Under Revision

Entrepreneurs, Optimism and the Competitive Edge

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Abstract: Unrealistic optimism in business can lead to a misallocation of resources and a reduction in welfare. But unrealistic optimism can also stimulate saving and investment and provide added incentives for hard work. In this paper, I explore the interaction of these two effects with respect to entrepreneurs who are overoptimistic about the productivity of their firms, and I examine the nature of the competition between such optimistic entrepreneurs and their realistic counterparts. I demonstrate that in some technological environments (such as those characterized by small firms with rapidly decreasing returns to scale) optimistic entrepreneurs may coexist with realists in competitive equilibria or even drive the realists out of business. Moreover, the resulting competitive equilibria will evince significant distortions. This is important because a large body of evidence in the psychology literature indicates that unrealistic optimism is a widespread human trait, one that may be common among business executives.

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Normal people believe to an unrealistic degree that the future holds a bounty of good things and few bad things. Depressed people are more realistic in their perceptions of the future. Shelley E. Taylor (1989, pp. 214)

1 The Prevalence of Unrealistic Optimism among Entrepreneurs

Within the last twenty years, evidence has appeared in the psychology literature indicating that unrealistic optimism is a widespread human trait. But modern psychologists were anticipated by Adam Smith. In *The Wealth of Nations*, Smith (1776) wrote:

> The overweening conceit which the greater part of men have of their abilities is an ancient evil remarked by the philosophers and moralists of all ages. Their absurd presumption in their own good fortune has been less taken notice of [but is], if possible, still more universal. There is no man living who, when in tolerable health and spirits, has not some share of it. The chance of gain is by every man more or less overvalued, and the chance of loss is by most men undervalued. . . [1]

More recently, Weinstein (1980, 1982) found that experimental subjects consistently rated their chances of experiencing positive events to be above average and their chances of experiencing negative events to be below average as compared with the chances of their peers. Other investigators have found a pattern of unrealistic optimism among business executives. For example, Kidd and Morgan (1969, pp. 162-64) found that electric utility management consistently underestimated the downtime of generating equipment. Larwood and Whittaker (1977) studied a sample of corporate presidents and found them to be unrealistic in their predictions of success. In her comprehensive book on this subject, psychologist Shelley E. Taylor (1989) asserts that unrealistic optimism is an indispensable trait of the healthy mind, and she summarizes the results of hundreds of empirical studies in the psychology literature to support her claim.

Of course, even if unrealistic optimism turns out to be common in the business community, this would not imply that the actions of executives or entrepreneurs are biased by unrealistic views. An important tenet of free-market economics holds that agents whose behavior is inconsistent with rational maximizing behavior cannot long persist. This idea is summed up in Friedman's well-known assertion:

[1] As evidence for these assertions, Smith cites the following facts pertaining to 18th century Britain: most houses and many ships at sea were uninsured, young people gave little weight to downside risk in their career choice and wages were not significantly elevated in certain classes of dangerous jobs. Smith offered one explanation of unrealistic optimism now confirmed by modern psychologists, namely, that people mentally exaggerate their ability to control future events. See Smith (1776) Book I, Chapter X.

> ... unless the behavior of businessmen in some way or other approximated behavior consistent with the maximization of returns, it seems unlikely that they would remain in business for long. Let the apparent immediate determinant of business behavior be anything at all—habitual reaction, random chance, or whatnot. Whenever this determinant happens to lead to behavior consistent with rational and informed maximization of returns, the business will prosper and acquire resources with which to expand; whenever it does not, the business will tend to lose resources and can be kept in existence only by the addition of resources from outside. [Friedman (1953, p.22)]

Friedman's argument would seem to imply that unrealistic optimists must either behave like realists or else be forced out of business. Yet, the following simple example shows that Friedman's claim must be interpreted with care. Consider a farmer who owns high-quality land but farms it with inefficient methods. Such a farmer will obtain a smaller net income than his efficient counterparts. But the farmer need not be forced out of business; indeed, his farm may well provide him with a substantial income. Here, the farmer implicitly uses up some of the rents from the land he owns to compensate for his inefficiency. These rents form the "resources from outside" that Friedman refers to.

It is true that an observant neighbor might attempt to lease the land of the inefficient farmer by offering a rental payment larger than the farmer's earnings. By farming efficiently, the neighbor could earn a tidy profit. Thus, the market can correct inefficient behavior by bribery if not by coercion. The inefficiency caused by unrealistic optimism, however, may be resistant to market correction. This is because an optimist may incorrectly anticipate that future returns will exceed the payments a realist would be willing to provide. Furthermore, the incentives created by optimism may increase the income that the optimist can use to compensate for his inefficiency.

In this paper, I model the behavior of entrepreneurs who are unrealistically optimistic about the productivity of their own firms. I show that under certain circumstances, the market favors these optimists over their realistic and informed counterparts, even in a purely competitive environment. If the two types coexist in the marketplace, the optimists may consistently earn more than realists and, in time, crowd some or all of the realists out of business.

Underlying such results is the fact that optimism about productivity leads agents to anticipate unrealistically high marginal products of their own inputs. This, in turn, creates an incentive for agents to increase their savings rates and work effort, which can have a positive effect on steady-state income (though the agent's utility will be reduced). But optimism may also tend to reduce income through a negative effect on economic efficiency. The sources of the two effects on income are distinct: the negative efficiency effect is associated with the overuse of external resources, such as hired labor and borrowed capital, whereas the positive incentive effect is associated with the overuse of resources internal to the entrepreneur, such as his personal savings and effort. One purpose of this paper is to explore how these two effects trade off against one another in various economic environments.[2]

To the extent that the incentive effect of optimism overbalances the inefficiency effect, we might view optimism as a psychological trait devised by nature for inducing more hard work and thrift, two behavioral attributes that undoubtedly increase the fitness of our species. Thus, one might look to human evolution for an explanation of the prevalence of unrealistic optimism in human populations reported by Taylor (1989) and many others. But this immediately raises the following question. Why wouldn't preference characteristics such as a low discount rate and a low disutility of effort dominate optimism as a means of producing the desired behavioral traits? After all, unlike optimism, such preference characteristics would not necessarily be associated with inefficiency.

One possible answer to this question is explored by M. Waldman (1994). Waldman shows that in a model of sexual reproduction, evolution would be unlikely to change two separately inherited characteristics simultaneously unless each change alone would increase fitness. This is because of the fact that if a mutant characterized by both changes mates with a non-mutant, only one quarter of the offsprings are likely to have both changes, while one half will have only one of the two changes. Waldman argues that unrealistic optimism and a preference for leisure over work form what he calls a 'second-best adaption,' meaning that optimism maximizes fitness given our preference for leisure and our preference for leisure maximizes fitness given our optimism. But this does not negate the possibility that a simultaneous change in both would increase fitness. A simultaneous decrease in the level of optimism and in the disutility of work, for example, would presumably increase fitness, but each of these changes taken separately would not. Thus, a human population characterized by unrealistic optimism along with a certain reluctance to save and work hard, might be evolutionarily stable.

There are many models in the literature in which agents with limited rationality learn over time. But in this study, in order to make the results as strong as possible, I model overoptimistic entrepreneurs who do not learn, or else learn too late, that their judgments about firm productivity are systematically erroneous. Using agents who are incorrigible in their lack of realism, I question Friedman's assertion that the market will tend to eliminate those whose behavior is inconsistent with the maximization of returns.

Many academic economists seem to feel that it is unreasonable to assume the perseverance of erroneous beliefs in the face of contrary evidence. But a substantial body of results in the psychology literature indicates that erroneous beliefs often survive devastating empirical challenge.[3] Moreover, if optimism is a normal human psychological trait that may carry an evolutionary advantage, it wouldn't make sense to adopt as an axiom the hypothesis that agents update optimistic beliefs in a logical fashion. Not even Friedman argues that irrational

[2] Like optimism, unrealistic pessimism would tend to reduce economic efficiency. But unlike optimism, pessimism would create incentives to reduce the use of internal resources and thus lower accounting profits. Consequently, the pessimistic entrepreneur would be unlikely to survive in the marketplace and is not of interest here.

[3] To observe this, one need only examine the beliefs of many of our colleagues in the economics profession. Ross and Anderson (1982, pp. 144-152) review recent research on the psychology of belief perseverance. For a good example of belief perseverance in the realm of economic activity, see Thaler's (1988) discussion of the "winner's curse."

or uninformed businessmen would learn from their mistakes or mend their ways. He argues only that they will be disciplined by the market.

The foregoing analysis does not require that most or even many people are overoptimistic, only that some are. And although I model optimists who live forever but never learn, the results may be expected to carry over to a more familiar scenario in which succeeding generations of entrepreneurs with finite lives have optimistic priors when young, but learn to temper their optimism as they grow older.

