t = (1 - \delta_e)K_t + \delta_e K_{0t}, \tag{28}$$

where we have included the capital stock brought by new entrants.

Define aggregate output and aggregate labor as $Y_t = \int_0^1 Y_t^j dj$ and $N_t = \int_0^1 Y_t^j dj$. By Proposition 1, all firms choose the same capacity utilization rate. Thus all firms have the same capital-labor ratio. By the linear homogeneity property of the production function, we can then show that

$$Y_t = (u_t X_t)^\alpha (A_t N_t)^{1-\alpha} . \tag{29}$$

As a result, the wage rate is given by

$$W_t = \frac{(1 - \alpha)Y_t}{N_t}, \tag{30}$$

Let B_t^a denote the total bubble in period t. Adding up the bubble of the firms of all ages and using (26) yield:

$$B_t^a = \sum_{\tau=0}^{t} (1-\delta_e)^\tau \delta_e \omega B_{t,\tau} \equiv m_t b_t^*, \tag{31}$$

where m_t satisfies the recursion,

$$m_t = m_{t-1}(1-\delta_e)\theta_{t-1} + \delta_e \omega, \ m_0 = \delta_e \omega. \tag{32}$$

The process $\{m_t\}$ is stationary in the neighborhood of the steady state as long as $(1-\delta_e)\bar\theta < 1$.

By equations (26) and (21),

$$b_t^* = \beta(1-\delta_e)\theta_t E_t \frac{\Lambda_{t+1}}{\Lambda_t} b_{t+1}^* (1+G_{t+1}). \tag{33}$$

This equation gives an equilibrium restriction on the size of the new bubble. Substituting (31) into the above equation yields:

$$B_t^a = \beta(1-\delta_e)\theta_t E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{m_t}{m_{t+1}} B_{t+1}^a (1+G_{t+1}). \tag{34}$$

This equation gives an equilibrium restriction on the value of the total bubble in the economy. The above two equations prevent any arbitrage opportunities for old and new bubbles. Equations (32) and (34) reveal that a sentiment shock affects the relative size m_t and hence the aggregate bubble B_t^a.

Aggregating all firm value in (15), we obtain the aggregate stock market value of the firm:

$$P_t^s = Q_t K_{t+1} + B_t^a.$$

This equation reveals that the aggregate stock price consists of two components: the fundamental $Q_t K_{t+1}$ and the bubble B_t^a.

Competitive financial intermediaries implies that the deposit rate is equal to the lending rate so that $R_{dt} = R_{ft}(1-\delta_e)$, where we have taken into account that firms die with probability δ_e. It follows from (23) and $G_{t+1} > 0$ that

$$\frac{1}{R_{dt}} = \frac{1}{(1-\delta_e)R_{ft}} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}(1+G_{t+1}) > \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}. \tag{35}$$

Thus households prefer to borrow until their borrowing constraints bind, i.e., $d_{t+1} = 0$. Without borrowing constraints, no arbitrage implies that $G_{t+1} = 0$. In this case equation (21) and the transversality condition would rule out bubbles.

By the market-clearing conditions for bank loans, $L_t = \int L_t^j dj = d_t = 0$ for all $t \geq 0$. This means that firms with high investment efficiency shocks borrow and invest, while all other firms

save and lend.

Let $I_t = \int I_t^j dj$ denote aggregate investment. Using Proposition 1 and adding up (18) for firms of all ages, we can use a law of large numbers to drive aggregate investment as

$$
\begin{aligned}
P_t I_t &= [(u_t R_t + \xi_t Q_t + \eta_t) X_t + B_t^a - L_t] \int_{\varepsilon > \frac{P_t}{Q_t}} d\Phi(\varepsilon) \\
&= [(u_t R_t + \xi_t Q_t + \eta_t) X_t + B_t^a] \int_{\varepsilon > \frac{P_t}{Q_t}} d\Phi(\varepsilon),
\end{aligned}
\tag{36}
$$

where in the second line we have used the fact that $L_t = 0$. Similarly, the aggregate capital stock evolves according to

$$
\begin{aligned}
K_{t+1} &= (1 - \delta_t) X_t + \int I_t^j \varepsilon_t^j dj \\
&= (1 - \delta_t) X_t + I_t \frac{\int_{\varepsilon > \frac{P_t}{Q_t}} \varepsilon d\Phi(\varepsilon)}{\int_{\varepsilon > \frac{P_t}{Q_t}} d\Phi(\varepsilon)},
\end{aligned}
\tag{37}
$$

where we have used a law of large numbers and the fact that I_t^j and ε_t^j are independent by Proposition 1.

The total capacity of external financing is given by

$$
\underbrace{\eta_t K_t}_{\text{new equity}} + \underbrace{\xi_t Q_t K_t + B_t^a}_{\text{debt}},
\tag{38}
$$

where we have used equations (11) and (17) to conduct aggregation. Then the fluctuation in this capacity reflects the overall financial market conditions. We can use a single shock defined as

$$
\zeta_t \equiv \eta_t / Q_t + \xi_t,
\tag{39}
$$

to capture the disturbance to the degree of the overall financial constraints and rewrite the total capacity of external financing as $\zeta_t Q_t K_t + B_t^a$. Assume that $\ln \zeta_t$ follows an AR(1) process. Using (39), equations (22) and (36) become

$$
Q_t = \beta(1 - \delta_e) E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[u_{t+1} R_{t+1} + Q_{t+1}(1 - \delta_{t+1}) + (u_{t+1} R_{t+1} + \zeta_{t+1} Q_{t+1}) G_{t+1} \right],
\tag{40}
$$

$$
P_t I_t = [(u_t R_t + \zeta_t Q_t) X_t + B_t^a] \int_{\varepsilon > \frac{P_t}{Q_t}} d\Phi(\varepsilon).
\tag{41}
$$

In Section 3 we will estimate the shock ζ_t instead of its two components η_t and ξ_t.

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The resource constraint is given by

$$C_t + \left[1 + \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)^2\right]\frac{I_t}{Z_t} = Y_t. \tag{42}$$

A competitive equilibrium consists of stochastic processes of 15 aggregate endogenous variables, $\{C_t,\ I_t,\ Y_t,\ N_t,\ K_t,\ u_t,\ Q_t,\ X_t,\ W_t,\ R_t,\ P_t,\ m_t,\ B_t^a,\ R_{ft},\ \Lambda_t\}$ such that 15 equations (42), (41), (29), (3), (37), (19), (40), (28), (30), (9), (27), (32), (34), (23) and (4) hold, where G_t satisfies (20) and $\delta_t = \delta(u_t)$.

There may exist two types of equilibrium: bubbly equilibrium in which $B_t^a > 0$ for all t and bubbleless equilibrium in which $B_t^a = 0$ for all t. A bubbly equilibrium can be supported by the belief that a new firm may bring a new bubble with a positive probability $\omega > 0$. A sentiment shock θ_t can generate fluctuations in the aggregate bubble B_t^a because households believe that the size of the old bubble relative to that of the new bubble fluctuates randomly over time. A bubbleless equilibrium can be supported by the belief that either old or new firms do not contain any bubble $(\omega = \theta_t = m_t = 0)$.

3.   Bayesian Estimation

Since the model has two unit roots, one in the investment-specific technology shock and the other in the TFP shock, we have to appropriately transform the equilibrium system into a stationary one. In Appendix B of the online supplemental material, we present the transformed equilibrium system and in Appendix C of the online supplemental material we show that the transformed equilibrium system has a nonstochastic bubbly steady state in which all the transformed variables are constant over time. While our model features various types of inequality constraints, we show that households are always borrowing constrained and the borrowing constraints are always binding for some firms but never binding for others. After aggregation, the equilibrium system does not involve any inequalities. We can solve the transformed system numerically by log-linearizing around the nonstochastic steady state. We seek saddle-path stable solutions. We shall focus on the bubbly equilibrium as our benchmark.

