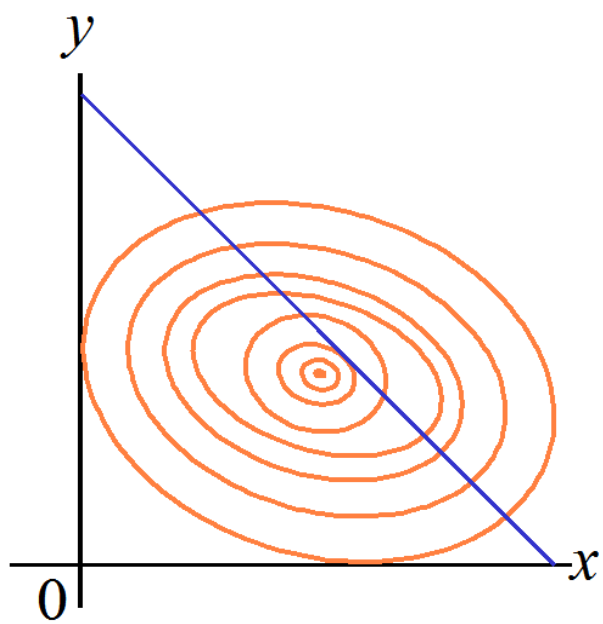
Problem Set 1. Due: Tuesday 9/20

Problem MM.1. Juanjo has the indicated indifference curves and budget set for American food (x) and Chinese food (y). First, in a few sentences, explain his tastes. Second, explain how much of each type of food he will buy.



Problem MM.2. Suppose Jeff consumes only two goods, mangos (x) and pears (y). His utility function is $U(x, y) = x^{10} + y^{10}$. Explain in words what this utility function says about his preferences. Sketch the indifference curve for $U = 1$[no need to find the exact values, but the general shape should be correct].

Problem MM.3. Ching-to likes to eat bread with olive oil. More bread or more olive oil always increases his satisfaction. When he has no olive oil, an additional piece of bread gives him only half the satisfaction as a tablespoon of olive oil. But when he already has a lot of olive oil for his bread, then even an additional liter of olive oil is not as satisfying as another piece of bread. Make a drawing of Ching-to's indifference curves, and construct a utility function that corresponds to your drawing.

Problem M-C.D.2. A consumer consumes one consumption good x and hours of leisure h. The price of the consumption good is p, and the consumer can work at a wage rate of $s = 1$. What is the consumer's budget set?

Problem MM.5. Juanito likes clocks and sugar. Clocks are indivisible (1.72 clocks has the same value to Juanito as 1 clock), but sugar can be used in any amount. Sketch reasonable indifference curves for this case, and construct a corresponding utility function.

Problem MM.6. Show that the utility function $U(x, y) \equiv \sqrt[3]{x} + \sqrt[3]{y}$ is homogeneous. What is the degree of homogeneity?

Problem M-C. Exercise 2.E.1. Suppose $x(p, w)$ is given by

$$x_1(p, w) = \frac{p_2}{p_1 + p_2 + p_3} \frac{w}{p_1},$$

$$x_2(p, w) = \frac{p_3}{p_1 + p_2 + p_3} \frac{w}{p_2},$$

$$x_3(p, w) = \frac{\beta p_1}{p_1 + p_2 + p_3} \frac{w}{p_3}.$$

Does $x(p, w)$ satisfy homogeneity of degree 0 and Walras Law when $\beta = 1$? when $0 < \beta < 1$?

Problem Set 2. Due: Tuesday 9/27

Problem MM.7. Consider the demand function

$$x(p,w) = \begin{bmatrix} \frac{2w}{3p_1} \\ \\ \frac{w}{3p_2} \end{bmatrix}.$$

i) Show that $x(p,w)$ is homogeneous of degree 0.

ii) Show that $x(p,w)$ satisfies Walras Law.

iii) Find a utility function that generates this demand function.

iv) Find the matrix ε of price elasticities of demand.

$$\varepsilon = \begin{bmatrix} -1 & ? \\ \\ ? & ? \end{bmatrix}$$

v) Derive the general formula for the Slutsky elasticity matrix s, that is for $\left.\frac{\partial x(p,w)}{\partial p}\right]_{\bar{w}}$ expressed in elasticity form. Explain the terms in the expression that you find.

vi) Complete the Slutsky elasticity matrix:

$$s = \begin{bmatrix} ? & ? \\ \\ \frac{2}{3} & ? \end{bmatrix}.$$

vii) Interpret the results you get for s. The off-diagonal elements of s are not equal to 0. Why not?