The central idea of this paper, then, is that optimistic agents may attempt to maximize utility, but because they save too much and hire too much labor, they systematically fail to do so. However, as they are unwittingly using their own internal resources to compensate for their mistakes, the unrealistic agents do "make money" and so are encouraged by the market. In a way then, we are turning Friedman's argument on its head; here "the addition of resources from outside" necessary to keep inefficient businesses afloat comes from the optimistic entrepreneurs themselves. And to the extent that the systematic mistakes of these entrepreneurs change prices, the economy as a whole may be distorted.

The ability to earn economic profits in a competitive environment indicates that a firm is making a positive contribution to social welfare. But the forces of market selection do not act on economic profits directly. The market leaves it up to the owners of firms to deduct the implicit costs of their own effort and capital from their net earnings. The impersonal forces of market selection tend to favor entrepreneurs with high accounting profits regardless of internal costs. The market does not discourage overwork.

2 The Economics Literature

There is a vast theoretical literature in which business decision-makers are assumed to be characterized by some form of limited or near rationality.

Hey (1984) incorporates optimism (and pessimism) into a general model of decision-making under uncertainty. Kahneman and Tversky (1979), Russell and Thaler (1985), Haltiwanger and M. Waldman (1987, 1989, and 1991), and other writers have explored the implications of various types of near-rational behavior. Near-rational agents can have a significant affect on the nature of economic equilibria. For example, Akerlof and Yellen (1985) show that when rational and near-rational agents (who suffer from slight inertia) coexist, the equilibrium can differ substantially from that of all rational agents.

Robert Frank has modeled agents whose decision-making rules are directly contrary to what economists view as rational. In his book, *Passions within Reason: The Strategic Role of Emotions,* Frank (1988) discusses the usefulness of emotions as a device for making credible commitments. And in Frank (1987), he describes why traits such as honesty increase evolutionary fitness.

Discussions of optimism and overconfidence (a form of optimism) frequently arise in the burgeoning field of behavioral finance. Optimism is an ingredient of the well-known papers of

De Long *et al* (1990, 1991). The authors show that if all traders are risk-averse, then optimistic "noise-traders" may be able to make enough money to retain their position in the market. More recently Daniel, Hirshleifer, and Subrahmanyam (1997) model the overconfidence of investors in financial markets, and De Meza and Southey (1996) and Manove and Padilla (1997) model the overconfidence of borrowers.

3 A Model of Entrepreneurial Optimism in a Dynamic Setting

As an example of perverse market encouragement of unrealistic optimists with resulting inefficiency, I formalize the following story. A purely competitive, free-market economy has only one production technique for which capital, labor, and one unit of entrepreneurial effort must be used as inputs. There are a large number of infinitely lived agents. Each agent may be either an entrepreneur who runs a business or a worker who sells his labor at the market wage.

Entrepreneurs start their businesses with an initial stock of wealth, and they remain in business as long as they choose, or until their wealth is exhausted. The model is dynamic: each period entrepreneurs allocate their wealth between saving and consumption and decide how much labor to hire for productive purposes. All entrepreneurs must self-finance their businesses; there is no capital market. But the labor market is perfectly competitive.

Agents, both entrepreneurs and workers, are of different types: they may be realists or optimists of varying degrees. All workers behave the same way, regardless of type, but different types of entrepreneurs behave differently. Realistic entrepreneurs know the true production function associated with their firms, whereas optimistic entrepreneurs believe their firms to be more productive than they really are. Otherwise, realists and optimists are perfectly rational and identical in every way. In each period, every entrepreneur accurately observes his current wealth, and given his perceptions of productivity, makes a rational decision about the intertemporal allocation of his resources. In the following period, an optimistic entrepreneur will find that he has less wealth than he had anticipated, but he infers nothing about the true nature of his production function from this observation.

For simplicity, we assume that the economy has only one produced good, which can be either consumed or held as productive capital. There are a large number of firms in the economy, each owned by one entrepreneur who may be of any type. Entrepreneurs pay workers in kind and are free to consume the residual, so that there is no product market in the model. All interaction between firms is intermediated by the market wage rate. Wage effects can occur through two different channels: through firm's derived demand for labor and through labor-supply decisions of agents, who must choose to be either workers or entrepreneurs.

The model is analyzed in several stages. In Section 3.1, we establish the framework of the model. In Section 3.2 we examine the entrepreneur's dynamic problem and solve for his resulting behavior. Then, in Section 3.3 we characterize the steady state of a designated entrepreneur with the wage rate exogenously specified. At this point, we will have the tools to study the entire economy in the steady state. In Section 3.4, we find the market-clearing wage and short-run competitive equilibrium associated with an exogenously specified distribution of entrepreneur types. Finally, in Section 3.5, we allow agents to choose between being workers and entrepreneurs, and we find the long-run equilibrium associated with an exogenous distribution of agent types. The characteristics of this long-run equilibrium are explored in Section 3.7.

We will show that in the context of this model, optimistic entrepreneurs may remain in business in the steady state, and we analyze their effect on the market in both short-run and long-run competitive equilibria. By bidding up the wage, optimists tend to increase the welfare of workers and lower that of other entrepreneurs. Their effect on output turns out to be ambiguous. In general, optimistic entrepreneurs utilize resources to excess. To the extent that they overutilize external, market-supplied resources such as hired labor, the ensuing distortion is likely to diminish output. But to the extent that internal resources, such as the owners' savings, are overutilized, output tends to be increased. It should not be surprising then, if in small businesses, where internal resources form a high share of value added, optimism increases output, while in large companies, the opposite may be true. This is an outcome that our model suggests.

In this model of the effects of entrepreneurial optimism, the important distinction among inputs is between internally and externally supplied factors of production, and not between labor and capital *per se*. But I have modeled labor as a purely external factor, available only in the labor market, while capital is viewed as internal and financed entirely from the savings of the entrepreneur. In the real world, of course, both factors come from both internal and external sources. Entrepreneurs can and do supply their own effort, which inevitably will be of great importance and value, especially to small startup firms. Furthermore, for most firms, and especially for large ones, capital can be obtained in a variety of external capital markets, as well as from the savings of owners. The specification of labor as externally obtained and capital as internally financed greatly simplifies this exposition without changing the spirit of the results.

3.1 Framework of the Model

We use a discrete-time structure. At the beginning of each period, a given entrepreneur has a stock s of the produced good. He immediately allocates a portion $k < s$ of this stock to serve as productive capital and consumes the rest, so that consumption is given by

$$c = s - k. \tag{1}$$

Each firm requires one entrepreneur and positive quantities of capital, k, and hired labor, l, in order for production to occur. The entrepreneur hires labor in a competitive labor market at the market wage w (measured in the produced good). Labor is paid at the end of the production period.

A entrepreneur of type θ perceives that production in his firm is governed by the relation:

$$\tilde{q} = \tilde{f}(k, l) \equiv \theta f(k, l), \tag{2}$$

where $\tilde{q}$ is anticipated output, $\tilde{f}$ is the *perceived* production function, f is the true production function for all firms, and $\theta \geq 1$ is the designated entrepreneur's optimism type. Entrepreneurs for whom $\theta = 1$ are realists, and those with $\theta > 1$ are optimists.[4] The production function f is characterized by decreasing returns to scale, technical complementarity of inputs, marginal products that go to infinity near the origin, and other standard properties.[5]

Let s' denote the stock of the produced commodity at the beginning of the period following the current one. Then, the evolution of commodity stocks over time is determined by the transition equation,

$$s' = k + f(k, l) - wl. \tag{3}$$

But the *anticipated* evolution of commodity stocks over time is given by the *perceived* transition equation,

$$s'_p = k + \theta f(k, l) - wl, \tag{4}$$

where s'_p is the stock that the entrepreneur anticipates for the following period.

3.2 The Entrepreneur's Behavior

Given their perceptions, all entrepreneurs are completely rational. The behavior of each entrepreneur is embodied in a policy function obtained by solving a dynamic planning problem.

Suppose all entrepreneurs have discount rate ρ, and let $\delta \equiv 1/(1+\rho)$ denote the common discount factor. We now analyze the planning problem of an entrepreneur of type θ when the market wage is constant and given by w.

The problem of the type-θ entrepreneur is to decide how much productive capital k to set aside and how much labor l to hire in each period. Because the entrepreneur is assumed to use the perceived production function for all decisions and all inferences about the future, the derivation of the entrepreneur's policy function must be based on (4) rather than (3). But because every entrepreneur is assumed to observe the size of his current stock correctly,[6] the argument of the policy function will be his current stock as determined by (3) applied to the stock of the previous period.

We assume that the entrepreneur wants to maximize the present discounted value of anticipated lifetime consumption. Let $t > 0$ index future time periods (say, one year in length), with $t = 0$ representing the current period. Then, a type-θ entrepreneur with a current stock s, faces the following dynamic planning problem:

$$\max_{\{k_t, l_t\}} \sum_{t=0}^{\infty} \delta^t c_t, \tag{5}$$

subject to the consumption level,

$$c_t = s_t - k_t \tag{6}$$

[4] For reasons explained in Footnote 2, we do not model $\theta < 1$, which would indicate pessimism.