3.1.   Shocks and Data

We use Bayesian methods to fit the log-linearized model to the U.S. data.[13]   Our model has six orthogonal shocks: persistent and transitory TFP shocks (λ_{at}, A_t^m), the investment-specific technology shock Z_t, the labor supply shock ψ_t, the financial shock ζ_t, and the sentiment shock θ_t. We need six data series to identify these shocks. We choose the following five quarterly U.S. time series data: the relative price of investment (P_t), real per capita consumption (C_t), real per

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[13]We use Dynare to conduct Bayesian estimation. See Adjemian et al (2011).

capita investment in consumption units (I_t/Z_t), per capita hours (N_t), and real per capita stock price index (defined as $P_t^s = Q_t K_{t+1} + B_t^a$ in the model). The first four series are taken from LWZ (2013), and the stock price data is the S&P composite index downloaded from Robert Shiller's website. We normalize it by the price index for non-durable goods and population. The sample period covers the first quarter of 1975 through the fourth quarter of 2010. More details about the data construction can be found in Appendix A in LWZ (2013).

The sixth data series is the Chicago Fed's National Financial Conditions Index (NFCI), which is used to identify the financial shock ζ_t. In Section 4.3 we will show that without including the NFCI data, the estimation would produce a counter-intuitive smoothed financial shock series. The NFCI is a comprehensive index on U.S. financial conditions in money markets, debt and equity markets, as well as the traditional and shadow banking systems. The NFCI is normalized to have mean zero and standard deviation of one over a sample period extending back to 1973. A positive (negative) number means tight (loose) financial conditions. The data extends back to 1973 and is available at quarterly frequency.[14] We have also tried several subindices of NFCI (other variation of the NFCI index) and the results are similar.

Besides the standard measurement equations, we include the following measurement equation:

$$NFCI_t = -f_1 \hat{\zeta}_t - f_2 \hat{Q}_t - f_3 \left(\hat{B}_t^a - \hat{K}_t \right), \tag{43}$$

where $f_1 > 0$, $f_2 > 0$, $f_3 > 0$, $\hat{\zeta}_t$ denotes log-deviation from the steady state, and $\hat{Q}_t$, $\hat{B}_t^a$, and $\hat{K}_t$ denote the log-deviations from the steady state for the corresponding detrended variables. This equation is motivated from (38) and (39). The total capacity of external financing is $\zeta_t Q_t K_t + B_t^a$ and its fluctuation depends on financial market conditions, represented by the NFCI. The preceding measurement equation relates the NFCI to the log-linearized expression of financing capacity normalized by capital K_t. The intuition is that an increase in either one of $\hat{\zeta}_t$, $\hat{Q}_t$, or $\hat{B}_t^a - \hat{K}_t$ will reduce the NFCI and hence reduce the tightness in the overall financial market as revealed in equation (38).

In principle, one could use the credit market data such as total debt to identify the credit shock ξ_t and use the equity market data such as aggregate new equity issuance to identify the equity issuance shock η_t. We have not followed this approach because aggregate debt is zero in our model, but firms can borrow and save among themselves. Our model is consistent with the empirical evidence documented by Chari, Christiano, and Kehoe (2008) and Ohanian (2010). They find that the corporate sector typically has substantial cash reserves and thus can be largely self-financing. In addition, our modeling of using one shock to describe the financial market conditions is parsimonious. Our purpose is not to identify all shocks that drive the financial market conditions, but to study how the sentiment shock and a single reduced-form financial shock to the financial market conditions affect the real economy.

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[14]See Brave and Butter (2011) for a detailed description on the construction of the NFCI index at http://qa.chicagofed.org/digital_assets/publications/economic_perspectives/2011/1qtr2011_part2_brave_butters.pdf

3.2. Parameter Estimates

We focus on the bubbly steady state for the stationary equilibrium in which the capacity utilization rate and the investment goods price are both equal to 1. Due to the log-linearization solution method, we do not need to parameterize the depreciation function $\delta(\cdot)$ and the distribution function $\Phi(\cdot)$. As shown in Appendices C and D of the online supplemental material, we only need to know the steady-state values of $\delta(1)$, $\delta'(1)$, $\delta''(1)$, $\Phi(\varepsilon^*)$, and $\mu \equiv \frac{\phi(\varepsilon^*)\varepsilon^*}{1-\Phi(\varepsilon^*)}$, where ε^* is the steady-state investment threshold for the idiosyncratic shock ε_t. We treat these values as parameters to be either estimated or calibrated.

We partition the model parameters into three subsets. This procedure could be viewed as a reasonable shortcut to that proposed by Del Negro and Schorfheide (2008). The first subset of parameters includes the structural parameters, which are calibrated using the steady-state relations. This subset of parameters is collected in $\Psi_1 = \{\beta, \alpha, \delta(1), \delta'(1), \delta_e, \bar{\psi}, \Phi(\varepsilon^*), g_\gamma, \bar{\lambda}_z, K_0/\tilde{K}, \bar{\theta}, \omega\}$, where $\bar{\psi}$ is the mean labor supply shock, g_γ is the steady-state gross growth rate of output, $\bar{\lambda}_z$ is the steady-state gross growth rate of IST, K_0 is the detrended capital stock endowed by the new entrants, and $\tilde{K}$ is the detrended steady-state aggregate capital stock. Note that the parameter ω does not affect the steady-state bubble-output ratio by the proposition in Appendix C of the online supplemental material. Appendix D of the online supplemental material also shows that it does not affect the log-linearized equilibrium system. Thus, it can take any positive value, say, $\omega = 0.5$.

As is standard in the literature, we fix the discount factor β at 0.99, the capital share parameter α at 0.3, and the steady-state depreciation rate $\delta(1)$ at 0.025. We can pin down $\delta'(1)$ to ensure that the steady-state capacity utilization rate is equal to one. We choose $\bar{\psi}$ such that the steady-state average hours are 0.25 as in the data. Using data from the U.S. Bureau of the Census, we compute the exit rate as the ratio of the number of closed original establishments with non-zero employment to the number of total establishments with non-zero employment. The average annual exit rate from 1990 to 2007 is 7.8 percent, implying about 2 percent of quarterly exit rate. Thus we set the exit rate δ_e at 0.02.[15] This number is consistent with the literature. For instance, Bilbiie, Ghironi and Melitz (2012) set the quarterly firm exit rate to be 0.025, Bernard, Redding and Schott (2010) find a quarterly 2.2 percent minimum production destruction rate. We can pin down $\Phi(\varepsilon^*)$ by targeting the steady-state investment-output ratio $(\tilde{I}/\tilde{Y})$ at 0.20 as in the data, given that we know the other parameter values. We set the growth rate of per capita output $g_\gamma = 1.0042$ and the growth rate of the investment-specific technology $\bar{\lambda}_z = 1.0121$ as in the data reported by LWZ (2013). We can then pin down the average growth rate of TFP, $\bar{\lambda}_a$. Dunne, Roberts and Samuelson (1988) document that the average relative size of entrants to all firms in the period 1972-1982 is about 0.20. We thus set the ratio of the initial capital stock of new entry firms to the average capital stock $K_0/\tilde{K}$ to 0.20. By (23) and (34), the growth rate of bubbles of the surviving firms in the steady state is given by $\bar{\theta} = R_f/g_\gamma$. We use this equation to pin down $\bar{\theta}$, the calibrated value is 0.9975.[16] In summary, Table 1 presents the values assigned to the calibrated parameters in Ψ_1.

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[15]Our results are not sensitive to this number.

[16]In particular, we use the 3-month treasury bill rates from 1975Q2-2010Q4 adjusted by the expected inflation rate

Table 1. Calibrated Parameters

Parameter	Value	Description
β	0.99	Subjective discounting factor
α	0.3	Capital share in production
$\delta(1)$	0.025	Steady-state depreciation rate
δ_e	0.020	Exit rate
N	0.25	Steady-state hours
g_γ	1.0042	Steady-state gross growth rate of output
$\bar{\lambda}_z$	1.0121	Steady-state gross growth rate of investment-specific technology
u	1	Steady-state capacity utilization rate
$\tilde{I}/\tilde{Y}$	0.2	Steady-state investment-output ratio
$K_0/\tilde{K}$	0.20	Ratio of capital endowment for an entrant to total capital stock
$\bar{\theta}$	0.9975	Relative size of the old bubble to the new bubble
ω	0.5	Fraction of entrants with bubbles

The second subset of parameters $\Psi_2 = \{h, \Omega, \delta''/\delta'(1), \overline{\zeta}, \mu, f_1, f_2, f_3\}$ includes the habit formation parameter h, the investment-adjustment cost parameter Ω, the capacity utilization parameter $\delta''/\delta'(1)$, the mean value of the financial shock $\overline{\zeta}$, the elasticity μ of the probability of undertaking investment at the steady-state cut-off, and the coefficients f_1, f_2, and f_3 in the measurement equation of the financial condition index. These parameter values are estimated by the Bayesian method.

