A Theory of Optimal Quality Reports with Inertia

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Abstract

This article considers quality reporting systems which imperfectly provide quality rating information to consumers. Such reporting systems attempt to correct the moral hazard problem from opportunistic firms when consumers cannot observe quality. I investigate the optimal rating accumulation rule which provides incentives for firms to choose the highest expected overall quality level in the market across periods. In a quality report, a firm's rating is determined by the expected quality of the firm's goods and a carried-over advantage from the firm's past rating. The size of this advantage is the system designer's control variable and models the degree of inertia in the system. I consider a two-period repeated contest model between two firms which try to get the relatively better rating since the better rating leads to a bigger market share. The results show that the optimal system allows greater rating accumulation in a market with a larger degree of uncertainty in quality evaluation or production. I also study the case in which the early winner may have an advantage in market share as the incumbent due to consumers' switching costs.

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1 Introduction

Quality reporting system (QRS) provide imperfect quality information to consumers and can help to reduce information asymmetries and hence moral hazard when product quality is observable to a firm but not its consumers. QRS is a type of evaluation system which observes the quality of each good in a market and credibly reports it to consumers. Once it is announced to the public, the evaluation affects consumers' decisions, therefore firms' incentives. Consumer Reports is one example of QRS that plays a major role in purchaser behavior and firm incentives. In this paper, I study QRS as a centralized system in that the information intermediation process is controlled and managed by an authority. In this setting, a natural question is what the optimal information to deliver is, in terms of the effect on firm incentives. This article focuses on a dynamic aspect of information intermediation, namely, how much past evaluations should affect the current quality rating.

When eating out in a restaurant, for example, people can judge the cleanliness only by what they see in the dining area, though the hygiene condition of the kitchen would be more relevant information for their health. People would prefer not to eat in a restaurant which has cockroaches in its kitchen. Such situations have moral hazard since it is costly for restaurants to maintain good hygiene and for potential customers to observe it. In Los Angeles, for example, the Department of Health Services carries out hygiene inspections of each restaurant in the county, issues hygiene grade cards, and mandates that restaurants to display the cards in the window. Jin and Leslie (2003) found that since the grade cards were implemented, a restaurant with a good grade receives higher revenue than when there were no grade cards. Recently, online networks enable consumers to access quality information more easily and this improvement in information for consumers enhances firms' awareness of their public evaluation.[1]

There is a growing literature on how to design report cards and how effective they are (Jin and Leslie (2003), Harrington et al. (2003), Glazer and McGuire (2006), Glazer et al. (2008)). Accumulation in ratings has not received much attention, however, though it may be especially important in a dynamic setting. I treat the degree of inertia in rating as a control variable by the reviewer and study how the inertia affects firms' incentives. I view the inertia

[1] For instance, the Hospital Quality Initiative (HQI) which is a program run by the Centers for Medicare and Medicaid Services (CMS) in conjunction with the Hospital Quality Alliance, makes quality information available to the public through a web site. On the website (www.hospitalcompare.hhs.gov), people can find a list of hospitals around their zipcode and can compare a summary score of each hospital's service quality side by side.

in ratings as modeling a typical property of reputation: a reputation is not easily shattered but is built up over time. For example, a nursing home may build a good reputation over many years. Once such a reputation is acquired, one bad experience may be viewed as an outlier rather than as a signal of bad quality. In QRS, the cumulative rating scheme may describe this property. For instance, when the Indiana State Department of Health calculates the overall evaluation score for all certified nursing facilities within Indiana, the overall score is the total of the scores for each of the three survey periods, weighted so that recent surveys count more than older surveys.[2] To model this feature in the simplest way, I introduce a two-period repeated contest model between two firms. When consumers are more likely to purchase from firms with higher scores, such a reporting system generates a sequence of virtual contests where the prizes are bigger market shares. I assume that a firm's current rating (or reputation) is computed from two parts: its current quality output evaluation and the carried-over advantage points from its past rating. Advantage point transfer is a form of a bonus score to the previously higher rated firm. A report system with a bigger bonus score to the previous winner is one in which there is bigger inertia in evaluation.

Economists have long been interested in reputation as a contract enforcement mechanism in the absence of the third-party enforcer. In many parts of the literature (Klein and Leffler (1981), Shapiro (1983), Allen (1984)), consumers are assumed to behave as if they know the cost functions of firms. Given the prices charged, they are thus able to infer indirectly the quality of goods a profit-maximizing firm would choose in a competitive market, even though they cannot directly observe the quality. However, QRS plays a useful role especially in a market where consumers face a high cost of acquiring information, thus they cannot infer the firm's quality level from its equilibrium behavior. This paper is related to work on certification intermediaries (Lizzeri 1999) in that there is a mediator who reveals information to uninformed parties and information revelation is treated as a control variable of the mediator. In the consumers' view, QRS is an information system which helps them make better choices. On the other hand, in the firms' view, they are an incentive mechanism which induces them to get better evaluations and hence higher profit. In the certification intermediary literature, the intermediaries are usually considered as information systems rather than incentive mechanisms. Therefore, many parts of the literature consider cases where firms have hidden characteristics but no hidden action. In this paper, I concentrate on the role of intermediaries as an incentive mechanism.

In this paper, I consider a market with a small number of players rather than perfect

[2] See http://www.state.in.us/isdh/reports/QAMIS/ltc/repcard/

competition. In many markets where QRS is used, such as public schools or health care market, there are usually a small number of players in any area, and these actors are large enough to each other that strategic effects are important. Also, I assume the price is exogenous. For example, firms often have little ability to control prices in many public sectors. In the health care market, even in cases where firms have the ability to raise prices, the prices they can affect are usually reimbursements, not consumers' out-of-pocket costs.

The results show that success should breed success under higher uncertainty. That is, ratings inertia should be larger in markets where the uncertainty in quality evaluation or production is high. The intuition is that when the luck component is more crucial to winning than improving quality, firms have little incentive to increase quality when it is costly to do so. Since the relative score advantage makes early contests more valuable to win, it enhances the marginal return on quality improvement in early periods but reduces the disincentive effect from high uncertainty. In sum, however, a first-order improvement in incentives outweighs a second-order loss in incentives.[3] I also examine how the rating inertia affects firm behavior over time. As I mentioned, to get the advantage score and hence a bigger market share in period 1, firms have a stronger incentive to provide high quality in period 1. However, once the advantage score is assigned, it predetermines the contest result at period 2 by the extent of the advantage, so competition is dampened. Moreover, when firms are not identical in their efficiency, the inertia of rating affects the firms asymmetrically. The more efficient firm chooses higher quality in the second period when it is the early loser compared to if it were the early winner. On the contrary, the less efficient firm chooses higher quality in the second period when it is the early winner compared to if it were the early loser. That is, the more efficient firm tries harder in the second period if it is behind than when it is ahead, while the less efficient firm does the opposite.

As a natural extension in a dynamic competition setting, I study how optimal ratings with inertia would be changed if some consumers have little ability to change firms to take advantage of quality differences as they encounter substantial switching costs in the second period (Klemperer (1987)). Switching costs can be defined as the costs involved in changing from one firm to another. In addition to objectively measurable monetary costs, switching costs may also pertain to time and psychological cost involved in facing the uncertainty of dealing with a new firm. It has been argued that switching costs for services that are

[3] Several papers analyze settings in which it is optimal for the designer to discriminate or introduce bias in favor of agents who achieve early success(Laffont and Tirole (1988), Meyer (1991, 1992)). This paper applies such ideas to characterize the incentive structure behind score accumulation. I consider optimal solutions with attention to important dynamic concerns in the QRS context.

intrinsically difficult to observe, or for which there is only a small number of suppliers, are high (Brown and Swartz (1989)). For example, many patients exhibit psychological inhibitions against changing doctors (Jones and Sasser (1995)). In a market with high switching costs, consumers are reluctant to change their previous choice even when they recognize there is an alternative with better quality. This allows the early winner to have an advantage in market share as the incumbent. I find that in a market where more consumers have high switching costs, the system needs less inertia in ratings. We can see the trade-off between the advantage in payoff (the market share advantage) and the advantage in win probability (the score advantage) in terms of incentives. Success should breed success to a greater extent when the incumbent has less advantage in market share.

2 Model

There are two firms, A and B, in a market over two periods. In each period $t \in \{1,2\}$, the firms simultaneously chooses quality investment levels $q_{it} \geq 0$ which is unobservable by consumers. Firms have complete information. I model consumers in a simple way which can be justified by the assumption that they have incomplete information, as I explain below. To reduce the information gap between informed firms and uninformed consumers, the society establishes a reliable information system, called a *reviewer,* which imperfectly evaluates q_{it}. Following Lazear and Rosen (1982), firm i's evaluated quality outcome, y_{it}, is given by:

$$y_{it} = q_{it} + \epsilon_{it}, \quad i \in \{A, B\}$$

where ϵ_{it} is the individual specific observation error with zero mean. Assume that $\epsilon_{At} - \epsilon_{Bt}$ is i.i.d $N(0, \sigma^2)$. Let $F(\cdot)$ denote its cumulative distribution function and $f(\cdot)$ the density.[4] The cost function has a quadratic form, $C_i(q_{it}) = c_i \frac{q_{it}^2}{2}$ where marginal cost satisfies $c_B \geq c_A > 0$.

Based on this current evaluation of quality, the reviewer determines the quality rating of each firm in each period t, called the *reputation score* at period t. The reputation score can be cumulative, specifically, the score at period 2 is the sum of the current quality outcome y_{i2} and some carried-over advantage score of the first-period winner, $\Delta \in \mathbb{R}$, called *the score advantage.* In sum, firm i's reputation score at period t will be

$$s_{it} = y_{it} + \Delta \cdot I(r)$$

[4] Symmetry about 0 ensures that within each period there is no systematic difference in evaluation between sellers, and independence ensures that any shock to relative evaluation output $y_{At} - y_{Bt}$ does not persist over time. We can also interpret ϵ_{it} as randomness in quality realization.

where $r \in \{h, l\}$ at period 2, $I(h) = 1$ and otherwise 0. Assume that $r = \varnothing$ at period 1. The relative reputation position r represents whether i was the winner or the loser in the previous period in the sense of having a higher or lower reputation score respectively, than the other firm. In other words, firm i gets a bonus score Δ in period 2 if and only if $y_{i1} > y_{j1}$ where j is the opponent of i.[5] Δ describes how much inertia in rating the reporting system uses. What the reviewer reports in public is the ranking of reputation scores. That is, with two firms, only the identity of the winner and loser is reported. In period 1, firm i is reported as the winner when $s_{i1} > s_{j1}$. In period 2, firm i is reported as the winner if $y_{i2} + \Delta \cdot I(r) > y_{j2} + \Delta \cdot I(r')$ where r' represents the opposite position of r.

In the text, I model consumer behavior in a very simple way. In the Appendix, I show that my model can be seen as the reduced form of a more complex model. The idea is that in each period, there is a fixed number of consumers. The utility of the consumer who chooses firm i is $u_{it} = q_{it} - p_t$ where p_t represents the price. The price is exogenous and constant across firms. Hence, consumers choose among firms based only on expectations of quality. I assume that consumers only receive quality information from the QRS. Such an assumption seems reasonable if, for example, word-of-mouth does not play its role efficiently when the communication cost among consumers is high.