[5] More formally, admissible functions f are assumed to be continuous, strictly increasing, twice-differentiable, strictly concave, with $f_{kl} > 0$, $f(k, 0) = f(0, l) = 0$, and $f_k, f_l \to \infty$ as $k, l \to 0$.

[6] I do not doubt that many individuals perceive their current stocks to be larger than they truly are, but I will not burden the reader by inserting this possibility in the model.

the perceived transition between states,

$$s_{t+1} = k_t + \theta f(k_t, l_t) - wl_t, \tag{7}$$

the initial condition,

$$s_0 = s, \tag{8}$$

and, the non-negativity conditions,

$$k_t, l_t, c_t \geq 0. \tag{9}$$

Problems of this general type have been treated extensively in the growth-model literature (see Stokey and Lucas (1989), Chapter 5, for a discussion and references). The solution of (5)—(9) is of the 'bang-bang' variety: the optimal investment policy is to save and invest at the maximum feasible level when stocks are low, and then to jump directly to a steady-state level of capital usage when stocks permit. Optimal labor use requires that the perceived marginal product of labor be set equal to the wage rate, no matter what the entrepreneur's stock.

We now proceed to define the solution of (5)—(9) more precisely. Let $\bar{k}$ and $\bar{l}$ denote the simultaneous solution for k and l of the first-order conditions,

$$\theta f_k(k, l) = \rho \tag{10}$$

and

$$\theta f_l(k, l) = w, \tag{11}$$

and let $\hat{l}(k)$ denote the solution of (11) for l given k. The properties of f guarantee that $\bar{k}$, $\bar{l}$, and $\hat{l}(k)$ exist and are unique. In Appendix A we prove:

Proposition 1 *The perceived optimal policy function is described by $k(s)$ for capital and $l(s)$ for labor, with corresponding consumption $c(s) = s - k(s)$ and* actual *production $q(s) \equiv f(k(s), l(s))$, as given in the following table:*

Interval

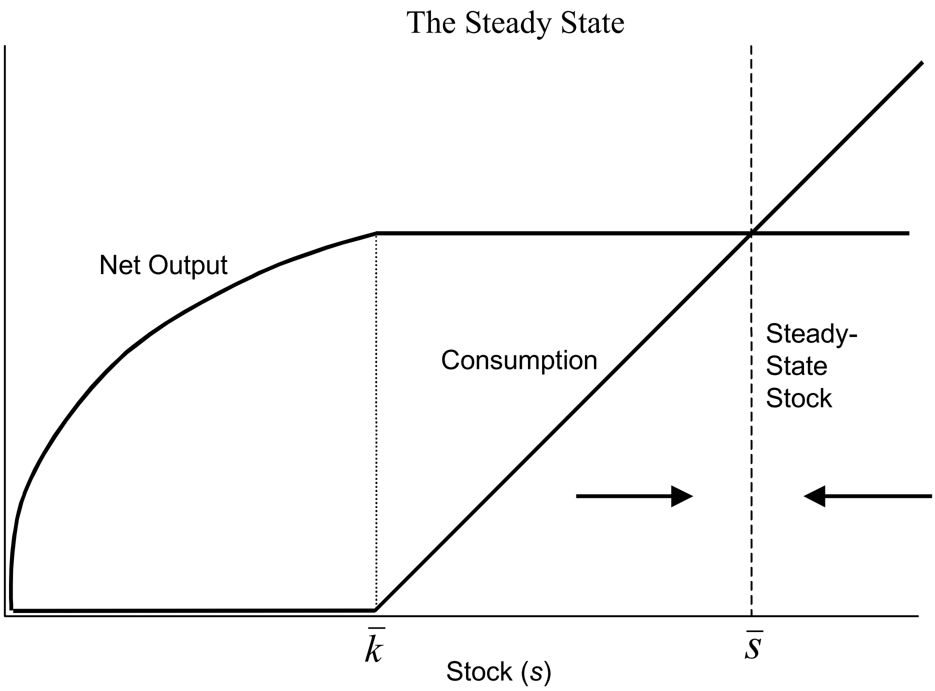
Function		$s < \bar{k}$	$s \geq \bar{k}$
	$k(s)$	s	$\bar{k}$
	$l(s)$	$\hat{l}(s)$	$\bar{l}$
	$c(s)$	0	$s - \bar{k}$
	$q(s)$	$f(s, \hat{l}(s))$	$f(\bar{k}, \bar{l})$

(12)

Inasmuch as $k(s)$ and $l(s)$ are continuous and monotonically increasing, so is $q(s)$, and as is apparent from its definition, $c(s)$ also has these properties.

The entrepreneur's behavior in every period is determined by the policy function (12), which he applies to the actual current value of his stock as given by (3). Because (12) reflects an incorrect specification of the production function, the entrepreneur in using it does not successfully maximize the objective function in (5).

Figure 1:



3.3 The Entrepreneur's Steady State

Let the wage w be given. The entrepreneur's policy function, (12), and the transition equation, (3), along with the initial stock and current parameter values, determine the trajectory of the entrepreneur's stock over time. We show that for θ not too large, each entrepreneur has a unique non-degenerate steady state, and we find its value.

From (3) and the identity $s \equiv c + k$, we see that the true change of state for the optimist is given by

$$\Delta s = s' - s = q(s) - w\,l(s) - c(s). \tag{13}$$

We define net production, $y(s)$ by

$$y(s) \equiv q(s) - w\,l(s). \tag{14}$$

From (13), we can say that stock increases when net production is greater than consumption and decreases when the opposite is true. A steady state exists where net production equals consumption, so that a necessary and sufficient condition for a steady state is given by

$$c(s) = y(s). \tag{15}$$

The steady state is illustrated in in Figure 1. In Appendix B, we demonstrate the following:

Proposition 2 *Let ρ and w be given. For some $\tilde{\theta} > 1$, an entrepreneur of any type $\theta \in [1, \tilde{\theta}]$ has a unique non-degenerate steady state. Steady-state capital and labor inputs are given by $\bar{k}$ and $\bar{l}$, which are increasing functions of θ. The steady-state stock is given by*

$$\bar{s} \equiv \bar{k} + f(\bar{k}, \bar{l}) - w\bar{l}. \tag{16}$$

This steady state is stable and affords the positive level of consumption $\bar{c}$ given by

$$\bar{c} \equiv f(\bar{k}, \bar{l}) - w\,\bar{l} > 0. \tag{17}$$

3.4 Short-Run Competitive Equilibria

We now describe the economy to which our agents belong, and we analyze the interaction of the agents in the steady state. Consider an economy with identical production opportunities, described by the production function $f(k, l)$, and a given distribution of agents by optimism-type. We assume that there is an upper bound $\tilde{\theta}$ on the degree of optimism possessed by agents that satisfies the restriction in Proposition (2). We describe the cumulative frequency distribution of agent types by a function $N = N(\theta)$, for $1 \leq \theta \leq \tilde{\theta}$, where the value of $N(\theta)$ is the number (mass) of agents of types less than or equal to θ. This function is assumed to be monotonically increasing, piecewise-continuous, and continuous on the right, conditions which allows us to consider a discrete distribution of types, a continuous one, or both combined. By definition, $N(1)$ is the number of realists in the economy; for $\theta' > \theta$, $N(\theta') - N(\theta)$ is the number of agents with types in the set $(\theta, \theta']$; and $n = N(\tilde{\theta})$ is the total number of agents in the economy. In the model that follows, a vector of the form $(\tilde{\theta}, N, f)$ will constitute a complete description of an economy.

We assume that in any given period, each agent may choose to work either as an entrepreneur or as a laborer, but not both. Let w denote the prevailing wage; $B(\theta)$, the cumulative frequency distribution of agents who currently are working as entrepreneurs; and $S(\theta)$, the stock held by agents of type θ.[7] The current state an economy $(\tilde{\theta}, N, f)$ is completely described by a triple of the form $\{w, B, S\}$.

Note that to make sense, the distribution of entrepreneurial types given by B must bear an appropriate relationship to the distribution of agent types N. In particular, observe that the cumulative frequency distribution of agent-types among laborers is given by $L(\theta) \equiv N(\theta) - B(\theta)$, so that the structure of B must be such that L is a monotonically increasing function of θ.

In what follows, we use Θ_n, Θ_b, and Θ_l, to denote the supports of $N(\theta)$, $B(\theta)$, and $L(\theta)$. As we have seen, the steady-state values $\bar{s}, \bar{k}, \bar{l},$ and $\bar{q} = f(\bar{k}, \bar{l})$, depend on θ and w, and for the purposes of this and later sections we will sometimes write the former as functions of the latter.

A short-run equilibrium of the economy described by $(\tilde{\theta}, N, f)$ is defined to be a state $\{w, B, S\}$ that satisfies the following two axioms:

1. each entrepreneur is in a stable steady state, and
2. the prevailing wage, w, clears the labor market.