Following LWZ (2013), we assume that the prior of h follows the Beta distribution with mean 0.33 and standard deviation 0.24. This prior implies that the two shape parameters in the Beta distribution are given by 1 and 2. The prior density declines linearly as h increases from 0 to 1. The 90 percent interval of this prior density covers most calibrated values for the habit formation parameter used in the literature (e.g., Boldrin, Christiano, and Fisher (2001) and Christiano, Eichenbaum and Evans (2005)). We assume that the prior for Ω follows the Gamma distribution with mean 2 and standard deviation 2. The 90 percent interval of this prior ranges from 0.1 to 6, which covers most values used in the DSGE literature (e.g., Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), Liu, Waggoner, and Zha (2012), and LWZ (2013)). For $\delta''/\delta'(1)$, we assume that the prior follows the Gamma distribution with mean 1 and standard deviation 1. The 90 percent interval of this prior covers the range from 0.05 to 3, which covers most calibrated values for $\delta''/\delta'(1)$ (e.g., Jaimovich and Rebelo (2009)). For $\overline{\zeta}$, we assume that the prior follows the Beta distribution with mean 0.3 and standard deviation 0.1. The 95 percent interval of this prior density ranges roughly from 0.1 to 0.5. Covas and den Hann (2011) document that $\overline{\zeta}$ ranges from 0.1 to 0.4 for various sizes of firms. Our prior covers their empirical estimates. We find that our estimate of $\overline{\zeta}$ is quite robust and not sensitive to the prior distribution. For μ, we assume that the prior follows the Gamma distribution with mean 2 and standard deviation 2. The 90 percent interval of this prior ranges from 0.1 to 6, which is wide enough to cover low to high

(from the University of Michigan's survey of consumer) and take the average to obtain the steady state R_f of 1.0017.

Table 2. Prior and Posterior Distributions

Parameter	Prior Distribution			Posterior Distribution			
	Distr.	Mean	St.Dev.	Mode	Mean	5%	95%
h	Beta	0.33	0.24	0.54	0.54	0.49	0.60
Ω	Gamma	2	2	0.03	0.03	0.01	0.06
δ''/δ'	Gamma	1	1	11.79	11.92	8.27	15.15
$\bar{\zeta}$	Beta	0.3	0.1	0.30	0.30	0.23	0.36
μ	Gamma	2	2	2.53	2.58	2.11	3.06
f_1	Gamma	1	1	0.05	0.04	0.01	0.07
f_2	Gamma	1	1	4.73	4.79	2.79	6.69
f_3	Gamma	1	1	0.41	0.33	0.00	0.57
ρ_a	Beta	0.5	0.2	0.97	0.96	0.94	0.99
ρ_{a^m}	Beta	0.5	0.2	0.96	0.96	0.95	0.98
ρ_z	Beta	0.5	0.2	0.36	0.34	0.22	0.46
ρ_θ	Beta	0.5	0.2	0.93	0.92	0.90	0.95
ρ_ψ	Beta	0.5	0.2	0.99	0.98	0.97	0.99
ρ_ζ	Beta	0.5	0.2	0.88	0.87	0.81	0.94
σ_a (%)	Inv-Gamma	1	Inf	0.22	0.24	0.18	0.29
σ_{a^m} (%)	Inv-Gamma	1	Inf	1.01	1.05	0.93	1.16
σ_z (%)	Inv-Gamma	1	Inf	0.59	0.60	0.53	0.66
σ_θ (%)	Inv-Gamma	10	Inf	18.39	19.25	12.26	26.03
σ_ψ (%)	Inv-Gamma	1	Inf	0.80	0.82	0.72	0.92
σ_ζ (%)	Inv-Gamma	1	Inf	0.77	0.83	0.48	1.18

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

elasticity used in the literature. For example, if we assume that ε follows the Pareto distribution $1 - \varepsilon^{-\varsigma}$, then $\mu = \varsigma$. Wang and Wen (2012b) estimate that ς is equal to 2.4, which lies in our range. For f_1, f_2, and f_3, we assume that the priors follow the Gamma distribution with mean 1 and standard deviation 1. The 90 percent interval of this prior covers fairly large range from 0.05 to 3. We find that our estimates of these parameters are quite robust and not sensitive to the prior distribution.

The third subset of parameters is $\Psi_3 = \{\rho_i, \sigma_i\}$ for $i \in \{a, z, a^m, \theta, \zeta, \psi\}$, where ρ_i and σ_i denote the persistence parameters and the standard deviations of the six structural shocks. Following Smets and Wouters (2007) and LWZ (2013), we assume that ρ_i follows a Beta distribution with mean 0.5 and standard deviation 0.2. The prior for σ_i follows inverse Gamma distribution with mean 1% and standard deviation ∞, except for σ_θ. For the sentiment shock θ_t, we assume that the prior mean of σ_θ is equal to 10%. The choice of this high prior volatility is based on the fact that the stock price is the main data used to identify the sentiment shock. Since we know that the stock market is very volatile, it is natural to specify a large prior volatility for the sentiment shock. As a robustness check, we also consider the prior mean 1% of σ_θ and find similar results (see Appendix E in the online supplemental material).

Table 2 presents the prior distributions of the parameters in groups two Ψ_2 and three Ψ_3. It

also presents the modes, means, and 5th and 95th percentiles of the posterior distributions for those parameters obtained using the Metropolis-Hastings algorithm with 200,000 draws.[17] In later analysis, we choose the posterior modes as the parameter values for all simulations.

Table 2 reveals that our estimates of most parameters are consistent with those in the literature (e.g., LWZ (2013)). We shall highlight some of the estimates. First, the sentiment shock is highly persistent and volatile. The posterior mode and mean of the AR(1) coefficient are equal to 0.93 and 0.92, respectively. The posterior mode and mean of the standard error are equal to 18.39% and 19.25%, respectively. Second, our estimated investment adjustment cost parameter is small. The posterior mode and mean of this parameter are equal to 0.03. This result is important because a large adjustment cost parameter is needed for most DSGE models in the literature to explain the variations in stock market prices or returns. But a large value is inconsistent with micro-level evidence (Cooper and Haltiwanger (2006)). By contrast, in our model the aggregate stock market value contains a separate bubble component. The movement of the stock market value is largely determined by the bubble component which is driven largely by the sentiment shock. According to our estimated parameter values, the bubble component accounts for about 14 percent of the stock market value in the steady state. We will show below that this small component plays a dominant role in explaining fluctuations in the stock market as well as macroeconomic quantities.

3.3. Model Evaluation

To evaluate our model performance, we compare with three alternative models estimated using Bayesian methods. The first alternative model (labeled "no sentiment") is derived from our baseline model presented in Section 2 after removing the sentiment shock in equation (25) and setting $\theta_t = \bar{\theta} = 0.9975$. In this model stock price bubbles still exist, but their fluctuations are driven by fundamental shocks only. In the second alternative model (labeled "no bubble"), we replace the credit constraint (12) with the Kiyotaki-Moore type constraint

$$\frac{L_{t+1}^j}{R_{ft}} \leq (1 - \delta_e)\xi_t Q_t K_t^j. \tag{44}$$

The resulting equilibrium is identical to the bubbleless equilibrium in our baseline model so that neither bubbles nor sentiments shocks play any role. In order to make the above two models flexible enough to fit the stock prices and to avoid the stochastic singularity problem, we add measurement errors in the observation equation for stock prices. In the third alternative model (labeled "no stock price"), we do not include the stock price data in the estimation. By comparing with this model, we intend to see how the stock price data is important to identify the sentiment shock and improve model performance.