The reduced form I use in the text is a market share structure $\alpha = \left(\alpha_1, \alpha_2^h, \alpha_2^l\right)$. Here, α_1 represents the market share for the winner at period 1, while α_2^r represents the market share for the second period winner when the firm's position in the first period was r. So, α_2^h is for the repeated winner and α_2^l for the new winner at period 2. It is natural to expect that the winner achieves a larger market share than the loser, so I assume $\alpha_t > \frac{1}{2}$. That is, the higher ranking enables the firm to achieve a larger market share. To make the model more general, I consider the case where consumers may not respond to the review sensitively. For example, some consumers may have a strong preference for a specific firm. The winner therefore may not take all of market, so $\alpha_t \leq 1$. Also, some consumers may face substantial costs of switching from a firm to its competitor. Then, though consumers have no ties to any particular firm in period 1, switching costs in period 2 are created by first-period choices. In this case, the new winner may take a smaller market share than the repeated winner, so I assume $\alpha_2^l \leq \alpha_2^h$. Note that the first period winner takes at least $1 - \alpha_2^l$ in period 2. A larger $1 - \alpha_2^l$ implies that the early winner has a greater overall advantage in market share.

[5] For its simplicity, I consider the case that Δ is constant rather than increasing in $y_{i1} - y_{j1}$. The core of our results will not be different in either case. And since the case of a tie has zero probability, we disregard it.

To focus on the incentive effect of switching costs, we will frequently use the loser's market share, $\beta_t^r \equiv 1-\alpha_t^{r'}$. Under the assumption that consumers have constant tastes over time, the difference $\beta_2^h - \beta_1$ describes the fraction of consumers who choose the current loser in period 2 due to switching costs. Since the difference is strictly increasing in β_2^h, I will use β_2^h as an indicator of switching costs and call it the *incumbency advantage.*

Note that the probability that firm A wins in period 1 is $\Pr(y_{A1} > y_{B1}) = F(q_{A1} - q_{B1})$.[6] In period 2, the probability of winning for the first period winner A will be $\Pr(y_{A2}^h + \Delta > y_{B2}^l) = F(q_{A2}^h - q_{B2}^l + \Delta)$ and that for the first period loser B will be $\Pr(y_{B2}^l > y_{A2}^h + \Delta) = F(q_{B2}^l - q_{A2}^h - \Delta)$ since the one's advantage becomes the other's disadvantage.[7] Normalize total market revenue to 1. Then, the expected revenue of firm i in period 1 is

$$R_{i1} = F(q_{i1} - q_{j1}) \cdot \alpha_1 + [1 - F(q_{i1} - q_{j1})] \cdot (1-\alpha_1).$$

In period 2, the expected revenue of the early winner, R_{i2}^h, is

$$R_{i2}^h = F(q_{i2}^h - q_{j2}^l + \Delta) \cdot \alpha_2^h + \left[1 - F(q_{i2}^h - q_{j2}^l + \Delta)\right] \cdot \beta_2^h.$$

Similarly, the expected revenue of the early loser, R_{i2}^l, is

$$R_{i2}^l = F(q_{i2}^h - q_{j2}^l - \Delta) \cdot \alpha_2^l + \left[1 - F(q_{i2}^h - q_{j2}^l - \Delta)\right] \cdot \beta_2^l.$$

Each firm's product will be consumed proportional to its market share which is determined by contest results. For example, if firm i wins in period 1 with $\Pr(y_{i1} > y_{j1})$, α_1 of consumers will choose firm i and experience q_{i1} in expectation since $E(y_{it}|\epsilon_{it}) = q_{it}$. Analogously, the remaining β_1 of consumers will enjoy q_{j1} in expectation.

The reviewer's concern is maximizing the expected overall quality consumed in the market over time. Therefore, *the expected overall quality*, Q is given by

$$\begin{aligned} Q = {} & \Pr(y_{A1} > y_{B1}) \cdot [\alpha_1 q_{A1} + \beta_1 q_{B1} \\ & + \Pr(y_{A2}^h + \Delta > y_{B2}^l) \cdot \left\{\alpha_2^h q_{A2}^h + \beta_2^l q_{B2}^l\right\} + \Pr(y_{A2}^h + \Delta < y_{B2}^l) \cdot \left\{\beta_2^h q_{A2}^h + \alpha_2^l q_{B2}^l\right\}] \\ & + \Pr(y_{A1} < y_{B1}) \cdot [\beta_1 q_{A1} + \alpha_1 q_{B1} \\ & + \Pr(y_{A2}^l > y_{B2}^h + \Delta) \cdot \left\{\alpha_2^l q_{A2}^l + \beta_2^h q_{B2}^h\right\} + \Pr(y_{A2}^l < y_{B2}^h + \Delta) \cdot \left\{\beta_2^l q_{A2}^l + \alpha_2^h q_{B2}^h\right\}]. \end{aligned}$$[8]

[6] $\Pr(y_{A1} > y_{B1}) = \Pr(q_{A1} + \epsilon_{A1} > q_{B1} + \epsilon_{B1}) = \Pr(\epsilon_{A1} - \epsilon_{B1} > -(q_{A1} - q_{B1})) = 1 - F(-(q_{A1} - q_{B1})) = F(q_{A1} - q_{B1})$ since F is symmetric.

[7] $\Pr(y_{A2}^h + \Delta > y_{B2}^l) = \Pr(q_{A2}^h + \epsilon_{A2} + \Delta > q_{B2}^l + \epsilon_{B2}) = \Pr(\epsilon_{A2} - \epsilon_{B2} > -(q_{A2}^h - q_{B2}^l + \Delta)) = 1 - F(-(q_{A2}^h - q_{B2}^l + \Delta)) = F(q_{A2}^h - q_{B2}^l + \Delta)$ since F is symmetric.

[8] Especially when we understand ϵ_{it} as the randomness in quality production and the reviewer's observation is perfect, it should be stressed out that this objective of reviewer is maximizing the overall quality *ex ante.*

Δ is "free" since it is not something real which directly improves actual quality but just a number. Notice that Δ generates a shift of winning probability toward the early winner. That is, the reviewer decides how much the early evaluation is carried over to evaluation in following period by choosing the size of Δ. The size describes how much rating inertia the reporting system contains. The timing of events is as follows:

- The reviewer announces the scoring rule (the size of Δ).
- In period 1, each firm chooses its quality, q_{A1} and q_{B1}.
- The reviewer reports the ranking, that is, whether or not $y_{A1} > y_{B1}$.
- A greater proportion of consumers chooses the winner's product than the loser's and firms' revenues are determined.
- In period 2, each firm chooses its quality, q_{A2}^r and $q_{B2}^{r'}$.
- The reviewer reports the new ranking, with Δ going to the early winner, e.g. whether or not $y_{i2}^r + \Delta \cdot I(r) > y_{j2}^{r'} + \Delta \cdot I(r')$.
- A greater proportion of consumers chooses the winner's product than the loser's and firms' revenues are determined.

3 Equilibrium analysis

I focus on pure strategy equilibria only. Proceeding by backward induction, consider the Nash equilibrium behavior of firms in period 2 first. Given market share structure α and score advantage Δ, firm i with the first period contest result r at period 2 solves

$$\max_{q_{i2}^r} \Pi_{i2}^r = R_{i2}^r - c_i \frac{\left(q_{i2}^r\right)^2}{2}$$

where $c_B \geq c_A > 0$. The early winner i and the early loser j's best responses will satisfy, respectively

$$q_{i2}^h(q_{j2}^l) = \frac{\alpha_2^h - \beta_2^h}{c_i} \cdot f(q_{i2}^h - q_{j2}^l + \Delta) \text{ and } q_{j2}^l(q_{i2}^h) = \frac{\alpha_2^l - \beta_2^l}{c_j} \cdot f(q_{j2}^l - q_{i2}^h - \Delta).^{9}$$

When $\mathbf{\Pi}_{i2}^r$ is the second period equilibrium expected profit, firm i's problem at period 1 will be

$$\max_{q_{i1}} \left[\Pi_{i1} + \Pr(y_{i1} > y_{-i1}) \cdot \mathbf{\Pi}_{i2}^h + \Pr(y_{i1} < y_{-i1}) \cdot \mathbf{\Pi}_{i2}^l\right].$$

[9] Since the early winner A's expected revenue at period 2 is $R_{A2}^h = F(q_{A2}^h - q_{B2}^l + \Delta) \cdot \alpha_2^h + [1 - F(q_{A2}^h - q_{B2}^l + \Delta) \cdot \beta_2^h]$, the first order condition for q_{A2}^l is $f(q_{A2}^h - q_{B2}^l + \Delta) \cdot \alpha_2^h - f(q_{A2}^h - q_{B2}^l + \Delta)\beta_2^h = c_A \cdot q_{A2}^h$. The early loser B's case is analogous. Just note that $\alpha_2^l - \beta_2^l = \left(1 - \beta_2^h\right) - \left(1 - \alpha_2^h\right) = \alpha_2^h - \beta_2^h$.

From the first order condition, firm i's best response at period 1 will be

$$q_{i1}\left(q_{j1}\right) = \frac{1}{c_i} \cdot f(q_{i1} - q_{j1}) \cdot \left[2\alpha_1 - 1 + \mathbf{\Pi}_{i2}^h - \mathbf{\Pi}_{i2}^l\right].$$

3.1 Benchmark Case

In this section, assume both firms are identical in the marginal cost of quality improvement, $c = c_A = c_B$. Then we will have a pure equilibrium if σ^2 is big enough and such equilibrium has the unique and symmetric quality choices in both periods.

Proposition 1 *There exists $\widehat{\sigma}^2$ such that there is a pure equilibrium for any $\sigma^2 > \widehat{\sigma}^2$. When there exists a pure equilibrium, it is unique and symmetric pure equilibrium with $\mathbf{q}_{i2}^r = \frac{\alpha_2^r - \beta_2^r}{c} \cdot f(\Delta)$ and $\mathbf{q}_{i1} = \frac{2}{c} \cdot f(0) \cdot \left[\alpha_1 + \beta_2^h - 1 + \left(\alpha_2^h - \beta_2^h\right) \cdot F\left(\Delta\right)\right]$ for any i and r.*

Proof. The best response function at period 1, $q_{i1}\left(q_{j1}\right)$, is strictly decreasing in q_{j1} when $q_{i1} < q_{j1}$, strictly increasing when $q_{i1} > q_{j1}$ and $\frac{\partial q_{i1}(q_{j1})}{\partial q_{j1}} = 0$ when $q_{i1} = q_{j1}$. The best response functions cross each other only once at 45 degree line, $\mathbf{q}_{i1} = \mathbf{q}_{j1} = \frac{1}{c} \cdot f(0) \cdot \left[2\alpha_1 - 1 + \mathbf{\Pi}_{i2}^h - \mathbf{\Pi}_{i2}^l\right]$. Let $\mathbf{\Pi}_{i1}\left(\widehat{q}_{i1}, \widehat{q}_{j1}\right)$ denote the expected profit evaluated at $(q_{i1}, q_{j1}) = (\widehat{q}_{i1}, \widehat{q}_{j1})$ and $\overline{q}_{i1}$ denote q_{i1} which satisfies $\Pi_{i1}\left(\overline{q}_{i1}, q_{j1}\right) = 0$ given q_{j1}. Note that, as σ^2 gets smaller, $f(0)$ gets bigger, so does $\mathbf{q}_{i1}$ and $C(\mathbf{q}_{i1})$ The revenue R_{i1} is, however, weakly less than α_1 even when $q_{i1} = \infty$. Thus, there exists big enough $\widehat{\sigma}_1^2$ such that $\mathbf{q}_{i1} < \overline{q}_{i1}$ given q_{j1} for any $\sigma^2 > \widehat{\sigma}_1^2$.