Axiom 1 implies

$$S(\theta) = \bar{s}(\theta, w) \text{ for all } \theta \in \Theta_b. \tag{18}$$

[7] For simplicity, we assume that all entrepreneurs of the same type hold the same stock.

As for the labor market, the quantity of workers supplied is given by $L(\tilde{\theta})$, whereas the quantity demanded is $\int \bar{l}(\theta, w)dB(\theta)$. Consequently, Axiom 2 implies:

$$\int \bar{l}(\theta, w)dB(\theta) = L(\tilde{\theta}). \tag{19}$$

3.5 Long-Run Equilibria

The economy is described as in Section 3.4. A long-run equilibrium is a short-run equilibrium that satisfies free-entry and exit conditions for firms. More formally, a long-run equilibrium is defined as a state of the economy $\{w, B, S\}$ that satisfies the axioms of short-run equilibrium and the following additional axioms as well:

1. no entrepreneur would choose to exit and become a laborer,
2. no laborer, having been given a stock however large, would choose to use part of it to become an entrepreneur.

Note that in our definition of a long-run equilibrium, it is not enough that no laborer in his current state wants to go into business. We require something stronger: in long-run equilibrium no stock of any size could induce a laborer to choose to go into business. The analogous rule for entrepreneurs in equilibrium can be proven true without an explicit assumption. Thus, we are requiring that an equilibrium be 'windfall-proof': if additional stocks were randomly distributed among agents, consumption would increase for one period, but the equilibrium state would remain unchanged in the future.

We now establish necessary and sufficient conditions for long-run equilibria. In order to interpret Axioms 1 and 2, it is useful to observe that anticipated steady-state *economic* profits are given by

$$\tilde{\pi}(\theta, w) \equiv \theta\bar{q}(\theta, w) - \rho\bar{k}(\theta, w) - w(1 + \bar{l}(\theta, w)) \tag{20}$$

where the first term of the definition is anticipated steady-state revenue, the second is the steady-state opportunity cost of capital, and the third is the opportunity cost of entrepreneurial labor and the explicit steady-state costs of external labor.

Axiom 1 requires that entrepreneurs at $\bar{s}(\theta, w)$ would not expect to attain a higher present discounted value of consumption by becoming laborers than by continuing to operate firms. The optimist at state $\bar{s}$ incorrectly believes that output will be $\theta f(\bar{k}, \bar{l})$. He expects to move to state $\bar{s}_e \equiv \theta f(\bar{k}, \bar{l}) + \bar{k} - w\bar{l}$ in one period, and he perceives $\bar{s}_e$ to be a steady state. Although he knows that his present consumption at $\bar{s}$ is $f(\bar{k}, \bar{l}) - w\bar{l}$, he believes that his future consumption at $\bar{s}_e$ will be $\theta f(\bar{k}, \bar{l}) - w\bar{l}$.

Thus at the steady state $\bar{s}$, the optimistic entrepreneur expects a stream of consumption whose present value is given by

$$v(\bar{s}) = f(\bar{k}, \bar{l}) - w\bar{l} + \frac{1}{\rho}(\theta f(\bar{k}, \bar{l}) - w\bar{l})$$

If the same entrepreneur were to close shop and become a laborer, he could consume all of his stock in the present period and earn the market wage thereafter, so that the present value of planned consumption would be $\bar{s} + w/\rho$. Axiom 1 thus requires that

$$\bar{s} + \frac{w}{\rho} \leq \bar{q} - w\bar{l} + \frac{\theta}{\rho}\bar{q} - \frac{w}{\rho}\bar{l}.$$

Applying (16) and (20), we have that Axiom 1 is equivalent to:

$$\tilde{\pi}(\theta, w) \geq 0 \quad \text{for all} \quad \theta \in \Theta_b, \tag{21}$$

i.e., anticipated economic profits for all entrepreneurial types in business must be non-negative.

Consider now an agent of type θ, currently working as a laborer. Assume that the agent has available a stock, s, sufficient to invest $\bar{k}(\theta, w)$ in a new firm without dropping below minimum consumption. Then the anticipated present value of consumption to be obtained by entry will be $s - \bar{k} + (1/\rho)(\theta f(\bar{k}, \bar{l}) - w\bar{l})$, while the anticipated present value of consumption to be obtained by remaining in the labor force will be $s + (1/\rho)w$. Thus Axiom 2 is equivalent to:

$$\tilde{\pi}(\theta, w) \leq 0 \quad \text{for all} \quad \theta \in \Theta_l, \tag{22}$$

i.e., that no laborer can anticipate positive economic profits as an entrepreneur.

We may conclude, now, that $\{w, B, S\}$ constitutes a long-run equilibrium if and only if it satisfies conditions (18)—(22).

3.6 Existence of Long-Run Equilibria

Because (10) and (11), the defining equations of $\bar{k}$ and $\bar{l}$, are the first-order conditions for $\max_{k,l}\{\theta f(k,l) - \rho k - wl\}$, we can rewrite (20) in the form

$$\tilde{\pi}(\theta, w) = \max_{k,l}\{\theta f(k,l) - \rho k - wl\} - w, \tag{23}$$

so that by the envelope theorem, $\partial\tilde{\pi}/\partial\theta = f(\bar{k}, \bar{l}) > 0$. By conditions (21) and (22) this immediately implies that in a long-run equilibrium, if any agent of type θ is in business, then all agents of $\theta' > \theta$, are in business as well. Let $\tilde{\theta}$ be highest agent type in the economy, let $n = N(\tilde{\theta})$ be the total number of agents, and let b represent the number of agents in business. Now the monotonicity property is equivalent to the following:

Proposition 3 *In equilibrium, there is a degree of optimism θ^* such that all agents of types $\theta < \theta^*$ are workers and all agents of types $\theta > \theta^*$ are entrepreneurs. Thus, in an equilibrium with b entrepreneurs in business, the frequency distribution of entrepreneurial types is given by*

$$B(\theta) = \max\{0, b - [n - N(\theta)]\}. \tag{24}$$

We now show:

Proposition 4 *For any admissible production function f and for $\tilde{\theta}$ sufficiently small, every economy $(\tilde{\theta}, N, f)$ has a unique long-run competitive equilibrium.*

The complete proof of this proposition can be found in Appendix C, but we outline the general argument here. We index potential equilibrium states by b, the number of entrepreneurs in business. By Proposition 3, precisely one equilibrium distribution of entrepreneurs is consistent with each postulated value of b. Because of the nature of that distribution, each postulated b uniquely divides the set of agents into sets of workers and entrepreneurs, where all workers have θ's as low or lower than all entrepreneurs. Furthermore, the distribution of entrepreneurs determined by b yields a unique demand function for labor (assuming all firms are in the steady state). Given b, the quantity of labor supplied must be $n - b$, so that one and only one equilibrium wage $w(b)$ is consistent with a postulated value of b. That wage determines one agent-type $\theta^*[w(b)]$ whose anticipated economic profits would be zero. Because anticipated economic profits are increasing in θ, we complete the proof with respect to Axioms 1, 2, 1, and 2, by showing that there is a unique value of b such $\theta^*[w(b)]$ separates the set of workers from the set entrepreneurs indicated by b.

3.7 Characteristics of Long-Run Equilibria

Slightly optimistic entrepreneurs consume more in equilibrium than realists. In the Appendix, we show:

Proposition 5 *If the economy includes optimistic types arbitrarily close to realists ($\theta = 1$), and if some realists are in business in equilibrium, then there are some optimistic types in business whose equilibrium consumption is greater than that of the realists.*

This proposition is important because it indicates that not only can optimistic entrepreneurs be present in equilibrium, but that some of them will be "doing better" than their realistic counterparts. The intuition for this result is easy to state: led on by optimistic expectations, the optimists have sacrificed more (saved more) early on, and despite a misallocation of resources, they reach a steady state with higher consumption.

A natural question to ask at this point is how changes in the degree of optimism of the various agents would effect the equilibrium. First, we demonstrate that increased optimism among a subset of entrepreneurs will raise the equilibrium wage and crowd out some marginal entrepreneurs:

Proposition 6 *Consider an economy $(\tilde{\theta}, N, f)$, and let θ^* be the degree of optimism that separates entrepreneurs from workers. If the distribution of agent types N is changed parametrically to $\hat{N}$ so as to make some entrepreneurs more optimistic (i.e. $\hat{N}(\theta) \leq N(\theta)$ for all θ, and $\hat{N}(\theta) < N(\theta)$ for some $\theta > \theta^*$), then the corresponding equilibrium wage must increase, and the number of entrepreneurs must fall.*

Let b^* and $\hat{b}^*$ and w^* and $\hat{w}^*$ denote the equilibrium number (mass) of entrepreneurs and equilibrium wage rates with distributions N and $\hat{N}$. And suppose that $\hat{w}^* < w^*$. Because perceived profits increase as θ increases and as w falls, anticipated profits of the least-profitable entrepreneurial type in the N-equilibrium must be positive in the $\hat{N}$-equilibrium. It follows that $\hat{b}^* \geq b^*$. From Proposition 2 we know that at any given wage w, the steady-state quantity of labor demanded is a strictly increasing function of θ, so that aggregate quantity of labor demanded in the new equilibrium must be greater than in the old, while the aggregate labor supplied, $n - \hat{b}^*$ is no larger. Thus the new market clearing wage $\hat{w}^*$ must be greater than w^*, a contradiction, and the proposition is proved.