In Table 1 of the online supplemental material, we present our baseline model's predictions re-

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[17]The MCMC univariate convergence diagnostic (Brooks and Gelman (1998)) shows that our posterior distribution of each parameter constructed from random draws converges to a stationary distribution.

garding standard deviations, correlations with output, and serial correlations of output, consumption, investment, labor hours, and stock prices. This table also presents results for the preceding three alternative models. The model moments are computed using the simulated data from the estimated model by taking the posterior modes as parameter values. Both simulated and actual data are in logs and HP-filtered.
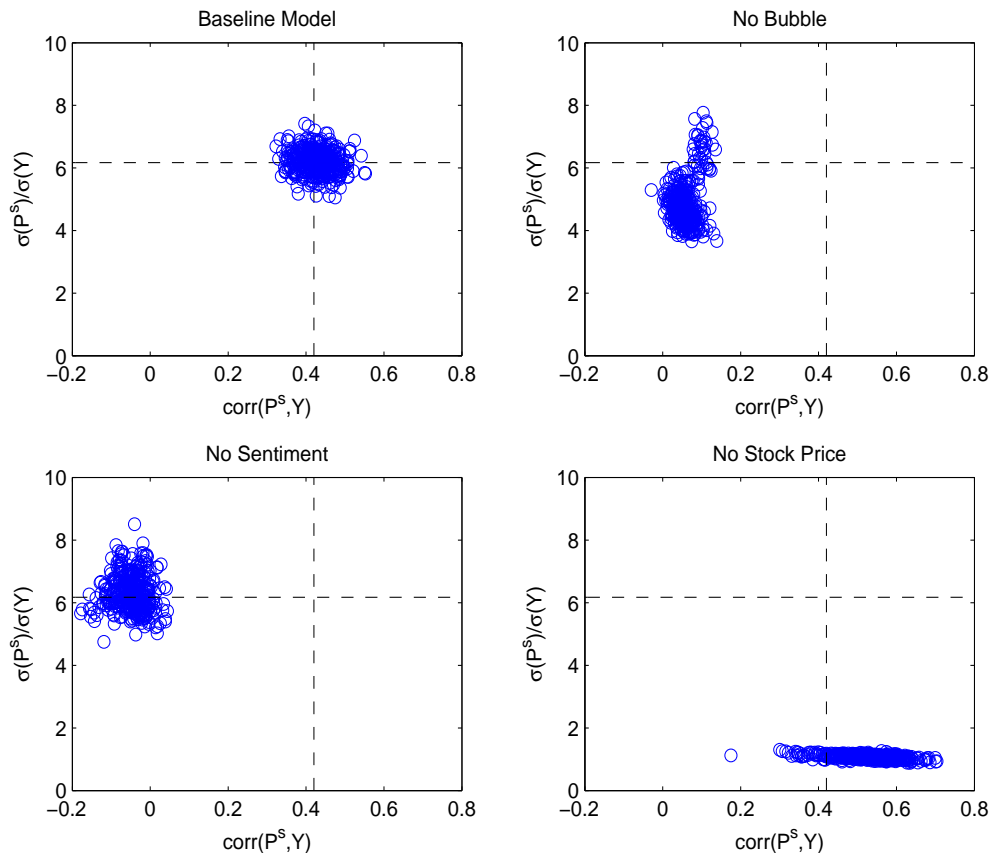

Figure 1: Posterior predictive checks. This figure depicts 400 draws from posterior predictive distribution of sample moments: relative volatility of stock prices to output $\sigma(P^s)/\sigma(Y)$ and correlation between stock prices and output $corr(P^s, Y)$. The intersection of dashed lines indicates the actual sample relative standard deviation and correlation.

We find that our baseline model matches the data closely and the three alternative models match the data reasonably well except for two moments: the volatility of stock prices relative to output and the correlation between stock prices and output. To further examine this issue, we use the posterior predictive checks discussed in An and Schorfheide (2007), Chang, Doh, and Schorfheide (2007), and Del Negro and Schorfheide (2011). In Figure 1 we plot 400 draws from the posterior predictive distribution of the sample relative volatility of stock prices and the correlation between stock prices and output. Each point in the plot is generated as follows: we take a draw from the posterior distribution of the DSGE model parameters, simulate 144 artificial observations from the linearized model conditional on the drawn parameter values, and compute sample moments. The

intersection of the dashed lines indicates the sample moments calculated from the actual U.S. data: the relative volatility of stock prices is 6.36 and the correlation between stock prices and output is 0.42. If the estimated model fits the data well, the actual sample moment should not lie too far in the tails of the posterior predictive distribution.

Figure 1 reveals that this is indeed the case for our baseline model, but not for the three alternative models. For the no sentiment model, the fluctuations of bubbles driven by fundamental shocks together with measurement errors can help match the stock price volatility, but the correlation between stock prices and output is too low compared with the data. For the no bubble model, both the stock price volatility and the correlation between stock prices and output are too low even though measurement errors are introduced. When the stock price data is removed in the estimation of the baseline model, the model generated stock price volatility is too low, but the correlation between between stock prices and output is moderately higher than the data.

To further compare the performance of our baseline model with that of the alternative models, we compute the marginal data density based on the harmonic mean estimator discussed in Herbst and Schorfheide (2015). We find that the log marginal data densities for our baseline model, the model without sentiment shocks, and the model without bubbles are equal to 2223.9, 2110.6, and 2090.2, respectively. This suggests that the data favor our baseline model if one assigns equal prior probabilities to the three models.

4.   Economic Implications

In this section we discuss the model's empirical implications based on the estimated parameters. We address the following questions: How much does each shock contribute to the variations in the stock market, output, investment, consumption, and hours? What explains the stock market booms and busts? Does the stock market affect the real economy? We then use our model to shed light on two major bubble and crash episodes in the U.S. economy: (i) the internet bubble during the late 1990s and its subsequent crash, and (ii) the recent stock market bubble in tandem with the housing bubble and the subsequent Great Recession.

4.1.   Relative Importance of the Shocks

Our estimated model can help us evaluate the relative importance of the shocks in driving fluctuations in the growth rates of stock prices and macroeconomic quantities by the variance decomposition. Table 3 reports this decomposition across the six structural shocks at the business cycle frequency for the baseline model, the three alternative models, and the extended model discussed in Section 5.[18] This table shows that the sentiment shock accounts for about 98 percent of the stock

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[18]We compute variance decomposition using the spectrum of the linearized models and an inverse first difference filter for stock prices, output, consumption, investment to reconstruct the levels. The spectral density is computed from the state space representation of the model with 2000 bins for frequencies covering that range of periodicities.

market fluctuations in the baseline model. The contributions of the other shocks are negligible. The sentiment shock is transmitted from the stock market to the real economy through the credit constraints. A sentiment shock causes the fluctuations in the credit limit and hence affects a firm's investment decisions. This in turn affects aggregate investment and aggregate output. Table 3 reveals that the sentiment shock explains about 20 and 31 percent of the fluctuations in investment and output, respectively. The sentiment shock is the dominating force driving the fluctuations in consumption, accounting for about 32 percent of its variation. This is due to the large wealth effect caused by the fluctuations in the stock market value.

The two TFP shocks are important in explaining variations in macroeconomic quantities as in the RBC literature, but they barely affect the stock market fluctuations. The permanent IST shock does not explain much of the fluctuations in investment, output, consumption, and hours. This is because our model is designed to fit the data of the relative price of the investment goods and the IST shock is tied to the fluctuations in the relative price of investment goods. This result is consistent with the findings reported in Justiniano, Primiceri, and Tambalotti (2011), LWZ (2013), Christiano, Motto and Rostagno (2014), and Liu, Waggoner, and Zha (2012). The labor supply shock accounts for most of the fluctuations in hours (about 72 percent). It also contributes to sizable fractions of fluctuations in output, investment and consumption. This shock is a reduced-form shock capturing the labor wedge. A similar finding is reported in LWZ (2013) and Justiniano, Primiceri, and Tambalotti (2011).