At period 2, the best response of firm i is $q_{i2}^r(q_{j2}^{r'}) = \frac{\alpha_2^r - \beta_2^r}{c} \cdot f(q_{i2}^r - q_{j2}^{r'} + \Delta \cdot (I\left(r\right) - I(r')))$. For each r, the best response functions of both firms are increasing until they cross $q_{i2}^r = q_{j2}^{r'} - \Delta \cdot \left(I\left(r\right) - I(r')\right)$ line. Note that $q_{i2}^r(0) > 0$ and $q_{i2}^h < q_{j2}^l$ when it passes $q_{i2}^h = q_{j2}^l - \Delta$ line. We can simply see a symmetric equilibrium $\mathbf{q}_{i2}^h = \mathbf{q}_{j2}^l = \frac{\alpha_2^r - \beta_2^r}{c} f(\Delta)$ which implies the best responses cross each other at least on $q_{i2}^h = q_{j2}^l$ line. Depending on its position r, a firm's best response crosses $q_{j2}^l = q_{i2}^h$ line asymmetrically. The early winner's best response, $q_{i2}^h(q_{j2}^l)$, crosses $q_{j2}^l = q_{i2}^h$ line while it is increasing and then reaches $q_{i2}^h = q_{j2}^l - \Delta$ line. The early loser's, however, $q_{j2}^l(q_{i2}^h)$, reaches $q_{i2}^h = q_{j2}^l - \Delta$ line first and then crosses $q_{j2}^l = q_{i2}^h$ line while it is decreasing. Again, we can find $\widehat{\sigma}_2^2$ which allows interior solution $\mathbf{q}_{i2}^r < \overline{q}_{i2}^r$. Therefore, there exist a pure equilibrium big for any $\sigma^2 > \widehat{\sigma}^2 = \max\left[\widehat{\sigma}_1^2, \widehat{\sigma}_2^2\right]$. Note that $q_{j2}^l(q_{i2}^h)$ cannot pass the point $\mathbf{q}_{i2}^h = \mathbf{q}_{j2}^l$ once it crosses $q_{i2}^h(q_{j2}^l)$ before it reaches $q_{j2}^l = q_{i2}^h$ line. Thus, when the eqilibrium exists, it is unique and symmetric. When we plug $\mathbf{q}_{i2}^h = \mathbf{q}_{j2}^l = \frac{\alpha_2^r - \beta_2^r}{c} f(\Delta)$ into $\mathbf{\Pi}_{i2}^h$ and $\mathbf{\Pi}_{i2}^l$, we can get $\mathbf{q}_{i1} = \frac{2}{c} \cdot f(0) \cdot \left[\alpha_1 + \beta_2^h - 1 + \left(\alpha_2^h - \beta_2^h\right) \cdot F\left(\Delta\right)\right]$. ■

The condition that σ^2 is large enough is necessary for the existence of pure equilibria. The reason is similar to why there is no pure equilibrium in all-pay auctions when valuations are symmetric (See Baye et al. 1996). When the uncertainty is small so that a marginal quality improvement is very decisive in determining the winner, there is no pure equilibrium. Since every condition is symmetric between the two firms in period 1, the symmetry of the equilibrium in period 1 is expected. However, the symmetry in period 2 is surprising. This result shows that quality does not depend on the firm's position when the probability distribution is symmetric around zero and firms are homogenous. To see why, it is important to remember that the equilibrium quality depends on the *marginal* probability of winning. Also note that the score advantage Δ is merely shifting the win probability of both firms in opposite directions depending on their positions. So, it makes the probability of winning for the early winner and the early loser different, but the marginal probabilities of winning remain equal if $f(\cdot)$ is symmetric. The probability of winning for the early winner is $F(q_{i2}^h - q_{j2}^l + \Delta)$ which is greater than that of the early loser $F(q_{j2}^l - q_{i2}^h - \Delta)$. But, both firms' marginal probability of winning are equal regardless of their positions $f(q_{i2}^h - q_{j2}^l + \Delta) = f(q_{j2}^l - q_{i2}^h - \Delta)$ by the symmetry of f.

From the above result, we can observe that Δ has a negative effect on the second period equilibrium quality, $\frac{\partial \mathbf{q}_{i2}^r}{\partial \Delta} < 0$, but has a positive effect on the first period equilibrium quality, $\frac{\partial \mathbf{q}_{i1}}{\partial \Delta} > 0$. Through the score advantage, the contest at period 1 is not only for market revenue at period 1 but also for that at period 2. If a firm wins the better reputation at period 1, she takes not only a bigger market share at period 1 but also an advantage in the winning probability at period 2. Therefore, both firms have a stronger incentive to be the early winner as the score advantage increases. In the second period, however, the contest result is predetermined by the extent of the advantage, so competition is dampened. Therefore, as the score advantage increases, both firms have more incentive to choose high quality in period 1 but less incentive in period 2.

Turning to the reviewer, the tension between the early period incentive gain and later loss provides us with an interesting result. For given α, the reviewer wishes to maximize the expected overall quality level $Q(\Delta; \alpha)$ by choosing the score advantage Δ. Since $\Pr(y_{i1} > y_{j1}) = F(q_{i1} - q_{j1})$ and $\Pr(y_{i2}^h + \Delta > y_{j2}^l) = F(q_{i2}^h - q_{j2}^l + \Delta)$,

$$Q(\Delta; \alpha) = \textstyle\sum_{i \in \{A,B\}} F(q_{i1} - q_{j1})[\alpha_1 q_{i1} + \beta_1 q_{j1}$$
$$+ F(q_{i2}^h - q_{j2}^l + \Delta) \cdot (\alpha_2^h q_{i2}^h + \beta_2^l q_{j2}^l) + F(-\left(q_{i2}^h - q_{j2}^l + \Delta\right)) \cdot (\beta_2^h q_{i2}^h + \alpha_2^l q_{j2}^l)]$$

By the fact that equilibrium quality is symmetric, we have $F(\mathbf{q}_{A1} - \mathbf{q}_{B1}) = \frac{1}{2}$ and $F(\mathbf{q}_{A2}^h - \mathbf{q}_{B2}^l + \Delta) = F(\Delta)$. So, we have

$$Q(\Delta; \alpha) = \mathbf{q}_{i1} + \mathbf{q}_{i2}^h = \tfrac{2}{c} f(0) \left[\alpha_1 + \beta_2^h - 1 + \left(\alpha_2^h - \beta_2^h\right) F(\Delta)\right] + \tfrac{\alpha_2^h - \beta_2^h}{c} f(\Delta).$$

One quick observation is that the score advantage Δ is beneficial to the expected overall quality level, in the sense that some score advantage is better than no score advantage. In other words, $Q(\Delta;\alpha) \geq Q(0;\alpha)$ for any Δ, α, strictly for $\Delta > 0$.[10] this is because the first order effect of Δ on the incentive gain at period 1 dominates the second order effect on the incentive loss at period 2 but this only suggests $Q(\Delta;\alpha) > Q(0;\alpha)$ for some small $\Delta > 0$. This does not mean, however, expected overall quality is monotonically increasing in the score advantage. When Δ is too big, the early loser has little hope to win at period 2 and reduces his quality. On the other hand, since the early winner is very likely to win again and the marginal return of quality improvement is small, it enjoys its high reputation and chooses low quality.

Proposition 2 *When $c_A = c_B$, there exists a unique optimal score advantage $\Delta^* = 2f(0)\sigma^2$, which is increasing in σ^2.*

Proof. $\frac{\partial Q(\Delta;\alpha)}{\partial \Delta} = \frac{2}{c} f(0) \left[f(\Delta) \left(\alpha_2^h - \beta_2^h \right) \right] + \frac{1}{c} \left(\alpha_2^h - \beta_2^h \right) f'(\Delta) \geq 0 \Leftrightarrow 2f(0)f(\Delta) - \frac{\Delta}{\sigma^2} f(\Delta) \geq 0$ since $f'(\Delta) \equiv \frac{\partial f(\Delta)}{\partial \Delta} = -\frac{\Delta}{\sigma^2} f(\Delta)$ when f is the density for $N(0,\sigma^2)$. Therefore, when $\Delta = 2f(0)\sigma^2$, $Q(\Delta;\alpha)$ is maximized for any α. ■

This proposition says that there exists an optimal degree of reputation score accumulation which is independent of α. As the advantage Δ gets bigger, the marginal incentive gain for period 1 gets smaller but the marginal incentive loss in period 2 gets bigger. Expected overall quality is maximized when the score advantage balances the marginal incentive gain in period 1 and the marginal incentive loss in period 2. It is noteworthy that the optimal score advantage is increasing in the variance of the shock in observation σ^2. The intuition is that bigger σ^2 reduces the marginal probability of winning from quality improvement and thus discourages firms from improving their quality. The accumulation of reputation score boosts the marginal probability of winning from improving quality at period 1, though it reduces the marginal probability of winning in period 2. Remember that the first order effect of Δ on the incentive gain in period 1 dominates the second order effect on the incentive loss in period 2. Thus, the score advantage reduces the negative incentive effect from σ^2.

The incumbency advantage in market share β_2^h has very similar effects on quality choice as the score advantage. Like Δ, β_2^h also plays a role as a link between the present and the future incentives. That is, bigger β_2^h boosts the first period incentive for high quality but

[10] The proof is simple. $Q(\Delta;\alpha) \geq Q(0;\alpha) \Leftrightarrow \frac{2}{c} f(0) \left[\alpha_1 + \beta_2^h - 1 + \left(\alpha_2^h - \beta_2^h \right) F(\Delta) \right] + \frac{\alpha_2^h - \beta_2^h}{c} f(\Delta) > \frac{2}{c} \left[\alpha_1 + \alpha_2^h - 1 \right] f(0) \Leftrightarrow \frac{f(\Delta)}{1-F(\Delta)} > 2f(0)$ for any $\Delta, \beta_2^h \geq 0$. The equality holds only when $\Delta = 0$. The same result holds when $c_A \neq c_B$. For the proof, see the Appendix.

dampens the second period incentive, $\frac{\partial \mathbf{q}_{i1}}{\partial \beta_2^h} > 0$ and $\frac{\partial \mathbf{q}_{i2}^r}{\partial \beta_2^h} < 0$. In words, when more consumers face high switching cost, firms have more incentive to win in period 1 by increasing their quality but less incentive once acquiring the advantage at period 2. The following proposition shows, however, that the incumbency advantage β_2^h affects the expected overall quality level differently from score advantage Δ.