An example in Section 4 demonstrates that increases in optimism will in some cases cause the national product (aggregate output) to fall. In other cases such increased optimism will cause the national product to rise. Which alternative materializes, a fall or a rise in national product, depends on the tradeoff between the incentive and efficiency effects of optimism as the degree of optimism parametrically increases. If an analogous model is constructed in which all agents are realists but with differing discount rates, then, inasmuch as the use of resources is always efficient, one would expect that the equilibrium national product would always rise as discount rates fall parametrically and agents accumulate larger capital stocks in the steady state. More formally, we have:

Proposition 7 *For some economies $(\tilde{\theta}, N, f)$ there are distributions $\hat{N}$, more optimistic than N (i.e. $\hat{N}(\theta) \leq N(\theta)$ for all θ), such that the equilibrium national product of $(\tilde{\theta}, \hat{N}, f)$ is less than that of $(\tilde{\theta}, N, f)$.*

Furthermore, one can show:

Proposition 8 *Suppose all agents are realists, but assume they differ with respect to their discount rates, ρ. Then, in a model analogous to the above, suppose that the distribution of agents with respect to their discount rates changes parametrically so as to make some entrepreneurs more patient without leaving any agents less patient. Then in long-run equilibrium, national product must rise.*

We will not present a formal proof of this proposition, as this would require us to reconstruct the entire model presented above and to prove analogous propositions. Rather, we present a outline of the proof. Differentiating the first-order conditions that define steady-state capital stocks and the demand for labor, we can see that the strict convexity of f implies that both increase as ρ falls. An argument parallel to that in the proof of Proposition 6 demonstrates that the equilibrium wage increases and the equilibrium number of entrepreneurs falls, increasing the number of workers. Since the wage has risen, and since the discount rate for the different entrepreneurial types has remained constant or fallen, the capital-labor ratio of each remaining entrepreneur must have risen. The exit into the labor force of entrepreneurs with the highest discount rates and thus the lowest capital-labor ratios enhances this effect. The result is that the aggregate capital-labor ratio is greater. Because steady-state labor usage is larger, so the capital stock must also be larger than before. Finally, inasmuch as resources are employed efficiently both before and after the change in discount rates, output must be larger as well. It is only this efficient use of resources that sets off the outcome of Proposition 7 from that of the current proposition.

Propositions 7 and 8 are of particular importance, because in the context of this model, they provide a clear indication that the behavior of optimists cannot be simulated merely by changing the utility functions of agents in a natural way. In particular,optimists fundamentally differ from realistic agents with low discount rates.

3.8 Implications for Evolution

Suppose that the following axioms hold for agents:

1. all agents die stochastically at a uniform rate;
2. each type begets its own kind; and
3. each type begets a number of offspring proportional to their steady-state consumption.

Note that (1) will not change the nature of the preceding calculations; the uniform death rate would merely be added to the discount rate. This suggests that a 'reasonable' degree of optimism would confer a fitness advantage. Over time, the number of optimists in the population would increase and the number of realists would decrease. See Waldman (1994) for an excellent formal treatment of the potential evolutionary value of "irrationality."

4 The Comparative Statics of Equilibria in a Special Class of Economies: An Example

In this sections we will explore a special class of economies: those with generalized Cobb-Douglas production functions and with only two types of entrepreneurs: realists and one optimistic type. So suppose that the true production function belongs to the family of functions specified by

$$f(k,l) = Ak^{\alpha}l^{\beta}, \quad \text{with} \quad \alpha + \beta \equiv \eta < 1, \tag{25}$$

a generalized Cobb-Douglas functions with decreasing returns to scale. Also suppose that the distribution of types belongs to the family of distributions given by

$$N(\theta) = \begin{cases} n_1 & \text{for} \quad 1 \leq \theta < \hat{\theta} \\ & \\ n_1 + n_2 & \text{for} \quad \theta \geq \hat{\theta} \end{cases}, \tag{26}$$

which describes an economy with n_1 realists and n_2 optimists of type $\hat{\theta}$.

We begin the analysis by finding steady-state input and output levels for an agent of type θ. The Cobb-Douglas factor-share identities give us $kf_k/f \equiv \alpha$ and $lf_l/f \equiv \beta$, and (10) and (11) yield:

$$\bar{k} = \frac{\alpha}{\rho}\theta\bar{q}, \tag{27}$$

and

$$\bar{l} = \frac{\beta}{w}\theta\bar{q}, \tag{28}$$

where

$$\bar{q} \equiv f(\bar{k}, \bar{l}), \tag{29}$$

so that steady-state consumption of the entrepreneur is given by

$$\bar{c} = \bar{q} - w\bar{l} = (1 - \beta\theta)\bar{q}. \tag{30}$$

If $\theta < \frac{1}{\beta}$, then (16) yields a stable steady state at

$$\bar{s} = (1 - \beta\theta + \alpha\theta/\rho)\bar{q}. \tag{31}$$

For use in the simulations below, we solve for $\bar{q}$ explicitly. Substituting (27) and (28) into (29), and solving for $\bar{q}$ yields

$$\bar{q} = B\theta^{\frac{\eta}{1-\eta}}/w^{\frac{\beta}{1-\eta}} \tag{32}$$

where

$$B \equiv A\left(\alpha^{\alpha}\beta^{\beta}/\rho^{\alpha}\right)^{\frac{1}{1-\eta}}$$

In this example, how does steady-state consumption depend on an entrepreneur's the degree of optimism? For $\theta < \frac{1}{\beta}$, the ratio r_c of steady-state consumption by the optimist to steady-state consumption by the realist is given by

$$r_c = \frac{1 - \beta\theta}{1 - \beta}\theta^{\frac{\eta}{1-\eta}} \tag{33}$$

It follows that steady-state consumption increases with the degree of optimism for $\theta \in [1, \frac{\eta}{\beta}]$ and decreases with the degree of optimism for $\theta \in [\frac{\eta}{\beta}, \frac{1}{\beta})$, so that maximum steady-state consumption occurs at $\theta = \frac{\eta}{\beta}$. Note that $\frac{\beta}{\eta}$ is the relative factor share of labor, the external factor. The "most fruitful" degree of optimism, then, is the reciprocal of the relative factor share of the external factor. From a qualitative point of view, this is not a surprising result. The advantages of optimism to the business are derived from overuse of the internal factor; whereas the disadvantages result from the overuse of the external factor. Because total availability of the internal factor is fixed, and because owners normally have to procure larger proportions of inputs on the market as a business grows, we might expect the relative factor share of external factors to be correlated with the size of businesses. This means that any given degree of optimism would be advantageous for a sufficiently small business but would be disadvantageous for a sufficiently large one.

Venture capitalists who finance a startup concern may allow the presumably optimistic "inventor" to run a business when it is small, but they often will move in pin-striped executives to run the business once it becomes larger. This may be one reason why.

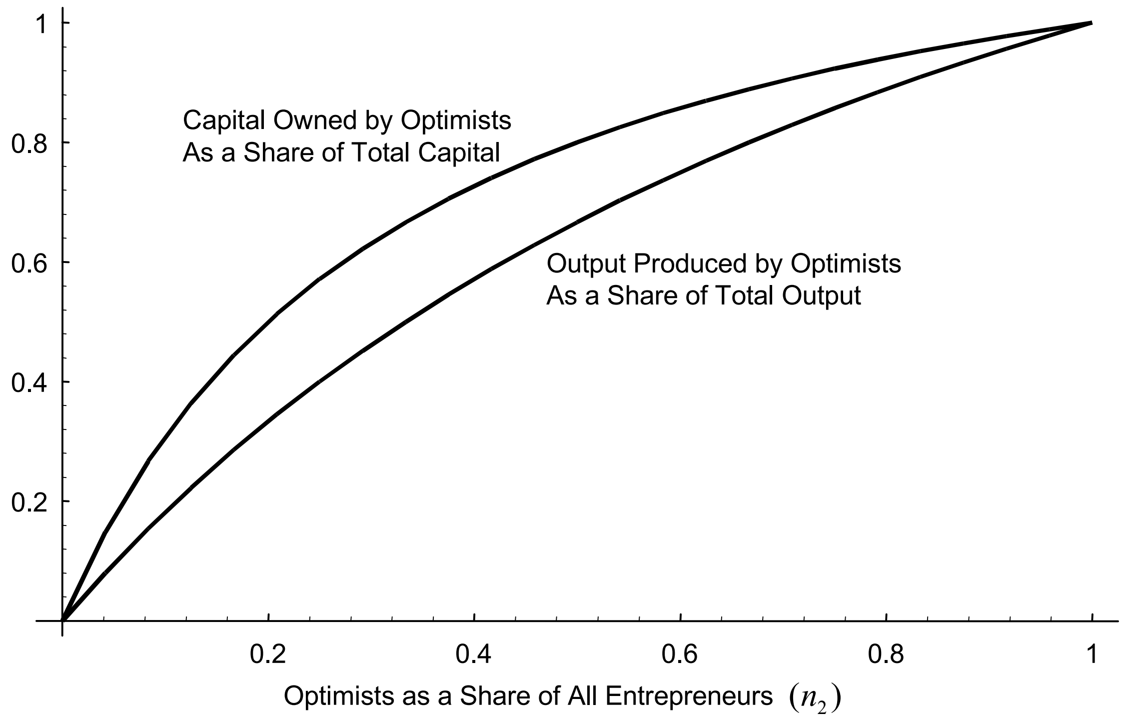
4.1 Short-Run Comparative Statics

Suppose that the number of agents in the economy is fixed at n, and let us set the degree of homogeneity of the production function to $\eta = .5$. The remaining notation is given by the following table:

	θ	Total Population	Number in Business	Steady-State Labor Demand	Steady-State Output
Realists	1	$n - n_2$	b_1	$\bar{l}_1$	$\bar{q}_1$
Optimists	$\hat{\theta}$	n_2	b_2	$\bar{l}_2$	$\bar{q}_2$

Figure 2:

Capital Owned and Output Produced by Optimists



In the short run $n_2, b_1,$ and b_2 are all exogenous.

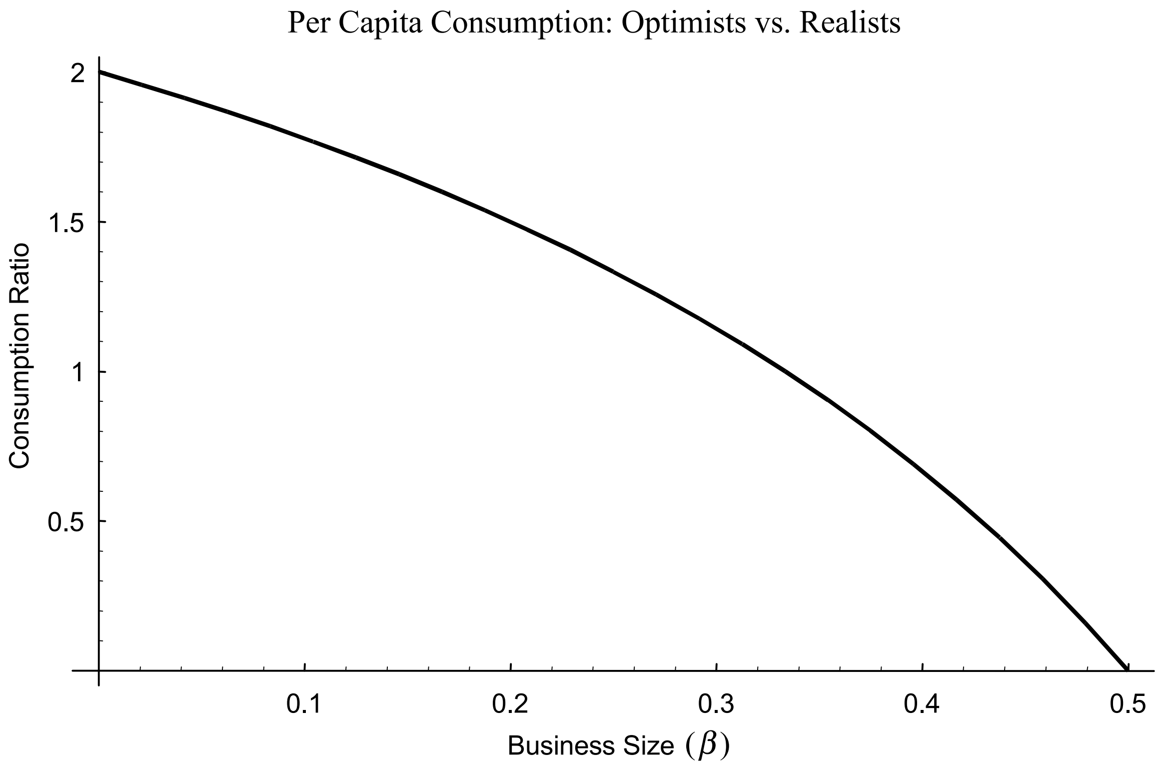
In order to perform short-run comparative statics, we hold the number of entrepreneurs constant, and we vary parametrically both the percentage of optimists, $b_2/(b_1 + b_2)$, among those in business and the factor share of hired labor, β, which is our proxy for business size.

The short-run results for this example are summarized in two graphs. Figure 2 shows the percentage of total output produced by optimists and the percentage of capital stocks held by optimists as a function of the share of optimists among all entrepreneurs. Both curves lie above the 45-degree line, a fact that implies that in short-run equilibrium the average optimistic entrepreneur produces more and holds more capital than his realistic counterparts. Figure 3 graphs the ratio of equilibrium consumption (net income) of optimists to that of realists as a function of business size, as described by β. As the share of the internal factor used for production goes to 1 (small business), optimists consume twice as much as realists in the steady state, but as the share of the internal factor goes to 0 (large business), so does the consumption of the optimists.

4.2 Long-Run Comparative Statics

We continue with the example of Section 4.1, noting that in the long-run b_1 and b_2 are endogenous. Otherwise we use the same parameter values as before, but now, in our long-run comparative statics, we vary parametrically the numbers of optimists and realists in the population,

Figure 3:



holding the total population fixed at n. We have from (20), (27), and (28) that anticipated profits are given by

$$\tilde{\pi}(\theta, w) = \frac{\theta \bar{q}}{2} - w \tag{34}$$

We look for a long-run equilibrium in which all optimists and some, but not all, realists are in business. Under these circumstances, $0 < b_1 < n_1$ and $b_2 = n_2$. Conditions (21) and (22) imply that $\tilde{\pi}(\hat{\theta}, w) \geq 0$ and $\tilde{\pi}(1, w) = 0$. The latter implies that $\bar{q}_1 = 2w$, and the Cobb-Douglas identities yield $\bar{l}_1 = \frac{\beta}{w}\bar{q}_1 = 2\beta$ and $\bar{l}_2 = \frac{\hat{\theta}^2 \beta}{w}\bar{q}_1 = 2\hat{\theta}^2\beta$. The labor-market equilibrium requirement reduces to

$$b_1 \bar{l}_1 + n_2 \bar{l}_2 = n - b_1 - n_2, \tag{35}$$

where the left-hand side is labor demand and the right-hand side is labor supply. Solving for b_1, the number of realists in business, we have

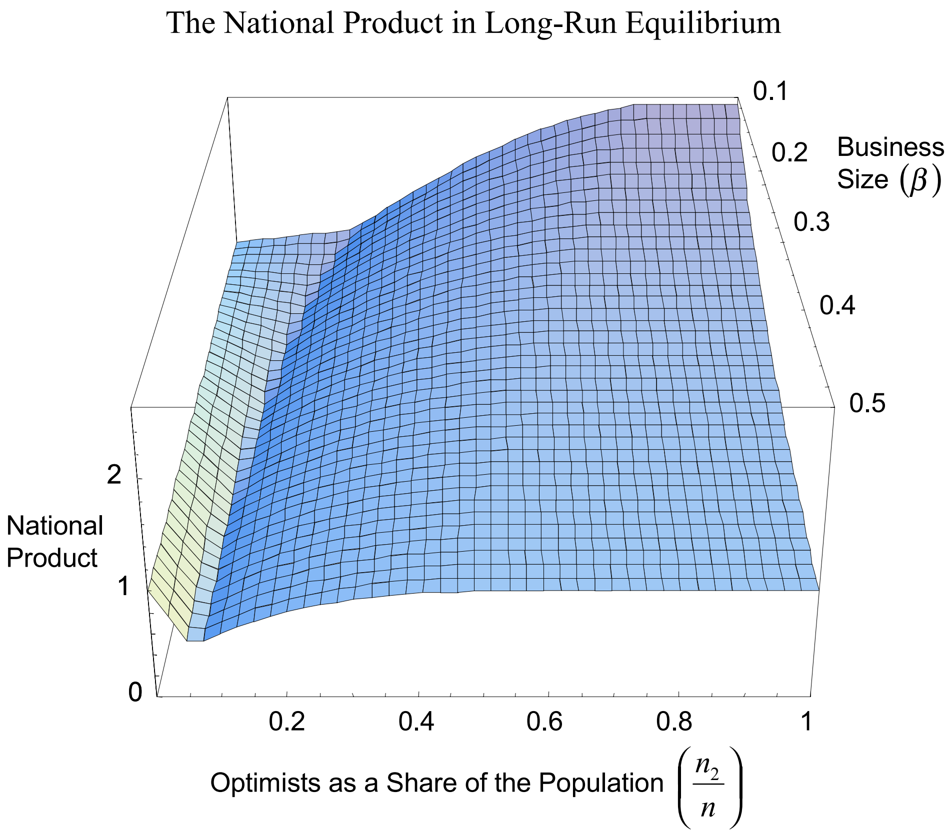
$$b_1 = \frac{n - n_2(1 + 2\beta\hat{\theta}^2)}{1 + 2\beta}, \tag{36}$$

which implies

$$\frac{\partial b_1}{\partial n_2} = -\frac{1 + 2\beta\hat{\theta}^2}{1 + 2\beta}, \tag{37}$$

We see immediately that $\partial b_1/\partial n_2$ is strictly negative, which means that as optimistic agents parametrically replace realistic agents in the population, the optimists will become entrepreneurs and crowd realistic entrepreneurs out of the market. When the degree of optimism is small (i.e.