Our estimated financial shock is highly persistent, but accounts for a negligible fraction of fluctuations in stock prices, investment, consumption, output, and hours. The intuition is that the sentiment shock and the financial shock work through a similar channel since both shocks affect the credit constraints. However, the sentiment shock displaces the financial shock once the stock price data is included in the estimation because only this shock can generate comovement between stock prices and macro quantities, as well as the excessive volatility of stock prices. Table 3 shows that when the stock price data is not included in the estimation, the re-estimated financial shock becomes much more important, explaining about 28, 14, 60 percent of the variations in stock prices, output, and investment, respectively. However, this re-estimated model cannot explain the stock market volatility (see Figure 1).

We find that the measurement errors explain almost all of the stock market volatility in the other two alternative models. In particular, they explain about 93 percent of the fluctuations in the stock prices in the alternative model without sentiment shocks. The IST shock and the two TFP shocks together explain about 87 percent of the investment fluctuation. The impact of the financial shock is still negligible as in our baseline model. The large impact of the measurement errors indicate that this model is misspecified.

Similar patterns emerge in the alternative model without bubbles as Table 3 reveals. In particular, the measurement errors now explain 94 percent of the fluctuation in the stock prices, and the IST shock and the two TFP shocks together explain about 80 of the fluctuation in the investment. Again, the financial shock plays a negligible role.

Table 3. Variance Decomposition at Business Cycle Frequencies

	Sentiment	Financial	IST	Agrowth	Atrans	Labor	MeaErr
			Stock Price				
Baseline	98.45	0.00	0.22	0.53	0.51	0.29	–
No Stock Price	0.70	27.61	2.43	23.32	5.94	39.99	–
No Bubble	–	0.01	1.70	3.11	0.42	1.21	93.55
No Sentiment	–	0.00	1.14	4.49	0.54	0.90	92.93
Extended	73.37	11.26	0.04	1.78	6.44	7.09	–
			Output				
Baseline	30.56	0.11	0.93	18.09	31.17	19.13	–
No Stock Price	14.29	14.48	1.21	35.48	2.34	32.21	–
No Bubble	–	0.46	21.45	31.00	15.03	32.05	0.00
No Sentiment	–	0.09	22.49	12.97	31.22	33.23	0.00
Extended	17.22	2.45	1.18	53.59	13.04	12.53	–
			Investment				
Baseline	19.54	0.72	4.17	36.22	25.31	14.04	–
No Stock Price	23.94	60.36	0.40	6.75	0.92	7.63	–
No Bubble	–	1.85	53.78	20.68	6.82	16.88	0.00
No Sentiment	–	0.49	55.95	20.73	10.27	12.56	0.00
Extended	10.49	1.16	6.27	37.47	36.18	8.43	–
			Consumption				
Baseline	31.68	0.03	0.38	18.68	29.78	19.45	–
No Stock Price	3.97	0.64	1.45	49.10	2.58	42.26	–
No Bubble	–	0.02	6.11	58.10	11.98	23.79	0.00
No Sentiment	–	0.00	7.70	36.37	27.80	28.13	0.00
Extended	19.92	3.67	0.52	57.25	3.62	15.02	–
			Hours				
Baseline	4.20	0.41	2.10	19.00	2.67	71.62	–
No Stock Price	4.42	19.30	0.04	1.65	1.68	72.91	–
No Bubble	–	0.30	21.07	21.24	10.36	47.04	0.00
No Sentiment	–	0.09	20.70	15.79	12.62	50.81	0.00
Extended	2.50	0.77	5.42	10.08	15.03	66.19	–

Note: "No Bubble" corresponds to the model without bubbles. "No Sentiment" corresponds to the baseline model without sentiment shocks. "No Stock Price" corresponds to the baseline model without using the stock price data in estimation. "Extended" corresponds to the model in Section 5. "MeaErr" denotes the measurement error in the measurement equation for the stock prices.

4.2. What Explains Stock Market Booms and Busts?

From the variance decomposition, we find that the sentiment shock is the most important driving force behind the fluctuation in the stock market.[19] Why are other shocks not important? To address this question, we derive the log-linearized detrended stock price as

$$\hat{P}_t^s = \frac{\tilde{K}\tilde{Q}}{\tilde{P}^s}(\hat{Q}_t + \hat{K}_{t+1}) + \frac{\tilde{B}^a}{\tilde{P}^s}\hat{B}_t^a, \tag{45}$$

where a variable with a tilde denotes its steady state detrended value and a variable with a hat denotes the relative deviation from the steady state. We can derive

$$\hat{B}_t^a = -\hat{\Lambda}_t + \left[1 - \beta(1 - \delta_e)\bar{\theta}\right]\varphi_G \sum_{j=1}^{\infty} E_t(\hat{P}_{t+j} - \hat{Q}_{t+j}) + \frac{1 - (1 - \delta_e)\bar{\theta}}{(1 - \delta_e)\bar{\theta}} \sum_{j=1}^{\infty} E_t\hat{m}_{t+j}, \tag{46}$$

where φ_G is a negative number given in the online supplemental material. Equation (45) shows that the variations in the stock price are determined by the variations in marginal Q, $\hat{Q}_t$, the capital stock, $\hat{K}_{t+1}$, and the bubble, $\hat{B}_t^a$. As is well known in the literature, the capital stock is a slow-moving variable and cannot generate large fluctuations in the stock price. The variation in marginal Q can be large if the capital adjustment cost parameter is large. But according to our estimation, this parameter is small and hence movements in marginal Q cannot generate large fluctuations in the stock price. Equation (46) reveals that the variation in the bubble is largely determined by the variation in the expected future relative size of the aggregate bubble to the new bubble, $\hat{m}_{t+j}$, because the variations in $\hat{\Lambda}_t$, $\hat{P}_{t+j}$ and $\hat{Q}_{t+j}$ are small. The variation in $\hat{m}_{t+j}$ is determined by the sentiment shock $\hat{\theta}_{t+j}$ as shown in equation (32). According to our estimation, the sentiment shock is the dominant driver of the stock market fluctuations, even though the bubble component accounts for a small share of the stock price ($\tilde{B}^a/\tilde{P}^s = 0.14$) in the deterministic steady state.

Why are the other shocks not important drivers of the stock market fluctuations? First, the IST shock cannot be the primary driver when we allow the model to fit both the stock price data and the relative price of investment goods data. This is because the price of the investment goods is countercyclical, but the stock market value is procyclical. A positive IST shock can reduce the price of the investment goods, but it also reduces the marginal Q and hence the stock market value.

The labor supply shock cannot be the primary driver either. Since it affects the marginal utility of leisure directly, it is an important shock to explain the variation in hours. However, it cannot generate large movements in the stock price because its impact on the marginal Q is small.

We next turn to the two TFP shocks, which are considered to be the main driver of the fluctuations in real quantities in the RBC literature. Figure 2 shows that a positive permanent shock cannot be an important driver of the stock market movements. A permanent positive TFP shock reduces marginal Q because it reduces the future marginal utility of consumption due to the