Proposition 3 *For any given* Δ, $\frac{\partial Q}{\partial \beta_2^h} \leq 0$ *with equality only when* $\Delta = 0$.

Proof. $\frac{\partial Q(\Delta;\alpha)}{\partial \beta_2^h} = \frac{2}{c} f(0) \times (1 - F(\Delta)) - \frac{1}{c} F(\Delta) \leq 0 \Leftrightarrow 2(1 - F(\Delta)) f(0) - F(\Delta) \leq 0$ $\Leftrightarrow \frac{F(\Delta)}{1-F(\Delta)} \geq 2f(0)$ for any Δ since the left hand side is increasing in Δ. The equality holds only when $\Delta = 0$. ■

When the early winner has a score advantage (i.e., when $\Delta > 0$), the expected overall quality level is always reduced by the incumbency advantage β_2^h. Unlike the score advantage, the incentive loss is first order but the gain is second order. Therefore, the gain is always dominated by the loss. Recall that $\beta_2^h = 0$ implies that the contest is *reset* each period. At the opposite extreme, $\beta_2^h = 1$ describes a one-shot contest and generates the smallest expected overall quality level. In a market where more consumers face high switching costs, given the same reputation score accumulation rate, the expected overall quality in the market is lower than that in the market with fewer consumers facing high switching costs. This is similar to the switching cost literature, where the switching costs reduce competition (see Klemperer 1987a).

3.2 Optimal Score Advantage and Incumbency Advantage

Recall that Δ^* is independent of α when firms are homogeneous. This is because the marginal probability of winning $f\left(\mathbf{q}_{it}^r - \mathbf{q}_{jt}^{r'} - \Delta \cdot (I(r) - I(r'))\right)$ depends on Δ but not on $\alpha_t^r - \beta_t^r$ since $\mathbf{q}_{it}^r = \mathbf{q}_{jt}^{r'}$ in the equilibrium. Since it appears on both sides of the marginal incentive gain and loss at the optimum of Δ, $\alpha_t^r - \beta_t^r$ term is canceled out. To investigate the relationship of the incumbency advantage with the score advantage, assume that firm A is more efficient than firm B in the sense that $c_A < c_B$. It is easy to show that the existence and uniqueness of equilibrium quality carries to the heterogeneous case. From the period 2 first order conditions, we can immediately observe that $\mathbf{q}_{B2}^{r'} = \frac{c_A}{c_B} \mathbf{q}_{A2}^r$ for each r where $\mathbf{q}_{A2}^r = \frac{\alpha_2^r - \beta_2^r}{c} \cdot f(\mathbf{q}_{A2}^r - \mathbf{q}_{B2}^{r'} + \Delta \cdot (I(r) - I(r')))$. So, we can describe the difference in equilibrium quality levels between firms at period 2, $\mathbf{q}_{A2}^h - \mathbf{q}_{B2}^l$ and $\mathbf{q}_{B2}^h - \mathbf{q}_{A2}^l$, in terms of firm A's equilibrium quality level, $\lambda \mathbf{q}_{A2}^h$ and $-\lambda \mathbf{q}_{A2}^l$ respectively, where $\lambda = 1 - \frac{c_A}{c_B} \in [0, 1)$ since $\frac{c_A}{c_B} \in (0, 1]$. This λ measures *the difference in efficiency* between the two firms. Using this, we can construct both firms' period 1 best response function, $\mathbf{q}_{i1} = \frac{1}{ci} f(\mathbf{q}_{i1} - q_{j1}) \left[2\alpha_1 - 1 + \mathbf{\Pi}_i^h - \mathbf{\Pi}_i^l\right]$ as in the homogenous case.

Proposition 4 *(i) For any t, r, $\mathbf{q}_{At}^r > \mathbf{q}_{Bt}^{r'}$*
(ii) $\mathbf{q}_{A2}^l > \mathbf{q}_{A2}^h$, $\mathbf{q}_{B2}^h > \mathbf{q}_{B2}^l$, and $\mathbf{q}_{A2}^l - \mathbf{q}_{A2}^h > \mathbf{q}_{B2}^h - \mathbf{q}_{B2}^l$.

Proof. See the Appendix. ■

The first part of the proposition is straightforward. The more efficient firm A chooses higher quality than the less efficient firm B regardless of their positions. For a given score advantage, the marginal probabilities of winning (therefore, marginal revenues) are equal across firms but the more efficient firm has smaller marginal cost, so it has more incentive to choose higher quality in the equilibrium than the less efficient firm.

From the second part, we can see how the score advantage affects the incentive of the early winner and the early loser differently depending on each firm's relative efficiency. The more efficient firm chooses higher quality when it is the early loser than when it is the early winner: $\mathbf{q}_{A2}^l > \mathbf{q}_{A2}^h$. To the contrary, the less efficient firm chooses higher quality when its is the early winner than when it is the early loser: $\mathbf{q}_{B2}^h > \mathbf{q}_{B2}^l$.

Recall that the expected score difference is determined by the quality difference between firms and the score (dis)advantage. From the first part of the Proposition 4, since the more efficient firm A always provides higher quality than firm B regardless of their positions, the investment difference in A's view is always positive, $\mathbf{q}_{At}^r - \mathbf{q}_{Bt}^{r'} > 0$ for any r. So, roughly speaking, the reputation score difference is larger in expectation when firm A has the score advantage. That is, $\mathbf{q}_{A2}^h - \mathbf{q}_{B2}^l + \Delta > \mathbf{q}_{A2}^l - \mathbf{q}_{B2}^h - \Delta$, which means $\mathbf{q}_{A2}^l - \mathbf{q}_{B2}^h - \Delta$ is closer to zero than $\mathbf{q}_{A2}^h - \mathbf{q}_{B2}^l + \Delta$.

The marginal probability of winning is bigger as the gap between two firm's score gets smaller. From the first part of the proposition, firm A always invests more and the score advantage it has makes the gap larger when firm A is the early winner. Therefore, A has a weaker incentive to choose higher quality when it is the early loser than when it is the early winner. Firm B's situation is exactly opposite. Since the quality difference in firm B's view is always negative, when it has the score advantage, the expected quality difference is closer to zero. Therefore, its marginal quality improvement has more effect when it is the early winner than when it is the early loser. As the reputation score gap between the two firms gets smaller, the marginal probability of winning from a quality improvement gets greater so the competition is more fierce. This result is consistent with a well-known result in R&D race literature, namely that competition is generally more fierce as the positional gap between two firms is narrower (see Aoki (1991)). From (ii), we can also see that the more efficient firm's quality choice is more sensitive to the change of reputation position.

With these equilibrium quality choices, we can characterize the optimal score advantage. First, note that the expected overall quality level in equilibrium is

$$\begin{aligned}Q &= F_1 \cdot [\alpha_1 \mathbf{q}_{A1} + \beta_1 \mathbf{q}_{B1} + F^h \cdot (\alpha_2^h \mathbf{q}_{A2}^h + \beta_2^l \mathbf{q}_{B2}^l) + \left(1 - F^h\right) \cdot (\beta_2^h \mathbf{q}_{A2}^h + \alpha_2^l \mathbf{q}_{B2}^l)] \\ &+ (1 - F_1) \cdot [\alpha_1 \mathbf{q}_{B1} + \beta_1 \mathbf{q}_{A1} + \left(1 - F^l\right) \cdot (\alpha_2^h \mathbf{q}_{B2}^h + \beta_2^l \mathbf{q}_{A2}^l) + F^l \cdot (\beta_2^h \mathbf{q}_{B2}^h + \alpha_2^l \mathbf{q}_{A2}^l)]\end{aligned}$$

where $F_1 = F(\mathbf{q}_{A1} - \mathbf{q}_{B1})$, $F^h \equiv F(\lambda \mathbf{q}_{A2}^h + \Delta)$ and $F^l \equiv F(\lambda \mathbf{q}_{A2}^l - \Delta)$. Recall that $\frac{c_A}{c_B} = 1 - \lambda$ when $\lambda = 1 - \frac{c_A}{c_B}$. Therefore, $\mathbf{q}_{B2}^h = (1 - \lambda)\, \mathbf{q}_{A2}^l$ and $\mathbf{q}_{B2}^l = (1 - \lambda)\, \mathbf{q}_{A2}^h$. We can rewrite $\mathbf{q}_{B2}^{r'}$ in terms of $\mathbf{q}_{A2}^r$, so

$$\begin{aligned}Q &= F_1 \cdot \left[\alpha_1 \mathbf{q}_{A1} + \beta_1 \mathbf{q}_{B1} + F^h \cdot \left(1 + \alpha_2^h \lambda - \lambda\right) \mathbf{q}_{A2}^h + \left(1 - F^h\right) \left(1 + \beta_2^h \lambda - \lambda\right) \mathbf{q}_{A2}^h\right] \\ &+ (1 - F_1) \cdot \left[\beta_1 \mathbf{q}_{A1} + \alpha_1 \mathbf{q}_{B1} + F^l \cdot \left(1 + \alpha_2^l \lambda - \lambda\right) \mathbf{q}_{A2}^l + \left(1 - F^l\right) \left(1 + \beta_2^l \lambda - \lambda\right) \mathbf{q}_{A2}^l\right]\end{aligned}$$

We can use the first-order condition for Δ to show the effect of the incumbency advantage β_2^h on the optimal score advantage Δ^*.

Proposition 5 *For large enough σ^2, there exists a unique Δ^*. And $\frac{\partial \Delta^*}{\partial \beta_2^h} < 0$.*

Proof. See the Appendix. ■

Similarly to the homogeneous case, it is easy to check that the score advantage is still increasing in σ^2 to reduce the incentive for firms to rely on luck more than quality improvement. In addition, we have a new observation in the heterogenous firm case. First, the optimal score advantage Δ is decreasing in β_2^h. That is, in a market where more consumers face high switching costs, optimal score accumulation is lower. As we saw in the previous section, Δ and β_2^h affect the marginal incentive gain and loss in the same directions ($\frac{\partial \mathbf{q}_{i1}}{\partial \Delta}, \frac{\partial \mathbf{q}_{i1}}{\partial \beta_2^h} > 0$ and $\frac{\partial \mathbf{q}_{i2}^r}{\partial \Delta}, \frac{\partial \mathbf{q}_{i2}^r}{\partial \beta_2^h} < 0$). In this sense, Δ acts as a substitute for β_2^h in producing expected overall quality. This result provides an interesting trade-off between an advantage in the probability of winning (Δ) and an advantage in market share (β_2^h). Intuitively, 'success breeds success' is more easily justified as the underlying payoff structure gets closer to symmetry between the early winner and the early loser. In sum, how much accumulation a quality report should have depends on the uncertainty in quality evaluation or production as well as the inequality in underlying market shares between the incumbent and the challenger.