Figure 4:



$\hat{\theta}$ is near 1), crowding out is close to one for one. But for larger degrees of optimism, one optimist can crowd out many realists. Also, the crowding-out effect is more pronounced when firms are large (large β) than when they are small.

This crowding-out can cause the national product to fall as the share of optimists in the population is increased. From (32) we see that with $\eta = 1/2$ the output of an optimist is $\hat{\theta}$ times that of a realist. But if $\hat{\theta} > \frac{1}{2\beta}$, then we see from (37) that the rate at which optimists crowd realists out of business is greater than the rate at which the optimists replace their production. Thus, national product falls.

Recall that just as optimists crowd realists out of business, so, in a similar world of realists with differing discount rates, agents with low discount rates would crowd out those with high discount rates. But Proposition 8 implies that the low-discount-rate types would replace the product of the high-discount-rate types faster than they crowd them out.

Figure 4 shows the national product in long-run equilibrium for this example as a function of business size and the share of optimists in the population.[8] From this graph it is evident

[8]There are three distinct regions in the domain of this function: the region in which some realists and all optimists are entrepreneurs, the region in which all realists are workers and all optimists are entrepreneurs, and the region in which all realists and some optimists are workers. The first-mentioned region is the only one in which national product might decrease as a function of the share of optimists: it is the region described in the above paragraphs. For the purposes of 4, the author used a symbolic-manipulation computer program to derive the equilibrium national product for each of the three regions and to construct the 3-dimensional graph.

both that the inefficiency effect of optimism can reduce the national product when firms are large and that the incentive effects of optimism can greatly increase the national product when firms are small.

5 Conclusion

In this paper, I have conducted a thought experiment. I have asked what would happen in the long run if some significant proportion of potential entrepreneurs were unrealistic optimists, who do not learn. The answers, I think, are surprising. First of all, the market will not necessarily force the optimists out of business. Indeed, unrealistic optimists may earn more than their realistic counterparts, and they may control a significant part of productive activity in long-run competitive equilibrium. Moreover, the behavior of optimistic entrepreneurs has a distortionary effect on the economy. In equilibrium, aggregate output may be reduced, a fact that rules out the suspicion that optimists are merely low-discount-rate types in disguise. Also, the presence of optimists may significantly change the distribution of income. Optimists tend to drive up wages, making workers better off and other entrepreneurs worse off.

These results depend on the proposition that the misguided addition of the internal resources of optimistic entrepreneurs can not only keep economically unprofitable businesses afloat, but can actually make their owners wealthy (if not happy). One might ask whether the value of such resources as entrepreneurial effort and self-financed capital can be sufficient to have a significant effect on the economy as a whole. I myself believe that a significant part of the value of the national product of free-market economies should be imputed back to such resources, but I shall not attempt to make that argument here.

The model constructed above, of course, is a very special case. But I do think its results are sufficient to raise some interesting questions: In a real free-market economy with its grinding competitive pressures, what kinds of entrepreneurs survive and prosper financially? What sorts of men and women run small firms and large companies? Is their outlook systematically biased away from realism? If so, what are the effects of their unrealistic behavior?

A Proof of Proposition 1

We begin by simplifying the statement of the proposition.[9]

We define net production as actual output less external labor costs. For an entrepreneur of type θ, maximum planned net production is given by

$$h(k) = \max_{l}\{\theta f(k,l) - wl\}. \tag{A1}$$

By the envelope theorem, we know that h inherits the strict convexity of f.[10] Because any choice of l that yielded less than the maximum net output could not be part of an optimal policy, problem (5)—(7) reduces to:

$$\max_{k_t} \sum_{t=0}^{\infty} \delta^t c_t \tag{A2}$$

such that:

$$s_t = c_t + k_t, \tag{A3}$$

$$s_{t+1} = k_t + h(k_t), \tag{A4}$$

$$s_0 = s \text{ given}, \tag{A5}$$

and,

$$c_t, k_t \geq 0. \tag{A6}$$

We define a candidate for the optimal policy function. Let $\bar{k}$ be the unique quantity of capital that satisfies

$$h'(k) = \rho, \tag{A7}$$

and let the candidate policy function $\tilde{g}$ be given by

$$\tilde{g}(s) = \begin{cases} s & \text{for} \quad 0 \leq s < \bar{k} \\ \bar{k} & \text{for} \quad s \geq \bar{k} \end{cases} \tag{A8}$$

We also define $\tilde{v}(s)$ to be the discounted utility obtained from repeated use of $\tilde{g}$ at an initial state s.

We now demonstrate that $\tilde{v}$ and $\tilde{g}$ are the value function and the unique optimal policy function for problem (A2)—(A6). To accomplish this, we show that $\tilde{v}$ satisfies the Bellman equation for this problem,

$$v(s) = \max_{k\in[0,s]}\{s - k + \delta v(k + h(k))\}, \tag{A9}$$

and that $\tilde{g}$ uniquely satisfies

$$g(s) \in \operatorname*{argmax}_{k\in[0,s]}\{s - k + \delta v(k + h(k))\}. \tag{A10}$$

[9] I owe much of this proof to Debraj Ray. He is to blame for any errors.

[10] The envelope theorem yields $h'(k) = \theta f_k(k, \hat{l}(k))$, where $\hat{l}(k)$ is defined as in Section 3.2. Thus, we have $h'' = \theta(f_{kk} + f_{kl}\hat{l}')$. The identity $\hat{l}' \equiv -f_{kl}/f_{ll}$ then gives us, $h'' = \theta(f_{kk}f_{ll} - f_{kl}^2)/f_{ll}$. Inasmuch as the strict convexity of f requires its Hessian matrix to be strictly negative-definite, we have immediately that $h'' < 0$.

Claim 9 *For $s \geq \bar{k}$, $\tilde{v}(s) = s - \bar{k} + h(\bar{k})/\rho$.*

Proof. If $s = \bar{k}$ and $\tilde{g}$ is applied, the agent will invest $\bar{k}$, consume nothing, and go to state $\bar{k} + h(\bar{k})$ in the next period. When $\tilde{g}$ is applied again, the agent will invest $\bar{k}$, consume $h(\bar{k})$, and remain at state $\bar{k} + h(\bar{k})$. This occurs every time $\tilde{g}$ is repeated. Thus, with initial state $\bar{k}$, the agent consumes nothing immediately, but does consume $h(\bar{k})$ at the beginning of every succeeding period. This yields discounted utility $h(\bar{k})/\rho$. If the agent has state $s > \bar{k}$ initially, he will consume $s - \bar{k}$ immediately, and then follow the same program as the agent at $\bar{k}$, yielding our result. ■

Claim 10 *$\tilde{v}(s)$ satisfies (A9) and $\tilde{g}(s)$ uniquely satisfies (A10) for $s \geq \bar{k}$.*

Proof. Replacing v by the value of $\tilde{v}$ for $s \geq \bar{k}$ in the right-hand side of (A9) yields the equation

$$v(s) = \max_{k}\{s - k + \delta[k + h(k) - \bar{k} + \frac{1}{\rho}h(\bar{k})]\}, \tag{A11}$$

and the first-order condition for this maximum is

$$-1 + \delta + \delta h'(k) = 0 \tag{A12}$$

which simplifies to $h'(k) = \rho$. From (A7) and the strict concavity of h, we have that $k = \bar{k}$ uniquely maximizes the maximand of (A11), and that the maximum value of that expression is the very definition of $\tilde{v}(s)$, which proves our claim. ■

Now assume that v is the true value function. We already have shown that $v(s) = \tilde{v}(s)$ for all $s \geq \bar{k}$. We proceed to show that $v(s) = \tilde{v}(s)$ on the entire domain.

Claim 11 *For states $z > y \geq 0$ we have*

$$v(z) - v(y) \geq z - y \tag{A13}$$

for the true value function v.