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[19]We use the Campbell-Shiller approximation (Campbell (1999)) to compute the stock return volatility and find that the sentiment shock explains more than 90 percent of the stock return volatility.
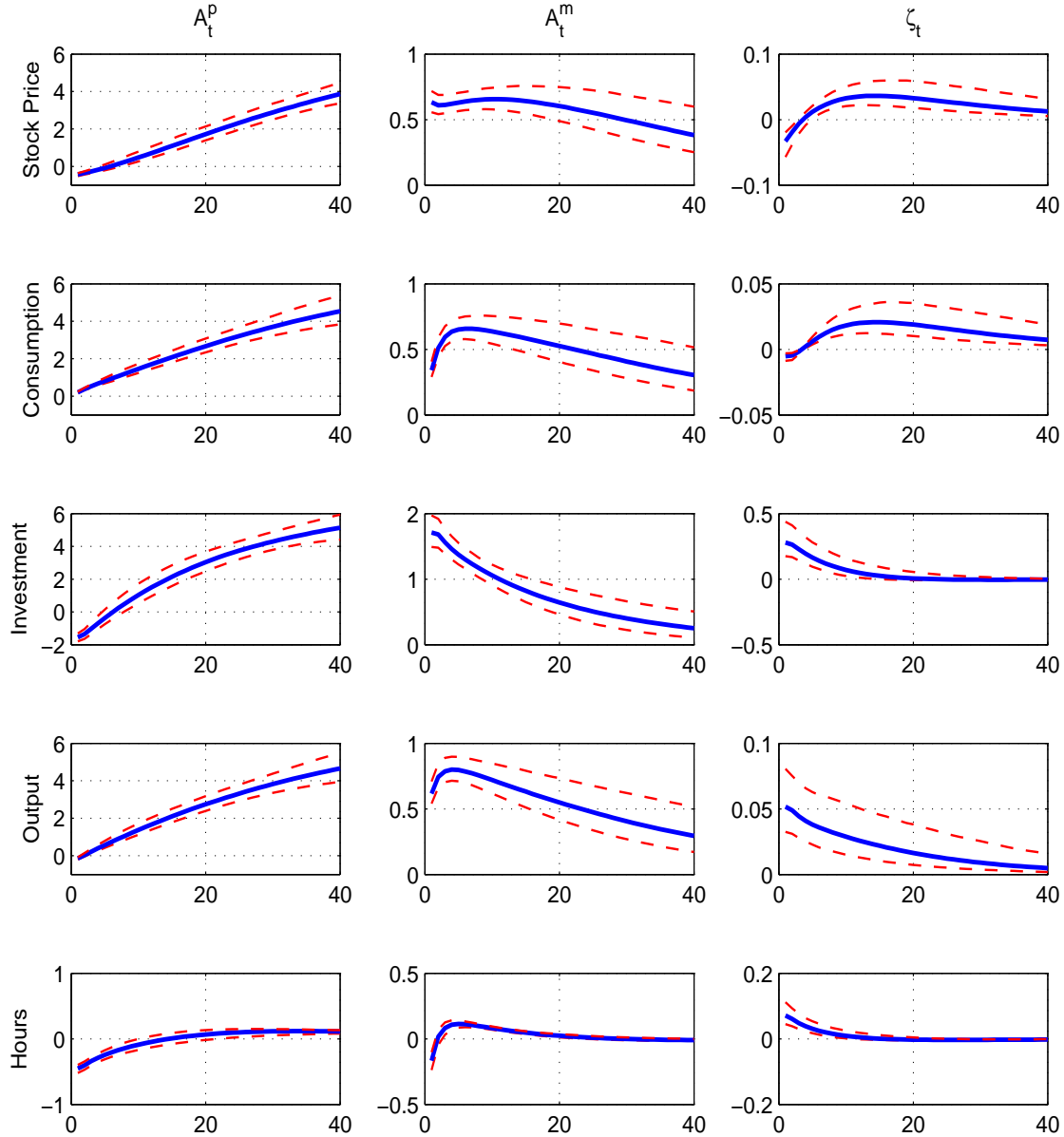
Figure 2: Impulse responses to a one-standard-deviation permanent TFP shock (A_t^p), transitory TFP shock (A_t^m), and financial shock (ζ_t) in the baseline model. All vertical axes are in percentage. We compute the responses for 20,000 draws from the posterior distributions. The solid line is the median value, the dashed lines indicate the 90 percent credible interval.

wealth effect. Though it raises the bubble in the stock price, the net impact on the stock price is negative and small. As Figure 2 shows, the impulse responses of output are similar to those of the stock price. This implies that the volatility of the stock market would be counterfactually similar to that of output growth if the permanent TFP shock were the driving force.

As illustrated in Figure 2, although a positive transitory TFP shock raises both marginal Q and the bubble, its impact on the stock price is small, compared to that on consumption, investment, and output. Thus it cannot explain the high relative volatility of the stock market.[20]

Recently, Jermann and Quadrini (2012) show that the financial shock is important for business cycles. LWZ (2013) find that the housing demand shock displaces the financial shock when the housing price data is included in estimation. The variance decomposition in Table 3 shows that once the stock market data is incorporated, the role of the financial shock is significantly weakened. The intuition is that an increase in the financial shock causes the credit constraints to be relaxed, thereby raising investment. Since it does not affect output directly, consumption falls on impact. Thus the financial shock cannot generate comovement between consumption and investment as shown in Figure 2. As capital accumulation rises, marginal Q falls, causing the fundamental value of the stock market to fall. In addition, the bubble component also falls on impact because there is no room for a bubble as the credit constraints are already relaxed. As a result, the net impact of an increase in the financial shock is to reduce the stock price, implying that the financial shock cannot drive the stock market cyclicality.

Now consider the impact of a sentiment shock presented in Figure 3. A positive sentiment shock raises the size of the bubble, causing the credit constraints to be relaxed. Thus firms make more investment. As capital accumulation rises, marginal Q falls so that the fundamental value of the stock market also falls. But this fall is dominated by the rise in the bubble component, causing the stock price to rise on impact. This in turn causes consumption to rise due to the wealth effect. The capacity utilization rate also rises due to the fall of marginal Q, causing the labor demand to rise. The rise in the labor demand is dominated by the fall in the labor supply due to the wealth effect, and hence labor hours fall on the impact period, but rises afterward. The increased capacity utilization raises output.

Notice that on impact the stock price rises by about 8 percent, which is much larger than the impact effects on output (0.2 percent), consumption (0.2 percent) and investment (0.3 percent). This result indicates that the sentiment shock can generate a large volatility of the stock market relative to that of consumption, investment, and output. The sentiment shock has a small impact on the price of investment goods. This allows the movements of the price of investment goods to be explained by the IST shock.