Problem M-C.3.B.3. Draw the indifference map of preferences that are locally nonsatiated but not monotone.

—Please Turn Over—

Problem M-C.3.C.6. Suppose that in a two-commodity world, the consumer'sutility function takes the form

$$U(x_1, x_2) = [\alpha_1 x_1^\rho + \alpha_2 x_2^\rho]^{\frac{1}{\rho}}.$$

This utility function is known as the **constant elasiticity of substitution (CES)** function.

a. Demonstrate that the utility functions in i), ii) and iii) below are equivalent in the limit to special cases of the CES function as ρ goes to $1, 0$ and $-\infty$, respectively.

i) $U_L(x_1, x_2) = \alpha_1 x_1 + \alpha_2 x_2$ [Linear]

ii) $U_C(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2}$ [Cobb-Douglas]

iii) $U_F(x_1, x_2) = \min\{x_1, x_2\}$ [Fixed-Proportions]

b. How would you have to change the formula for the CES utility function in order to get $U(x_1, x_2) = \min\{a_1 x_1, a_2 x_2\}$ as ρ goes to $-\infty$?

Problem MM.8. Find the demand functions implied by the linear and fixed-proportions utility functions.

Problem M-C.3.D.5.d. In the context of demand, the elasticity of substitution between two goods is defined informally by

$$\sigma \equiv -\frac{\%\Delta \frac{x_1}{x_2}}{\%\Delta \frac{p_1}{p2}},$$

that is, the percentage change in the ratio of the two goods demanded caused by a 1% change in their relative price. Show that the CES utility function defined in problem M-C.3.C.6, above, has a constant elasticity of substitution given by

$$\sigma = \frac{1}{1-\rho}.$$

[Note: CES is the abbreviation for "Constant Elasticity of Substitution."] What elasticities of substitution are implied by the linear, Cobb-Douglas and Fixed-Proportions utility functions?

Problem Set 3. Due: Tuesday 10/4

Problem MM.9. Let f be any function (not necessarily continuous) and let Y and Z be any sets in the codomain of f. Let S_C denote the complement of S. Prove the following:

i) If $f^{-1}(Y)$ and $f^{-1}(Z)$ are closed sets, then $f^{-1}(Y \cup Z)$ is a closed set.

ii) $f^{-1}(Y_C) = [f^{-1}(Y)]_C$

Problem MM.10. (From Midterm Exam 1, Fall 2004) Ma Ching-to consumes only one good: rice. Let x be the amount of rice he consumes, p the price of rice, and w his wealth. His utility function is given by

$$u(x) = 20x - x^2,$$

for all $x \geq 0$.

a) Illustrate the graph of his utility function. [This is important. Make sure you have the right shape.]

b) Is his utility function characterized by local nonsatiation? Explain.

c) Describe the indifference curve and upper contour set for $u = 64$. Is his utility function quasiconcave? Explain.

d) Assume that Ma must consume whatever he buys [no free disposal]. Find demand for $p, w \geq 0$. Explain your reasoning. Is demand a funtion?

e) Find Ma's indirect utility function $v(p, w)$.

f) Find Hicksian compensated demand $h(p, u)$. Is $h(p, u)$ defined for all values of u? Explain.

g) Find Ma's expenditure function. Explain.

Problem MM.11. Find the indirect utility functions that correspond to the linear, Cobb-Douglas, fixed-proportions and CES direct utility functions.