3.3 Extension: the persistence of quality

So far, I have assumed that quality investment is not accumulative. As an extension, I consider a case where the first period investment affects the second period quality as well as the first period quality. For example, when a hospital purchases new MRI machines, the investment increases service quality both in current year and in the future. Assume now that the actual quality of firm i at period 2 is determined by its second-period quality investment and its first-period investment, $\widetilde{q}_{i2}^{r} = q_{i2}^{r} + \delta q_{i1}$ where δ captures the degree of persistence. Then, the probability of winning by firm i with position r at period 2 will be $F_{it}^{r} = F(\widetilde{q}_{it}^{r} - \widetilde{q}_{it}^{r'} + \Delta \cdot (I(r) - I(r')))$ instead of $F(q_{it}^{r} - q_{it}^{r'} + \Delta \cdot (I(r) - I(r')))$. That is, $\Pi_{it}^{r} = F_{it}^{r}\alpha_t^r + (1 - F_{it}^{r})\,\beta_t^r - c_i \frac{\left(q_{it}^{r}\right)^2}{2}$. It is straightforward to characterize firm choices as

$$\mathbf{q}_{i1} = \tfrac{1}{c_i} f_{i1} \cdot \left[2\alpha_1 - 1 + \mathbf{\Pi}_i^h - \mathbf{\Pi}_i^l\right] + \delta \tfrac{\alpha_2^h - \beta_2^h}{c}\left[F_{i1} \cdot f_{i2}^h + (1 - F_{i1}) \cdot f_{i2}^l\right] \text{ and } \mathbf{q}_{i2}^r = \tfrac{\alpha_2^r - \beta_2^r}{c} f_{i2}^r.$$

There is also a minor change in the reviewer's problem:

$$\begin{aligned} \max\nolimits_{\Delta} \widetilde{Q} = \textstyle\sum_{i \in \{A,B\}} \Pr(y_{i1} > y_{j1})[\alpha_1 q_{i1} + \beta_1 q_{j1} \\ + \Pr(\widetilde{y}_{i2}^h + \Delta > \widetilde{y}_{j2}^l) \cdot \{\alpha_2^h \widetilde{q}_{i2}^h + \beta_2^l \widetilde{q}_{j2}^l\} + \Pr(\widetilde{y}_{i2}^h + \Delta < \widetilde{y}_{j2}^l) \cdot \{\beta_2^h \widetilde{q}_{i2}^h + \alpha_2^l \widetilde{q}_{j2}^l\}] \end{aligned}$$

where $\widetilde{y}_{i2}^r = \widetilde{q}_{i2}^r + \epsilon_{i2}$ for each r. A quick observation is that $\frac{\partial \mathbf{q}_{i1}}{\partial \delta} > 0$. As the quality gets more persistent, a bigger investment in period 1 is made since it can create an advantage in subsequent periods. To focus on the effect of δ on Δ^*, suppose firms are homogeneous. Then,

$$\mathbf{q}_{i1} = \tfrac{2}{c} f(0)\left[\alpha_1 + \beta_2^h - 1 + \left(\alpha_2^h - \beta_2^h\right) F(\Delta)\right] + \delta \tfrac{\alpha_2^h - \beta_2^h}{c} f(\Delta) \text{ and } \mathbf{q}_{i2}^r = \tfrac{\alpha_2^r - \beta_2^r}{c} f(\Delta).$$

Now the expected overall quality $\widetilde{Q} = \mathbf{q}_{i1} + \widetilde{\mathbf{q}}_{i2}^r = (1+\delta)\,\mathbf{q}_{i1} + \mathbf{q}_{i2}^r$ since $F(\mathbf{q}_{i1} - \mathbf{q}_{j1})\left(f^h + f^l\right) = f(\Delta)$. From the first order condition for Δ, $\frac{\partial \widetilde{Q}}{\partial \Delta} = (1+\delta)\,\frac{2}{c} f(0)\left(\alpha_2^h - \beta_2^h\right) f(\Delta) - [(1+\delta)\,\delta + 1]\,\frac{\Delta}{\sigma^2} \frac{\alpha_2^h - \beta_2^h}{c} f(\Delta) = 0$,

$$\Delta^* = \tfrac{2(1+\delta)}{1+\delta+\delta^2} f(0)\sigma^2.$$

Therefore, it is still true that the optimal score advantage is increasing in the uncertainty in evaluation or production, $\frac{\partial \Delta^*}{\partial \sigma^2} > 0$. Now we have another new observation: the optimal score advantage is decreasing in the degree of quality persistence.

Proposition 6 $\frac{\partial \Delta^*}{\partial \delta} < 0$

Proof. $\frac{\partial\left(\frac{2(1+\delta)}{1+\delta+\delta^2}\right)}{\partial\delta} = \frac{2}{\left(1+\delta+\delta^2\right)^2} - \frac{2(1+\delta)}{\left(1+\delta+\delta^2\right)^2}(1+2\delta) = -\frac{2\delta(2+\delta)}{\left(1+\delta+\delta^2\right)^2} < 0.$ ■

In a market where learning-by-doing is significant or investment is durable, there should be little inertia in rating. The role of δ is similar to that of Δ in that both create inertia in the probability of winning.[11]. δ decreases the marginal incentive gain of Δ and increases the marginal incentive loss of Δ. At the optimum, therefore, Δ^* is smaller as δ gets larger.

4 Discussion

If we think of the QRS as a means of *designing* a reputation for each firm, the inertia I focus on can be thought of as modeling a typical aspect of reputation: a reputation is not easily shattered but is usually built up. In reputation building, we should distinguish accumulation from persistence. Accumulation is about the degree of transfer from the past to the present, but persistence is about the duration of the effect of this transfer. As the first step toward reputation design, this paper studies the accumulation part since accumulation is necessary for something to be built-up, but persistence is not.

When the reviewer is a benevolent social planner, its objective arguably should include firm profits as well as the overall quality level. However, then the reviewer's objective is still a simple quadratic combination qualities since firms' profits are also a function of quality. Therefore, the socially optimal inertia level in this case simply depends on the social weight on firms' payoff. That is, Δ^* is still interior when the weight on firms' profits is low enough. We have used a specific production function, $y_{it} = h(q_{it}) + \epsilon_{it}$ where $h(q_{it}) = q_{it}$. It is easy to show that as long as $h' > 0 \geq h''$, the above results hold.[12] Finally, I focus on linear accumulation scoring rules. One may consider other types of accumulation, but linear forms are the simplest case and hence a natural starting point.

5 Concluding Remarks

In this paper, I have analyzed the optimal accumulation of reputation score as a means of giving incentives to opportunistic firms to provide high quality in a two-period repeated contest model. I characterize the unique equilibrium in firms' quality choices over time and across positions given the advantage in the probability of winning. One role of the inertia

[11] With the assumption $\widetilde{q}^r_{i2} = q^r_{i2} + \delta q_{i1}$, $\frac{\partial \mathbf{q}^r_{i2}}{\partial\delta} = 0$. However, when the second period quality is the weighted average of the second and the first period quality, $\widetilde{q}^r_{i2} = (1-\delta)\, q^r_{i2} + \delta q_{i1}$, we can observe $\frac{\partial \mathbf{q}^r_{i2}}{\partial\delta} < 0$.

[12] More generally, the result holds as long as $\frac{c'(q_{it})}{h'(q_{it})}$ is strictly increasing in q_{it}. Obviously, when $\frac{c'(q_{it})}{h'(q_{it})}$ is strictly decreasing in q_{it}, the result will be the opposite; Δ^* should decrease in σ^2.

in ratings is to cancel out the negative effect of the uncertainty. Thus, the quality reporting system should allow for a greater degree of reputation score accumulation, the greater is the degree of uncertainty in evaluation or production. Moreover, I consider two types of inertia from underlying market characteristics. One type is from the demand side, the incumbency advantage due to the switching costs of consumers. The other is from the supply side, the quality persistence. When there exists another type of inertia in the market, less inertia in ratings is required. That is, a smaller score advantage is optimal as the incumbency advantage or the degree of quality persistence increases.

My model is a step toward building a general theory of designing optimal quality reporting systems as an incentive mechanism, but more work is needed. First, the model assumes that the price is exogenous and equal across firms. When price is allowed to be a control variable of the firms, we may get a better understanding of the relationship between reputation and price premia.[13] Second, richer models of consumer learning are worth exploring since reputation is an information system as well as an incentive system. Third, extending to three or more periods may provide a better understanding of the optimal persistence of the score advantage.

[13] In a slightly different setting, I discuss how inertia in evaluation affects firms' incentives to choose their price and quality level. For details, see Suh (2008).

Appendix

Proof of Proposition 4

At period 2, we have $\mathbf{q}_{B2}^{l}=\frac{c_A}{c_B}\mathbf{q}_{A2}^{h}$ and $\mathbf{q}_{B2}^{h}=\frac{c_A}{c_B}\mathbf{q}_{A2}^{l}$ by the symmetry of f. Since $\frac{c_A}{c_B}<1$, we have $\mathbf{q}_{B2}^{r'}<\mathbf{q}_{A2}^{r}$ for any r. Then, for each r, $\mathbf{q}_{A2}^{r}-\mathbf{q}_{B2}^{r'}=\mathbf{q}_{A2}^{r}-(1-\lambda)\,\mathbf{q}_{A2}^{r}=\lambda\mathbf{q}_{A2}^{r}$ where $\lambda=1-\frac{c_A}{c_B}$. Let $F_1=F\left(\mathbf{q}_{A1}-\mathbf{q}_{B1}\right)$ and $f_1=f\left(\mathbf{q}_{A1}-\mathbf{q}_{B1}\right)$,

$F^{r}=F\left(\mathbf{q}_{A2}^{r}-\mathbf{q}_{B2}^{r'}+\Delta\cdot\left(I\left(r\right)-I\left(r'\right)\right)\right)=F\left(\lambda\mathbf{q}_{A2}^{r}+\Delta\cdot\left(I\left(r\right)-I\left(r'\right)\right)\right)$

and $f^{r}=f\left(\mathbf{q}_{A2}^{r}-\mathbf{q}_{B2}^{r'}+\Delta\cdot\left(I\left(r\right)-I\left(r'\right)\right)\right)$ since $f\left(\lambda\mathbf{q}_{A2}^{l}-\Delta\right)=f\left(-\lambda\mathbf{q}_{A2}^{h}+\Delta\right)$. We can simplify each firm's expected equilibrium profit at period 2 in terms of $\mathbf{q}_{A2}^{r}$,

$\mathbf{\Pi}_{A}^{h}-\mathbf{\Pi}_{A}^{l}=\beta_2^h-\beta_2^l+\left(\alpha_2^h-\beta_2^h\right)\left(F^h-F^l\right)+\frac{c_A}{2}\left(\left(\mathbf{q}_{A2}^{l}\right)^2-\left(\mathbf{q}_{A2}^{h}\right)^2\right)$ and

$\mathbf{\Pi}_{B}^{h}-\mathbf{\Pi}_{B}^{l}=\beta_2^h-\beta_2^l+\left(\alpha_2^l-\beta_2^l\right)\left(F^h-F^l\right)-(1-\lambda)\frac{c_A}{2}\left(\left(\mathbf{q}_{A2}^{l}\right)^2-\left(\mathbf{q}_{A2}^{h}\right)^2\right)$ since

$\mathbf{\Pi}_{A}^{h}=F^h\cdot\alpha_2^h+\left(1-F^h\right)\cdot\beta_2^h-c_A\frac{\left(\mathbf{q}_{A}^{h}\right)^2}{2}$, $\mathbf{\Pi}_{A}^{l}=F^l\cdot\alpha_2^l+\left(1-F^l\right)\cdot\beta_2^l-c_A\frac{\left(\mathbf{q}_{A2}^{l}\right)^2}{2}$,

$\mathbf{\Pi}_{B}^{h}=\left(1-F^l\right)\cdot\alpha_2^h+F^l\cdot\beta_2^h-c_B\frac{\left(\mathbf{q}_{B2}^{h}\right)^2}{2}$, $\mathbf{\Pi}_{B}^{l}=\left(1-F^h\right)\cdot\alpha_2^l+F^h\cdot\beta_2^l-c_B\frac{\left(\mathbf{q}_{B2}^{l}\right)^2}{2}$, $\left(\mathbf{q}_{B2}^{r'}\right)^2=\left(\frac{c_A}{c_B}\mathbf{q}_{A2}^{r}\right)^2$, and $\frac{c_B}{2}(1-\lambda)^2=\frac{c_B}{2}\left(\frac{c_A}{c_B}\right)^2=(1-\lambda)\frac{c_A}{2}$.