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[20]Note that both permanent and transitory TFP shocks can generate a fall in hours on impact. This is due to the presence of habit formation utility and investment adjustment costs (see Francis and Ramey (2005) and Smets and Wouters (2007)).
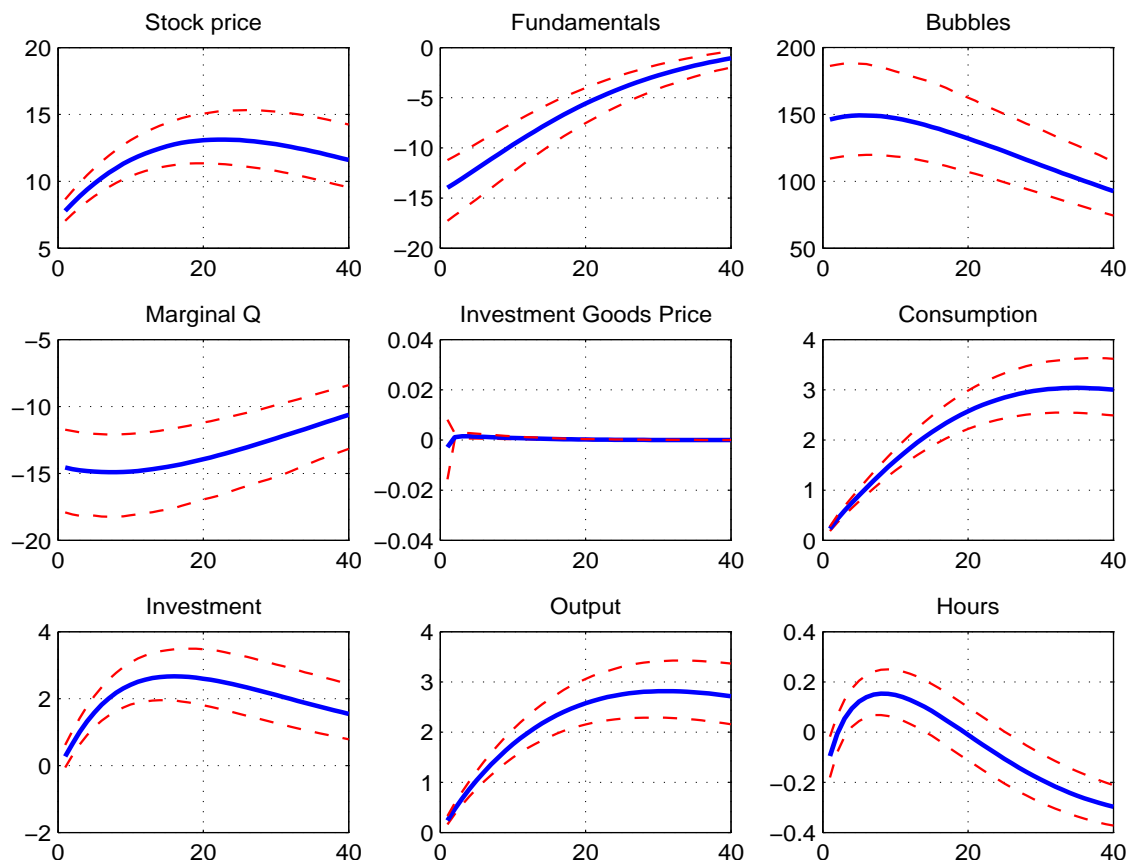
Figure 3: Impulse responses to a one-standard-deviation sentiment shock in the baseline model. All vertical axes are in percentage. We compute the responses for 20,000 draws from the posterior distributions. The solid line is the median value, the dashed lines indicate the 90 percent credible intervals.
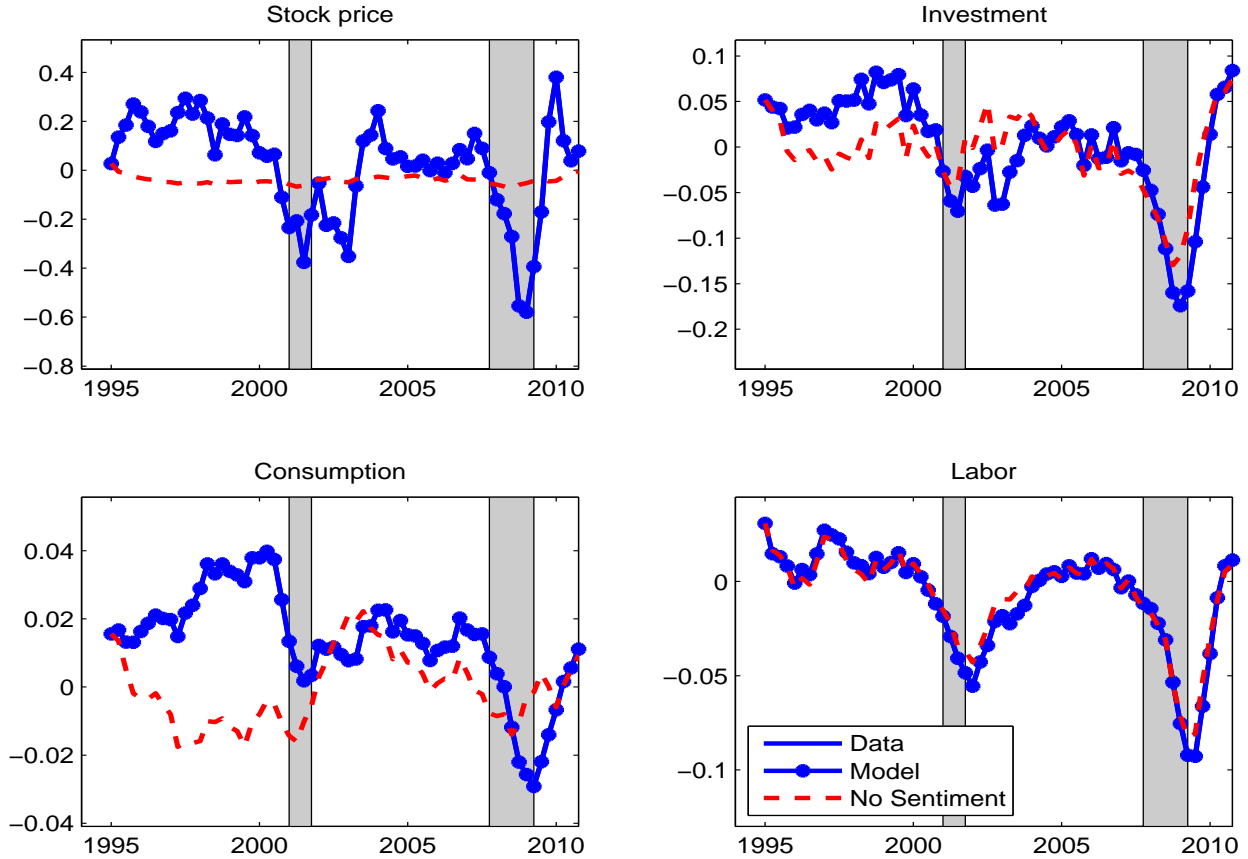
Figure 4: The internet bubble and Great Recession episodes. This figure plots the year-on-year growth rates of stock prices, investment, consumption, and labor hours. The shaded areas represent NBER recession bars. Data: actual data. Model: model fitted data when all shocks are turned on. No Sentiment: model fitted data when the sentiment shock is shut down.

4.3. Understanding Major Bubble and Crash Episodes

The U.S. economy has experienced two major bubble and crash episodes: (i) the internet bubble during the late 1990s and its subsequent crash, and (ii) the recent stock market bubble in tandem with the housing bubble and the subsequent Great Recession. Can our model help understand these two episodes? To address this question, we compute the paths of stock prices, business investment, consumption, and labor hours implied by our estimated model when all shocks are turned on and when the sentiment shock alone is turned off. We then compare these paths with the actual data during these two episodes.

Figure 4 shows that our estimated DSGE model fits the actual data almost exactly. In particular, the sentiment shock plays the single most important role in explaining the stock price fluctuation. Without the sentiment shock, the fitted stock prices are are almost flat. We also find that there are sizable gaps between the actual consumption and investment data and the simulated data when the sentiment shock is shut down during the internet bubble and the Great Recession. This suggests that the sentiment shock contributes a sizable share to the fluctuation in consumption

and investment. The last panel of Figure 4 shows that the sentiment shock does not contribute much to the fluctuation in labor hours. As Table 3 shows, the labor supply shock accounts for most of the variation in labor hours. The labor supply shock captures the labor wedge and may be interpreted as a reduced form representation of the labor market friction. Our result suggests that labor market frictions played a significant role in accounting for drops in hours growth especially during the Great Recession. Modeling such frictions is an interesting future research topic, which is beyond the scope of this paper.

What is the role of the financial shock? The top panel of Figure 5 plots the smoothed sentiment shock and financial shock, indicating that there was a large negative financial shock that tightened credit constraints during the Great Recession. It also shows that the IT bubble in 1990s and the subsequent crash were not quite related to financial shocks because the smoothed financial shock moved countercyclically during that period. Instead, the sentiment shock rose in the late 1990s and declined in the early 2000s. Thus this shock is the most important shock to explain that episode.
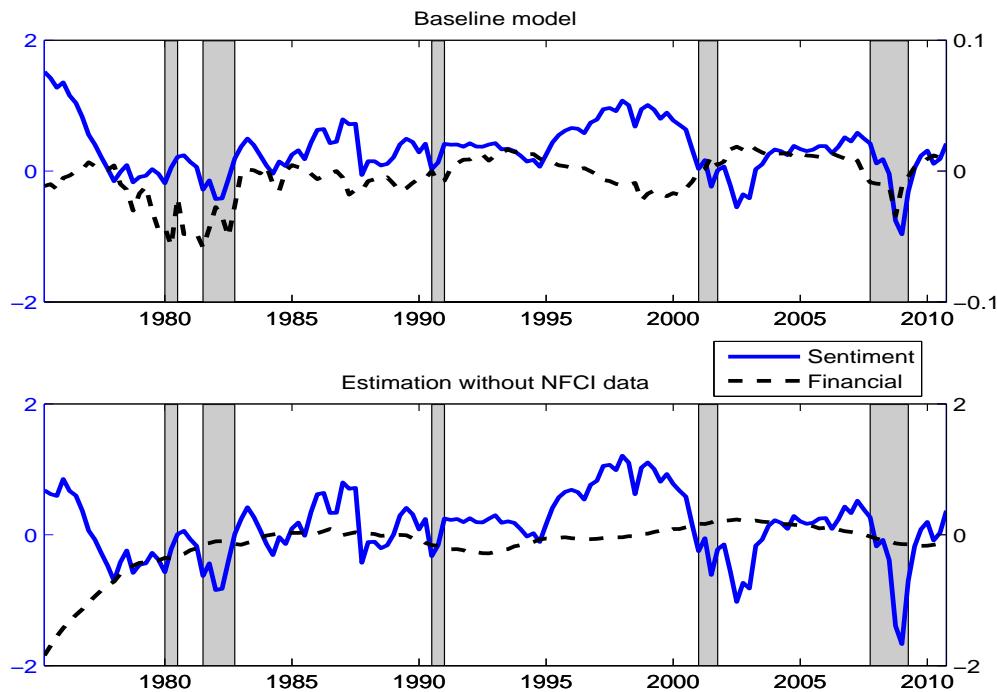

Figure 5: Smoothed financial shocks $\hat{\zeta}_t$ (right vertical axis) and sentiment shocks $\hat{\theta}_t$ (left vertical axis). The shaded areas represent NBER recession bars.

