Name: | | | | | | | | | | | | | | | | | | |

[Last], [First]

EC 701 **Microeconomics**

Michael Manove **20 December 2004**

Final Examination

Instructions: *Answer two of the three parts of Problem 1 (but do* **not** *answer all three parts). Then answer Problems 2, 3, 4 and 5. Think before you write. Do not spend too much time on any one problem. If you have to leave the room for any reason, please give the instructor your examination on the way out. You will have 180 minutes to complete a 150-minute exam. I suggest that you do not exceed the recommended times for each question until you have answered all questions. Then you can use the extra time to improve your answers. If you finish before 11:45 pm, you may leave, but be extremely quiet on the way out and in the hallway!*

Problem 1. *[20 minutes, total]* Prove any two of the following three propositions:

a) *[10 minutes]* ***Proposition (Slutsky equation for Hicksian demand).*** If $U(x)$ is a strictly quasiconcave and well-behaved utility function and $V(p, w)$ is the corresponding indirect utility function, then

$$\frac{\partial x_i(p,w)}{\partial p_j} = \frac{\partial h_i(p,u)}{\partial p_j} - \frac{\partial x_i(p,w)}{\partial w} x_j(p,w)$$

where $x_i(p, w)$ is ordinary demand and $h_i(p, u)$ is Hicksian demand and where $u = V(p, w)$. *[You are free to cite theorems about the expenditure function $e(p, u)$ in your proof.]*

b) *[10 minutes]* ***Proposition (Law of Supply).*** Suppose $y(\cdot)$ is a supply (derived demand) function. Then for any two price vectors p' and $p \gg 0$,we have

$$(p' - p)(y(p') - y(p)) \geq 0.$$

c) *[10 minutes]* ***Proposition (Walras' Law for markets).*** Consider a pure exchange economy in which Walras' law holds for demand. If $p \gg 0$, and if all but one market clears, then the last market must clear also.

Problem 2. *[45 minutes, total]* Andrew and Shenyi are the only individuals in a pure-exchange economy with two goods, bread (b) and noodles (n). Andrew and Shenyi *each* have the same initial allocation: $\omega_b = 60$ and $\omega_n = 30$ (total b and n available in the economy are twice these amounts). Andrew's utility function is

$$u_a = \max\{b, n\}$$

and Shenyi's is

$$u_s = \min\{b, n\}.$$

Let p_b and p_n denote the prices of bread and noodles.

*[**Warning:** There was a question on a past Micro Qualifier with the same utility functions as these. But the qualifier question was different from this question in important ways. Please answer this question, not the qualifier question.]*

a) *[12 minutes]* Describe Andrew's buying habits (in words, without math). Then find Andrew's *excess demand relation.*

b) *[8 minutes]* Describe Shenyi's buying habits (in words, without math). Then find Shenyi's *excess demand relation.*

c) *[10 minutes]* Does this economy have a competitive equilibrium? If your answer is "yes," find the equilibrium prices and allocation. If your answer is "no," explain why it doesn't.

d) *[15 minutes]* Find all Pareto Optimal Allocations for this economy. What allocations are not Pareto Optimal? Explain carefully.

Problem 3. *[40 minutes, total]* Suppose there are two brothers, Jorge and Pedro, and suppose 1000 dollars is divided between them. Let m_j denote the amount of money given to Jorge and m_p the amount given to Pedro, where $m_j + m_p = 1000$. Their utility functions are as follows:

$$u_j(m_j, m_p) = \begin{cases} m_j - 2(m_p - m_j) & \text{for } m_p > m_j \\ m_j & \text{for } m_p \leq m_j \end{cases}$$

and

$$u_p(m_j, m_p) = \begin{cases} m_p - 2(m_j - m_p) & \text{for } m_j > m_p \\ m_p & \text{for } m_j \leq m_p \end{cases}$$

a) *[5 minutes]* What do these utility functions imply about the character of the brothers? What is in their relationship besides love?

b) *[10 minutes]* Suppose that Jorge and Pedro are expected utility maximizers, and suppose that $m_j = m_p = 500$. The father of Jorge and Pedro suggests a bet. He will flip a coin. If he gets heads, then Jorge will win 100 from Pedro; if he gets tails, then Pedro will win 100 from Jorge. Will the brothers agree to this bet? Explain.

c) *[15 minutes]* Suppose that the father tells Jorge and Pedro that they must play the ultimatum game. Jorge will offer Pedro an amount $x \in [0,\ 1000]$. If Pedro accepts, then he gets $m_p = x$ and Jorge gets $m_j = 1000 - x$. But if Pedro rejects the offer, then both get 0 ($m_j = m_p = 0$). Assuming that Jorge and Pedro want to maximize utility, not money, find a subgame-perfect equilibrium of this game. What strategies will Jorge and Pedro use in that equilibrium? Is the subgame-perfect equilibrium unique? Explain.

d) *[10 minutes]* Find a Nash-equilibrium strategy profile that yields the offer of $x = 500$ as an outcome of the game. Is your equilibrium subgame perfect? Explain.

Problem 4. *[20 minutes, total]* Bart is an expected-utility maximizer. He has a [Bernoulli] utility function given by

$$u = \sqrt{w},$$

where w is the total value of his assets.

a) *[5 minutes]* Show that Bart has constant relative risk aversion. Find the degree of his risk aversion. Show your work.

b) *[15 minutes]* Suppose that Bart has a house worth 5 million dollars and 4 million in other assets. There is a .02 (2 percent) probability that his house will burn down with all of its value lost. How much would Bart be willing to pay for fire insurance that covers the full value of the house? Explain, and show all your work.

Problem 5. *[25 minutes, total]* Suppose that output y is produced with labor ℓ. Let p denote the price of the output and w the wage rate. Suppose that the profit function is given by

$$\pi^*(p,w) = \begin{cases} p\log\frac{p}{w} - (p-w) & \text{for} \quad w \leq p \\ 0 & \text{for} \quad w > p \end{cases}.$$

a) *[10 minutes]* Find supply $y^*(p,w)$ and the derived demand for labor $\ell^*(p,w)$.

b) *[15 minutes]* Find a production function for y that is consistent with the profit function specified above. Show all of your work.