Therefore,

$\mathbf{q}_{A1}=\frac{1}{c_A}f_1\left[2\alpha_1-1+\beta_2^h-\beta_2^l+\left(\alpha_2^h-\beta_2^h\right)\left(F^h-F^l\right)+\frac{c_A}{2}\left(\left(\mathbf{q}_{A2}^{l}\right)^2-\left(\mathbf{q}_{A2}^{h}\right)^2\right)\right]$ and

$\mathbf{q}_{B1}=\frac{1}{c_B}f_1\left[2\alpha_1-1+\beta_2^h-\beta_2^l+\left(\alpha_2^h-\beta_2^h\right)\left(F^h-F^l\right)-(1-\lambda)\frac{c_A}{2}\left(\left(\mathbf{q}_{A2}^{l}\right)^2-\left(\mathbf{q}_{A2}^{h}\right)^2\right)\right]$.

Since $\left(\mathbf{q}_{A2}^{l}\right)^2-\left(\mathbf{q}_{A2}^{h}\right)^2>0$ and $\frac{1}{c_B}<\frac{1}{c_A}$, $\mathbf{q}_{A1}>\mathbf{q}_{B1}$. Therefore, $\mathbf{q}_{At}^{r}>\mathbf{q}_{Bt}^{r'}$ for any t, r.

To see the second part of proposition, rearrange the first order conditions at period 2 as

$$\frac{\mathbf{q}_{A2}^{h}}{f(\lambda\mathbf{q}_{A2}^{h}+\Delta)}=\frac{\mathbf{q}_{A2}^{l}}{f(-\lambda\mathbf{q}_{A2}^{l}+\Delta)}=\frac{\alpha_2^h-\beta_2^h}{c_A} \tag{1}$$

Suppose $\mathbf{q}_{A2}^{h}>\mathbf{q}_{A2}^{l}$. Then, to satisfy (1) for given $\frac{\alpha_2^h-\beta_2^h}{c_A}$, $f(\lambda\mathbf{q}_{A2}^{h}+\Delta)>f(-\lambda\mathbf{q}_{A2}^{l}+\Delta)\Leftrightarrow\lambda\mathbf{q}_{A2}^{h}+\Delta<\left|-\lambda\mathbf{q}_{A2}^{l}+\Delta\right|$. When $-\lambda\mathbf{q}_{A2}^{l}+\Delta>0$, $\lambda\mathbf{q}_{A2}^{h}+\Delta<\left|-\lambda\mathbf{q}_{A2}^{l}+\Delta\right|$ becomes $\lambda\mathbf{q}_{A2}^{h}+\Delta<-\lambda\mathbf{q}_{A2}^{l}+\Delta$ which is impossible for any $\mathbf{q}_{A2}^{h}$, $\mathbf{q}_{A2}^{l}$, $\Delta\geq0$. When $-\lambda\mathbf{q}_{A2}^{l}+\Delta<0$, $\lambda\mathbf{q}_{A2}^{h}+\Delta<\left|-\lambda\mathbf{q}_{A2}^{l}+\Delta\right|$ becomes $\lambda\mathbf{q}_{A2}^{h}+\Delta<\lambda\mathbf{q}_{A2}^{l}-\Delta$ which is contradicted by $\mathbf{q}_{A2}^{h}>\mathbf{q}_{A2}^{l}$ for any $\Delta\geq0$. Therefore, $\mathbf{q}_{A2}^{h}<\mathbf{q}_{A2}^{l}$. Moreover, $\mathbf{q}_{A2}^{h}<\mathbf{q}_{A2}^{l}\Leftrightarrow\frac{c_B}{c_A}\mathbf{q}_{B2}^{l}<\frac{c_B}{c_A}\mathbf{q}_{B2}^{h}\Leftrightarrow\mathbf{q}_{B2}^{l}<\mathbf{q}_{B2}^{h}$.

Next, let's prove $\mathbf{q}_{A2}^{l}-\mathbf{q}_{A2}^{h}>\mathbf{q}_{B2}^{h}-\mathbf{q}_{B2}^{l}$.

$\mathbf{q}_{A2}^{l}-\mathbf{q}_{A2}^{h}=\frac{\alpha_2^h-\beta_2^h}{c_A}\cdot\left(f_A^l-f_A^h\right)$ and $\mathbf{q}_{B2}^{h}-\mathbf{q}_{B2}^{l}=\frac{\alpha_2^h-\beta_2^h}{c_B}\cdot\left(f_B^h-f_B^l\right)$. Note that $f_i^r=f_j^{r'}$. Therefore, $\mathbf{q}_{B2}^{h}-\mathbf{q}_{B2}^{l}=\frac{\alpha_2^h-\beta_2^h}{c_B}\cdot\left(f_B^h-f_B^l\right)=\frac{\alpha_2^h-\beta_2^h}{c_B}\cdot\left(f_A^l-f_A^h\right)=\frac{c_A}{c_B}\left(\mathbf{q}_{A2}^{l}-\mathbf{q}_{A2}^{h}\right)<\mathbf{q}_{A2}^{l}-\mathbf{q}_{A2}^{h}$ since $c_A<c_B$.

Proof of Proposition 5

Recall that $\mathbf{q}_{A2}^{r}=\frac{\alpha_2^r-\beta_2^r}{c_A}f^r$, $\mathbf{q}_{B2}^{r}=\frac{\alpha_2^r-\beta_2^r}{c_B}f^r$,

$\mathbf{q}_{A1}=\frac{1}{c_A}f_1\left[2\alpha_1-1+\beta_2^h-\beta_2^l+\left(\alpha_2^h-\beta_2^h\right)\left(F^h-F^l\right)+\frac{c_A}{2}\left(\left(\mathbf{q}_{A2}^{l}\right)^2-\left(\mathbf{q}_{A2}^{h}\right)^2\right)\right]$, and

$\mathbf{q}_{B1}=\frac{1}{c_B}f_1\left[2\alpha_1-1+\beta_2^h-\beta_2^l+\left(\alpha_2^h-\beta_2^h\right)\left(F^h-F^l\right)-(1-\lambda)\frac{c_A}{2}\left(\left(\mathbf{q}_{A2}^{l}\right)^2-\left(\mathbf{q}_{A2}^{h}\right)^2\right)\right]$.

$\mathbf{q}_{A1}-\mathbf{q}_{B1}=\lambda\mathbf{q}_{A1}+\Lambda$ where $\Lambda\equiv\frac{(2-\lambda)(1-\lambda)}{2}f_1\left(\left(\mathbf{q}_{A2}^{l}\right)^2-\left(\mathbf{q}_{A2}^{h}\right)^2\right)$.

Rewrite the expected overall quality as

$Q=F_1\cdot\left[\alpha_1\mathbf{q}_{A1}+\beta_1\mathbf{q}_{B1}+F^h\cdot\left\{\alpha_2^h\mathbf{q}_{A2}^h+\beta_2^l\mathbf{q}_{B2}^l\right\}+\left(1-F^h\right)\cdot\left\{\beta_2^h\mathbf{q}_{A2}^h+\alpha_2^l\mathbf{q}_{B2}^l\right\}\right]$

$+(1-F_1)\cdot\left[\beta_1\mathbf{q}_{A1}+\alpha_1\mathbf{q}_{B1}+F^l\cdot\left\{\alpha_2^l\mathbf{q}_{A2}^l+\beta_2^h\mathbf{q}_{B2}^h\right\}+\left(1-F^l\right)\cdot\left\{\beta_2^l\mathbf{q}_{A2}^l+\alpha_2^h\mathbf{q}_{B2}^h\right\}\right]$

$=F_1\cdot\left[\alpha_1\mathbf{q}_{A1}+\beta_1\mathbf{q}_{B1}+F^h\cdot\left(1+\alpha_2^h\lambda-\lambda\right)\mathbf{q}_{A2}^h+\left(1-F^h\right)\left(1+\beta_2^h\lambda-\lambda\right)\mathbf{q}_{A2}^h\right]$

$+(1-F_1)\cdot\left[\beta_1\mathbf{q}_{A1}+\alpha_1\mathbf{q}_{B1}+F^l\cdot\left(1+\alpha_2^l\lambda-\lambda\right)\mathbf{q}_{A2}^l+\left(1-F^l\right)\left(1+\beta_2^l\lambda-\lambda\right)\mathbf{q}_{A2}^l\right]$

By using $\alpha_1\mathbf{q}_{A1}+\beta_1\mathbf{q}_{B1}=\left(1-\beta_1\right)\mathbf{q}_{A1}+\beta_1\mathbf{q}_{B1}=\mathbf{q}_{A1}-\beta_1\left(\mathbf{q}_{A1}-\mathbf{q}_{B1}\right)=\left(1-\beta_1\lambda\right)\mathbf{q}_{A1}-\beta_1\Lambda$. Similarly, $\beta_1\mathbf{q}_{A1}+\alpha_1\mathbf{q}_{B1}=\left(1-\alpha_1\lambda\right)\mathbf{q}_{A1}-\alpha_1\Lambda$,

$Q=F_1\cdot\left[\left(1-\beta_1\lambda\right)\mathbf{q}_{A1}-\beta_1\Lambda+\left\{F^h\cdot\left(\alpha_2^h-\beta_2^h\right)\lambda+\left(1+\beta_2^h\lambda-\lambda\right)\right\}\mathbf{q}_{A2}^h\right]$

$+(1-F_1)\cdot\left[\left(1-\alpha_1\lambda\right)\mathbf{q}_{A1}-\alpha_1\Lambda+\left\{F^l\cdot\left(\alpha_2^l-\beta_2^l\right)\lambda+\left(1+\beta_2^l\lambda-\lambda\right)\right\}\mathbf{q}_{A2}^l\right]$

$=\left[1-\alpha_1\lambda+F_1\cdot\left(\alpha_1-\beta_1\right)\right]\mathbf{q}_{A1}+\left[F_1\cdot\left(\alpha_1-\beta_1\right)-\alpha_1\right]\Lambda$