Problem Set 4. Due: Tue 10/12

Problem MM.12. Suppose Lucia consumes only two goods, candy x (kilograms) and leisure h (hours). Her utility function is $U(x,h) = \min\{x,h\}$. Let p denote the price of candy and s her wage rate. Her consumption of leisure per day is 24 minus the number of hours she works.

a) Find Lucia's budget set for one 24-hour day. Explain carefully. Sketch her budget set when $p = 1$ and $s = 1$, and when $p = 2$ and $s = 1$ (an exact graph is not required).

b) Explain in words what this utility function says about her preferences. Add a sketch Lucia's indifference curves to the sketch of her budget sets.

c) Find Lucia's demand functions for candy and leisure. Find x and h when $p = s = 1$. Note that income (wealth) is not an argument of the demand functions. Explain why not.

d) If $s = 1$ and $p = 1$ and then the price of candy increases to $p = 2$, what is the size of the income and substitution effects? Explain your answer. [Use common sense, not mathematics.]

e) Derive the Slutsky compensated demand function for candy $x_s(p, s, x_0, h_0)$ that corresponds to initial consumption of $x_0 = h_0 = 12$.

f) Find the Slutsky substitution-effect derivative of the demand for candy with respect to its own price p and the wage rate s. What is the size of the substitution effect for small changes in price and wage rate?

g) Suppose Lucia receives a gift of money m from her mother. Find $\partial x/\partial m$, the income-effect derivative of the demand for candy. Verify the Slutsky equation for $\partial x/\partial p$.

Problem MM.13. Riccardo can work between 0 and 8 hours per day at \$40 per hour and after that he can work as much as he wants at \$20 per hour. Other than leisure, Riccardo consumes only one good, priced at \$1 per unit. Illustrate Riccardo's budget set for goods and leisure. Find the Kuhn-Tucker necessary first order conditions for the optimality of working exactly 8 hours per day.

Problem MM.14. Verify Roy's Identity for the Cobb-Douglas function, and use the expenditure function to find the corresponding Hicksian compensated demand function.

Problem MM.15. Dieter loves beer (x) and schnaps (y), the more the better. But he always drinks them in the ratio of 2 to 1. If he has more beer than twice the amount of his schnaps he pours the excess beer down the drain. If he has more schnaps than half the amount of his beer, he pours the excess schnaps down the drain. Find a utility function that would explain Dieter's behavior. Also find Dieter's expenditure function and his [Hicksian] compensated demand function. Illustrate with graphs.

Problem Set 5. Due: Tuesday 10/19

Problem MM.15 (continued). Dieter loves beer (x) and schnaps (y), the more the better. But he always drinks them in the ratio of 2 to 1. If he has more beer than twice the amount of his schnaps he pours the excess beer down the drain. If he has more schnaps than half the amount of his beer, he pours the excess schnaps down the drain. For $p_y = 4$ and $w = 60$ find the compensating variation, the equivalent variation and the change in consumer surplus if p_x changes from 5 to 3.

Problem Set 6. Due: Tuesday 10/26

Problem MM.16. Illustrate the following proposition on a graph (but do not prove it formally).

Suppose $f : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous and twice differentiable with $f(0) = 0$ and $f''(x) < 0$ for all $x > 0$ (strict concavity). Then, for any $\bar{x}$, we have

$$\alpha f(\bar{x}) < f(\alpha\bar{x}) \text{ for } \alpha < 1$$

and

$$\alpha f(\bar{x}) > f(\alpha\bar{x}) \text{ for } \alpha > 1$$

Problem MM.17. Consider the production function $f(x_1, x_2) = \delta \ln(x_1+1)+(1-\delta)\ln(x_2+1)$, and define the production set

$$Y = \left\{ \begin{bmatrix} -x_1 \\ -x_2 \\ x_3 \end{bmatrix} \middle| \; x_1, x_2 \geq 0 \text{ and } x_3 \leq f(x_1, x_2) \right\}$$

i) Prove that Y shows decreasing returns to scale. [Hint: use the proposition in the previous problem.]

ii) Prove that Y is convex.

iii) If $p \gg 0$ denotes the vector of commodity prices, we can define the profit function, $\pi : Y \to \mathbb{R}$ by $\pi(p) = \max\{py \,|\, y \in Y\}$, which represents the maximum profits that can be earned from any production activity in Y.