To see why the NFCI data is important to identify the financial shock, we estimate the model without the NFCI data and obtain the smoothed financial shock on the bottom panel of Figure 5. We find that the correlation between the smoothed financial shock and the NFCI is only -0.03, whereas the correlation in the baseline estimation is around -0.88, indicating that the NFCI data is informative for identifying the financial shock. But the NFCI data is not informative for identifying the sentiment shock because the smoothed sentiment shocks in the two panels of Figure 5 are very

similar. The bottom panel of Figure 5 shows that there was an increasing sequence of financial shocks during the IT bubble period. The shock series is relatively smooth and there was no significant drop during the Great Recession. All these counterintuitive features are in contrast to those shown on the top panel of Figure 5 and are inconsistent with the common view.

5. An Extended Model with Consumer Sentiment Index

In our model the sentiment shock is an unobserved latent variable. We infer its properties from our six time series of the U.S. data using an estimated model. We find that the consumer sentiment index (CSI) published monthly by the University of Michigan and Thomson Reuters is highly correlated with our smoothed sentiment shock as illustrated in Figure 6.[21] The correlation is 0.61. We now incorporate this data in the estimation and consider the following measurement equation:

$$CSI_t = CSI + b_1\hat{\theta}_t + b_2\Delta\hat{Y}_t + b_3\Delta\hat{Y}_{t-1} + b_4\Delta\hat{Y}_{t-2} + b_5\Delta\hat{Y}_{t-3} + \varepsilon_{cci,t}^{err},$$

where Δ denotes the first difference operator. In this equation we allow for measurement errors $\varepsilon_{cci,t}^{err}$ and the correlation between CSI and business cycles (i.e., output growth in the past four quarters). This specification captures the fact that CSI may be influenced by current and past GDP growth. We also allow the sentiment shock to be correlated with other shocks in the model such that

$$\hat{\theta}_t = \hat{\theta}_{1t} + \hat{\theta}_{2t}, \quad \hat{\theta}_{1t} = \rho_\theta\hat{\theta}_{1t-1} + \hat{\varepsilon}_{\theta,t},$$
$$\hat{\theta}_{2t} = a_1\hat{\zeta}_t + a_2\hat{A}_t^m + a_3\hat{\lambda}_{a,t} + a_4\hat{\lambda}_{z,t} + a_5\hat{\psi}_t,$$

where $\lambda_{a,t} = A_t^p/A_{t-1}^p$ and $\lambda_{z,t} = Z_t/Z_{t-1}$. We call this model "the extended model."

The variance decomposition for the extended model is presented in Table 3.[22] This table shows that the impact of the sentiment shock is weakened compared to the baseline model. But it is still the dominant force driving the stock market fluctuations, explaining about 73 percent of the variation. It also explains a sizable fraction of the variations in real quantities. In particular, it explains about 17, 10 and 20 percent of the variations in output, investment and consumption, respectively. The two TFP shocks are the most important force in explaining these quantities. But they are still not important in explaining the stock market fluctuations. Table 1 in the online supplemental material shows that the extended model and the baseline model perform almost equally well in explaining business cycle statistics.

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[21] This index is normalized to have a value of 100 in December 1964. At least 500 telephone interviews are conducted each month of a continental United States sample (Alaska and Hawaii are excluded). Five core questions are asked. An important objective of this index is to judge the consumer's level of optimism/pessimism.

[22] The parameter estimates are presented in Table 2 of the online supplemental material.
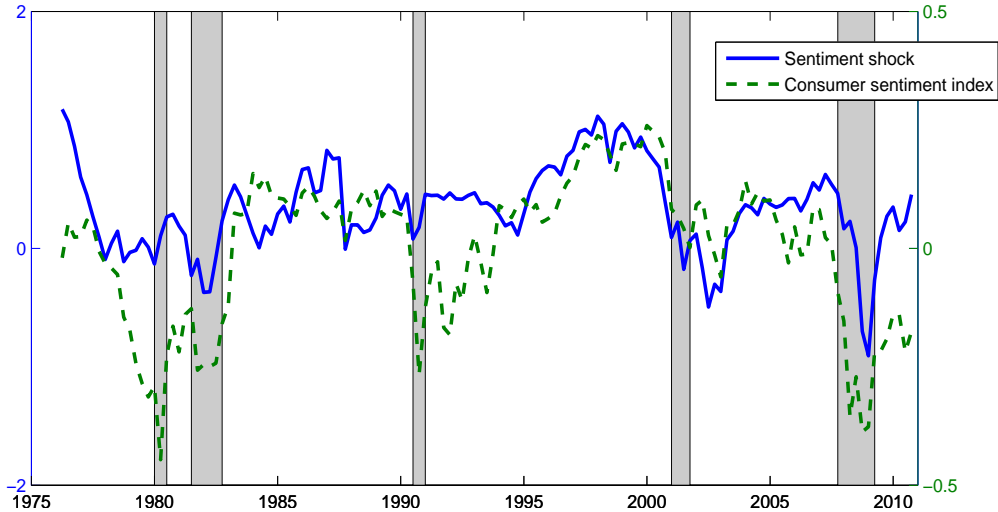
Figure 6: This figure plots the sentiment shock (left vertical axis) estimated from the baseline model and the consumer sentiment index (right vertical axis) downloaded from the University of Michigan. Both series are measured as the deviation from the mean divided by the mean. The shaded areas represent NBER recession bars.

6.   Conclusion

Stock markets are highly volatile and it is challenging to explain their movements entirely by fundamentals. Many people believe that bubbles, fads or irrationality may play an important role in determining stock prices. This idea has been developed extensively in the theoretical literature. However, the development of the empirical literature is hindered by the lack of identification of bubbles using the VAR approach or other reduced-form regression analysis. As a result, the empirical importance of bubbles for the stock market and for the real economy is unclear.

Our main contribution is to provide a Bayesian DSGE model of stock market bubbles and business cycles. We identify a sentiment shock that drives the movements of bubbles and hence stock prices. Unlike many other demand side shocks such as news shocks and uncertainty shocks, the sentiment shock can generate comovements among consumption, investment, hours, output and stock prices. Our Bayesian estimation shows that the sentiment shock explains most of the stock market volatility and a sizable fraction of the variations in investment, consumption, and output. We find that the sentiment shock in our baseline model explains about 98 and 31 percent of the variations in the stock market volatility and output, respectively. This effect is weakened to 73 and 17 percent in our extended model. We still need further research to understand why fundamental shocks do not explain much of the stock market volatility and the comovement between the stock market and the real economy. We hope our analysis can stimulate further empirical studies to provide a robust estimate of the quantitative impact of bubbles and sentiment shocks. In addition to the empirical contribution, our paper also makes a theoretical contribution to the literature on rational bubbles by modeling recurrent bubbles in an infinite-horizon DSGE framework. Our

theoretical model is useful to address many other quantitative or empirical questions. For example, our model focuses on the real side and does not consider inflation and monetary policy. Should monetary policy respond to asset price bubbles? Miao, Wang and Xu (2012b) study this question by embedding the present model in a dynamic new Keynesian framework.

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