$+F_1\left\{F^h\cdot\left(\alpha_2^h-\beta_2^h\right)\lambda+\left(1+\beta_2^h\lambda-\lambda\right)\right\}\mathbf{q}_{A2}^h+(1-F_1)\left\{F^l\cdot\left(\alpha_2^l-\beta_2^l\right)\lambda+\left(1+\beta_2^l\lambda-\lambda\right)\right\}\mathbf{q}_{A2}^l$

When we plug $\{\mathbf{q}_{At}^r\}_{t,r}$ into it,

$Q=\frac{f_1}{c_A}\left[1-\alpha_1\lambda+F_1\cdot\left(\alpha_1-\beta_1\right)\right]$

$\times[2\alpha_1-1+\beta_2^h-\beta_2^l+\left(\alpha_2^h-\beta_2^h\right)\left(F^h-F^l\right)+\frac{\left(\alpha_2^h-\beta_2^h\right)^2}{2c_A}(\left(f^l\right)^2-\left(f^h\right)^2)]$

$+\left[F_1\cdot\left(\alpha_1-\beta_1\right)-\alpha_1\right]\Lambda+F_1\left\{F^h\cdot\left(\alpha_2^h-\beta_2^h\right)\lambda+\left(1+\beta_2^h\lambda-\lambda\right)\right\}\frac{\alpha_2^h-\beta_2^h}{c_A}f^h$

$+(1-F_1)\left\{F^l\cdot\left(\alpha_2^l-\beta_2^l\right)\lambda+\left(1+\beta_2^l\lambda-\lambda\right)\right\}\frac{\alpha_2^l-\beta_2^l}{c_A}f^l$

Note that $\mathbf{q}_t^r\sim o\left(\frac{1}{\sigma^2}\right)^{\frac{1}{2}}$ since $f_t^r\sim o\left(\frac{1}{\sigma^2}\right)^{\frac{1}{2}}$. Therefore,

$\frac{\partial f_1}{\partial\Delta}=-\frac{\lambda q_1}{\sigma^2}f_1\cdot\frac{\partial q_1}{\partial\Delta}\sim o\left(\frac{1}{\sigma^2}\right)^{\frac{5}{2}}$ and $\frac{\partial f_2^r}{\partial\Delta}=-\frac{1}{\sigma^2}f_2^r\cdot\left[\Delta\cdot\left(I(r)-I(r')\right)+o\left(\frac{1}{\sigma^2}\right)^{\frac{1}{2}}\right]\sim o\left(\frac{1}{\sigma^2}\right)^{\frac{3}{2}}$.

$\frac{\partial F_1}{\partial\Delta}=f_1\cdot\frac{\partial q_1}{\partial\Delta}\lambda\sim o\left(\frac{1}{\sigma^2}\right)^3$ and $\frac{\partial F_2^r}{\partial\Delta}=f_2^r\cdot\left[1+\frac{\partial q_2^r}{\partial\Delta}\lambda\right]=f_2^r+o\left(\frac{1}{\sigma^2}\right)^2\sim o\left(\frac{1}{\sigma^2}\right)^{\frac{1}{2}}$.

When we take a derivative to get the first order condition,

$\frac{\partial Q}{\partial\Delta}=\frac{f_1}{c_A}\left[1-\alpha_1\lambda+F_1\cdot\left(\alpha_1-\beta_1\right)\right]\left(\alpha_2^h-\beta_2^h\right)\left(f^h+f^l\right)$

$+F_1\cdot\left(\alpha_2^h-\beta_2^h\right)\lambda\frac{\alpha_2^h-\beta_2^h}{c_A}\left(f^h\right)^2-(1-F_1)\left(\alpha_2^l-\beta_2^l\right)\lambda\frac{\alpha_2^l-\beta_2^l}{c_A}\left(f^l\right)^2$

$-\frac{\Delta}{\sigma^2}F_1\left\{F^h\cdot\left(\alpha_2^h-\beta_2^h\right)\lambda+\left(1+\beta_2^h\lambda-\lambda\right)\right\}\frac{\alpha_2^h-\beta_2^h}{c_A}f^h$

$-\frac{\Delta}{\sigma^2}\left\{F^l\cdot\left(\alpha_2^l-\beta_2^l\right)\lambda+\left(1+\beta_2^l\lambda-\lambda\right)\right\}\frac{\alpha_2^l-\beta_2^l}{c_A}f^l+M.$

where M is terms with lower order of $o\left(\frac{1}{\sigma^2}\right)$.

After cancelling out $\frac{\alpha_2^h-\beta_2^h}{c_A}$ and collecting terms,

$$\Delta^* = \left(\frac{K_2}{K_1} + M'\right)\sigma^2 \text{ where}$$

$K_1 = F_1\left\{F^h\cdot\left(\alpha_2^h-\beta_2^h\right)\lambda+\left(1+\beta_2^h\lambda-\lambda\right)\right\}f^h+(1-F_1)\left\{F^l\cdot\left(\alpha_2^l-\beta_2^l\right)\lambda+\left(1+\beta_2^l\lambda-\lambda\right)\right\}f^l$

$K_2 = \left[1-\alpha_1\lambda+F_1\cdot(\alpha_1-\beta_1)\right]f_1\left(f^h+f^l\right)+F_1\cdot\left(\alpha_2^h-\beta_2^h\right)\lambda\left(f^h\right)^2-(1-F_1)\left(\alpha_2^l-\beta_2^l\right)\lambda\left(f^l\right)^2.$

Note that $K_1, K_2 > 0$. Obviously $\frac{\partial\Delta^*}{\partial\sigma^2} > 0$.

Next, look at the effect of β_2^h on Δ^*. Since $\frac{\partial\Delta^*}{\partial\beta_2^h} = \frac{\partial\left(\frac{K_2}{K_1}\right)}{\partial\beta_2^h} = \frac{\frac{\partial K_2}{\partial\beta_2^h}}{K_1} - \frac{K_2\frac{\partial K_1}{\partial\beta_2^h}}{K_1^2} = \frac{\frac{\partial K_2}{\partial\beta_2^h}K_1-\frac{\partial K_1}{\partial\beta_2^h}K_2}{K_1^2}$, its sign depends on whether $\frac{\partial K_2}{\partial\beta_2^h}K_1 - \frac{\partial K_1}{\partial\beta_2^h}K_2 > 0$.

$\frac{\partial K_1}{\partial\beta_2^h} = F_1\left(-F^h\lambda+\lambda\right)f^h > 0$ and

$\frac{\partial K_2}{\partial\beta_2^h} = -F_1\lambda\left(f^h\right)^2 + (1-F_1)\lambda\left(f^l\right)^2 < 0.$

Therefore, $\frac{\partial\Delta^*}{\partial\beta_2^h} > 0$ for any $\lambda > 0$.

Proof of $Q(\Delta) > Q(0)$ for any $\Delta > 0$.

Recall that, for given any α, $Q(\Delta)$ is increasing in Δ when $\Delta < \Delta^*$ and decreasing when $\Delta > \Delta^*$. Therefore, $Q(\Delta) > Q(0)$ for any $\Delta > 0$ if $\lim_{(\Delta\to\infty)} Q(\Delta) > Q(0)$.

When $\Delta\to\infty$, $F^h\to 1$, $F^l\to 0$, $f^h = f^l \to 0$, and $\Lambda\to 0$. Therefore,

$$\lim_{(\Delta\to\infty)} Q(\Delta) = \frac{f_1}{c_A}\left[1-\alpha_1\lambda+F_1\cdot(\alpha_1-\beta_1)\right]\left[2\alpha_1-1+\beta_2^h-\beta_2^l+\alpha_2^h-\beta_2^h\right].$$

When $\Delta = 0$, $F_1 = F^h = F^l$, $f_1 = f^h = f^l$, and $\Lambda = 0$. Therefore,

$$\begin{aligned} Q(0) = &\frac{f_1}{c_A}\left[1-\alpha_1\lambda+F_1\cdot(\alpha_1-\beta_1)\right]\left[2\alpha_1-1+\beta_2^h-\beta_2^l\right] \\ &+\left[F_1\cdot\left(\alpha_2^h-\beta_2^h\right)\lambda+1-\lambda+\beta_2^l\lambda\right]\left(\alpha_2^h-\beta_2^h\right)\frac{f_1}{c_A} \end{aligned}$$

Then, $\lim_{(\Delta\to\infty)} Q(\Delta) - Q(0)$

$= \left(\alpha_2^h-\beta_2^h\right)\frac{f_1}{c_A}\left[1-\alpha_1\lambda+F_1\cdot(\alpha_1-\beta_1)-\left\{F_1\cdot\left(\alpha_2^h-\beta_2^h\right)\lambda+1-\lambda+\beta_2^l\lambda\right\}\right] > 0$

if $1-\alpha_1\lambda+F_1\cdot(\alpha_1-\beta_1) > F_1\cdot\left(\alpha_2^h-\beta_2^h\right)\lambda+1-\lambda+\beta_2^l\lambda$

$\Leftrightarrow (\alpha_1-\beta_1)F_1 > \left[\alpha_1-(1-F_1)\alpha_2^h-F_1\beta_2^h\right]\lambda.$

Suppose $\alpha_1 < (1-F_1)\alpha_2^h - F_1\beta_2^h$. Then, $\left[\alpha_1-(1-F_1)\alpha_2^h-F_1\beta_2^h\right]\lambda$ is always weakly less than 0 and equal to 0 when $\lambda = 0$. Since $(\alpha_1-\beta_1)F_1 > 0$, $\lim_{(\Delta\to\infty)} Q(\Delta) - Q(0) > 0$. Now suppose $\alpha_1 > (1-F_1)\alpha_2^h - F_1\beta_2^h$. Then $\left[\alpha_1-(1-F_1)\alpha_2^h-F_1\beta_2^h\right]\lambda$ is at most $\alpha_1 - (1-F_1)\alpha_2^h - F_1\beta_2^h$ achieving this value as $\lambda\to 1$. $(\alpha_1-\beta_1)F_1 > \alpha_1-(1-F_1)\alpha_2^h-F_1\beta_2^h \Leftrightarrow (1-F_1)\alpha_2^h+F_1\beta_2^h > (1-F_1)\alpha_1+F_1\beta_1$ which is true since $\alpha_2^h > \alpha_1$ and $\beta_2^h > \beta_1$. Therefore, $\lim_{(\Delta\to\infty)} Q(\Delta) > Q(0)$ for given any α.

Consumer's behavior and market share structure

In this section, I give a more detailed model of consumers which generates the behavior assumed in the text. First, I describe the model for the special case where $\alpha_t = 1$ and $\beta_t = 0$. The utility of the consumer who chooses firm i is $u_{it} = q_{it} - p_t$ where p_t is exogenous. Hence, a consumer chooses i in period t if i has higher expected quality than j given the consumer's information. I assume consumers know all of the model in the text but do not know which firm is more efficient. Specifically, they know the marginal cost of one firm is $\underline{c}$ and that of the other is $\overline{c}$ where $\underline{c} < \overline{c}$, but they do not know whether $c_A =\underline{c}$ and $c_B = \overline{c}$ or the reverse. Assume that the consumer's prior belief that $c_A < c_B$ (that is, that $c_A =\underline{c}$ and $c_B = \overline{c}$) is $\frac{1}{2}$. Though I assume $c_A \leq c_B$ in the text, in the informed consumers' view, it is possible that $c_A > c_B$ in their beliefs. From Proposition 4., we can see that the optimal quality choices by firms are $q^r_{At}(c_A, c_B) > q^{r'}_{Bt}(c_A, c_B)$ if and only if $c_A < c_B$ for any r. That is, the more efficient firm always chooses higher quality than the less efficient one.