- **a)** Given Y as defined above, show that for $p \gg 0$, $\pi(p) \geq 0$.
- **b)** For what values of p is $\pi(p) > 0$? When does $\pi(p) = 0$? [Hint: Use the directional-derivative form of Kuhn-Tucker]

iv) Construct a production set that is a convex cone by adding a “hidden input” to Y.

Problem MM.18. **Extra Credit** (Not Required) Provide a formal proof of MM.16.

Problem Set 7. Due: Tuesday, 11/1

Problem MM.19. Olive oil can be produced with Spanish workers or Portuguese workers or any combination of the two types of workers, but the production process is always characterized by decreasing returns to scale.

i) Construct a production function in which the Spanish and Portuguese work well together and increase each other's productivity.

ii) Construct a production function in which the Spanish and Portuguese do not work well together and reduce each other's productivity.

iii) Prove that each of your production functions has decreasing returns to scale.

iv) Are either of your two production functions concave? Provide proof.

v) For each of your production functions, find the associated profit function and supply relation [Use Kuhn-Tucker if necessary].

—*Please Turn Over*—

Problem MM.21. Consider the production function

$$f(L) = \begin{cases} \sqrt[2]{L} & \text{for} \quad 0 \leq L \leq 1 \\ \\ \sqrt[3]{L} & \text{for} \quad L > 1 \end{cases}$$

where L denotes labor input. Assume that the price of output is 1 and the wage rate is w.

i) Prove that f is continuous and strictly concave.

ii) Find the profit function.

iii) Find the derived demand for labor.

iv) Are there wage rates at which no production would occur? Explain.

v) Suppose we introduce the hidden input, say managers, M, and suppose that managers earn the same wage rate as laborers, w. Assume that the production function $f(L)$ is the output of a firm with one manager.

- **a)** Construct the production function $F(L, M)$ for a firm with any number of workers and managers.
- **b)** Show that F has constant returns to scale.
- **c)** If production occurs, what will be the ratio of workers to managers? Why?
- **d)** What is the maximum wage rate at which production will occur?
- **e)** Now suppose that each firm must hire an integral number of workers and managers (firms are not allowed to cut workers or managers into pieces), and suppose that a firm can hire at most 2 managers. How many workers and managers will each firm hire as a function of the wage rate w.

Problem Set 8. Due: Tuesday, 11/8

Problem MM.22. Suppose that the preference relation $\succsim$ is defined by the expected utility of lotteries (that is $x \succsim x'$ whenever $\mathrm{EU}[x] \geq \mathrm{EU}[x']$). Show that $\succsim$ is rational, continuous and satisfies the independence axiom.

Problem MM.23. Consider the coefficient of relative risk aversion.

i)) Show that a person with the utility function

$$u(q) = \frac{1}{1-r} q^{1-r}$$

has constant relative risk aversion r.

ii)) Show that a person with the utility function $u(q) = \log(q)$ has constant relative risk aversion. Explain the relation between

$$u(q) = \log(q) \text{ and } u(q) = \frac{1}{1-r} q^{1-r}?$$

Problem MM.24. Consider two cdfs, F and G, with the same mean, μ.

i) Show that if F second-order dominates G and the variance σ^2 exists for both functions, then $\sigma^2(F) \leq \sigma^2(G)$.

ii) Find an example of cdfs F and G without a finite variance but where, nevertheless, F second-order dominates G.

iii) Find an example of cdfs F and G with $\sigma^2(F) \leq \sigma^2(G)$ but where F does `not` second-order dominate G.

Problem Set 9. Due: Tuesday, 11/15

Problem MM.22. The Cournot Duopoly. Suppose that in France there are exactly two profit-maximizing firms, L'Eau and and N'Eau, that produce bottled water. These firms face zero costs (except for marketing, which we can ignore). The French population has a demand curve for bottled water given by

$$x(p) = 60 - p.$$

In what follows let x_L represent the production of L'Eau and x_N, the production of N'Eau.

i) Why do madrileños spend good money on bottled water, given that the water from the faucet (del grifo) in Madrid is delicious? [Hint: the French spend money on bottled water because all French are silly (tontos).]

ii) Suppose that both L'Eau and N'Eau decide how much to produce simultaneously.