Proof. Let $k_y \in [0, y]$ be a value of k that satisfies (A9) when $s = y$, so that

$$v(y) = y - k_y + \delta v(k_y + h(k_y)). \tag{A14}$$

Then, from (A9) with $s = z$ we know that

$$v(z) \geq z - k_y + \delta v(k_y + h(k_y)), \tag{A15}$$

and the result follows immediately. ■

Claim 12 *For $s \in (0, \bar{k}]$ and $k < s$, and for the true value function v, we have*

$$s - k + \delta v(k + h(k)) < \delta v(s + h(s)) \tag{A16}$$

so that $k = s$ uniquely maximizes the maximand in (A9).

Proof. By (A13) and the strict concavity of h, we know that

$$v(s+h(s))-v(k+h(k))\geq s-k+(h(s)-h(k))>s-k+h'(\bar{k})(s-k)=(1+h'(\bar{k}))(s-k),$$

so that from the definition of $\bar{k}$ it follows that

$$\delta[v(s+h(s))-v(k+h(k))]>\delta(1+h'(\bar{k}))(s-k)=s-k.$$

The result follows immediately. ■

It follows directly from Claim 12 that $\tilde{g}$ is unique the optimal policy function for $s<\bar{k}$, so that $v=\tilde{v}$ on this part of the domain as well.

B Proof of Proposition 2

We know $y(s)=y(\bar{k})$ is constant for all $s>\bar{k}$, and from (12) we have that $c(s)=s-\bar{k}>0$ for $s>\bar{k}$ and $c(s)=0$ for $0\leq s\leq\bar{k}$. Therefore, as is evident from Figure 1, (15) has exactly one solution in the region of positive consumption if $y(\bar{k})>0$, and no solutions in that region otherwise. Furthermore if $y(\bar{k})>0$, it follows from $y(0)=0$ and the convexity of y that $y(k)>0$ for all $0<k<\bar{k}$. This means that there can be no equilibria with $c(s)=0$ aside from the degenerate equilibrium at $s=0$.

Thus to prove existence and uniqueness, we need only argue that $y(\bar{k})>0$. So suppose that $\theta=1$. Then economic profits are maximized when $k=\bar{k}$ and $l=\bar{l}$, and these profits must be positive because profits are positive for sufficiently small k and l. Furthermore, $y(\bar{k})$, which represents accounting profits at $k=\bar{k}$, must be greater than economic profits there, and it follows that $y(\bar{k})>0$ when $\theta=1$. Since, $\bar{k}$ and $y(\bar{k})$ are continuous in θ, it follows that $y(\bar{k})>0$ for all θ sufficiently close to 1.

To show that $\bar{k}$ and $\bar{l}$ are increasing functions of θ we calculate

$$\frac{\partial\bar{k}}{\partial\theta}=\frac{f_{kl}f_l-f_{ll}f_k}{\theta(f_{kk}f_{ll}-f_{kl}^2)}$$

and

$$\frac{\partial\bar{l}}{\partial\theta}=\frac{f_{kl}f_k-f_{kk}f_l}{\theta(f_{kk}f_{ll}-f_{kl}^2)}.$$

The strict concavity of f implies that the denominators are positive, and we have $f_k,f_l,f_{kl}>0$ and $f_{kk},f_{ll}<0$, so that both expressions are positive. The value of $\bar{s}$ is derived from (14) and (15), and stability follows from (13). ■

C Proof of Proposition 4

By Proposition 3, we can restrict our attention to the frequency distribution $B(\theta)$ given by (24).

Claim 13 *For every $b\in(0,n)$, there is a unique wage $w=w(b)$ that clears the labor market, and $w(b)$ is strictly increasing in b.*

Given b, the quantity of labor supplied is $n - b$, and equilibrium labor demand is $\int \bar{l}(\theta, w) dB(\theta)$. Inasmuch as labor demand is continuous, and because $\bar{l}$ goes to zero for w sufficiently large and to infinity for w sufficiently small, it follows that the quantity of labor demanded takes the value of $n - b$ for some w, so that a market-clearing wage must exist for each b. Furthermore,

$$\frac{\partial \bar{l}}{\partial w} = \frac{f_{kk}}{\theta(f_{kk} f_{ll} - f_{kl}^2)} < 0$$

because of the assumed strict concavity of f, so that the market-clearing wage is unique. As b increases, the postulated quantity of labor supplied decreases and labor demand shifts out. Because labor demand is downward-slopping, it follows that $w(b)$ is strictly increasing in b.

beginclaim For every $w \geq 0$, there exists a unique non-negative θ^* such that $\tilde{\pi}(\theta^*, w) = 0$, and θ^* is strictly increasing in w.

Consider $\tilde{\pi}$ as defined in (23). We know that $\tilde{\pi}(0,0) = 0$. For any $w > 0$, $\tilde{\pi}(0, w) < 0$ while $\tilde{\pi}(\theta, w) > 0$ for sufficiently large θ, so that θ^* with $\tilde{\pi}(\theta^*, w) = 0$ must exist. That this θ^* is unique follows from the fact that $\partial \tilde{\pi}/\partial \theta > 0$. Finally, we have

$$\frac{d\theta^*}{dw} = -\frac{\partial \tilde{\pi}/\partial w}{\partial \tilde{\pi}/\partial \theta} > 0,$$

and the claim is proved.

Given b agents in business, we let $\bar{\theta}(b)$ denote the type of the least optimistic among them, i.e. $\bar{\theta}(b) = \inf\{\theta \mid n - N(\theta) < b\}$. The function $\bar{\theta}(b)$ is flat where $N(\theta)$ has discontinuities, and it is discontinuous where $N(\theta)$ is flat, but $\bar{\theta}(b)$ is weakly decreasing and continuous on the left. Because $\theta^*(w(b))$ is continuous and strictly increasing, we know that $\theta^*(w(b))$ cuts $\bar{\theta}(b)$ at a unique value of b given by $\bar{b} = \sup\{b \mid \theta^*(w(b)) \leq \bar{\theta}(b)\}$. Defining the frequency distribution of the $\bar{b}$ entrepreneurs by $\bar{B}(\theta) = \max\{0, \bar{b} - [n - N(\theta)]\}$ as given in (23), and the stock of each type of entrepreneur by $S(\theta) = \bar{s}(\theta, w(\bar{b}))$, we can show:

Claim 14 *For any distribution of agents $N(\theta)$ with $\tilde{\theta}$ sufficiently small, the state $\{w(\bar{b}), \bar{B}(\cdot), S(\cdot)\}$ is a unique long-run equilibrium.*

The definition of $\bar{\theta}(b)$ implies that given b, the measure of workers of types greater than $\bar{\theta}(b)$ is zero, and the measure of entrepreneurs of types less than $\bar{\theta}(b)$ is also zero.

Now consider $\bar{\theta}(\bar{b})$. The properties of the functions θ^* and $\bar{\theta}$ and the definition of $\bar{b}$ imply that $\theta^*(w(\bar{b})) \leq \bar{\theta}(\bar{b})$. This means that if $\bar{b}$ is the measure of entrepreneurs, the measure of entrepreneurs of types less than $\theta^*(w(\bar{b}))$ is zero. To show that the measure of workers of types greater than $\theta^*(w(\bar{b}))$ is also zero, we need show only that $N(\theta^*(w(\bar{b}))) \geq n - \bar{b}$. But otherwise we would have $\bar{b} < n - N(\theta^*(w(\bar{b})))$. Then, because N is continuous on the right, we know that for some $b' > \bar{b}$, $b' < n - N(\theta^*(w(b')))$, an assertion that contradicts the definition of $\bar{b}$. Thus we have demonstrated that if the measure of entrepreneurs is $\bar{b}$, then 1 and 2 must be satisfied.

D Proof of Proposition 5

Let $\bar{c}(\theta)$ denote steady state consumption in business for an agent of type θ. The proposition follows from a demonstration that $\bar{c}'(1) > 0$.

From (15), (14), and (12), we have that

$$\bar{c}(\theta) = f(\bar{k}, \bar{l}) - w\bar{l} \tag{D17}$$

With all of the steady-state variables treated as function of θ, differentiation with respect to θ yields

$$\bar{c}'(\theta) = f_k(\bar{k}, \bar{l})\frac{\partial \bar{k}}{\partial \theta} + f_l(\bar{k}, \bar{l})\frac{\partial \bar{l}}{\partial \theta} - w\frac{\partial \bar{l}}{\partial \theta}. \tag{D18}$$

And from (10) and (11), the defining equations of $\bar{k}$ and $\bar{l}$, we have

$$\bar{c}'(\theta) = \frac{\rho}{\theta}\frac{\partial \bar{k}}{\partial \theta} + \frac{w}{\theta}\frac{\partial \bar{l}}{\partial \theta} - w\frac{\partial \bar{l}}{\partial \theta}, \tag{D19}$$

so that

$$\bar{c}'(1) = \rho\frac{\partial \bar{k}}{\partial \theta} \tag{D20}$$

From Appendix B, we know that $\partial \bar{k}/\partial \theta > 0$. ∎

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