Note that Proposition 4 and the following discussion does not depend on whether $\alpha^r_t > \frac{1}{2}$ (that is, $\alpha^r_t > \beta^{r'}_t$) or not.

In the first period, if the reviewer reports $y_{A1} > y_{B1}$, then the expected quality of firm A's product is

$$\mathbf{E}(q_{A1}|y_{A1} > y_{B1}) = \Pr\left[c_A = \underline{c}|y_{A1} > y_{B1}\right] q_{A1}(\underline{c}, \overline{c}) + \Pr\left[c_A = \overline{c}|y_{A1} > y_{B1}\right] q_{A1}(\overline{c}, \underline{c}).$$

The consumers buy from firm A if this is larger than the analogous expression for firm B. Since $q_{A1}(\underline{c}, \overline{c}) = q_{B1}(\overline{c}, \underline{c})$ and $\Pr\left[c_A = \overline{c}|y_{A1} > y_{B1}\right] = \Pr\left[c_B = \underline{c}|y_{A1} > y_{B1}\right]$, we see that the consumers buy from firm A if

$$\left\{\Pr\left[c_A = \underline{c}|y_{A1} > y_{B1}\right] - \Pr\left[c_A = \overline{c}|y_{A1} > y_{B1}\right]\right\}\left[q_{A1}(\underline{c}, \overline{c}) - q_{B1}(\overline{c}, \underline{c})\right] > 0.$$

As noted above, the lower cost firm always produces higher quality, so the second term is strictly positive. Hence this holds if and only if

$$\Pr\left[c_A = \underline{c}|y_{A1} > y_{B1}\right] > \tfrac{1}{2}$$

or, equivalently,

$$\Pr\left[c_A < c_B|y_{A1} > y_{B1}\right] > \tfrac{1}{2}.$$

$\Pr[c_A < c_B | y_{A1} > y_{B1}] = \frac{\Pr[y_{A1}>y_{B1}|c_A<c_B]\cdot\Pr[c_A<c_B]}{\Pr[y_{A1}>y_{B1}]}$

$= \frac{\Pr[y_{A1}>y_{B1}|c_A<c_B]\cdot\Pr[c_A<c_B]}{\Pr[y_{A1}>y_{B1}|c_A<c_B]\cdot\Pr[c_A<c_B]+\Pr[y_{A1}>y_{B1}|c_A>c_B]\cdot\Pr[c_A>c_B]} = \frac{\Pr[y_{A1}>y_{B1}|c_A<c_B]}{\Pr[y_{A1}>y_{B1}|c_A<c_B]+\Pr[y_{A1}>y_{B1}|c_A>c_B]}$

The last line comes from the fact $\Pr[c_A < c_B] = \Pr[c_A > c_B] = \frac{1}{2}$. Therefore, the consumers buy from firm A when it wins the first period competition if

$$\Pr[y_{A1} > y_{B1}|c_A < c_B] > \Pr[y_{A1} > y_{B1}|c_A > c_B].$$

This is equivalent to

$$\Pr[y_{A1} > y_{B1}|c_A < c_B] > \tfrac{1}{2}$$

as

$$\Pr[y_{A1} > y_{B1}|c_A > c_B] = 1 - \Pr[y_{A1} > y_{B1}|c_A < c_B]$$

from the symmetry of the equilibrium quality choices. But

$$\Pr[y_{A1} > y_{B1}|c_A < c_B] = \Pr[q_{A1}(\underline{c},\overline{c}) - q_{B1}(\underline{c},\overline{c}) > \epsilon_{B1} - \epsilon_{A1}].$$

So the fact that $q_{A1}(\underline{c},\overline{c}) - q_{B1}(\underline{c},\overline{c}) > 0$ implies that this probability is greater than 1/2. Of course, the analogous conclusions follow when firm j is the first period winner.

The second period is more complex since the firms are no longer symmetric. Essentially the same reasoning works if the asymmetry induced by Δ is not too large. One can find reasonable conditions on $\underline{c}$ and $\overline{c}$ such that consumers will buy from the second period winner. Here I prove the existence of such $(\underline{c},\overline{c})$ for given any Δ. Since it is straightforward when the reviewer reports $s_{A1} > s_{B1}$ and $s_{A2} > s_{B2}$, let's focus on the case when the reviewer reports $s_{A1} < s_{B1}$ and $s_{A2} > s_{B2}$. In such a case, the expected quality of firm A's product is

$$\mathbf{E}(q^l_{A2}|Y) = \Pr[c_A = \underline{c}|Y]\, q^l_{A1}(\underline{c},\overline{c}) + \Pr[c_A = \overline{c}|Y]\, q^l_{B2}(\overline{c},\underline{c})$$

where $Y = y_{A1} > y_{B1}$ and $y^l_{A2} - \Delta > y^h_{B2}$. Thus, the consumers buy from firm A if $\mathbf{E}(q^l_{A2}|Y) - \mathbf{E}(q^h_{B2}|Y) > 0$ which is equivalent to

$$\Pr[c_A = \underline{c}|Y]\left\{q^l_{A2}(\underline{c},\overline{c}) - q^h_{B2}(\overline{c},\underline{c})\right\} - \Pr[c_i = \overline{c}|Y]\left\{q^h_{A2}(\underline{c},\overline{c}) - q^l_{B2}(\overline{c},\underline{c})\right\} > 0.$$

Let $\underline{q}_2^r$ and $\overline{q}_2^r$ denote $q_{A2}^r(\underline{c},\overline{c}) = q_{B2}^r(\overline{c},\underline{c})$ and $q_{A2}^r(\overline{c},\underline{c}) = q_{B2}^r(\underline{c},\overline{c})$, respectively. When I rearrange this

$$\begin{aligned}
\frac{\Pr\left[c_A = \underline{c}|Y\right]}{\Pr\left[c_B = \overline{c}|Y\right]} &> \frac{\underline{q}_2^h - \overline{q}_2^l}{\underline{q}_2^l - \overline{q}_2^h} \Leftrightarrow \frac{\Pr\left[Y|c_A < c_B\right]}{\Pr\left[Y|c_A > c_B\right]} > \frac{\lambda \underline{q}_2^h}{\lambda \underline{q}_2^l} \text{ where } \lambda = \frac{\overline{c} - \underline{c}}{\overline{c}} \\
&\Leftrightarrow \frac{\left[1 - F\left(\underline{q}_1 - \overline{q}_1\right)\right] F(\underline{q}_2^l - \overline{q}_2^h - \Delta)}{F\left(\underline{q}_1 - \overline{q}_1\right)\left[1 - F(\underline{q}_2^h - \overline{q}_2^l + \Delta)\right]} > \frac{f(\lambda \underline{q}_2^h + \Delta)}{f(\lambda \underline{q}_2^l - \Delta)} \\
&\Leftrightarrow \frac{F(\underline{q}_2^l - \overline{q}_2^h - \Delta)}{1 - F(\underline{q}_2^h - \overline{q}_2^l + \Delta)} f(\lambda \underline{q}_2^l - \Delta) > \frac{F(\underline{q}_1 - \overline{q}_1)}{1 - F(\underline{q}_1 - \overline{q}_1)} f(\lambda \underline{q}_2^h + \Delta).
\end{aligned}$$

In the first line, I use the fact $\overline{q}_2^r = \frac{\underline{c}}{\overline{c}} \underline{q}_2^{r'}$ at the equilibrium. And in the second line, I use that $\epsilon_{At} - \epsilon_{Bt}$ is independent across t and $\underline{q}_2^r = \frac{1}{\underline{c}} f(\lambda \underline{q}_2^r + \Delta)$ at the equilibrium. Notice that, in the third line, the left hand side is equal to the right hand side when $\lambda = 0$. Let's consider the case when λ is a little bigger than zero. Since we can focus on the lowest order of $\left(\frac{1}{\sigma^2}\right)$ terms when σ^2 is big enough, we have

$$\begin{aligned}
\frac{\partial\left[\frac{F(\underline{q}_2^l - \overline{q}_2^h - \Delta)}{1 - F(\underline{q}_2^h - \overline{q}_2^l + \Delta)} f(\lambda \underline{q}_2^l - \Delta)\right]}{\partial \lambda}\Big|_{\lambda \to 0} &\to \frac{f(\Delta)}{1 - F(\Delta)} \left(\underline{q}_2^l + \underline{q}_2^h\right) f(-\Delta) \text{ and} \\
\frac{\partial\left[\frac{F(\underline{q}_1 - \overline{q}_1)}{1 - F(\underline{q}_1 - \overline{q}_1)} f(\lambda \underline{q}_2^h + \Delta)\right]}{\partial \lambda}\Big|_{\lambda \to 0} &\to \frac{f(0)}{1 - F(0)} \left(\underline{q}_1 + \underline{q}_1\right) f(\Delta).
\end{aligned}$$

By the normality, it is true that $\frac{[f(\Delta)]^2}{(1-F(\Delta))F(\Delta)} > \frac{[f(0)]^2}{(1-F(0))F(0)}$ for any $\Delta > 0$. Thus, we can see that

$$\frac{f(\Delta)}{1-F(\Delta)} \underline{q}_2^l > \frac{f(0)}{1-F(0)} \underline{q}_1$$

since $\underline{q}_2^l = \frac{1}{\underline{c}} f(\Delta)$ and $\underline{q}_1 = \frac{2}{\underline{c}} F(\Delta)$ at the equilibrium. Therefore, there exists small enough λ (or $\overline{c} - \underline{c}$) such that $\mathbf{E}(q_{i2}^l|Y) > \mathbf{E}(q_{j2}^h|Y)$ for any given $\Delta > 0$. That is, when the production cost difference between firms is not large, rational consumers would follow the reviewer's recommendation.

For the case where $0 < \beta_t^r < \alpha_t^r < 1$, we need to introduce heterogeneity among the consumers. To model this, consider a Hotelling model (see Klemperer 1987b for the details). Initially, consumers for one unit of service are arrayed with symmetric density along the line segment $(0, 1)$ with firm A and B at 0 and 1, respectively. A consumer at x has a transport cost of x of using A's service or $(1 - x)$ of using B's service. If consumers trade-off quality against transportation cost, then even when consumers agree on which firm has the highest expected quality, not all of them will purchase from that firm. Also, assume consumers have a switching cost s of purchasing a product that they have not previously purchased. Assume that $s \in (0, 1)$, so that at least some consumers' preferences for underlying product characteristics can outweigh their switching costs. Thus, the market share for the firm that wins in the second period will be higher if he won in the first period than if he lost in the first period, $\alpha_2^h > \alpha_2^l$.

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