- **a)** Construct a normal-form game that represents this market. Who are the players? What is their strategy space?
- **b)** Find all the Nash equilibria of this game. Is $x_L = 60$ and $x_N = 0$ a Nash equilibrium? Explain.
- **c)** Construct an extensive-form game that represents this market. Describe the nodes, branches and information sets carefully. What are actions are allowed? Describe the strategy spaces of each firm?
- **d)** Find all the Nash equilibria of the extensive-form game.

iii) Suppose now that L'Eau decides how much to produce first. N'Eau observes L'Eau's decision and then decides how much he will produce.

- **a)** Construct an extensive-form game that represents this market. Describe the nodes, branches and information sets. What actions are allowed? Carefully describe the strategy space of each firm.
- **b)** Is "produce 20 under all circumstances" a strategy in N'Eau's strategy space? How would you write that strategy formally? Is there a Nash equilibrium in which N'Eau adopts that strategy?
- **c)** Find a subgame-perfect equilibrium of this game. Explain your results.

Problem MM.23. Suppose Ignacio (I) and his brother, Javier (J) have 1000 euros to divide between them. First, Ignacio must offer an amount $x \in [0, 1000]$ to Javier. If Javier accepts the offer he receives x and Ignacio receives $1000 - x$. If Javier rejects the offer, then 400 euros disappear and 600 remain. Javier must then offer Ignacio $y \in [0, 600]$. If Ignacio accepts the offer, he receives y and Javier receives $600 - y$. If Ignacio rejects the offer all of the money disappears and both brothers receive 0.

i) Draw an extensive-form game tree that represents this process. Include the nodes, branches, histories and information sets.

ii) Carefully describe the strategy space of each each brother.

iii) Find a **Nash equilibrium** in which Ignacio receives 1000 and Javier receives 0. Explain carefully.

iv) Find a **subgame-perfect equilibrium** of this game. How much do Ignacio and Javier receive in that subgame-perfect equilibrium? Show your work.

Problem MM.24. Compute a mixed strategy equilibrium for the coordination game discussed in class:

		MIGUEL H	MIGUEL T
Vanesa	*H*	1 *2*	0 *0*
	T	0 *0*	2 *1*

.

Problem Set 10. Due: Tuesday, December 6

Problem MM.28. Competitive equilibria. Suppose that:

- there are two goods, m and x;
- there are I households, each with the utility function $u_i(x_i, m_i) = m_i + \ln(x_i)$;
- initial endowments of x are given by $\hat{x}_i = 0$ for $i = 1, ..., I-1$ and $\hat{x}_I = I$;
- initial endowments of m are given by $\hat{m}_i = 2$ for all i;
- this is a pure-exchange economy without production.

Let p denote the price of good x, and assume that the price of m is 1. Unless otherwise specified, the subscript i denotes a household other than household I.

i) Find a competitive equilibrium of this economy.

ii) Prove that the equilibrium you found is a Pareto optimum. [Hint: difficult.]

iii) Show that if the total initial endowment of x is given by

$$x \equiv \sum_i \hat{x}_i = I,$$

then the competitive allocation of x does not depend on the original distribution of the endowment of x among the households (that is, it does not demand on the values of $\hat{x}_i$).

iv) Suppose now that the problem is the same as before, but that the initial endowment of m is 1/2 rather than 2 for all households. Find the new competitive equilibrium. [Hint: think.]

v) How will the competitive equilibrium in iv change if production can take place according to the production function $x = f(m)$?

Problem MM.29. Let η_D and η_S represent the price elasticity of demand and supply in the market for gasoline.

i) What is the incidence [percentage paid by producers and consumers] of a small excise tax on gasoline?

ii) Suppose all gasoline is imported from OPEC countries, and suppose that the government gives zero weight to the utility of owners of the OPEC oil resources. Under what conditions should the government lower the tax on gasoline when a supply shock has raised the price? Explain.